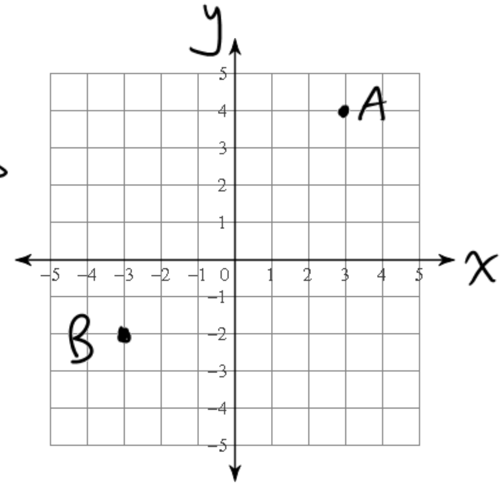
Unit 1: Systems of Linear Equations

Review of the Key Terms:

Cartesian Plane: x and y plane



Coordinates: Points on the Cartesian plane.

Ex. Plot the points A (3,4) and B (-3,-2)

x y   x y

Intercepts: where a line crosses the x or y axis.

Domain: all possible x-values

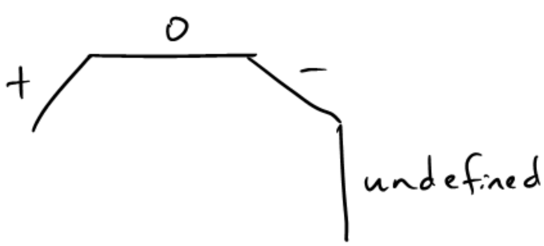
Range: all possible y-values

Table of Values:

ex

x	y
-2	
-1	
0	
1	
2	

Slope:



Slope Formula: $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$

Ex. What is the slope of a line that passes through (-2,5) and (-6, 2)

$x_1\ y_1 \quad x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-6--2} = \frac{-3}{-4} = \boxed{\frac{3}{4}}$$

Equation of a Line: $y = mx + b$: Slope-intercept form

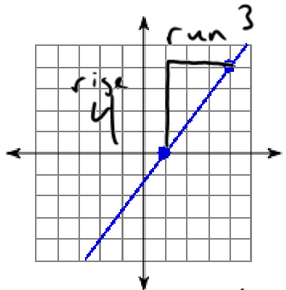
m : slope

b : y-intercept

Standard form for the Equation of a Line is $Ax + By + C = 0$

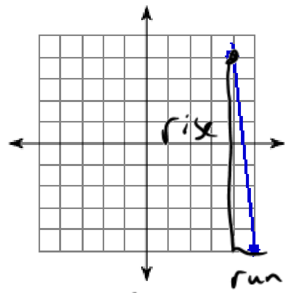
Ex. Find the slope of the following lines

1)



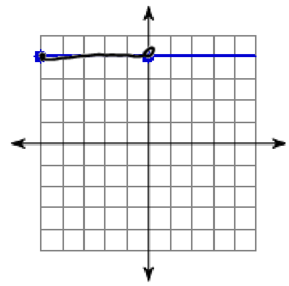
$m = \frac{rise}{run} = \frac{4}{3}$

2)



$m = \frac{rise}{run} = \frac{-9}{1} = -9$

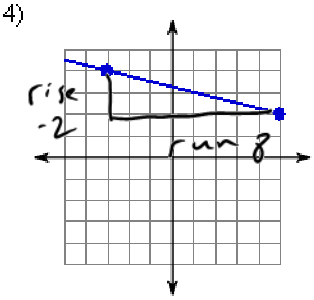
3)



$m = 0$

or $\frac{0}{5} = 0$

4)



$m = \frac{-2}{8} = -\frac{1}{4}$

Graph the following lines given their equation:

$y = 2x - 3$

$y = mx + b$

$m = \frac{2}{1}$ 2 up, 1 right

$b = -3$ y-int

$y = \boxed{-\frac{4}{3}}x \boxed{+4}$

b

$m = \frac{-4}{3}$ 4 down, 3 right

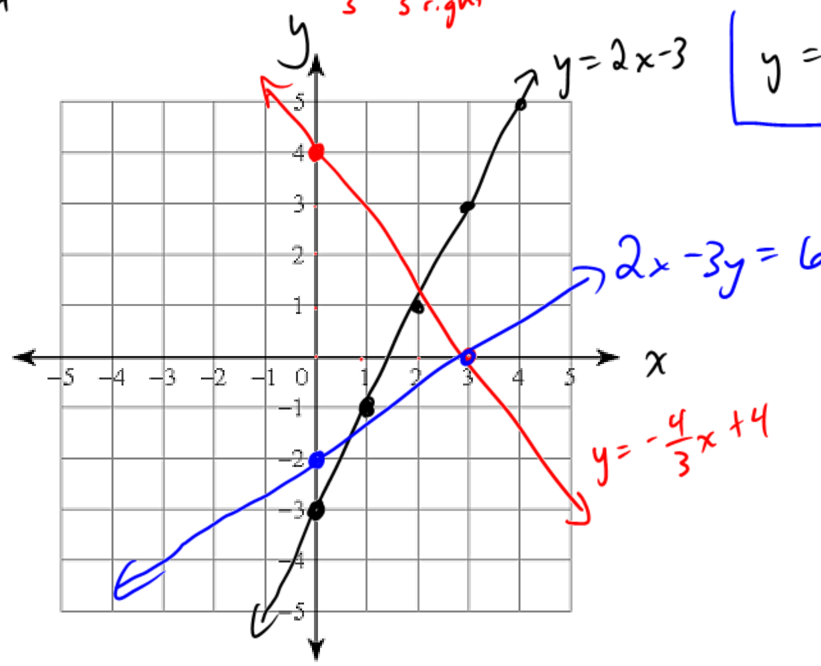
$2x - 3y = 6$

$\overset{-2x}{\curvearrowright}$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$

$\boxed{y = \boxed{\frac{2}{3}}x \boxed{-2}}$

$m = \frac{2}{3}$ $b = -2$



Steps to Graphing a Linear Equation:

1. Put the equation in $y = mx + b$ form
2. Plot the y-intercept on the graph
3. Plot the slope and draw the line

Do #1-10

Writing the Equation of a Line

Method 1: When you know the y-intercept, use the Slope-Intercept Form, $y = mx + b$

1. Identify the y-intercept (**b**) and slope (**m**)
2. Write the equation replacing the b and m

Method 2: When you have 2 points, neither which are the y-intercept, you can use Slope-Point form

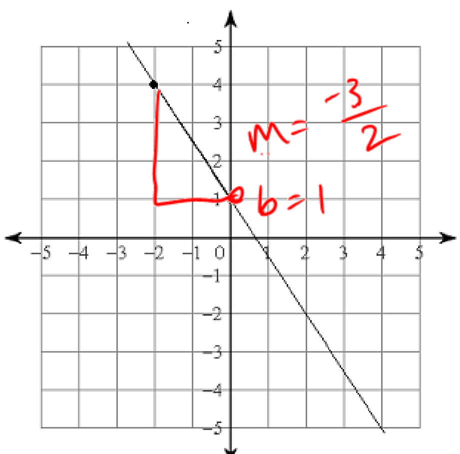
$$y - y_1 = m(x - x_1)$$

1. Use the 2 points to calculate the slope (**m**)
2. Sub in **m** and **a point for (x_1,y_1)** into the Slope-point form
3. Reorder the equation into Slope-Intercept form OR Standard form

Method 3: When you have 2 points, neither which are the y-intercept, you can use Slope-Intercept Form, $y = mx + b$

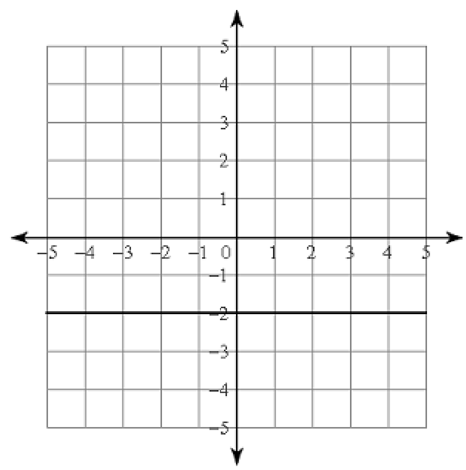
1. Use the 2 points to calculate the slope
2. Sub in **m** and **a point for (x,y)**
3. Solve for b
4. Write the equation replacing the b and m

Examples:

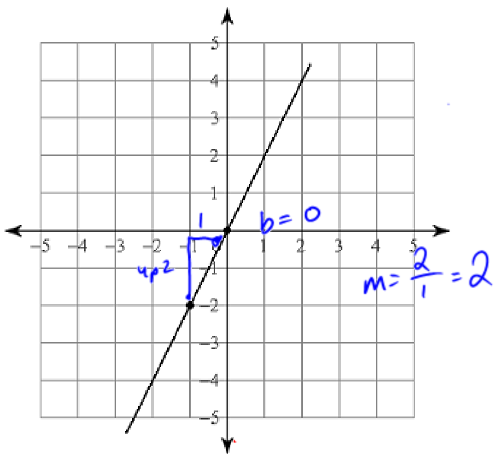


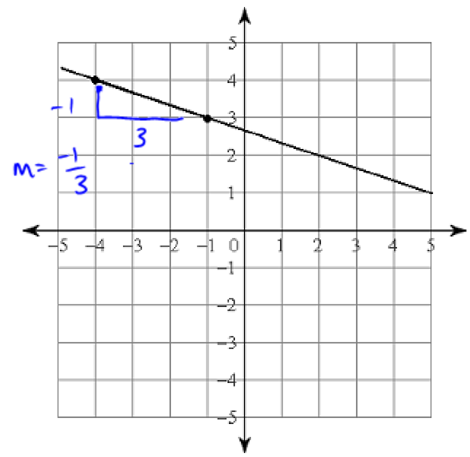
$y = mx + b$

$y = -\frac{3}{2}x + 1$



$\boxed{x = -2}$





$y = 2x + 0$

$y = 2x$

method 2

plug in $m = -\frac{1}{3}$ pt $(-1, 3)$ x_1, y_1

$y - y_1 = m(x - x_1)$

$y - 3 = -\frac{1}{3}(x + 1)$

$y - 3 + 3 = -\frac{1}{3}x - \frac{1}{3} + 3$

$y = -\frac{1}{3}x + \frac{8}{3}$

method 3

$m = -\frac{1}{3}$ pt $(-1, 3)$ x y

$y = mx + b$

$3 = -\frac{1}{3}(-1) + b$

$-\frac{1}{3}$ $3 = \frac{1}{3} + b$ $-\frac{1}{3}$

$\frac{8}{3} = b$

$y = -\frac{1}{3}x + \frac{8}{3}$

Solving for the Unknown

Consider this to be "undoing" whatever has been done to the unknown, often *x*. Solve for *x* in the following 2 equations.

$\frac{x}{9} + 3 \overset{-3}{=} 2 - 3$

$9\left(\frac{x}{9}\right) = (-1)\,9$

$x = -9$

$(\frac{x}{2})(\frac{3}{5}) - \frac{x}{3} = 5$

$\frac{3x}{10} - \frac{x}{3} = 5$ Common multiple of 3 and 10 is 30

$30\left(\frac{3x}{10}\right) - 30\left(\frac{x}{3}\right) = 30(5)$

$\frac{90x}{10} - \frac{30x}{3} = 150$

$9x - 10x = 150$

$-x = 150$

$x = -150$

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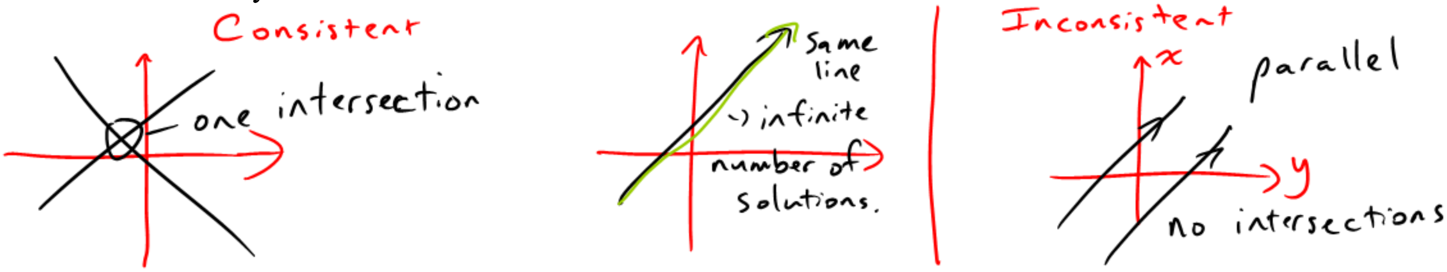
Solving Linear Systems

"Solving" a Linear System means finding the point at which 2 lines intersect. A Linear System can intersect in 3 ways:



In this unit, we will learn 3 ways to Solve a Linear System:

1. Solving by Graphing
2. Solving by Substitution
3. Solving by Elimination

Method 1: Solve by Graphing:

Steps:

1. Graph the 2 linear systems
2. State the POI (Point of Intersection) by stating "The POI is (,)"

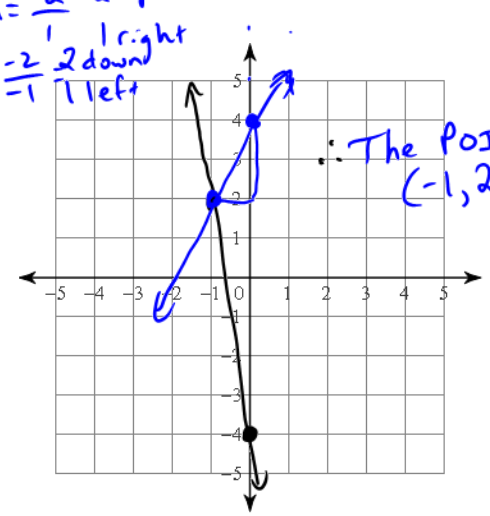
Examples

$y = -6x - 4$ → $m = \frac{-6}{1}$ → 6 down, 1 right $= \frac{6}{-1}$ 6 up, 1 left

$y - 2x = 4$ → $b = -4$

$y = 2x + 4$ → $b = 4$

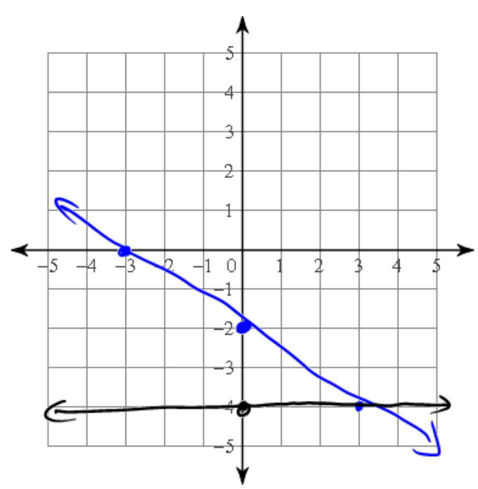
$m = \frac{2}{1}$ 2 up, 1 right $= \frac{-2}{-1}$ 2 down, 1 left



∴ The POI is $(-1, 2)$

$y = -\frac{2}{3}x - 2$ → $m = \frac{-2}{3}$ or $m = \frac{2}{-3}$

$y = -4$



∴ The POI is $(3, -4)$

up to #56 but 48 is a good goal.

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Method 2: Solve by Substitution:

Steps:

1. Isolate a variable (choose the easiest one!), let's call this the 1st equation
2. Replace the solution to Step 1 into the 2nd equation and solve the equation
3. Use the value from Step 2 and substitute it into the 1st equation to solve for the other variable
4. State the POI (Point of Intersection) by stating "The POI is (,)"

Examples

$y-3x=5$ $\quad {}^{+3x}\quad{}^{+3x}$ → 1. $y=5+3x$

$y+x=3$

2: $(5+3x)+x=3$

$5+4x=3$

$4x=3-5$

$\frac{4x}{4}=\frac{-2}{4}$

$x=-0.5$

3. $y=5+3x$

$y=5+3(-0.5)$

$y=5+(-1.5)$

$y=3.5$

4. ∴ The POI is $(-0.5, 3.5)$

$4x+y=11$ → 1. $y=11-4x$

$x+2y=8$

2. $x+2(11-4x)=8$

$x+22-8x=8$

$-7x=8-22$

$\frac{-7x}{-7}=\frac{-14}{-7}$

$x=2$

3. $y=11-4x$

$y=11-4(2)$

$y=11-8$

$y=3$

4. ∴ The POI is $(2,3)$

Method 3: Solve by Eliminaton:

Steps:

1. Place both equations in Pseudo Standard form, Ax + By = C
2. Get 2 coefficients to have the same value (ex. 3 and 3) or opposite value (ex. 3 and -3)
3. Add or subtract the two equations to eliminate the one variable
4. Solve
5. Use the value from Step 4 and substitute it into an equation to solve for the other variable
6. State the POI (Point of Intersection) by stating "The POI is (,)"

Examples

1.
$$7x+18y=-5$$
$$[-3x+9y=3]\times 2$$

2.
$$7x+18y=-5$$
$$-[-6x+18y=6]$$

3. Subtract
$$13x+0y=-11$$
$$\frac{13x}{13}=\frac{-11}{13}$$

4.
$$x=-\frac{11}{13}$$

5.
$$7x+18y=-5$$
$$7\left(-\frac{11}{13}\right)+18y=-5$$
$$-\frac{77}{13}+18y=-5$$
$$18y=-5+\frac{77}{13}$$
$$18y=\frac{12}{13}\div 18$$
$$\frac{18y}{18}$$
$$y=\frac{2}{39}$$

6. ∴ The POI is $\left(-\frac{11}{13}, \frac{2}{39}\right)$

$$4x=-2y+24$$
$$-x-3y=-21$$

1.
$$4x+2y=24$$
$$[-x-3y=-21]\times 4$$

2.
$$4x+2y=24$$
$$+[-4x-12y=-84]$$

Add
$$0x-10y=-60$$
$$\frac{-10y}{-10}=\frac{-60}{-10}$$
$$y=6$$

5.
$$4x+2(6)=24$$
$$4x+12=24$$
$$-12 \quad -12$$
$$\frac{4x}{4}=\frac{12}{4}$$
$$x=3$$

6. ∴ The POI is $(3,6)$