

Application Problems Involving Linear Systems

Single Equation

Frank is a hot dog vendor. He pays the city \$50 everyday to set up in front of the town hall. He makes \$3 on each hot dog.

- Write an equation to describe his earnings.
- Use the equation to find out how many hot dogs he needs to sell to break-even $\Rightarrow \text{Earnings} = 0$
- How many hot dogs does he need to sell in order to make \$190?

a) $\text{Earnings} = 3x - 50$
 x is the number of hot dogs

b) $0 = 3x - 50$
 $50 = 3x$
 $\frac{50}{3} = \frac{3x}{3}$
 $16.\bar{6} = x$ He needs to sell 17 hot dogs

c) $190 = 3x - 50$
 $\quad \quad \quad \leftarrow +50$

$$\frac{240}{3} = \frac{3x}{3}$$

$80 = x$
 He needs to sell 80 hot dogs.

SOLEs – Systems of Linear Equations

The difference of two numbers is 5. Their sum is 27. What are the numbers?

Let x be the 1st number

Let y be the 2nd number

$$\begin{array}{r} x - y = 5 \\ - [x + y = 27] \\ \hline 0x - 2y = -22 \end{array}$$

$$\begin{array}{r} 0x - 2y = -22 \\ -\frac{2y}{-2} = \frac{-22}{-2} \end{array}$$

$$\Rightarrow \boxed{y = 11}$$

$$\begin{array}{r} x - y = 5 \\ x - 11 = 5 \\ \hline \boxed{x = 16} \end{array}$$

The numbers are 16 and 11.

Trevon and Jose are selling pies for a school fundraiser. Customers can buy cherry pies and lemon meringue pies. Trevon sold 8 cherry pies and 8 lemon meringue pies for a total of \$152. Jose sold 4 cherry pies and 6 lemon meringue pies for a total of \$100. What is the cost each of one cherry pie and one lemon meringue pie?

Let c be the cost of cherry pies and m be the cost of lemon meringue.

$$\begin{array}{r} 8c + 8m = 152 \rightarrow 8c + 8m = 152 \\ [4c + 6m = 100] \times 2 \rightarrow [8c + 12m = 200] \\ \hline -4m = -48 \\ \frac{-4m}{-4} = \frac{-48}{-4} \\ m = 12 \end{array}$$

The cherry pie costs \$7 and the lemon meringue costs \$12

$$\begin{array}{r} \therefore 8c + 8(12) = 152 \\ 8c + 96 = 152 \\ 8c = 56 \\ c = 7 \end{array}$$

The school that Shreya goes to is selling tickets to a fall musical. On the first day of ticket sales the school sold 9 senior citizen tickets and 1 child ticket for a total of \$122. The school took in \$113 on the second day by selling 6 senior citizen tickets and 7 child tickets. Find the price of a senior citizen ticket and the price of a child ticket.

Let x be the price of the senior citizen ticket
and y be the price of the child ticket.

$$\text{Day ①: } 9x + y = 122 \rightarrow y = -9x + 122$$

$$\text{Day ②: } 6x + 7y = 113$$

$$6x + 7(-9x + 122) = 113$$

$$6x - 63x + 854 = 113$$

$$\begin{array}{r} -57x = -741 \\ \underline{-57} \end{array}$$

$$x = \$13$$

$$\rightarrow y = -9(13) + 122$$

$$y = \$5$$

The price of a senior ticket
is \$13, and a child is \$5.

Jennifer and Adam are selling pies for a school fundraiser. Customers can buy apple pies and pumpkin pies. Jennifer sold 1 apple pie and 5 pumpkin pies for a total of \$120. Adam sold 14 apple pies and 11 pumpkin pies for a total of \$500. Find the cost each of one apple pie and one pumpkin pie.

Let a be the cost of an apple pie, and p be the cost of a pumpkin pie.

$$a + 5p = 120 \checkmark$$

$$14a + 11p = 500 \checkmark \text{ now, solve}$$

One type of granola is 30% fruit and another type is 15% fruit. What mass of each type of granola should be mixed to make 600 g of granola that is 21% fruit?

Let x be the grams of 30% fruit
and y the grams of 15% fruit

$$x + y = 600 \rightarrow x = 600 - y$$

$$0.30x + 0.15y = 0.21(600)$$

$$\therefore 0.30(600 - y) + 0.15y = 126$$

$$180 - 0.3y + 0.15y = 126$$

$$\begin{array}{r} -0.15y = -54 \\ \underline{-0.15} \end{array}$$

$$y = 360 \rightarrow x = 600 - 360$$

$$x = 240$$

\therefore They need
240 grams of 30%
and 360 grams
of 15%.

A chemistry teacher needs to make 10L of 42% sulphuric acid solution. The acid solutions available are 30% sulphuric acid and 50% sulphuric acid, by volume. How many litres of each solution must be mixed to make the 42% solution?

50%
40%
30%

Let x be the Litres of 50%

Let y be the Litres of 30%

$$\begin{aligned} [x + y = 10] \times 0.5 &\Rightarrow 0.50x + 0.50y = 5 \\ 0.50x + 0.30y &= 0.42(10) \Rightarrow 0.50x + 0.30y = 4.2 \\ \hline 0x + 0.20y &= 0.8 \\ \frac{0.20}{0.20}y &= \frac{0.8}{0.20} \\ y &= 4 \end{aligned}$$

$$\begin{aligned} x + 4 &= 10 \\ x &= 6 \end{aligned}$$

We need 6 L. of 50% and 4 L. of 30% acid.

Typical Equations:

$$x + y = \text{sum}$$

$$\frac{\quad}{\quad}x + \frac{\quad}{\quad}y = \quad \text{(total)}$$

$$\frac{\quad}{\quad}x + \frac{\quad}{\quad}y = \quad \text{(%/\$ of total)}$$



Dollars for each amount
Percent of each amount

$$y = \frac{\quad}{\quad}x + b \quad \leftarrow \text{Starting Value/y-intercept}$$

Percent (commission)
Cost per $\frac{\quad}{\quad}$ (ex. square foot)

Challenge Problem: The sum of the digits of a certain two-digit number is 13. When you reverse its digits you increase the number by 27. What is the number?