Binomial Products

A binomial is two terms. For example: $2x+3y$, $2x^2+x$

$3x+1$

When finding the product of two binomials, we can find the product by either of the following methods:

A. FOIL - First Outside Inside Last $(ax+b)(cx+d)$

B. Multiply all of the terms in the first brackets, by all of the terms in the second brackets.

Examples:

$(2x-3)(4x+1)$

$= 8x^2 + 2x - 12x - 3$

$= 8x^2 - 10x - 3$

$(5a+4)(7a+2)$

$= 35a^2 + 10a + 28a + 8$

$= 35a^2 + 38a + 8$

$(3x-2y)(3x+8y)$

$= 9x^2 + 24xy - 6xy - 16y^2$

$= 9x^2 + 18xy - 16y^2$

$(2x-3)^2$ → Many mortels mess this up! You won't! → multiply by itself

$\neq \cancel{4x^2+9}$

$= (2x-3)(2x-3)$

$= 4x^2 - 6x - 6x + 9$

$= 4x^2 - 12x + 9$

$(2x-3)(3x^2-4x+1)$

$= 6x^3 - 8x^2 + 2x - 9x^2 + 12x - 3$

$= 6x^3 - 17x^2 + 14x - 3$

$\boxed{1-50}$ → Left side only

1, 3, 5, 7, etc.

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Common Factoring

When common factoring, you must look for the Greatest Common Factor (GCF). An algebraic expression has been fully factored once 1 or -1 is the only common factor left.

w/ numbers

ex 48

$$48 = 2 \cdot 24, \quad 24 = 2 \cdot 12, \quad 12 = 2 \cdot 6, \quad 6 = 2 \cdot 3$$

$48 = (2)^4 \cdot 3$

$-30x^2y^2 + 20x$

$= -10x(3xy^2 - 2)$

Check: Expand

$5xy + 4x^3 + 32x^2y$

$= x(5y + 4x^2 + 32xy)$

$-14n^6 - 20n^5 + 12n^3$

$= -2n^3(7n^3 + 10n^2 - 6)$

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Factoring Quadratics: $\boxed{x^2 + bx + c}$: D_o: 1 – 25 odd only
↳ 27 → end odd only

A quadratic expression in the form $x^2 + bx + c$ can be factored into two binomials of the form $(x + r)$ and $(x + s)$, when $r + s$ = b and $r \times s$ = c.

Why does this work? Look at what happens when we expand the following expression:

$(x + 4)(x - 5)$

$= x^2 - 5x + 4x - 20$

$= x^2 - x - 20$

$4 + -5$ $\quad$ $4 \cdot (-5)$

★ ALWAYS COMMON FACTOR FIRST

Factor the following expressions:

$x^2 + 9x + 20$

$4 + 5 = 9$

$4 \times 5 = 20$

1 20
2 10
4 5

$= (x + 4)(x + 5)$

Check: expand → $x^2 + 5x + 4x + 20$
$= x^2 + 9x + 20$

$2n^2 - 22n - 24$

$= 2[n^2 - 11n - 12]$

$-12 + 1 = -11$

$-12 \times 1 = -12$

$\boxed{= 2(n - 12)(n + 1)}$

Check: $2(n-12)(n+1)$
$= 2(n^2 + n - 12n - 12)$
$= 2(n^2 - 11n - 12)$
$= 2n^2 - 22n - 24$ ✓

$a^2 - 10a + 24$

$-4 + -6 = -10$

$-4 \times -6 = 24$

1 24
2 12
3 8
4 6

$= (a - 4)(a - 6)$ ✓

Note: You can use the signs of b and c to determine r and s

Trinomial	Factors
b and *c* are positive.	$(x + r)(x + s)$
b is negative, and *c* is positive.	$(x - r)(x - s)$
b and *c* are negative.	$(x - r)(x + s)$, where $r > s$
b is positive, and *c* is negative.	$(x + r)(x - s)$, where $r > s$

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Special Cases

A. Differences of Squares

Let's look at expanding first:

$(5n - 3)(5n + 3)$

$= 25n^2 + 15n - 15n - 9$

$= 25n^2 - 9$

$(7x - 4)(7x + 4)$

$= 49x^2 + 28x - 28x - 16$

$= 49x^2 - 16$

In the examples above, try figuring out how to work backwards from the last step to the first step.

Difference of Squares - the polynomial must be a binomial
- the 1st and 2nd terms must be perfect squares
- there must be a minus sign between the two terms
- Answer to $a^2 - b^2$ is $(a-b)(a+b)$

$49n^2 - 4$

$= (7n-2)(7n+2)$

$x^2 - 25$

$= (x-5)(x+5)$

$75n^4 - 27$

common factor first

$= 3(25n^4 - 9)$

$= 3(5n^2-3)(5n^2+3)$

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B. Perfect Squares

Let's look at expanding First:

$(5n + 3)^2$

$= (5n+3)(5n+3)$

$= 25n^2 + 15n + 15n + 9$

$= 25n^2 + 30n + 9$

$(7x - 4)^2$

$= (7x-4)(7x-4)$

$= 49x^2 - 28x - 28x + 16$

$= 49x^2 - 56x + 16$

In the examples above, try figuring out how to work backwards from the last step to the first step.

- the polynomial must be of the form $a^2x^2 + 2abx + b^2$ Answer: $(a+b)^2$

  or $a^2x^2 - 2abx + b^2$ Answer: $(a-b)^2$
- Remember to check $2ab$ by expanding

$4k^2 - 12k + 9$

$\sqrt{\ } = 2k$ $\quad \sqrt{\ } = 3$

$= (2k-3)^2$

Check: $2(2k)(-3)$

$= -12k$ ✓

$16m^2 + 8m + 1$

$\sqrt{\ } = 4m$ $\quad \sqrt{\ } = 1$

$= (4m+1)^2$

check: $2(4m)(1)$

$= 8m$ ✓

$9a^2 + 42a + 49$

$\sqrt{\ } = 3a$ $\quad \sqrt{\ } = 7$

$= (3a+7)^2$

Check: $2(3a)(7)$

$= 42a$ ✓

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Factoring Quadratics: $ax^2 + bx + c$

A quadratic expression in the form $ax^2 + bx + c$, where $a \neq 1$ can be factored using

Decomposition

When using decomposition, you need to look for two numbers, p and q, that add to b and multiply to $a \times c$.

Example:

$20x^2 - 7x - 6$

1. $-15 + 8 = -7$
$-15 \times 8 = -120$

120
1 120
2 60
3 40
4 30
5 24
6 20
8 15

2. $= 20x^2 - 15x + 8x - 6$

3. $= 5x(4x - 3) + 2(4x - 3)$

4. $= (5x + 2)(4x - 3)$

Doubt it? → Check it! Expand.

Steps:

1. Find two numbers, p and q, that multiply to (*ac*) and add to *b*
2. Decompose the middle term into the two numbers px and qx
3. Common factor the 1st and 2nd half
4. Leftovers and Brackets

$7v^2 + 34v - 5$

$10a^2 - 73a - 56$

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$\cancel{4x^2 - 12xy - 27y^2}$

33) $24m^2 + 2m - 126$

$= 2(\underline{12m^2 + m - 63})$

↳ Aside:

$\underline{28} + \underline{-27} = 1$

$\underline{28} \times \underline{-27} = -756$

$12m^2 + 28m - 27m - 63$

$= 4m(3m+7) - 9(3m+7)$

$= (4m-9)(3m+7)$

$= 2(4m-9)(3m+7)$

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Factoring: Choosing Your Method

① Common Factor

② Binomial → Difference of Squares

ex $4x^2 - 81$

$= (2x-9)(2x+9)$

② Trinomial →

ⓐ Perfect Square

$4x^2 - 36x + 81$

$= (2x-9)^2$

Check: $2(2x)(-9) = -36x$ ✓

ⓑ Simple Trinomial $a = 1$

$x^2 + 3x + 2$

$= (x+1)(x+2)$

$1 + 2 = 3$

$1 \times 2 = 2$

ⓒ Decomposition $a \neq 1$

ex $8x^2 - 10x - 3$

$-12 + 2 = -10$

$-12 \times 2 = -24$

$= 8x^2 - 12x + 2x - 3$

$= 4x(2x-3) + 1(2x-3)$

$= (4x+1)(2x-3)$

72. $x^2 - 6xy - 72y^2$ Notice: Like: $x^2 - 6x - 72$

$\underline{-12y} + \underline{6y} = -6y$

$\underline{-12y} \times \underline{6y} = -72y^2$

$= (x - 12y)(x + 6y)$

Check!