Exploring Quadratic Relations

Linear Equation: $0 = 2(0) + 3$ Degree: 1

Quadratic Equation: $0 = x^{(2)} + 2x - 3$ (standard form) Degree: 2

Parabola – a visual representation of a quadratic equation. Looks like a "U" right-side up or upside down. ~~It is called a~~ ________________

A graph of any quadratic relation of the form $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola.

Parabolas have a vertical $\boxed{\text{axis}}$ of symmetry with equation $x = h$

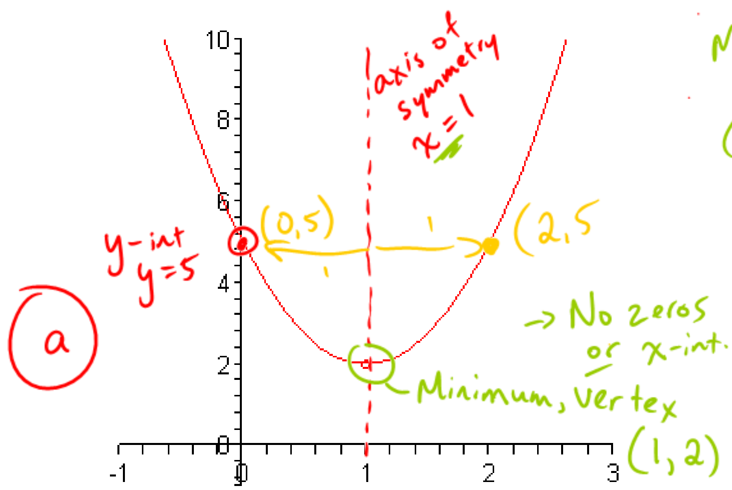
a reveals the direction of opening of the parabola.

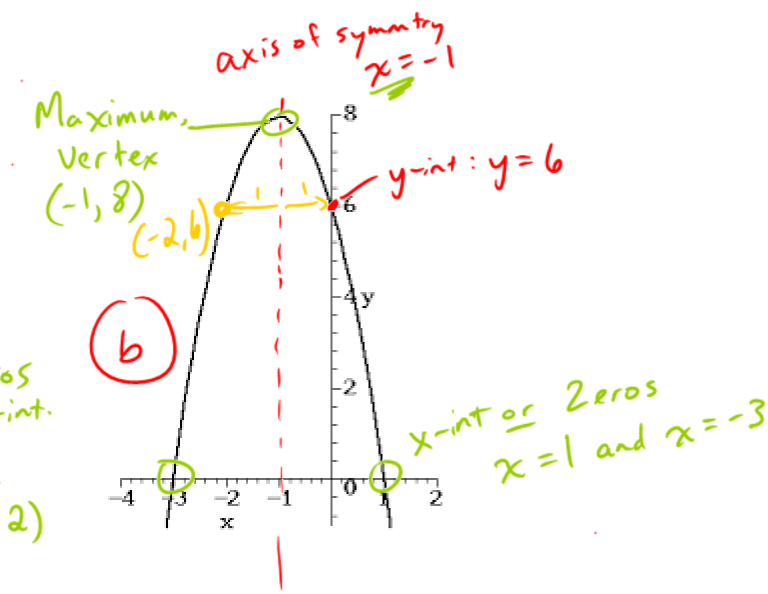
When $a > 0$, the parabola opens up.

When $a < 0$, the parabola opens down.

The constant c is the value of the y-intercept of the parabola.

Important points on a Parabola:





Minimum/Maximum Value vs. Minimum/Maximum point:

↳ just the y-value

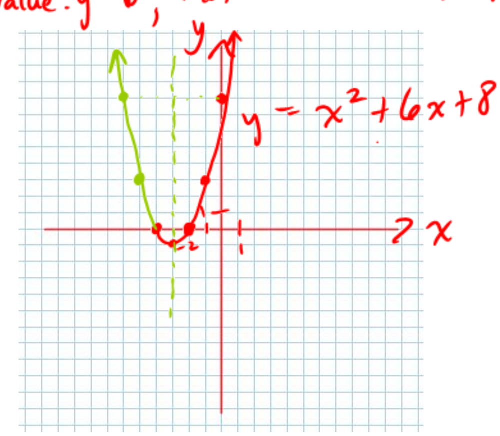
(a) Minimum value: $y=2$; Minimum Point: $(1,2)$

(b) Maximum value: $y=8$; Maximum Point: $(-1,8)$

Make a table of values and then graph the following equation:

$y = x^2 + 6x + 8$. Label ALL important points.

x	y
-2	$(-2)^2 + 6(-2) + 8 = 0$ $(-2,0)$
-1	$(-1)^2 + 6(-1) + 8 = 3$ $(-1,3)$
0	$(0)^2 + 6(0) + 8 = 8$ $(0,8)$
1	$(1)^2 + 6(1) + 8 = 15$ $(1,15)$
~~X~~	
-3	$(-3)^2 + 6(-3) + 8 = -1$ $(-3,-1)$
-4	$(-4)^2 + 6(-4) + 8 = 0$ $(-4,0)$



Factored Form of a Quadratic Relation

Side Note: Standard $y = ax^2 + bx + c$

Factored form of a Quadratic Equation is $y = a(x-s)(x-t)$
where s and t are the zeros or x-intercepts and a is the stretch factor and direction of opening.

1. For the following quadratic relations:
   i) Determine the zeros → How? factored form
   ii) Determine the y-intercept (c) or when $x = 0$
   iii) Determine the equation of the axis of symmetry → $x = \frac{s+t}{2}$
   iv) Determine the coordinates of the vertex → plug in the a.o.s.
   v) Sketch the graph

$y = x^2 + 10x + 16$

$8 + 2 = 10$
$8 \times 2 = 16$

i) $y = (x+8)(x+2)$
$y = 1(x-(-8))(x-(-2))$
$y = a(x-s)(x-t)$
zeros are -8 and -2

ii) $y = (0)^2 + 10(0) + 16$
$y = 16$ y-int is 16

iii) $x = \frac{-8 + -2}{2} = \frac{-10}{2}$
$\boxed{x = -5}$

iv) plug in $x = -5$
$y = (-5+8)(-5+2)$
$y = (3)(-3)$
$y = -9$ vertex $(-5, -9)$

v)

$y = 7x^2 - 53x - 24$

$-56 + 3 = -53$
$-56 \times 3 = -168$

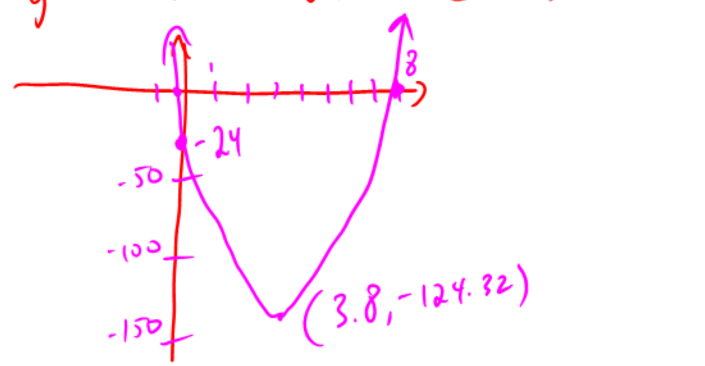
i) $y = 7x^2 - 56x + 3x - 24$
$y = 7x(x-8) + 3(x-8)$
$y = (7x+3)(x-8)$ $y = a(x-s)(x-t)$
$y = 7(x + \frac{3}{7})(x-8)$
zeros are $-3/7$ and 8.

ii) $x = 0$: $y = 7(0 + 3/7)(0-8) = -24$ → y-int

iii) $x = \frac{-3/7 + 8}{2} = 3.8$
$\boxed{x = 3.8}$

iv) $y = 7(3.8 + 3/7)(3.8 - 8)$
$y = -124.32$ vertex $(3.8, -124.32)$

v)



The ***axis of symmetry*** can be found using the following formula: $x = \frac{s+t}{2}$
Zeros occur when y = 0. *y-intercepts* occur when x = 0
The ***vertex*** can be found by plugging in the *x* value of the axis of symmetry and solving for y

Do #1 – 10 of the Zeros/Factored Form.

2. A parabola has zeros at -3 and 1. There is a y-intercept of -2. What is the equation of the parabola?

$y = a(x-s)(x-t)$ s, t are the zeros / x-int.

$s = -3$, $t = 1$

$y = a(x - -3)(x-1)$

$y = a(x+3)(x-1)$

Plug in $(0, -2)$

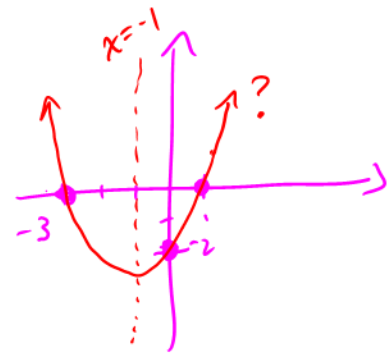
$-2 = a(0+3)(0-1)$

$-2 = a(3)(-1)$

$\frac{-2}{-3} = \frac{-3a}{-3}$

$\frac{2}{3} = a$

$y = \frac{2}{3}(x+3)(x-1)$



#11 $s = 1$, $t = 3$

$y = a(x-1)(x-3)$

plug in a point (not a zero)

$(2, 1)$ x y

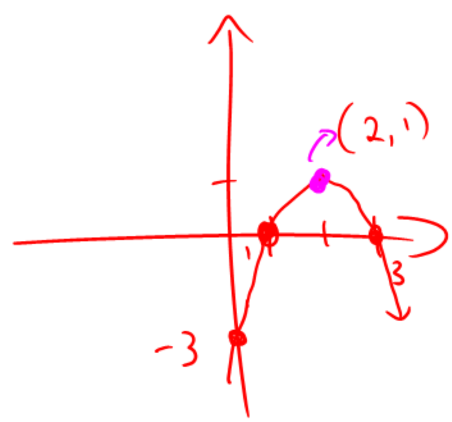
$1 = a(2-1)(2-3)$

$1 = a(1)(-1)$

$\frac{1}{-1} = \frac{-1a}{-1}$

$-1 = a$

$y = -1(x-1)(x-3)$



Note: ① Plug in s and t.

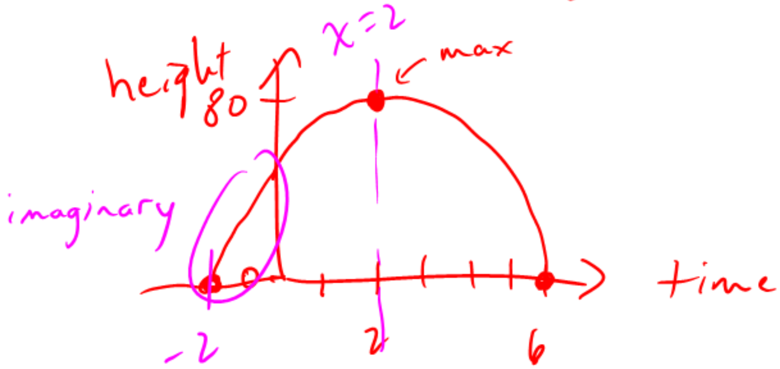
② Plug in any other point on the graph for x & y

③ Solve for "a"

④ State the equation.

Review Question # 16

2 sec. the ball is at 80 m.
It hits the ground at 6 seconds.



$y = a(x-6)(x+2)$

plug in (2,80)

$80 = a(2-6)(2+2)$

$80 = a(-4)(4)$

$\frac{80}{-16} = \frac{-16a}{-16}$

$-5 = a$

$y = -5(x-6)(x+2)$

b) y-int : $x = 0$

$y = -5(0-6)(0+2)$

$y = -5(-6)(2)$

$\boxed{y = 60}$

c) (6,0) -> when the ball hit the ground

(-2,0) -> imaginary (if the ball was thrown from the ground)

Stretching and Reflecting Quadratic Relations

Introducing.....Vertex Form!

The most basic quadratic relation is $\boxed{y=x^2}$ (aka the parent function)

To understand the concept of Stretch and Reflecting, we will be comparing it to $y = ax^2$

We call “a” a stretch factor.

Let's compare the Table of Values for $y = x^2$ and $y = 2x^2$.

$y = x^2$

x	y	
-2	$(-2)^2 =$	4
-1	$(-1)^2 =$	1
0	$(0)^2 =$	0
1	$(1)^2 =$	1
2	$(2)^2 =$	4

×2

$y = 2x^2$

x	y	
-2	$2(-2)^2 =$	8
-1	$2(-1)^2 =$	2
0	$2(0)^2 =$	0
1	$2(1)^2 =$	2
2	$2(2)^2 =$	8

The a value transforms the basic quadratic by multiplying the y-values. We call this a stretch by a factor of a.

Make tables of values for $y=\frac{1}{2}x^2$ and $y=-2x^2$. Then, make a sketch of the graphs.

$y = \frac{1}{2}x^2$

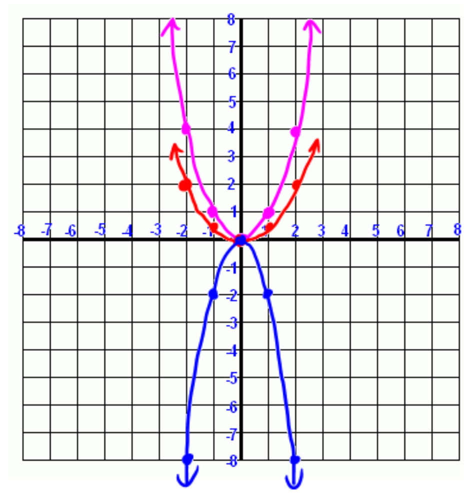
x	y		$y=x^2$
-2	$\frac{1}{2}(-2)^2 =$	2	4
-1	$\frac{1}{2}(-1)^2 =$	0.5	1
0	$\frac{1}{2}(0)^2 =$	0	0
1	$\frac{1}{2}(1)^2 =$	0.5	1
2	$\frac{1}{2}(2)^2 =$	2	4

$\times \frac{1}{2}$

$y = -2x^2$

x	y
-2	$-2(-2)^2 = -8$
-1	-2
0	0
1	-2
2	-8

× -2



Summary:

If $a < 0$ (negative) $\rightarrow$ opens down

$a > 0$ (positive) $\rightarrow$ opens up

$0 < a < 1$ (ex. 0.5) $\rightarrow$ parabola is wide (compressed)

$a > 1$ (ex. 3) $\rightarrow$ parabola is skinny (stretched)

Translations of Quadratic Functions

More of: Introducing.....Vertex Form!

Another word for translations is shifts.

Make a table of values and make a sketch of the graph for the following equations:

$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y=x^2-2$

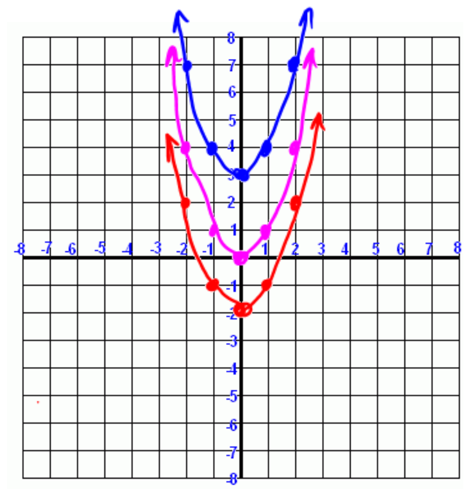
x	y
-2	2
-1	-1
0	-2
1	-1
2	2

- same shape
- down 2

$y=x^2+3$

x	y
-2	7
-1	4
0	3
1	4
2	7

- same shape
- up 3



Make a table of values and make a sketch of the graph for the following equations:

$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y=(x-2)^2$

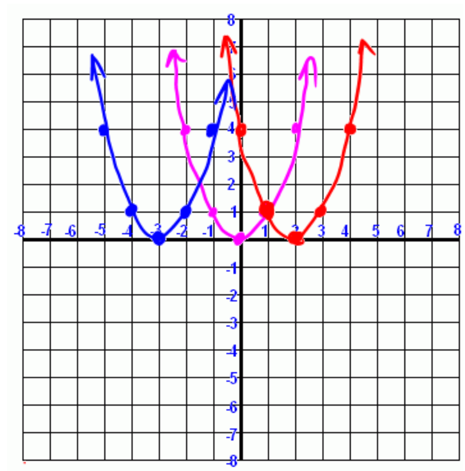
x	y
-2	16
-1	9
0	4
1	1
2	0
3	1
4	4

- same shape
- right 2

$y=(x+3)^2$

x	y
-2	1
-1	4
0	9
1	16
2	25
-3	0
-4	1
-5	4

- same shape
- left 3



Summary:

$y = x^2 + k \rightarrow$ $k>0$ we move up

$k<0$ we move down

$y = (x-h)^2 \rightarrow$ moves to the left/right

The general form of a "shifted" quadratic is:

$$y = (x-h)^2 + k$$

Describe the translations for $y = (x-1)^2 + 3$. Then make a sketch of the relation.

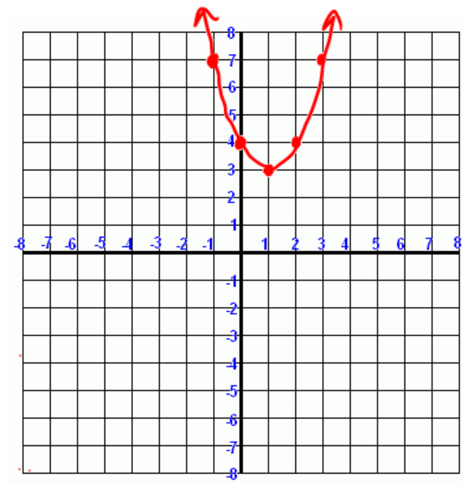
$\boxed{y=x^2}$ (x) right 1 +1 up 3 (y) (+3)

+1

x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-1	7
0	4
1	3
2	4
3	7

+3



Describe the translations for $y = (x+2)^2 - 4$. Then make a sketch of the relation.

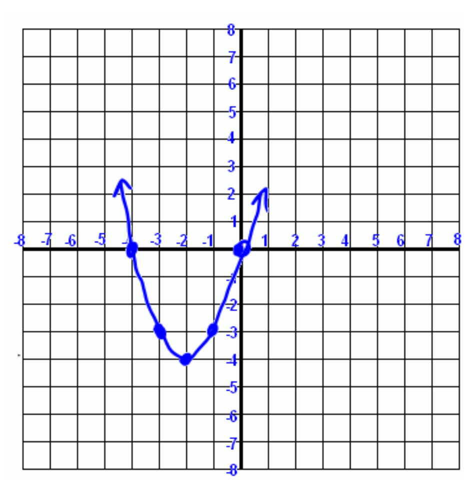
$\boxed{y=x^2}$ left 2 -2 down 4 -4

-2

x	y
-2	4
-1	1
0	0
1	1
2	4

x	y
-4	0
-3	-3
-2	-4
-1	-3
0	0

-4



Putting it All Together: Vertex Form of a Quadratic Relation

And finally, give it up for.....Vertex Form!

The general form for a quadratic in Vertex Form is:

$$y = a(x-h)^2 + k$$

where a is the stretch factor (and/or flip)
and the point (h,k) is the vertex of the parabola.

Fill in the chart

	$a(x-h)^2+k$ $y = -2(x-1)^2 + 4$	$y = 1(x-3)^2 + 0$	$y = 3(x+4)^2 + 2$
Expand ← standard form	$y=-2x^2+4x+2$	$y=x^2-6x+9$	$y=3x^2+24x+50$
stretch factor a	-2	1	3
Vertical shift k	up 4	none	up 2
Horizontal shift h	right 1	right 3	left 4
vertex (h,k)	$(1,4)$	$(3,0)$	$(-4,2)$
Equation of the Axis of symmetry $x=h$	$x=1$	$x=3$	$x=-4$
~~x-intercepts (zeros)~~			
y-intercept : when $x=0$	2	9	50
look at a Max/min value	max 4	min 0	min 2
Max/min point (h,k)	max $(1,4)$	min $(3,0)$	min $(-4,2)$

When graphing, you must STRETCH first, then SHIFT.

$y=-2(x-1)^2+4$
$y=-2(x-1)(x-1)+4$
$y=-2(x^2-x-x+1)+4$
$y=-2(x^2-2x+1)+4$
$y=-2x^2+4x-2+4$
$y=-2x^2+4x+2$

Graphing using a Table of Values

Example: Make a table of values for $y=x^2$ then apply the transformations for $y=2(x-1)^2-4$ to the table of values. Finally, graph it.

a stretch of $\times 2$ (y)

shift right +1 (x's)

shift 4 down −4 (y's)

$y=x^2$

x	y
−2	4
−1	1
0	0
1	1
2	4

×2

x	y
−2	8
−1	2
0	0
1	2
2	8

+1 −4

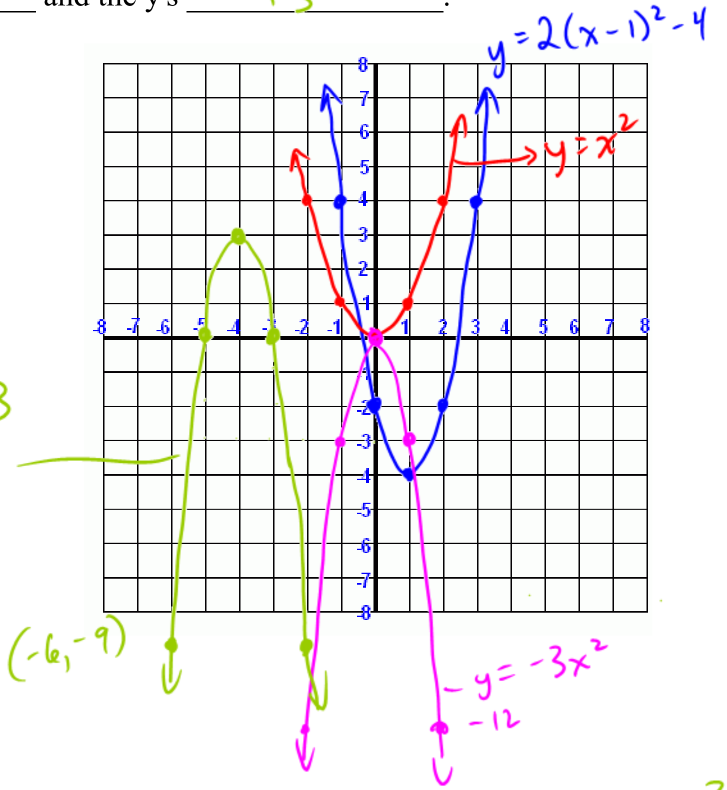
x	y
−1	4
0	−2
1	−4
2	−2
3	4

Must know This!

Graphing using the Sketch of the Graph

visual

To sketch $y=-3(x+4)^2+3$, start by drawing a sketch of $y=x^2$. Then resketch the graph applying the stretch. In this case, a is $\boxed{-3}$. Take this new sketch and then apply the shifts. In this case shift the x's ___−4___ and the y's ___+3___.



$y=-3(x+4)^2+3$

Homework: 1-6 of "Graphing $y=a(x-h)^2+k$"

Vertex Form

$x = \frac{s+t}{2}$

Convert from Standard Form to Vertex Form by finding the zeros, $\boxed{\text{AoS}}$, and the vertex.

↳ factor ↳ plug in the AoS

$y = 5x^2 + 60x + 175$

① $y = 5(x^2 + 12x + 35)$

$7 + 5 = 12$

$7 \times 5 = 35$

$y = 5(x+7)(x+5)$

$y = a(x - s)(x - t)$

zeros

Zeros: -7 and -5

② A.O.S.: $x = \frac{s+t}{2}$

$x = \frac{-7 + -5}{2} = \frac{-12}{2} = -6$

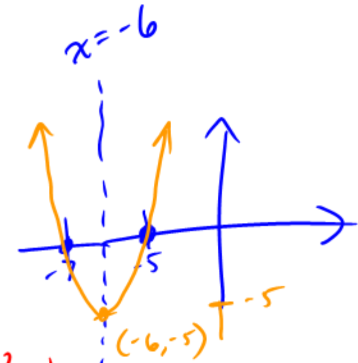
$\boxed{x = -6}$

③ $y = 5(x+7)(x+5)$ plug in $x = -6$

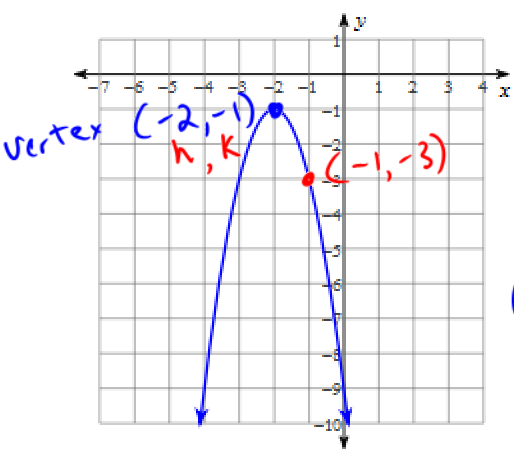
$y = 5(-6+7)(-6+5)$

$y = 5(1)(-1)$

$y = -5$



Find the Equation of the following parabola.



Steps

① plug in the vertex into vertex form

② plug in one other point

③ solve for a

④ State the equation

① $y = a(x-h)^2 + k$

$y = a(x+2)^2 - 1$

② $(-1, -3)$ x y

③ $-3 = a(-1+2)^2 - 1$

$-3 = a(1)^2 - 1$

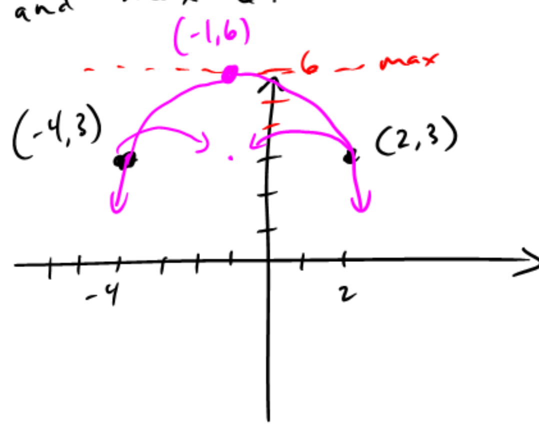
$-3 + 1 = 1a - 1 + 1$

$\frac{-2}{1} = \frac{1a}{1}$

$-2 = a$

④ $y = -2(x+2)^2 - 1$

3. Passes through $(2, 3)$ and $(-4, 3)$ and max 6.



$$y = a(x+1)^2 + 6$$

plug in $(-4, 3)$
x y

$$3 = a(-4+1)^2 + 6$$

$$3 = a(-3)^2 + 6$$

$$3 = 9a + 6$$

$$-3 = 9a$$

$$-\frac{1}{3} = a \rightarrow \quad y = -\frac{1}{3}(x+1)^2 + 6$$

Goal: Take $y=ax^2+bx+c$ and put into $y=a(x-h)^2+k$

Completing the Square

from section 6.3

$y=ax^2+bx+c$ can be written in vertex form $y=a(x-h)^2+k$ by creating a **perfect square** – called **Completing the Square.**

Algebraically, we consider squares to look like this: $5^2, 25$

24 is NOT a square, but we can add 1 to make it a square like this:

$24=(24+1)-1=5^2-1$

Consider: $(x+2)^2=x^2+4x+4$

↳ Note: $(x+2)(x+2)$

x^2+4x is NOT a square, what is missing? $+4$

$(x^2+4x+4)-4=(x+2)^2-4$

Example

$y=2x^2+12x-3$

① $y=2(x^2+6x)-3$

② ↳ $\left(\frac{6}{2}\right)^2=(3)^2=9$

$y=2(x^2+6x+9-9)-3$

③ $y=2(x^2+6x+9)-18-3$

④ $y=2(x+3)^2-21$ v$(-3,-21)$

↳ Note: $(x+3)(x+3)$

Steps to Completing the Square

1. Factor the a from the x^2 and x terms
2. Use the coefficient of the 2nd term, divide it by 2 and square it. Rewrite the equation by adding and subtracting this term in the brackets
3. Move the subtracted square outside the brackets by multiplying it by the a
4. Factor the perfect square and collect like terms

To do 1–4 of Completing the Square.

Examples – Complete the Square and State the Vertex

$y=1x^2+8x+15$

① $y=1(x^2+8x)+15$

② ↳ $\left(\frac{8}{2}\right)^2=(4)^2=16$

$y=1(x^2+8x+16-16)+15$

③ $y=1(x^2+8x+16)-16+15$

④ $y=1(x+4)^2-1$ v$(-4,-1)$

$y = \boxed{-2}x^2 + 12x - 7$

① $y = -2(x^2 - 6x) - 7$

② $\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$y = -2(x^2 - 6x + 9 - 9) - 7$

③ $y = -2(x^2 - 6x + 9) + 18 - 7$

④ $y = -2(x \boxed{-3})^2 + 11$ $v(3, 11)$

$y = \frac{1}{2}x^2 + 6x + 5$

$6 \div \frac{1}{2}$?

$6 \times \frac{2}{1} = 12$

$y = \frac{1}{2}(x^2 + 12x) + 5$

$\left(\frac{12}{2}\right)^2 = (6)^2 = 36$

$y = \frac{1}{2}(x^2 + 12x + 36 - 36) + 5$

$y = \frac{1}{2}(x^2 + 12x + 36) - 18 + 5$

$y = \frac{1}{2}(x + 6)^2 - 13$ $v(-6, -13)$

Questions 5 - 10