Name: ______________________________

AΩ∞Ω

MATH@TD

Similar Triangles

and

Trigonometry

Unit Outline:

a. Review of Angle Theorems
b. Similar Triangles
c. Right Angle Triangle Ratios
d. Primary Trigonometric Ratios SOH CAH TOA
e. Sine Law
f. Cosine Law

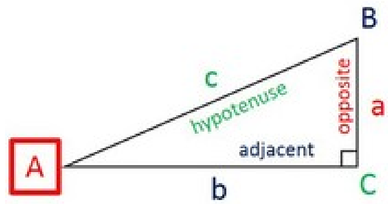
Formulas you will learn

Right Angle Triangles SOHCAHTOA

$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

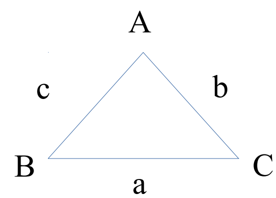
$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$



Sine Law

For the Sine Law we need: - An angle and its opposite side, and one other piece of information.
- OR SAS, ASA [and ASS (use with caution!)]

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



The length of any side, divided by the Sine of its opposite angle is the same for all three pairs
If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
If we are trying to **find the length** of a side, use the second for of the law (**with sides on top**)

Cosine Law

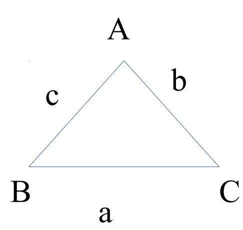
For the Cosine Law we need: - 2 sides and the included angle.
- 3 sides

To find a side (have SAS):

$$a^2 = b^2 + c^2 - 2\text{bc}\, cosA$$

$$b^2 = a^2 + c^2 - 2\text{ac}\, cosB$$

$$c^2 = a^2 + b^2 - 2\text{ab}\, cosC$$



To find an angle (have SSS):

$$\cos A = \frac{a^2 - b^2 - c^2}{-2\text{bc}}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2\text{ac}}$$

$$\cos C = \frac{c^2 - a^2 - b^2}{-2\text{ab}}$$

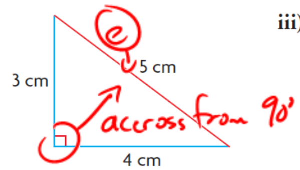
7.1 Similar Triangles and Trigonometry

Getting Started

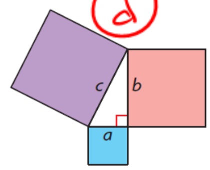
Terms you should already know. Match the term with the example:

a) Ratio
b) Proportion
c) Congruent Triangles
d) Pythagorean Theorem
e) Hypotenuse
f) Acute Angle

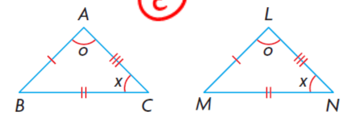
i) e



iii) d



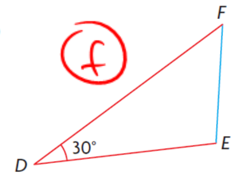
v) c



ii) $\frac{3}{5}$ or 3:5 or 3 to 5 a

iv) $\frac{x}{5} = \frac{3}{15}$ b

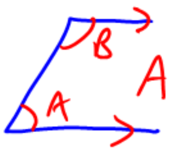
vi) f



Angle Theorems you should already know:

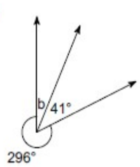
a Supplementary angles add to 180^0
b Complementary add to 90^0
c Opposite angles are equal → OAT: Opposite Angle Theorem
d Sum of interior angles in a triangle (180^0) ASTT: Angle Sum of a Triangle theorem

Parallel line relationships:
e Alternate angles are equal (Z-pattern) Z
f Corresponding angles are equal (F-pattern) F
g Co-interior angles add to 180^0 (C-pattern)



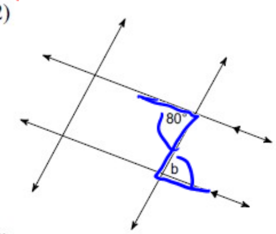
A + B = 180°

1)



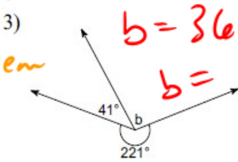
b = 360 − 41 − 296
b = 23°

2)



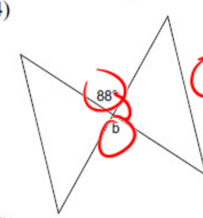
b = 80°

3)



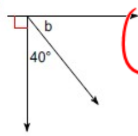
b = 360 − 41 − 221
b = 68°

4)



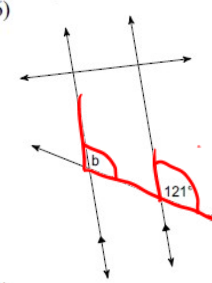
c b = 88°

5)



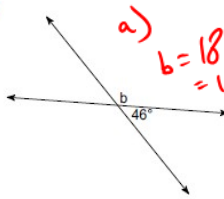
b 90 − 40 = b
50° = b

6)



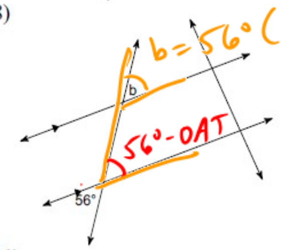
b = 121°

7)



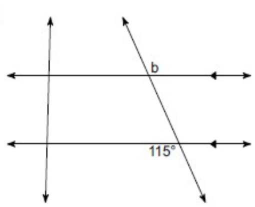
a) b = 180 − 46
= 134°

8)

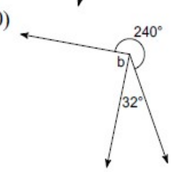


b = 56° (F pat)

9)

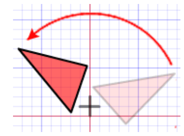


10)

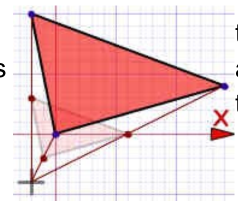


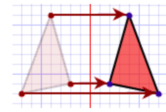
7.1 Congruence and Similarity in Triangles

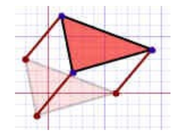
Congruent Triangles exist when one shape can become another using Rotations (turn), Reflections (flip) or Translations (slide). Corresponding sides have the same lengths and corresponding angles the same angles.



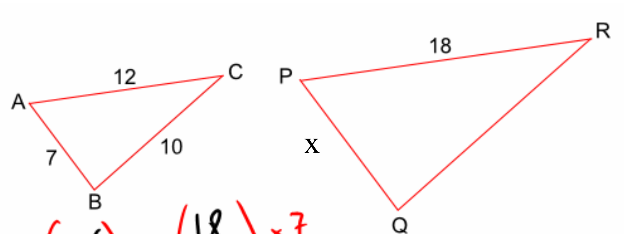
Similar triangles are triangles which have the exact same angle measures but the side lengths may be resized. The triangles are then related by a scale factor which is a ratio of corresponding side lengths stating how much bigger (or smaller) the second triangle is through proportions, you can solve for the side lengths.





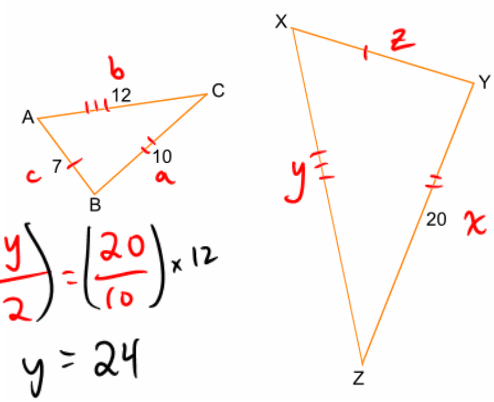


The following are similar triangles. Solve for *x* and *y*.



$$7\times\left(\frac{x}{7}\right)=\left(\frac{18}{12}\right)\times 7$$

$$x = 10.5$$



$$12\times\left(\frac{y}{12}\right)=\left(\frac{20}{10}\right)\times 12$$

$$y = 24$$

What is the scale factor between ΔABC and ΔPQR?

$$\frac{18}{12} = 1.5$$

∴ ΔPQR is $\frac{3}{2}$ times bigger than ΔABC

Notation:

The ~ symbol is used to indicate similarity. The ≅ symbol is used to indicate congruence.

When naming triangles that are congruent or similar, the corresponding vertices must be listed in the same order.

For example, if ∠A=∠D, ∠B=∠E, and ∠C=∠F, then ABC≅DEF. (ABC ≠ EDF)

△ABC ~ △RST. Complete each statement.

a) ∠ABC = ∠RST **d)** △STR ~ ΔBCA

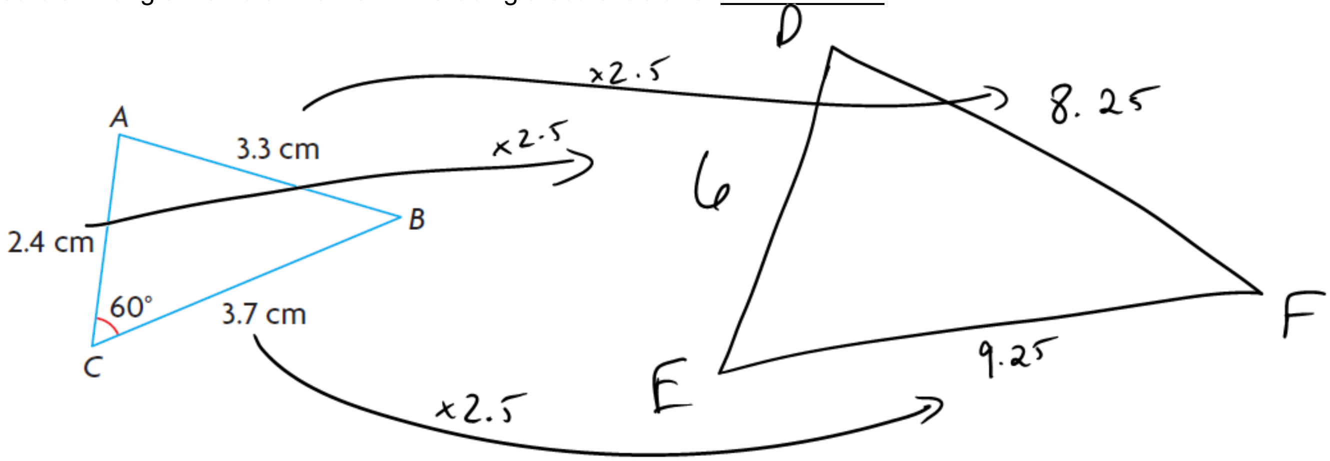
b) ∠BCA = ∠STR **e)** $\frac{ST}{BC} = \frac{RS}{AB} = \frac{RT}{AC}$

c) $\frac{AB}{RS} = \frac{BC}{ST} = \frac{AC}{RT}$ **f)** ∠SRT = ∠BAC

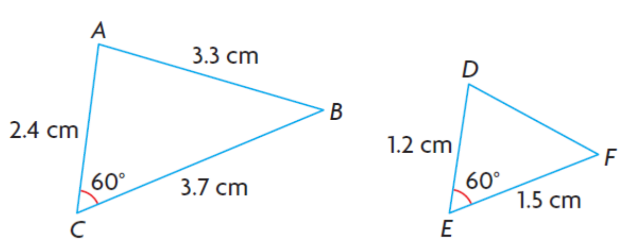
How do you know they are Congruent or Similar Triangles?

Congruence Proofs	
Side-Side-Side (SSS)	If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
Side-Angle-Side (SAS)	If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent. Fear: ASS
Angle-Side-Angle (ASA)	If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
Angle-Angle-Side (AAS)	If two angles and the non-included side of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
Similarity Proofs	
Angle-Angle (AA)	If two angles of one triangle are congruent to two angles of another triangle, the triangles are similar.
SSS proportional	If the three sets of corresponding sides of two triangles are in proportion, the triangles are similar.
SAS proportional	If an angle of one triangle is congruent to the corresponding angle of another triangle and the lengths of the sides including these angles are in proportion, the triangles are similar.

Create a Triangle that is similar to ΔABC using a scale factor of 2.5.



Is ΔDEF similar to ΔABC? Provide Proof.



Statement	*Reason*
$\angle C = \angle E$	given
$\frac{AC}{DE} = \frac{2.4}{1.2} = 2$	Not similar.
$\frac{BC}{EF} = \frac{3.7}{1.5} \neq 2$	

Homework: Chapter ~~7.1 pg 379: 4—7, 8acd, 11, 12, 13~~

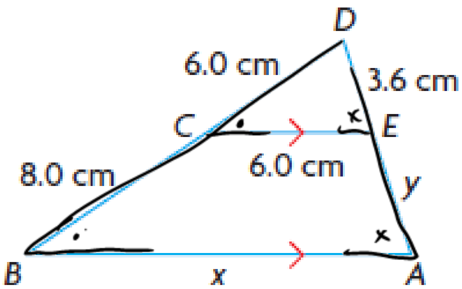
pg 372: 6 ; pg 378: 4, 7, 8, 11.

Example 7.2.1

a) Show that the two triangles to the right are similar, with reasons.

b) Determine *x and y* ✓ ✓

$\frac{6}{14} = \frac{3}{7}$



a) Statement	*Reason*
$\angle DCE = \angle DBA$	F pat.
$\angle DEC = \angle DAB$	F pat.

a) $\triangle DEC \sim \triangle DAB$ (AA~) why?

b) $\frac{x}{6} = \frac{14}{6}$

$x = 14$

$3.6 \times \left(\frac{AD}{3.6}\right) = \left(\frac{14}{6}\right) \times 3.6$

$AD = 8.4$

$\therefore y = 8.4 - 3.6 = 4.8$

Example 7.2.2

A new bridge is going to be built across a river, but the width of the river cannot be measured directly. Surveyors set up posts at points A, B, C, D and *E*. Then they took measurements relative to the posts.

What is the width of the river?

a) Show that the two triangles in this diagram are similar.

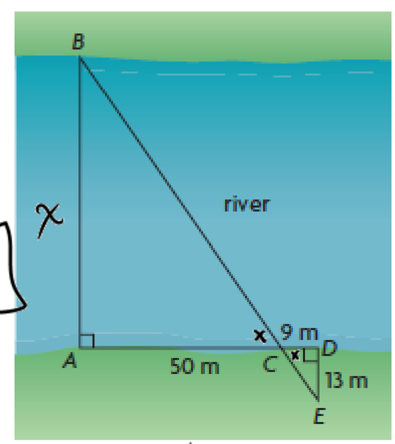
b) Determine the width of the river

a) Statement	*Reason*
$\angle A = \angle D$	90° (given)
$\angle ACB = \angle DCE$	O.A.T.

$\triangle ACB \sim \triangle DCE$ (AA~)

b) $13 \times \left(\frac{x}{13}\right) = \left(\frac{50}{9}\right) \times 13$

$\boxed{x = 72.2 \text{ m}}$



In order to solve “real world problems” you have to be SURE that the triangles you are working with are similar.

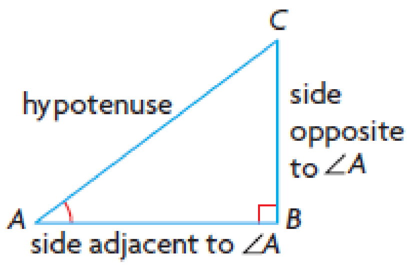
All that is needed for proof of similarity is AA similarity.

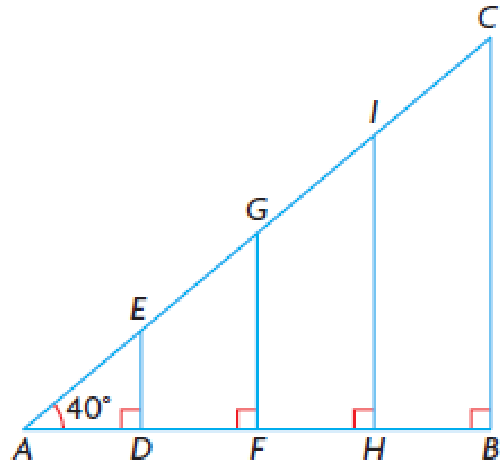
Homework Chapter 7.2 pg. 386: ~~4, 6, 9, 10, 11, 12, 14 (toughy!!)~~

4, 5, 6, 8, 9, 11 → tough.

7.3 Exploring Similar Right Angle Triangles

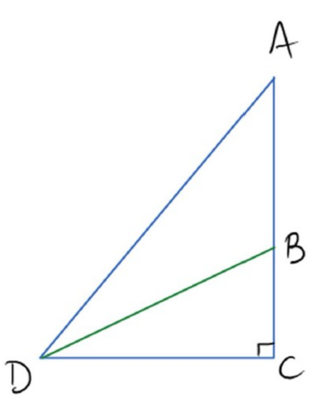
$\Delta ADE \sim \Delta AFG$ therefore $AD \sim AF$
$AE \sim AG$
$DE \sim FG$





Use a ruler and measure the side lengths then calculate the ratios.

				Trigonometric Ratios		
Triangle	Side OPPOSITE to <A	Side ADJACENT to <A	HYPOTENUSE	<u>OPPOSITE</u> HYPOTENUSE	<u>ADJACENT</u> HYPOTENUSE	<u>OPPOSITE</u> ADJACENT
ΔABC						
ΔADE						
ΔAFG						
ΔAHI						



Consider the picture:
Notice that $BC < AC$, and that

$\angle BDC < \angle ADC$

This suggests that there is a mathematical relationship in triangles between lengths of sides and size of angles That relationship can be described by what we call **TRIGONOMETRY!!**

7.4 The Primary Trigonometric Ratios

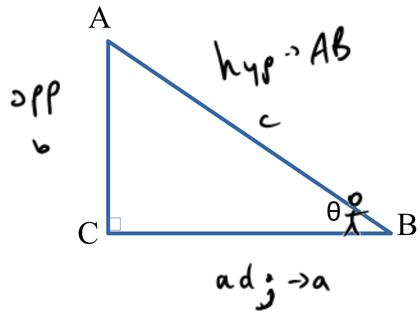
Given the Right ΔABC

We use θ, Theta to indicate angle in geometry.
For ∠B or θ we call side:

b -

a -

c -



The Trig Ratios

Sine: $\sin(B) = \frac{opp}{hyp}$

Cosine: $\cos(B) = \frac{adj}{hyp}$

Tangent: $\tan(B) = \frac{opp}{adj}$

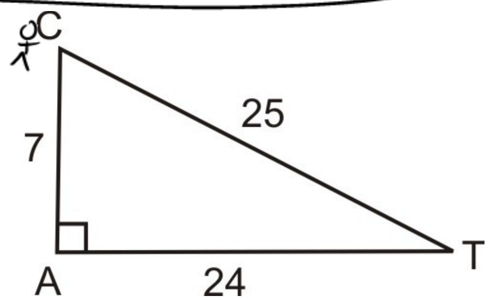
Examples ΔACT

Calculator Test: Sin 30 = 0.5
or 30, sin = 0.5

Sine: $\sin(T) = \frac{opp}{hyp} = \frac{7}{25}$

Cosine: $\cos(T) = \frac{24}{25}$

Tangent: $\tan(C) = \frac{opp}{adj} = \frac{24}{7}$



SOH CAH TOA

7.5 Solving for Sides using the Primary Trigonometric Ratios

Solve for the unknown in the following:

$8\times(\sin 35)=\left(\frac{x}{8}\right)\times 8$

$8\times\sin 35 = x$

Note! $8\sin 35 \neq \sin 280$

$4.6 = x$

$y\times(\tan 62)=\left(\frac{3}{y}\right)\times y$

$\frac{y\tan 62}{\tan 62} = \frac{3}{\tan 62}$

$y = \frac{3}{\tan 62}$ $\therefore y = 1.6$

Memorize: when the unknown is on the bottom: do the switch

ex $\sin 20 = \frac{4}{x}$

$x = \frac{4}{\sin 20}$

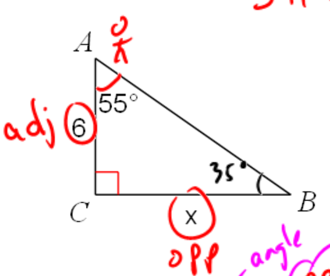
Notice: Pay close attention to when the unknown is in the numerator and when the unknown is in the denominator.

Steps to Solve: 1. Identify the given acute angle ~~you are solving~~.
2. Identify 1 known side and one unknown side.
3. Write the appropriate Trig Ratio using #1 and 2 and solve

Right Angled Triangles

Solve for the unknown side in the following examples

SOH CAH TOA

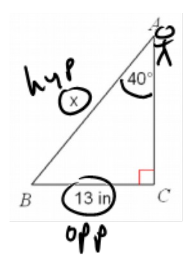


Tan θ = $\frac{opp}{adj}$ angle sides

$6\times(\tan 55)=\left(\frac{x}{6}\right)\times 6$

$6\tan 55 = x$

$8.6 = x$



SOH

$\sin\theta = \frac{opp}{hyp}$

$\sin 40 = \frac{13}{x}$

$x = \frac{13}{\sin 40}$

$x = 20.2$ in

11-20

7.5 Solving for Angles using the Primary Trigonometric Ratios

To solve for the angle, you must use the Inverse function, which is $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$

Solve for θ in the following examples

$\sin\theta=0.4782$

$\theta = \sin^{-1}(0.4782)$

$\theta = 28.6°$

$\tan\theta=2.01$

$\theta = \tan^{-1}(2.01)$

$\theta = 63.5°$

$\cos\theta=\frac{3}{5}$

$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

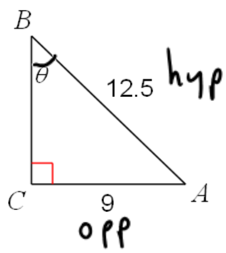
$\theta = 53.1°$

Steps to Solve:

1. Identify the angle you are solving.
2. Identify 2 known sides.
3. Write the appropriate Trig Ratio using #2 and solve

Right Δ's

S$\frac{O}{H}$ C$\frac{A}{H}$ T$\frac{O}{A}$



angle

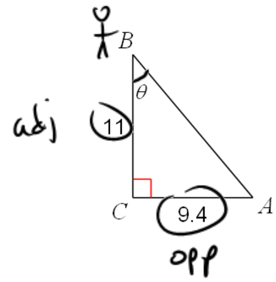
sides

$\sin\theta = \frac{opp}{hyp}$

$\sin B = \frac{9}{12.5}$

$\angle B = \sin^{-1}\left(\frac{9}{12.5}\right)$

$\angle B = 46°$



$\tan\theta = \frac{opp}{adj}$

$\tan B = \frac{9.4}{11}$

$\angle B = \tan^{-1}\left(\frac{9.4}{11}\right)$

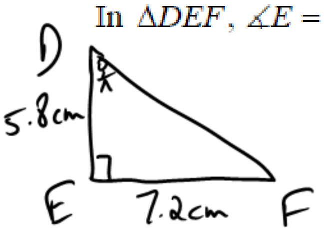
$\angle B = 40.5°$

$\boxed{21-34}$

Example 1.

In ΔDEF, $\measuredangle E = 90°$, $d = 7.2cm$, and $f = 5.8cm$. Solve the triangle. (pictures are your friends!)

↳ find all angles and sides



T$\frac{O}{A}$

$\tan D = \frac{7.2}{5.8}$

$\angle D = \tan^{-1}\left(\frac{7.2}{5.8}\right)$

$\angle D = 51°$

D = 51°	d = 7.2cm
E = 90°	e = 9.2cm (P.T.)
F = 39° (ASTT)	f = 5.8cm

Example 2.

A $6m$ ladder is leaning against a house. If the bottom of the ladder is $1.2m$ from the house, determine the angle the ladder makes with the ground.

C$\frac{A}{H}$

$\cos A = \frac{1.2}{6}$

$\angle A = \cos^{-1}\left(\frac{1.2}{6}\right)$

$\angle A = 78.5°$

The angle of the ladder to the ground is 78.5°

Homework Chapter 7.5 pg. ~~404: 2, 3, 8, 9, 11, 13~~

difficult.

412: 1, 2, 9-10, 11, 13, 16

7.6 Solving Right Triangle Real World Problems

Angle of Elevation: the angle up from a horizontal

Angle of Depression: the angle down from a horizontal

7.6 Examples

#4 A tree that is 9.5 m tall casts a shadow that is 3.8 m long. What is the angle of elevation of the Sun?

$T\frac{O}{A}$

$\tan A = \frac{9.5}{3.8}$

$\angle A = \tan^{-1}\left(\frac{9.5}{3.8}\right)$

$\angle A = 68.2°$

The angle of elevation is 68.2°

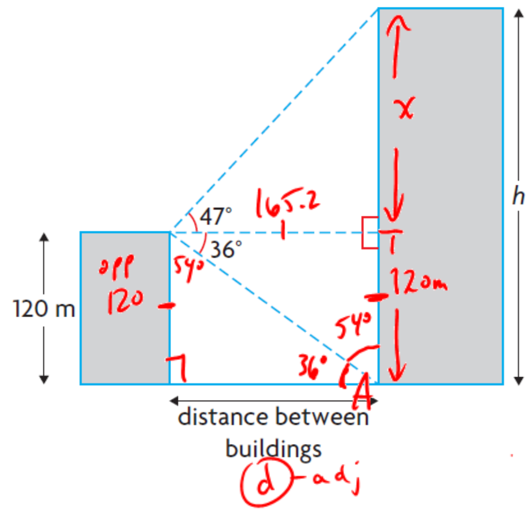
#6 A building code states that a set of stairs cannot rise more than 72 cm for each 100 cm of run. What is the maximum angle at which the stairs can rise?

#8 Firefighters dig a triangular trench around a forest fire to prevent the fire from spreading. Two of the trenches are 800 m long and 650 m long. The angle between them is 30°. Determine the area that is enclosed by these trenches.

#15 A video camera is mounted on top of a building that is 120 m tall. The angle of depression from the camera to the base of another building is 36°. The angle of elevation from the camera to the top of the same building is 47°

a) How far apart are the two buildings?

b) How tall is the building viewed by the camera?



a) $\tan 36° = \frac{120}{d}$

$d = \frac{120}{\tan 36}$

$d = 165.2\text{m}$

The distance b/w is 165.2m

b) $\tan 47 = \frac{x}{165.2}$

$165.2 \tan 47 = x$

$177.2\text{m} = x$

$h = 120 + 177.2 = 297.2$

The building is 297.2m tall!

Homework Chapter 7.6 pg. 412, # 5, 7, 9-13, 16, 17

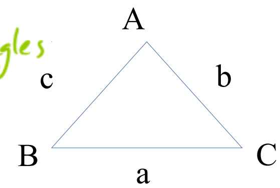
Understand that solving problems involves drawing a picture and then developing a plan to solve for the unknown. This may take several steps, PATIENCE, and PRACTICE.

8.1 and 8.2 Sine Law – Barbell Method

For the Sine Law we need: - An angle and its opposite side, and one other piece of information.
- OR SAS, ASA [and ASS (use with caution!)]

The Sine Law:

sides $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ or $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ – angles



~~sin 10~~
~~sin 170~~
~~sin 20~~
~~sin 160~~

The length of any side, divided by the Sine of its opposite angle is the same for all three pairs
If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
If we are trying to **find the length** of a side, use the second for of the law (**with sides on top**)

Ex 1. Calculate

$$a=\left(\frac{3}{\sin 60}\right)\sin 72$$

Type 3 ÷ sin60 enter
× sin 72

a = 3.3

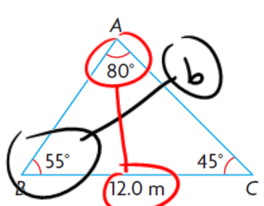
Ex 2. Calculate:

$$\sin A=\left(\frac{\sin 72}{15}\right)12$$

(sin 72) enter, ÷ 15 enter, × 12 enter

sin A = 0.7608
∠A = sin⁻¹(0.7608) → better: plug in the Answer
∠A = 49.5°

Ex. 3 – Find the Length of Side b to the nearest tenth



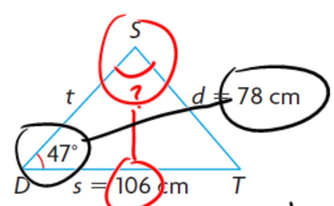
Start with the unknown:

$$\sin 55\left(\frac{b}{\sin 55}\right)=\left(\frac{12}{\sin 80}\right)\sin 55$$

$$b=\left(\frac{12}{\sin 80}\right)\sin 55$$

$$b = 9.98\text{ m}$$

Ex. 4 – Find <S to the nearest degree.



Start with the unknown.

$$106\times\left(\frac{\sin S}{106}\right)=\left(\frac{\sin 47}{78}\right)\times 106$$

$$\sin S=\left(\frac{\sin 47}{78}\right)\times 106$$

$$\sin S = 0.9939$$

$$\angle S = 83.67°$$

A telephone pole is supported by two wires on opposite sides. At the top of the pole, the wires form an angle of 60°. On the ground, the ends of the wires are 15.0 m apart. One wire makes a 45° angle with the ground. How long are the wires, and how tall is the pole?

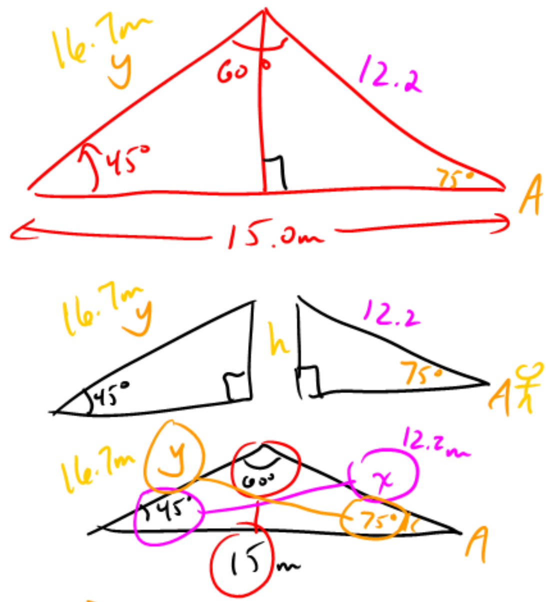
$$\sin 45\left(\frac{x}{\sin 45}\right) = \left(\frac{15}{\sin 60}\right)\sin 45$$

$$\boxed{x = 12.2\text{m}}$$

$$\angle A = 180 - 60 - 45 = 75°$$

$$\sin 75\left(\frac{y}{\sin 75}\right) = \left(\frac{15}{\sin 60}\right)\sin 75$$

$$\boxed{y = 16.7\text{m}}$$



$$S\frac{O}{H}: \quad 12.2\times(\sin 75) = \left(\frac{h}{12.2}\right)\times 12.2$$

$$\boxed{11.8\text{m} = h}$$

Make sentence.

Note: Sine law

$$\frac{h}{\sin 75} = \frac{12.2}{\sin 90}$$

$$\frac{h}{\sin 75} = 12.2$$

Homework: Pg. 433 – 434 #2b, 3acdf, 4 – 8, 10, 11

Do most

Pg 434 #10

$\tan P = \frac{8}{6}$

$\angle P = \tan^{-1}\left(\frac{8}{6}\right)$

$\angle P = 53.1°$

$6^2 + 8^2 = q^2$

$10 = q$

$\sin P = \frac{8}{10}$

$\angle P = 53.1°$

$8\left(\frac{\sin P}{8}\right) = \left(\frac{\sin 90}{10}\right)8$

$\sin P = \frac{8}{10}$

$\angle P = 53.1°$

8.3 Cosine Law

Which Law do I Use?

Do I have a right angle triangle? **YES** - use SOH CAH TOA

↓ **NO**

Do I have a CORRESPONDING angle and side pair in the triangle? **YES** - use SINE LAW

↓ **NO**

Cannot use SOHCAHTOA or SINE Law   Must use the **The COSINE LAW**

For the Cosine Law we need: - 2 sides and the included angle. SAS
- 3 sides

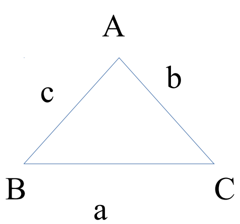
The cosine law is an extension of the Pythagorean theorem to triangles with no right angle.

To find a side (have SAS):

Exam $\boxed{a^2 = b^2 + c^2 - 2bc\cos A}$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



ΔXYZ $x^2 = y^2 + z^2 - 2yz\cos X$

To find an angle (have SSS):

$$\boxed{\cos A = \frac{a^2 - b^2 - c^2}{-2bc}}$$

$$\cos B = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$$

$$\cos X = \frac{x^2 - y^2 - z^2}{-2yz}$$

Ex 1. Calculate *a* if

$$a^2 = \boxed{4^2 + 6^2 - 2(4)(6)\cos 56}$$

use your calculator!

$\sqrt{a^2} = \sqrt{25.16\ldots}$

$a = 5.0$

~~or $a^2 = 16 + 36 - 48\cos 56$~~

Ex 2. Calculate <X if:

$$\cos X = \frac{4.5^2 - 3.2^2 - 4.6^2}{-2(3.2)(4.6)}$$] - type in] - type in

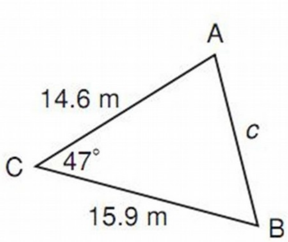
$\cos X = \frac{-11.15}{-29.44}$

$\angle X = \cos^{-1}\left(\frac{-11.15}{-29.44}\right)$

$\angle X = 68°$

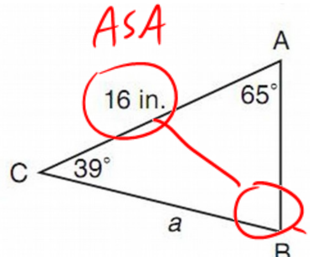
Determine if you need to use the SINE law or the COSINE law for the following triangles:

a)



SAS → cosine Law

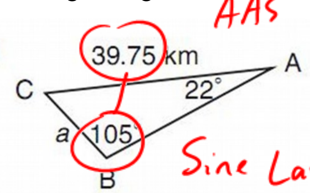
b) ASA



180-39-65

Sine Law

c) AAS



Sine Law

Ex. 3 – Find <A to the nearest tenth if a = 3m, b = 5m, and c = 4.7m

SSS

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos A = \frac{3^2 - 5^2 - 4.7^2}{-2(5)(4.7)}$$

$$\cos A = \frac{-38.09}{-47}$$

$$\angle A = \cos^{-1}\left(\frac{-38.09}{-47}\right) \rightarrow \angle A = 35.9^\circ$$

Ex. 4 – Find side b, to the nearest tenth if <B = 68°, a = 17 cm, and c = 22 cm

SAS - cosine Law.

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$b^2 = 17^2 + 22^2 - 2(17)(22)\cos 68$$

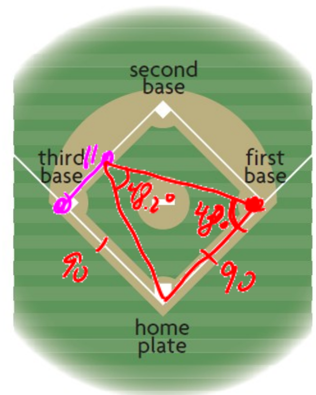
$$b^2 = 492.8$$

$$\boxed{b = 22.2\text{ cm}}$$

Ex. 5 The bases in a baseball diamond are 90 ft apart. A player picks up a ground ball 11 ft from third base, along the line from second base to third base. Determine the angle that is formed between first base, the player's present position, and home plate.

Choose your own adventure!

$\angle ? = 48^\circ$



Homework: pg 443: 2, 3a, 4b, 5bc, 7, 9, 12

Homework 2: Solving Problems using Trigonometry 449: 1 – 10, 15