ANSWERS

CHAPTER 1

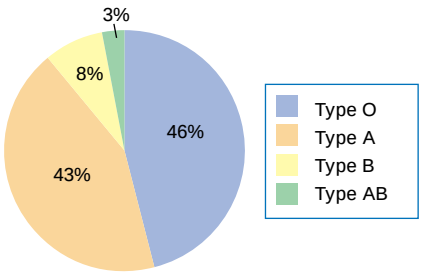
Review of Prerequisite Skills, pp. 4–5

1. a) -26 **b)** 35 **c)** 1 **d)** 3

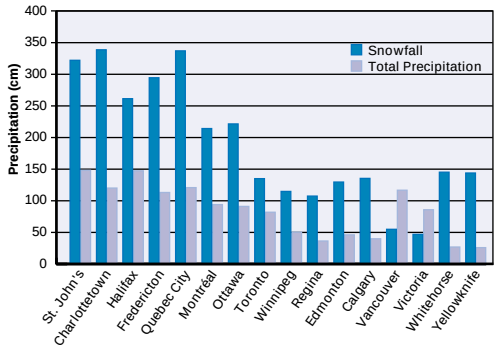
2. a) 4 **b)** 3 **c)** 44 **d)** 0 **e)** 30 **f)** 7

3. a) 5 **b)** -2 **c)** 20 **d)** 1 **e)** ± 5 **f)** 5 **g)** -5 **h)** 9

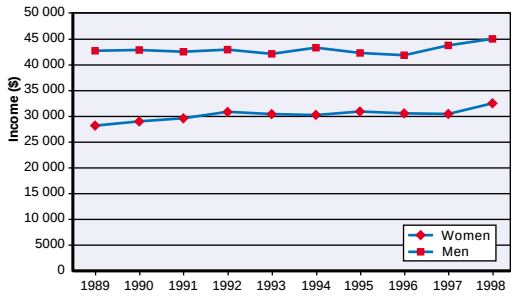
4.



5.



6.



7. a) A specific cell is referred to with a letter for the column and a number for the row. For example, cell A1.

b) In Microsoft® Excel, specify the first and last cells in the range, with a colon between them. For example, A1:A10. In Corel® Quattro® Pro, specify the first and last cells in the range, with two periods between them. For example, A1..A10.

c) Move the cursor to the cell containing the data. Select the Copy command from the Edit menu. Move the cursor to the cell into which you want to copy the data. Select the Paste command from the Edit menu.

d) Move the cursor to the cell containing the data. Point the mouse at the border of the cell, so that the mouse cursor appears as an arrow. Left click the mouse and hold. Drag the data to the new cell, and release the mouse.

e) Move the mouse to where the column letters are displayed near the top of the screen. Point the mouse directly at the border between the column you want to expand and the column to the immediate right. Left click the mouse and hold. Drag to the right until the column is the desired width, and release.

f) Move the cursor to any cell in the column to the immediate right of where you wish to add the new column. Select Columns from the Insert menu.

g) In Microsoft® Excel, the symbol = must precede a mathematical expression. In Corel® Quattro® Pro, the symbol + must precede a mathematical expression.

8. $\triangle ABC \sim \triangle HJG$ (SSS)

9. a) Subtract 3 to get subsequent terms; 56, 53, 50.

b) Divide by 2 to get subsequent terms; 12.5, 6.25, 3.125.

c) Divide by -2 to get subsequent terms; $\frac{1}{16}, -\frac{1}{32}, \frac{1}{64}$.

d) The number of factors of a increases by 1 and the number of factors of b doubles, to get subsequent terms: $aaaaa$, $bbbbbbbbbbbbbbb$, $aaaaaa$.

10. a) 4:1

11. a) $x^2 - 2x + 1$ **b)** $2x^2 - 7x - 4$ **c)** $-5x^2 + 10xy$

d) $3x^3 - 6x^2y + 3xy^2$ **e)** $9x^3 - 9x^2y$ **f)** $ac - ad + bc - bd$

12. a) 0.25 **b)** 0.46 **c)** $0.\overline{6}$ **d)** 11.5 **e)** $0.\overline{857\,142}$

f) 0.73

13. a) 46% **b)** 80% **c)** $3.\overline{3}\%$ **d)** 225% **e)** 137.5%

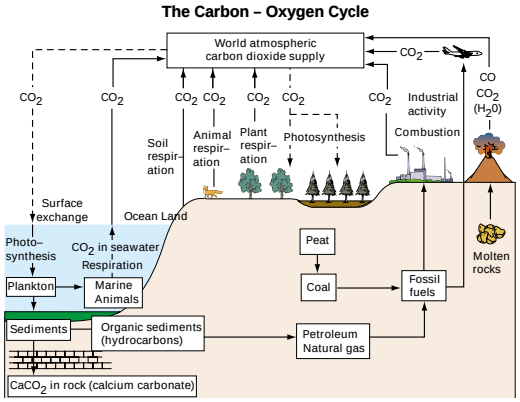
Section 1.1, pp. 10–13

Practise

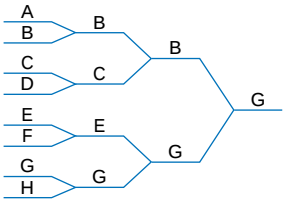
1. a) iterative **b)** iterative **c)** non-iterative

d) iterative **e)** non-iterative **f)** iterative

2.



3.



The order of winners may vary.

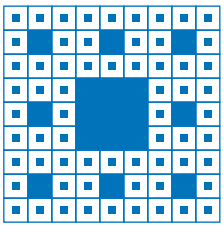
Apply, Solve, Communicate

4. Answers will vary.

5. The tracing shows irregularities according to the pulse rate. Sketches may vary.

6. a) Select an item to begin the process. Compare the ranking of this item with the ranking of a second item. The item with the higher ranking is now ranked first. Now, compare the next item's ranking with the first ranked item. The higher ranking of these two is now ranked first. Continue in this fashion with subsequent items to find the item with the highest rank. Once this item is found, work with the remaining items to find the item with the second highest ranking by selecting one of the remaining items to begin the process as outlined above. Continue in this fashion until there are no remaining items.

b) Compare the first two data, then the second and third, and then the third and fourth, and so on, interchanging the order of the data in any pair where the second item is ranked higher. Repeat this process until no data are interchanged.

7.  ; $1 - \left(\frac{8}{9}\right)^n$

8. a) 48 **b)** 192 **c)** $3 \times 4^{n-1}$

9. 10, 88, 7714, 59 505 532

10. a) i) Begin with a segment 1 unit long. Branch off at 90° with two segments each one half the length of the previous branch. Repeat the process.

ii) Begin with a segment 1 unit long. Branch off with 3 segments, each of which are one third as long and separated by 60°. Repeat the process.

iii) Begin with a vertical segment 1 unit long. Branch off the top to the left at 120° with a 1-unit segment. Continue upwards on the first branch with a 1-unit segment. Branch off the top right at 120° with a 1-unit segment. Iteration: Go up one, branch left and repeat everything that is one step below. Go up one and branch left. Go up one, branch right and repeat what is on the left, two steps below. Repeat.

b) i) 5 units **ii)** 3 units **iii)** 64 units

c) no **d)** Answers may vary.

11. a) Answers may vary. **b)** Answers may vary.

c) Answers may vary.

12. a) 0, 1, $\frac{1}{2}$, $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{\sqrt{2}}}$, $\frac{1}{\sqrt[4]{\sqrt{2}}}$, ..., $\frac{1}{\sqrt[2^n]{\sqrt{2}}}$

b) 256, 16, 4, 2, $\sqrt{2}$, $\sqrt{\sqrt{2}}$, ..., $\sqrt{\ldots\sqrt{2}}$

c) 2, $\frac{1}{2}$, 2, $\frac{1}{2}$, ...

14. Answers may vary.

15. $\frac{\cos\theta}{1 - \cos\theta}$

16. Answers may vary.

17. a) Set up a table with two columns. Write the number in base 10 in the first column. If 2 divides the number with no remainder, write 0 in the second column. If 2 divides the number with remainder 1, write 1 in the second column. Below the first number in the first column, write the value of the first number divided by 2, rounding down, if necessary. If 2 divides the second number in the first column with no remainder, write 0 opposite it in the second column. Otherwise, write a 1. Continue this process until the number in the first column is a 0. Then stop. Read the numbers in the second column, from bottom to top. That is the original number's representation in base 2. For example, converting 23 in base 10 to a number in base 2 would result in the following table:

23	1
11	1
5	1
2	0
1	1
0	

Thus, 23 is 10111 in base 2.

b) i) 10000 **ii)** 10101 **iii)** 100101 **iv)** 10000010

c) i) 10 **ii)** 32 **iii)** 58 **iv)** 511

Section 1.2, pp. 22–23

Practise

1. a) Student marks: 30, 43, 56, 56, 65, 72, 74, 76, 80, 81, 88, 92, 99; Appointment times: 30, 32, 38, 40, 40, 41, 45, 45, 45, 60

b) Student marks: 70.2; Appointment times: 41.6

c) Student marks: 74; Appointment times: 40.5

d) Student marks: 56; Appointment times: 45

2. a) Microsoft® Excel: SUM(A1:A9);
Corel® Quattro® Pro: @SUM(A1..A9)

b) Microsoft® Excel: MAX(F3:K3);
Corel® Quattro® Pro: @MAX(F3..K3)

c) Microsoft® Excel: MIN(A1:K4);
Corel® Quattro® Pro: @MIN(A1..K4)

d) Microsoft® Excel: SUM(A2,B5,C7,D9);
Corel® Quattro® Pro: @SUM(A2,B5,C7,D9)

e) Microsoft® Excel: AVERAGE(F5:M5), MEDIAN(F5:M5), MODE(F5:M5);
Corel® Quattro Pro®: @AVG(F5..M5), @MEDIAN(F5..M5), @MODE(F5..M5)

f) Microsoft® Excel: SQRT(A3);
Corel® Quattro® Pro: @SQRT(A3)

g) Microsoft® Excel: B6^3;
Corel® Quattro® Pro: B6^3

h) Microsoft® Excel: ROUND(D2,4);
Corel® Quattro® Pro: @ROUND(D2,4)

i) Microsoft® Excel: COUNT(D3:M9);
Corel® Quattro® Pro: @COUNT(D3..M9)

j) Microsoft® Excel: PRODUCT(A1,B3,C5:C10);
Corel® Quattro® Pro: A1*B3*C5*C6*C7*C8*C9*C10 or @MULT (A1, B3, C5..C10)

k) Microsoft® Excel: PI;
Corel® Quattro® Pro: @PI

Apply, Solve, Communicate

5. a) To a maximum of 4 decimal places: 0, 1, 0.5, 0.7071, 0.6125, 0.6540, 0.6355, 0.6437, 0.6401, 0.6417

b) To a maximum of 4 decimal places: 256, 16, 4, 2, 1.4142, 1.1892, 1.0905, 1.0443, 1.0219, 1.0109

c) 2, 0.5, 2, 0.5, 2, 0.5, 2, 0.5, 2, 0.5

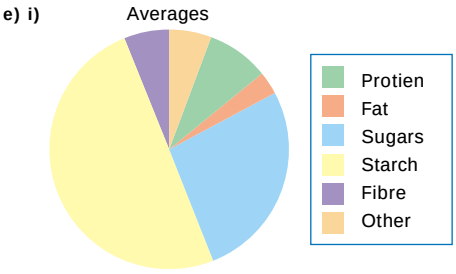
6. a), b), c)

Name	Protein	Fat	Sugars	Starch	Fibre	Other	TOTALS
Alphabits	2.4	1.1	12.0	12.0	0.9	1.6	30.0
Bran Flakes	4.4	1.2	6.3	4.7	11.0	2.4	30.0
Cheerios	4.0	2.3	0.8	18.7	2.2	2.0	30.0
Crispix	2.2	0.3	3.2	22.0	0.5	1.8	30.0
Froot Loops	1.3	0.8	14.0	12.0	0.5	1.4	30.0
Frosted Flakes	1.4	0.2	12.0	15.0	0.5	0.9	30.0
Just Right	2.2	0.8	6.6	17.0	1.4	2.0	30.0
Lucky Charms	2.1	1.0	13.0	11.0	1.4	1.5	30.0
Nuts 'n Crunch	2.3	1.6	7.1	16.5	0.7	1.8	30.0
Rice Krispies	2.1	0.4	2.9	22.0	0.3	2.3	30.0
Shreddies	2.9	0.6	5.0	16.0	3.5	2.0	30.0
Special K	5.1	0.4	2.5	20.0	0.4	1.6	30.0
Sugar Crisp	2.0	0.7	14.0	11.0	1.1	1.2	30.0
Trix	0.9	1.6	13.0	12.0	1.1	1.4	30.0
AVERAGES	2.52	0.93	8.03	14.99	1.82	1.71	
MAXIMUM	5.1	2.3	14.0	22.0	11.0	2.4	
MINIMUM	0.9	0.2	0.8	4.7	0.3	0.9	

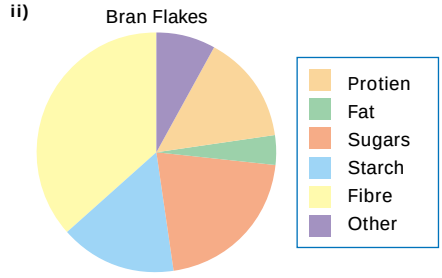
d)

Name	Protein	Fat	Sugars	Starch	Fibre	Other	TOTALS
Bran Flakes	4.4	1.2	6.3	4.7	11.0	2.4	30.0
Shreddies	2.9	0.6	5.0	16.0	3.5	2.0	30.0
Cheerios	4.0	2.3	0.8	18.7	2.2	2.0	30.0
Just Right	2.2	0.8	6.6	17.0	1.4	2.0	30.0
Lucky Charms	2.1	1.0	13.0	11.0	1.4	1.5	30.0
Sugar Crisp	2.0	0.7	14.0	11.0	1.1	1.2	30.0
Trix	0.9	1.6	13.0	12.0	1.1	1.4	30.0
Alphabits	2.4	1.1	12.0	12.0	0.9	1.6	30.0
Nuts 'n Crunch	2.3	1.6	7.1	16.5	0.7	1.8	30.0
Crispix	2.2	0.3	3.2	22.0	0.5	1.8	30.0
Frosted Flakes	1.4	0.2	12.0	15.0	0.5	0.9	30.0
Froot Loops	1.3	0.8	14.0	12.0	0.5	1.4	30.0
Special K	5.1	0.4	2.5	20.0	0.4	1.6	30.0
Rice Krispies	2.1	0.4	2.9	22.0	0.3	2.3	30.0
AVERAGES	2.52	0.93	8.03	14.99	1.82	1.71	
MAXIMUM	5.1	2.3	14.0	22.0	11.0	2.4	
MINIMUM	0.9	0.2	0.8	4.7	0.3	0.9	

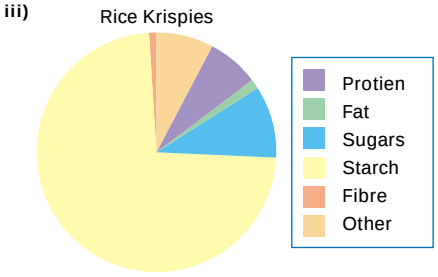
e) i)



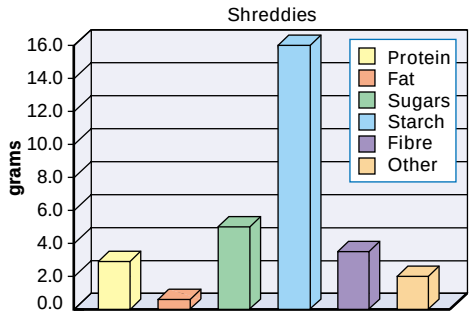
ii)



iii)



f)



7. a) 7.998 **b)** 424 488

8. Look for a menu item such as "Freeze Panes."

9. Enter 1 in cell A1, and the formula A1+1 in cell A2. Copy this formula to the *n*th row in column A. Enter 1 in cell B1, and formula B1*A2 in cell B2. Copy this formula to the *n*th row in column B. Cell B*n* will contain the value $n \times (n-1) \times \ldots \times 3 \times 2 \times 1$.

Section 1.3, pp. 31–32

Practise

1. a), b), c), d), and f) would be considered databases.

Apply, Solve, Communicate

Answers to questions 2 to 8 will vary.

Section 1.4, p. 40

Practise

1. a) Answers may vary. For example, on the TI-83 Plus: randInt(1,25,100).

b) Answers may vary. For example, on the TI-83 Plus: randInt(0,40,24)–20.

2. a) Microsoft® Excel: 1 + 24*RAND().
Corel® Quattro® Pro: 1 + 24*@RAND.
Copy this formula to fill 100 cells.

b) Microsoft® Excel: 1 + ROUND(24*RAND(),0).
Corel® Quattro® Pro: @RANDBETWEEN(1,25)
Copy this formula to fill 100 cells.

c) Microsoft® Excel: –40 + ROUND(80*RAND(),0).
Corel® Quattro® Pro: @RANDBETWEEN(–40,40)
Copy this formula to fill 16 cells.

d) COUNTIF(C10:V40,42.5)

Apply, Solve, Communicate

3. Answers will vary.

4. Answers will vary.

5. a) Answers will vary.

b) Random integers between 0 and 9 could be generated by recording the last digit of phone numbers.

6. Answers may vary.

7. Answers will vary.

9. Answers will vary.

Section 1.5, pp. 49–52

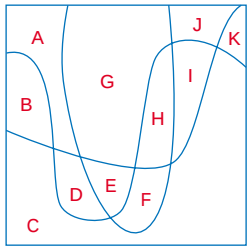
Practise

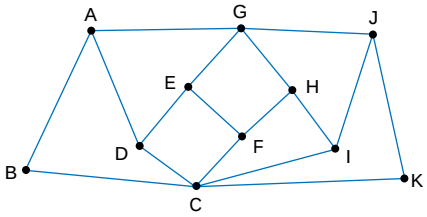
1. a) i) A: 2, B: 2, C: 3, D: 2, E: 3 **ii)** traceable

b) i) P: 4, Q: 5, R: 3, S: 4, T: 5, U: 3 **ii)** not traceable

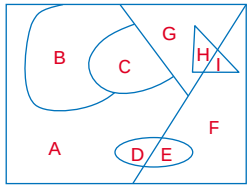
2. Label the regions of the maps as indicated.

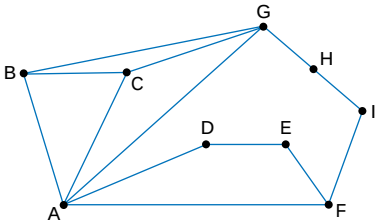
1. d)





1. e)



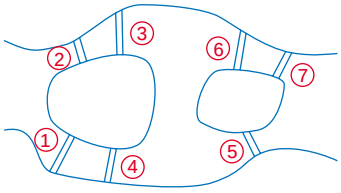


3. a) 3 **b)** 4

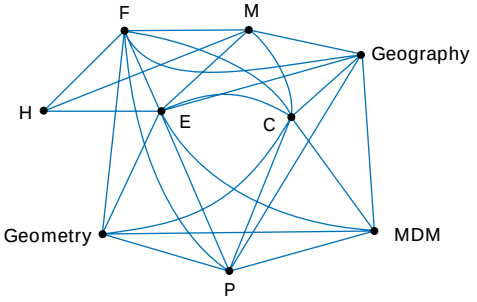
Apply, Solve, Communicate

4. The given map requires only 2 colours.

5. Yes; add the additional bridge, as shown. Several routes are possible.



6. a)



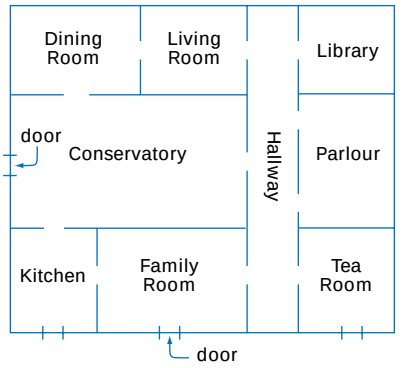
b) Answers may vary. Time Slot 1: English; Time Slot 2: Geography, Geometry; Time Slot 3: Calculus, History; Time Slot 4: Physics, Music; Time Slot 5: French, Mathematics of Data Management

7. a) no **b)** yes

8. a) A: 4, B: 2, C: 3, D: 3 **b)** 12 **c)** 6

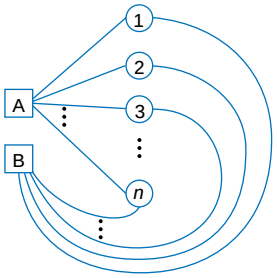
d) The sum of the degrees divided by 2 equals the number of edges.

9. a)



b) Answers will vary.

10. a) no **b)**



11. no

12. inside

13. a) North Bay to Kitchener, Hamilton to Windsor; both links could be backed up by linking Windsor to Thunder Bay.

b) Charlottetown to Halifax, Halifax to Montréal, Montréal to Toronto, Vancouver to Edmonton, Edmonton to Winnipeg, Winnipeg to Saskatoon; all links could be backed up by linking Saskatoon to Vancouver and Toronto to Charlottetown.

14. a) yes **b)** yes **c)** 285 km

15. a) Thunder Bay to Sudbury to North Bay to Kitchener to Windsor

b) Hamilton to Ottawa to North Bay to Sudbury

c) Answers may vary.

16. $1856

19. not possible

20. a) yes **b)** no

21. no

22. no

23. a)

b) three **c)** Answers may vary.

Section 1.6, pp. 60–62

Practise

1. a) 2×3 **b)** 1×3 **c)** 4×3

2. a) i) 6 **ii)** -4 **iii)** 2

b) i) a_{31} **ii)** a_{33} **iii)** a_{42}

3. a) MATRICES ARE FUN **b)** I LOVE MATH

c) WOW THIS IS FUN

4. Answers may vary.

a) $[2 \ \ -1]$, $[1 \ \ 2 \ \ 0]$, $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$

b) 1×2, 1×3, 2×1, 3×1

5. Answers may vary. Examples are given.

a) $\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ **b)** 2×2, 3×3

6. a) $\begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$ **b)** $\begin{bmatrix} 3 & 2 & 3 & 4 \\ 2 & 3 & 6 & 8 \\ 3 & 6 & 3 & 12 \\ 4 & 8 & 12 & 3 \end{bmatrix}$

7. a) $w = -2$, $x = 3$, $y = 5$, $z = 2$

b) $w = 2$, $x = \pm 3$, $y = 2$, $z = -5$

8. a) $\begin{bmatrix} 5 & 3 \\ -3 & 10 \\ 13 & 2 \\ -5 & -4 \end{bmatrix}$ **b)** $\begin{bmatrix} 5 & 3 \\ -3 & 10 \\ 13 & 2 \\ -5 & -4 \end{bmatrix}$ **c)** not possible

d) $\begin{bmatrix} 6 & -3 \\ 9 & 27 \\ 15 & 0 \\ -12 & 3 \end{bmatrix}$ **e)** $\begin{bmatrix} -\frac{3}{2} & -2 \\ 3 & -\frac{1}{2} \\ -4 & -1 \\ \frac{1}{2} & \frac{5}{2} \end{bmatrix}$ **f)** $\begin{bmatrix} 2 & 10 \\ -18 & -16 \\ 6 & 4 \\ 6 & -12 \end{bmatrix}$

g) $\begin{bmatrix} 0 & -11 \\ 21 & 25 \\ -1 & -4 \\ -10 & 13 \end{bmatrix}$

9. a) Both sides equal $\begin{bmatrix} 8 & -7 \\ 3 & 2 \\ 5 & 2 \end{bmatrix}$. **b)** Both sides equal $\begin{bmatrix} 10 & -4 \\ 11 & -4 \\ 9 & 3 \end{bmatrix}$.

c) Both sides equal $\begin{bmatrix} 40 & -35 \\ 15 & 10 \\ 25 & 10 \end{bmatrix}$.

10. $w = 11$, $x = 8$, $y = 3$, $z = 9$

11. a) $\begin{bmatrix} 4 & -2 & 6 \\ -6 & 3 & -10 \end{bmatrix}$ **b)** 2

Apply, Solve, Communicate

12. a)

Thunder Bay	Sault Ste. Marie	North Bay	Ottawa	Toronto	
0	710	1135	1500	1365	Thunder Bay
710	0	425	790	655	Sault Ste. Marie
1135	425	0	365	350	North Bay
1500	790	365	0	400	Ottawa
1365	655	350	400	0	Toronto

b) same result as in part a)

c) equal

13. a)

U.S.A.	U.K.	Germany	France	Sweden	
67	21	20	12	4	physics
43	25	27	7	4	chemistry
78	24	16	7	7	physiology/medicine
10	8	7	12	7	literature
18	13	4	9	5	peace
25	7	1	1	2	economic sciences

b) U.S.A.: 241; U.K.: 98; Germany: 75; France: 48; Sweden: 29

14. a)

Field of Study	1997	
	Males	Females
Social Sciences	28 421	38 244
Education	8 036	19 771
Humanities	8 034	13 339
Health	3 460	9 613
Engineering	10 125	2 643
Agriculture	4 780	6 995
Mathematics	6 749	2 989
Fine & Applied Arts	1 706	3 500
Arts & Sciences	1 730	3 802

Field of Study	1998	
	Males	Females
Social Sciences	27 993	39 026
Education	7 565	18 391
Humanities	7 589	13 227
Health	3 514	9 144
Engineering	10 121	2 709
Agriculture	4 779	7 430
Mathematics	6 876	3 116
Fine & Applied Arts	1 735	3 521
Arts & Sciences	1 777	3 563

b)

Field of Study	Males
Social Sciences	56 414
Education	15 601
Humanities	15 623
Health	6 974
Engineering	20 246
Agriculture	9 559
Mathematics	13 625
Fine & Applied Arts	3 441
Arts & Sciences	3 507

c)

Field of Study	Females
Social Sciences	77 270
Education	38 162
Humanities	26 566
Health	18 757
Engineering	5 352
Agriculture	14 425
Mathematics	6 105
Fine & Applied Arts	7 021
Arts & Sciences	7 365

d)

Field of Study	Average for Females
Social Sciences	38 635
Education	19 081
Humanities	13 283
Health	9 378.5
Engineering	2 676
Agriculture	7 212.5
Mathematics	3 052.5
Fine & Applied Arts	3 510.5
Arts & Sciences	3 682.5

15. a)

Matrix 1		Matrix 2	
Age Group	Males	Age Group	Females
0–4	911 028	0–4	866 302
5–9	1 048 247	5–9	996 171
10–14	1 051 525	10–14	997 615
15–19	1 063 983	15–19	1 007 631
20–24	1 063 620	20–24	1 017 566
25–29	1 067 870	25–29	1 041 900
30–34	1 154 071	30–34	1 129 095
35–39	1 359 796	35–39	1 335 765
40–44	1 306 705	40–44	1 304 538
45–49	1 157 288	45–49	1 162 560
50–54	1 019 061	50–54	1 026 032
55–59	769 591	55–59	785 657
60–64	614 659	60–64	641 914
65–69	546 454	65–69	590 435
70–74	454 269	70–74	544 008
75–79	333 670	75–79	470 694
80–84	184 658	80–84	309 748
85–89	91 455	85–89	190 960
90+	34 959	90+	98 587

b)

Age Group	Total
0–4	1 777 330
5–9	2 044 418
10–14	2 049 140
15–19	2 071 614
20–24	2 081 186
25–29	2 109 770
30–34	2 283 166
35–39	2 695 561
40–44	2 611 243
45–49	2 319 848
50–54	2 045 093
55–59	1 555 248
60–64	1 256 573
65–69	1 136 889
70–74	998 277
75–79	804 364
80–84	494 406
85–89	282 415
90+	133 546

c)

Age Group	Total
0–4	1 803 990
5–9	2 073 084
10–14	2 079 877
15–19	2 102 680
20–24	2 112 404
25–29	2 141 417
30–34	2 317 413
35–39	2 735 994
40–44	2 650 412
45–49	2 354 646
50–54	2 075 769
55–59	1 578 577
60–64	1 275 422
65–69	1 153 942
70–74	1 013 251
75–79	816 439
80–84	501 822
85–89	286 651
90+	135 549

16. a)

K	L	M	NF	O	S	T	W	
0	0	1	0	1	0	1	0	Kingston
0	0	0	0	0	0	1	1	London
1	0	0	0	1	0	1	0	Montréal
0	0	0	0	0	0	1	0	Niagara Falls
1	0	1	0	0	0	1	0	Ottawa
0	0	0	0	0	0	1	0	Sudbury
1	1	1	1	1	1	0	1	Toronto
0	1	0	0	0	0	1	0	Windsor

b) no direct connection between Niagara Falls and Montréal
c) no direct connection between Montréal and Niagara Falls
d) a direct connection would go both ways
e) the number of direct connections to Kingston
f) the number of direct connections from Kingston
g) direct connections go both ways

Section 1.7, pp. 74–77

Practise

1. a) $\begin{bmatrix} 12 & 47 \\ 21 & -7 \end{bmatrix}$ **b)** $\begin{bmatrix} -13 & -27 \\ -31 & 18 \end{bmatrix}$ **c)** $\begin{bmatrix} -59 & 18 \\ -14 & -63 \end{bmatrix}$

d) not possible **e)** $\begin{bmatrix} 18 & 20 & 4 \\ -16 & -17 & 4 \end{bmatrix}$ **f)** $\begin{bmatrix} -13 \\ 14 \\ -31 \end{bmatrix}$

g) $\begin{bmatrix} -15 & -26 & 1 \\ -7 & -11 & 5 \end{bmatrix}$

4. a) Both sides equal $\begin{bmatrix} 20 & 5 \\ 10 & -5 \end{bmatrix}$.

b) Both sides equal $\begin{bmatrix} 15 & 95 \\ 8 & 32 \end{bmatrix}$.

c) $AB = \begin{bmatrix} 15 & 20 \\ 8 & 8 \end{bmatrix}$, $BA = \begin{bmatrix} 23 & -4 \\ -10 & 0 \end{bmatrix}$

5. a) $\begin{bmatrix} 2 & 0.5 \\ -1 & 0 \end{bmatrix}$ **b)** no inverse **c)** $\begin{bmatrix} \frac{1}{3} & 0 \\ 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & 1.5 \\ 2 & -2.5 \end{bmatrix}$ **e)** no inverse

6. a) $\begin{bmatrix} 0.6 & -0.2 & -2 \\ 0 & 0 & -1 \\ 0.4 & 0.2 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 0.0851 & 0.4255 & 0.1064 \\ -0.0638 & -0.3191 & 0.1702 \\ 0.2340 & 0.1702 & 0.426 \end{bmatrix}$

c) $\begin{bmatrix} \frac{1}{6} & -\frac{1}{12} & -\frac{1}{4} & \frac{1}{6} \\ -\frac{1}{3} & \frac{1}{6} & -\frac{1}{2} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 & -\frac{2}{3} \\ \frac{1}{6} & \frac{5}{12} & \frac{1}{4} & \frac{1}{6} \end{bmatrix}$

Apply, Solve, Communicate

7. a) $A^{-1} = \begin{bmatrix} 2.5 & 2 \\ 1 & 1 \end{bmatrix}$, $(A^{-1})^{-1} = \frac{1}{2.5 - 2} \begin{bmatrix} 1 & -2 \\ -1 & 2.5 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & -2 \\ -1 & 2.5 \end{bmatrix}$

$= \begin{bmatrix} 2 & -4 \\ -2 & 5 \end{bmatrix}$

$= A$

b) Both sides equal $\begin{bmatrix} -3 & -1 \\ 2.5 & 1 \end{bmatrix}$.

c) Both sides equal $\begin{bmatrix} 2.5 & 1 \\ 2 & 1 \end{bmatrix}$.

8. Downtown store: \$6180, Northern store: \$4370, Southern store: \$4600

9. a)

	Mon	Tues	Wed	Thurs	Fri	Sat	Sun	
$A =$	0	8	0	8	8	0	0	Chris
	4	4	0	0	6.5	4	4	Lee
	0	4	4	4	4	8	8	Jagjeet
	0	3	3	3	3	8	0	Pierre
	8	8	8	8	0	0	0	Ming
	0	0	3	5	5	8	0	Bobby
	3	3	3	3	3	0	0	Ni1e
	8	8	8	8	8	0	0	Louis
	8	0	0	8	8	8	8	Glenda
	3	4.5	4	3	5	0	0	Imran

b)

7.00	Chris
6.75	Lee
7.75	Jagjeet
6.75	Pierre
11.00	Ming
8.00	Bobby
7.00	Ni1e
12.00	Louis
13.00	Glenda
7.75	Imran

c) Chris: \$168.00; Lee: \$151.88; Jagjeet: \$248.00; Pierre: \$135.00; Ming: \$352.00; Bobby: \$168.00; Nicole: \$105.00; Louis: \$480.00; Glenda: \$520.00; Imran: \$151.13

d) \$2479.01

10. Create a 10×1 matrix to represent the percent of males for each sport and multiply by the number of males. Do a similar scalar multiplication for females.

Males	Females	
1 325 007	480 597	Golf
1 432 440	61 615	Ice Hockey
954 960	382 013	Baseball
429 732	690 088	Swimming
549 102	234 137	Basketball
393 921	345 044	Volleyball
549 102	184 845	Soccer
429 732	221 814	Tennis
346 173	320 398	Skiing
358 110	246 460	Cycling

11. a)

Total Cloth (m^2)	Total Labour (\$)	
1080	10 200	small
1800	13 500	med
3750	25 000	LG
4000	22 000	XLG

b) box of 100

\$9 064	xsmall
\$9 662	small
\$10 386	med
\$11 645	LG
\$13 030	XLG

c) \$611 599

12. a) –50, –42, 86, 82, 13, –67, –11, 117, –22, –58, 58, 98, –4, –15, 40, 33, –67, –76, 117, 128

b) 23, –65, –13, 115, –27, 39, 65, –37, –14, 30, 50, –22, –40, 23, 68, –5, –46, –62, 86, 114

13. a) LUNCH TODAY AT JOES

b) HOW ARE YOU TODAY

14. Answers will vary.

15. a)

A	G	P	S	SI	T	
0	1	0	0	1	1	Administration
1	0	0	0	0	1	Guidance
0	0	0	1	0	1	Parents
0	0	1	0	0	1	Students
1	0	0	0	0	0	Superintendent
1	1	1	1	0	0	Teachers

b) If $a_{ij} = 1$, there is open communication between i and j; if $a_{ij} = 0$, there is no open communication between i and j.

c) the number of open links of communication between

parents and others

d)

	A	G	P	S	SI	T	
	3	1	1	1	0	1	Administration
	1	2	1	1	1	1	Guidance
	1	1	2	1	0	1	Parents
	1	1	1	2	0	1	Students
	0	1	0	0	1	1	Superintendent
	1	1	1	1	1	4	Teachers

e) 1; Parents-Teachers-Administration

f)

	A	G	P	S	SI	T	
	3	2	1	1	1	2	Administration
	2	2	1	1	1	2	Guidance
	1	1	2	2	0	2	Parents
	1	1	2	2	0	2	Students
	1	1	0	0	1	1	Superintendent
	2	2	2	2	1	4	Teachers

Each a_{ij} represents the number of lines of communication with no intermediary links and exactly one intermediary link between i and j.

16. a) 2; Honey Bridge-Green Bridge-Connecting Bridge, Honey Bridge-Blacksmith Bridge-Connecting Bridge

b) 3 **c)** yes

17. a) 14 **b)** 18

18. Answers will vary.

19. Rewrite the system $ax_1 + bx_2 = c$, $dx_1 + ex_2 = f$ as a matrix product $\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c \\ f \end{bmatrix}$. Then, multiply both sides on the left by the inverse of the matrix $\begin{bmatrix} a & b \\ d & e \end{bmatrix}$.

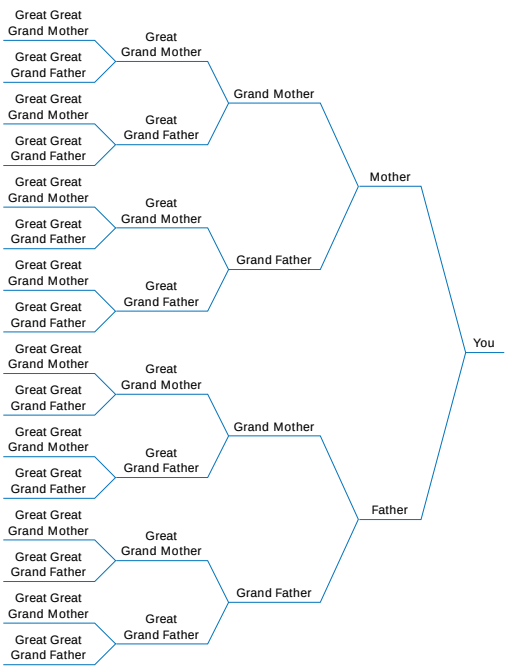
20. Answers will vary.

21. Answers will vary.

Review of Key Concepts, pp. 78–81

1.1 The Iterative Process

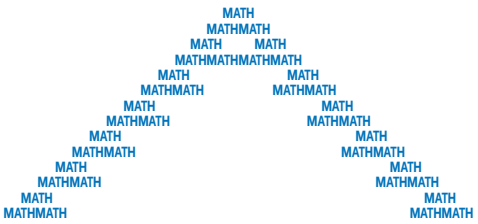
1. a)



b) 30

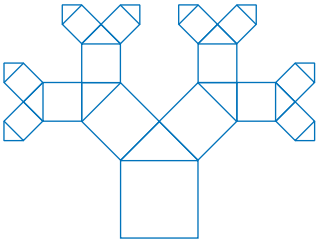
2. a) Beginning with the 3-word MATH triangle, place two copies of this triangle on each subsequent row; the copy to the extreme left has the T in the top word of the triangle centred under the first M of the previous line and the copy to the extreme right has the M in the top word of the triangle centred under the last T of the previous line.

b)



c) The repeated word MATH forms a pyramid.

3. a)



b) 16 cm^2 **c)** 64 cm^2

4. Answers may vary.

1.2 Data Management Software

5. Answers may vary; for example, Microsoft® Excel: spreadsheet functions; Fathom™: graphing; Microsoft® Access: sorting.

6. a) 0 **b)** 30 **c)** 12

7. Corel® Quattro® Pro: enter A:A3 in cell B2 in Sheet B and copy this formula through cell B9; Microsoft® Excel: enter Sheet1!A3 in cell B2 in Sheet2 and copy this formula through cell B9.

8.

	A	B
1	Celsius	Fahrenheit
2	-30	-22.0
3	-29	-20.2
4	-28	-18.4
5	-27	-16.6
6	-26	-14.8
7	-25	-13.0
8	-24	-11.2
9	-23	-9.4
10	-22	-7.6
11	-21	-5.8
12	-20	-4.0
13	-19	-2.2
14	-18	-0.4
15	-17	1.4
16	-16	3.2
17	-15	5.0
18	-14	6.8
19	-13	8.6
20	-12	10.4
21	-11	12.2
22	-10	14.0
23	-9	15.8
24	-8	17.6
25	-7	19.4
26	-6	21.2
27	-5	23.0

Use a filter to select the even Celsius degrees.

1.3 Databases

Answers to questions 9 to 12 will vary.

1.4 Simulations

13. Answers may vary.

14. a) randInt(18,65)

b) Microsoft® Excel: 18 + ROUND(47*RAND(), 0)
Corel® Quattro® Pro: @RANDBETWEEN (18, 65)

15. Remove 2 cards from a standard deck of 52 playing cards. To simulate a repeated brainteaser, draw 5 cards, one at a time with replacement, and reshuffle after every draw. If the same card appears more than once, this will count as a success—getting two chocolate bars with the same brainteaser. This experiment must be repeated many times to obtain an approximation to the probability of the actual event.

16. For each of 500 simulations, draw 5 random numbers from 1 to 50—if any two are the same, this counts as a success—getting two chocolate bars with the same brainteaser.

1.5 Graph Theory

17. a) 3 **b)** 3

18. a) i) yes **ii)** yes **iii)** yes
b) i) yes **ii)** no **iii)** no
c) i) yes **ii)** no **iii)** no

20. Time Slot 1: *Gone With the Wind*, *The Amazon Queen*; Time Slot 2: *Curse of the Mummy*, *West Side Story*, *Ben Hur*; Time Slot 3: *Citizen Kane*, *Jane Eyre*

21. a)

	A	D	M	P	S	T	
	0	0	0	1	0	1	Afra
	0	0	1	0	1	1	Deqa
	0	1	0	1	1	0	Mai
	1	0	1	0	1	0	Priya
	0	1	1	1	0	1	Sarah
	1	1	0	0	1	0	Tanya

b) no **c)** Sarah **d)** Afra

1.6 Modelling With Matrices

22. a) 4×3

b) i) -8 **ii)** 5 **iii)** -2

c) i) a_{23} **ii)** a_{42} **iii)** a_{12}

23. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \\ 4 & 8 & 12 \end{bmatrix}$

24. a) $\begin{bmatrix} 9 & 3 & -5 \\ -12 & 9 & 5 \end{bmatrix}$ **b)** not possible **c)** not possible

d) $\begin{bmatrix} 12 & 9 \\ -3 & 21 \\ 18 & 6 \end{bmatrix}$ **e)** $\begin{bmatrix} -3 & -\frac{1}{2} & 2 \\ \frac{5}{2} & -\frac{9}{2} & 0 \end{bmatrix}$ **f)** $\begin{bmatrix} 36 & 3 \\ 6 & 33 \\ 24 & 21 \end{bmatrix}$

g) $\begin{bmatrix} 11 & -9 \\ 5 & 4 \\ 1 & 10 \end{bmatrix}$ **h)** not possible

25. a)

15	10	17	basketballs
17	3	13	volleyballs
4	15	17	footballs
15	20	12	baseballs
8	12	12	soccer balls
12	5	16	pk tennis balls
10	15	23	pk golf balls

b)

8	basketballs
7	volleyballs
2	footballs
23	baseballs
8	soccer balls
1	pk tennis balls
2	pk golf balls

c)

6	basketballs
6	volleyballs
2	footballs
18	baseballs
6	soccer balls
1	pk tennis balls
2	pk golf balls

26. Answers will vary.

1.7 Problem Solving With Matrices

27. a) $\begin{bmatrix} -21 & 35 \\ -21 & 21 \end{bmatrix}$ **b)** $\begin{bmatrix} 4 & 5 \\ -62 & -4 \end{bmatrix}$ **c)** $\begin{bmatrix} -14 & 35 \\ 42 & -21 \end{bmatrix}$

d) not possible **e)** $\begin{bmatrix} 26 & 20 & 13 \\ -14 & 4 & 30 \\ -59 & -46 & 61 \end{bmatrix}$

28. a) $A^t = \begin{bmatrix} 1 & 8 \\ 5 & -2 \end{bmatrix}$, $B^t = \begin{bmatrix} 0 & 6 \\ 4 & -1 \end{bmatrix}$

b) Both sides equal $\begin{bmatrix} 30 & -12 \\ -1 & 34 \end{bmatrix}$.

29. a) XYZ: \$7350, YZX: \$4080, ZXY: \$12 400

b) Find the column sum of the product matrix $\begin{bmatrix} 7\,350 \\ 4\,080 \\ 12\,400 \end{bmatrix}$.

30. Answers may vary.

31. a) $A \times A^{-1} = I$

c) $\begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{bmatrix}$

32. a)

$$A = \begin{array}{c} \begin{matrix} B & LF & P & SF & S \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} \text{Bacteria} \\ \text{Large Fish} \\ \text{Plants} \\ \text{Small Fish} \\ \text{Snails} \end{matrix}$$

b) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **c)** 2

d) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 2 & 0 & 1 & 3 \\ 3 & 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

Each a_{ij} represents the number of routes from i to j with at most one intermediary link.

e) $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

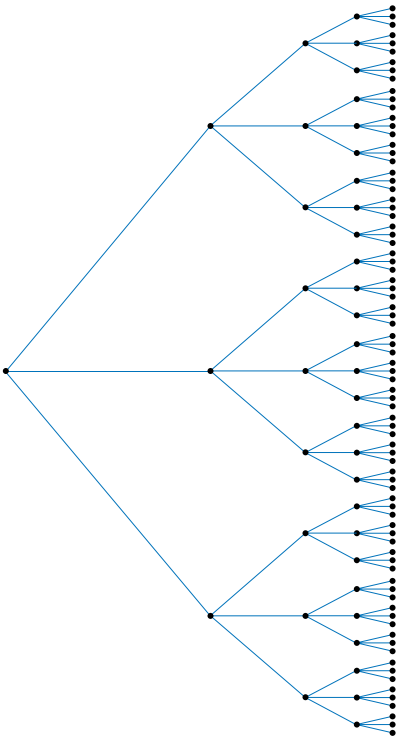
f) Plants-Small Fish-Large Fish-Bacteria; Plants-Small Fish-Snails-Bacteria; Plants-Large Fish-Snails-Bacteria

Chapter Test, pp. 82–83

1. a) Answers will vary. **b)** Answers will vary.

2. $0, \frac{1}{2}, \frac{2}{5}, \frac{5}{12}, \frac{12}{29}, \frac{29}{70}, \ldots$ After the first term, the numerator plus the denominator of the previous term added to the numerator of the following term gives the denominator of the following term.

3. a)



b) 6

4. a) C1+C2+C3+C4+C5+C6+C7+C8 or SUM(C1:C8)

b) MIN(A5:G5) **c)** (5–SQRT(6))/(10+15)

5. a)

Month	Balance	Payment	Interest	Principal	New Balance
February	1000.00	88.88	5.00	83.88	916.12
March	916.12	88.88	4.58	84.30	831.82
April	831.82	88.88	4.16	84.72	747.10
May	747.10	88.88	3.74	85.14	661.96
June	661.96	88.88	3.31	85.57	576.38
July	576.38	88.88	2.88	86.00	490.39
August	490.39	88.88	2.45	86.43	403.96
September	403.96	88.88	2.02	86.86	317.10
October	317.10	88.88	1.59	87.29	229.80
November	229.80	88.88	1.15	87.73	142.07
December	142.07	88.88	0.71	88.17	53.90
January	53.90	88.88	0.27	88.61	-34.71

In the spreadsheet, cell D2 contains the formula 0.005*B2, cell E2 contains the formula C2–D2, cell F2 contains the formula B2–E2, and cell B3 contains the formula F2. These cells are copied down to 12 months.

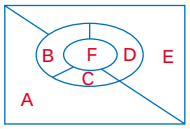
b) A final payment of \$88.88 overpays the loan by \$34.71 and so the final payment is \$88.88 – \$34.71 = \$54.17.

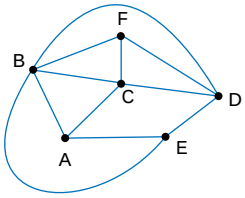
c) Highlight the balance column, and from the Insert menu, select Chart. From this menu, select line graph.

6. Answers may vary.

7. Use the TI-83 Plus: randInt(1, 50); use a spreadsheet function such as INT(RAND()*51); write the integers from 1 to 50 on equal-sized slips of paper and draw slips at random from a hat, replacing the slip after each draw.

8. **a)** Label the regions as shown.





b) 4 colours

9. **a)** No; more than two vertices have an odd degree.
b) Pinkford-Brownhill-Whiteford-Redville-Blueton-Greenside-Blacktown-Orangeton-Pinkford

10. Yes; there are exactly two vertices with an odd degree (in the associated network diagram)

11. **a)** 4×3 **b)** 9 **c)** a_{12}
d) No; the inner dimensions do not match.

12. **a)** $\begin{bmatrix} 10 & 9 \\ 19 & -5 \end{bmatrix}$ **b)** not possible **c)** not possible

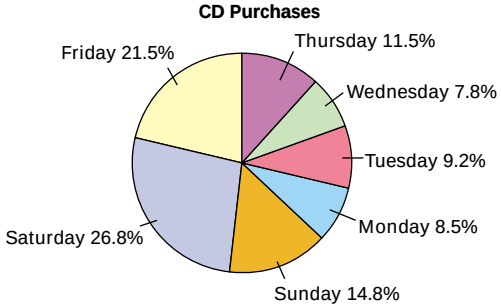
d) $\begin{bmatrix} 22 & 70 \\ 20 & 40 \\ -11 & -35 \end{bmatrix}$ **e)** $\begin{bmatrix} 8 & 5 & -4 \\ -2 & 0 & 1 \end{bmatrix}$

13. $4175

CHAPTER 2

Review of Prerequisite Skills, p. 90

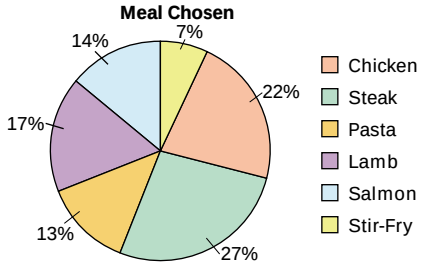
1. **a)** $79 **b)** $16.99 **c)** $479 **d)** $64.69
2. **a)** $13.50/h **b)** $0.83
3. 30%
4. $188.89
5. **a)** mean: 25.8, median: 26, mode: 26
b) mean: 21, median: 21, mode: no mode
c) mean: 20.3, median: 18, mode: 10, 18
d) mean: 43.2, median: 41, mode: 70
e) mean: 242.2, median: 207.5, mode: no mode
f) mean: 33.2, median: 33.5, mode: 32
6. **a)** approximately $1.44 **b)** 1997
c) yearly increases in price
d) 10.4% **e)** domain: {1996–2001}, range: {1.44–1.59}
7.



Section 2.1, pp. 101–103

Practise

1. **a)** Some intervals have common endpoints; a 38-year-old could be placed in either of two intervals.
b) The intervals 81–85 and 86–90 are omitted.
2. **a)** bar graph **b)** histogram
c) bar graph **d)** histogram
3. **a)**



b) 19.4% **c)** Answers will vary.
d) Less than 50% of the respondents order red meat.
4. Answers will vary.

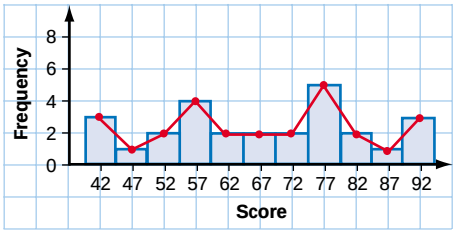
Apply, Solve, Communicate

5. **a)** 53 **b)** size: 5; number 11

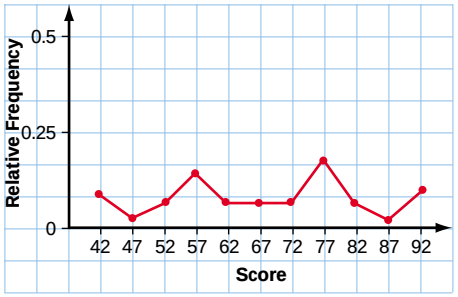
c)

Score	Tally	Frequency
39.5–44.5	III	3
44.5–49.5	I	1
49.5–54.5	II	2
54.5–59.5	IIII	4
59.5–64.5	II	2
64.5–69.5	II	2
69.5–74.5	II	2
74.5–79.5	𝍸	5
79.5–84.5	II	2
84.5–89.5	I	1
89.5–94.5	III	3

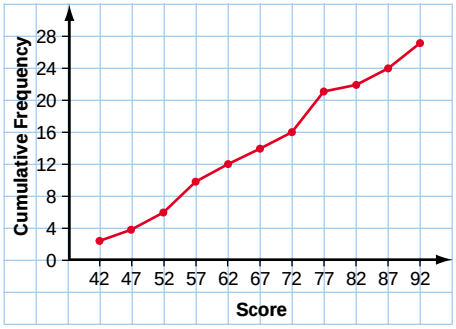
d)



e)

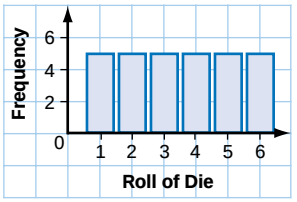


f)



g) Frequency polygon: shows the changes in frequency from one interval to the next. Relative frequency polygon: shows the changes in frequency relative to the total number of scores. Cumulative frequency: shows the rate of change of frequency from one interval to the next and the total number of scores.

6. a)

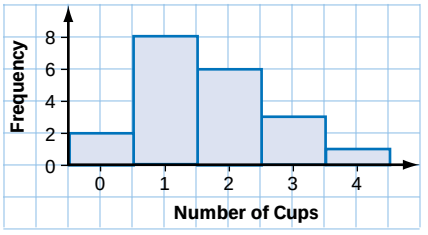


d) Discrepancies are due to chance variation.

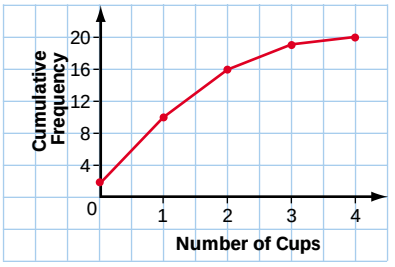
7. a) ungrouped

b)

Number of Cups	Tally	Frequency
0	II	2
1	𝍸 III	8
2	𝍸 I	6
3	III	3
4	I	1



c)

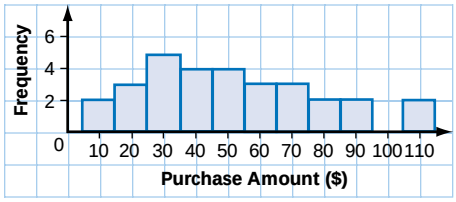


d) Answers may vary.

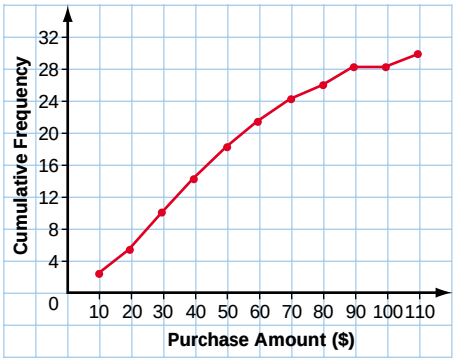
8. a) grouped

b)

Purchase Amount ($)	Tally	Frequency
5.00–14.99	II	2
15.00–24.99	III	3
25.00–34.99	𝍸	5
35.00–44.99	IIII	4
45.00–54.99	IIII	4
55.00–64.99	III	3
65.00–74.99	III	3
75.00–84.99	II	2
85.00–94.99	II	2
95.00–104.99		0
105.00–114.99	II	2



c)

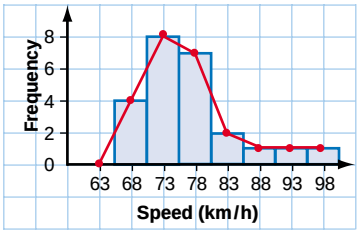


d) Answers will vary.

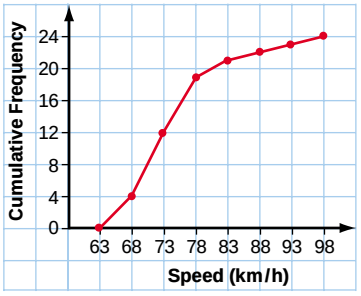
9. a)

Speed (km/h)	Tally	Frequency
61–65		0
66–70	IIII	4
71–75	𝍸 III	8
76–80	𝍸 II	7
81–85	II	2
86–90	I	1
91–95	I	1
96–100	I	1

b)



c)

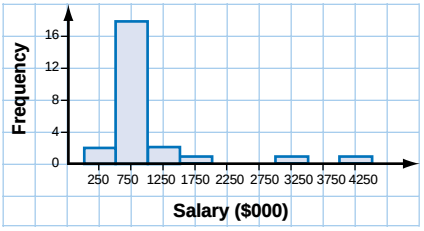


d) 12 **e)** 5

10. a)

Salary	Tally	Frequency
0 – 499 999	II	2
500 000 – 999 999	𝍸 𝍸 𝍸 II	17
1 000 000 – 1 499 999	II	2
1500 000 – 1 999 999	I	1
2 000 000 – 2 499 999		0
2 500 000 – 2 999 999		0
3 000 000 – 3 499 999	I	1
3 500 000 – 3 999 999		0
4 000 000 – 4 499 999	I	1

b)



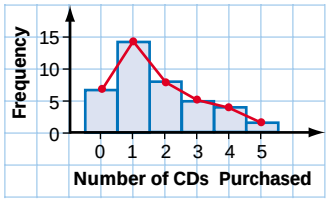
11. a) 1

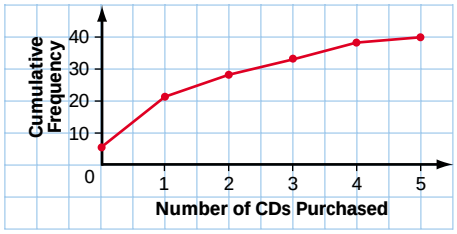
b) Relative frequencies are a percent of the total, and so their sum is 100%, or 1.

12.

Score	Frequency
29.5–39.5	1
39.5–49.5	2
49.5–59.5	4
59.5–69.5	8
69.5–79.5	5
79.5–89.5	3
89.5–99.5	2

13.

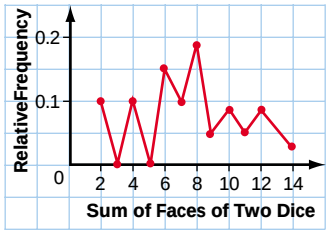




The data does support the theory since fewer than 50% of those surveyed purchased 2 or more of the band's CDs. However, this evidence is far from conclusive.

14. a) 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14

b)



e) chance variation

15. a) 54.5–64.5; this interval has the steepest line segment.

b) 8

16. Answers may vary.

Section 2.2, pp. 109–112

Practise

1. a) 11.5 **b)** 11.5% **c) i)** 8.14 **ii)** $57.80

Apply, Solve, Communicate

2. a) to have a collection of items that are representative of purchases of typical Canadians

b) no, the changes in all prices are used to compute an average

3. a) 1996 **b)** 0 **c)** 1999–2001 **d)** no

4. a) an increase in prices and a decrease in purchasing power

b) They give a measure of the increase in prices.

c) In general, an increase in one of these indices would coincide with an increase in the other.

d) Housing will have a significant weighting in the CPI.

5. a) scale on vertical axis; period of time over which the graph ranges; scale on horizontal axis

b) Trends may be observed over a longer period of time. A period of time over which the index has doubled is shown.

c) Detailed month-by-month variations are not exhibited. Year-by-year growth rates are not easily calculated.

d) 1980

e) The average retail price has doubled from 1980 to 1992.

6. a) The mean annual change is lower for each graph.

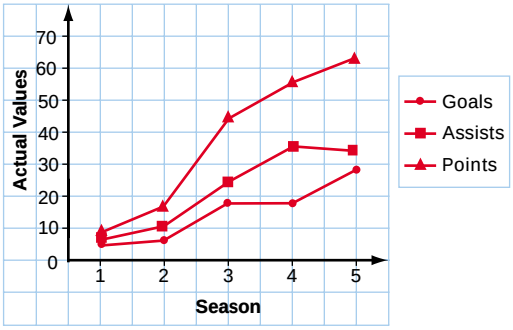
b) Energy costs have increased more rapidly than the average cost of other goods and services.

c) the percent change in the CPI due to energy costs alone

d) about 8%

e) significantly lower

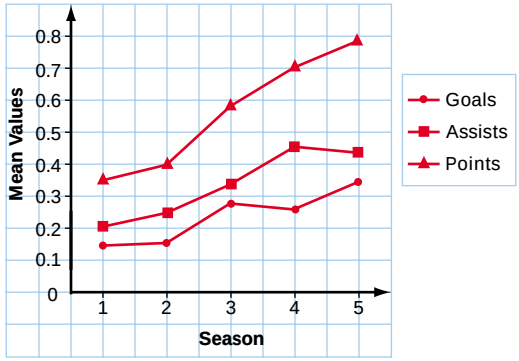
7. a)



b)

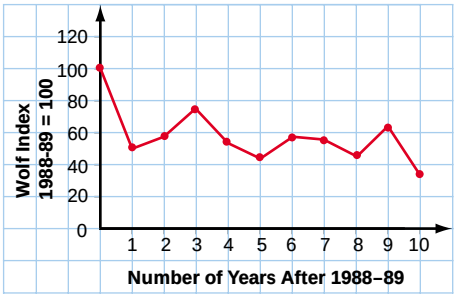
Season	Means Values Per Game		
	Goals	Assists	Points
1	0.15	0.20	0.35
2	0.16	0.24	0.40
3	0.25	0.33	0.58
4	0.24	0.46	0.70
5	0.34	0.44	0.78

c)



d) The graph of the actual number of points produces the most steeply increasing curve and so is likely the one that the agent would use during negotiations. The manager will likely use the graphs of means.

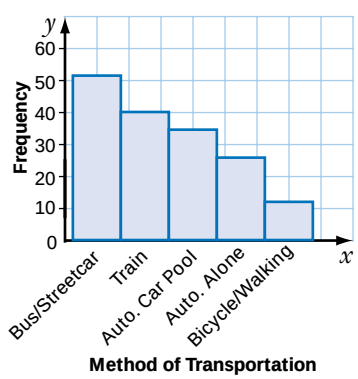
8. a)



b) Overall, there is a downward trend in the annual change; however, a year in which there is a decrease is frequently followed by a year in which there is an increase, and vice-versa.

9. Answers may vary.

10. Answers may vary.
11. Answers may vary.
12. Answers may vary.
13. a) Ontario and Alberta
b) No; the four larger circles have areas roughly proportional to the emissions they represent, but the area of the circle representing 199 100 is not $\frac{199\,100}{638} \doteq 312$ times the area of the circle representing 638.
c) Advantage: easy visual comparisons can be made. Disadvantage: precise values are difficult to determine.
d) Prince Edward Island, New Brunswick, Nova Scotia.
e) Answers may vary.
15. a)



b) Answers will vary.
16. Answers will vary.
17. Answers will vary.

Section 2.3, pp. 117–118

Practise

1. a) students in the particular school
b) grade 10 students in the particular school
c) you
d) all those who have listened to the Beatles' music
e) all those who have tried the new remedy
2. a) voluntary-response sample **b)** stratified sample
c) convenience sample **d)** systematic sample
e) multi-stage sample **f)** cluster sample
3. a) stratified sample
b) simple random sample, convenience sample
c) voluntary-response sample

Apply, Solve, Communicate

4. sample $\frac{12}{37} \doteq 32\%$ of the members of each group; 4 children, 3 teens, 5 adults
5. Obtain a list of the students in the school. Calculate $\frac{n}{20}$ to the nearest integer, where n is the number of names on the list. Choose a name at random from the first $\frac{n}{20}$ names. Then, select every $\frac{n}{20}th$ name.
6. Answers may vary.
7. a) cluster sample
b) No, not every member of the community had an equal chance of being surveyed.
c) To the same extent as the member of the community centre was randomly selected.
8. a) convenience sample
b) i) yes **ii)** no
c) In part ii), a simple random sample should be used.
9. a) voluntary-response sample
b) yes
c) not if the host claims that the callers are representative of the population
10. Answers will vary.
11. Answers will vary.
12. Answers will vary.

Section 2.4, pp. 123–124

Practise

1. a) sampling bias **b)** response bias
c) measurement bias—loaded question
d) sampling bias—the cluster may not be representative

Apply, Solve, Communicate

2. a) A random sample of area residents should be taken.
b) The responses could be returned anonymously.
c) Remove the preamble to the question and the phrase "forward-thinking."
d) A random sample of area residents should be taken.
3. Answers may vary.
a) Which party will you vote for in the next federal election?
b) same as a)
c) Do you think first-year calculus is an easy or difficult course?
d) Who is your favourite male movie star?
e) Do you think that fighting should be eliminated from professional hockey?
4. Answers may vary.
6. Not statistically valid; the sample may contain response bias and also sampling bias. The radio station may have a conservative audience.
7. Answers may vary.
8. Answers may vary.

Section 2.5, pp. 133–135

Practise

1. a) 2.58, 2.4, 2.4 **b)** 14.6, 14.5, 14 and 18
2. Answers may vary.
a) 1, 2, 3, 4, 5, 6, 7, 8 **b)** 1, 2, 3, 4, 6, 7, 8, 9
c) 1, 1, 1, 2, 3, 4, 4, 4 **d)** 1, 2, 3, 4, 4, 5, 6, 7

Apply, Solve, Communicate

3. 82
4. a) mode **b)** mean or median
c) median **d)** mean
5. Enzo

6. The two outliers lower the mean mark for all the students by 3.3, as opposed to lowering the mean mark for Class B by 5.7. This is due to the much larger size of the group of all students—the additional higher marks reduce the effect of the outliers.

7. **a)** 77.95% **b)** 74%

8. **a)** Paulo: 3.75; Janet: 4.25; Jamie: 4
   b) Paulo: 3.625; Janet: 4; Jamie: 4.125
   c) Jamie

9. **a)** mean: 8, median: 7.5, mode: 7
   b) mode **c)** 10
   d)

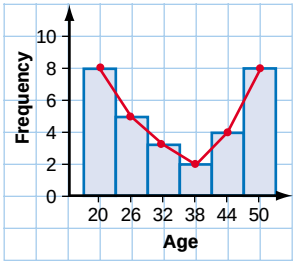
10. **a)** 90 **b)** $\frac{x_1 + \ldots + x_{15}}{15} = 6 \Rightarrow x_1 + \ldots + x_{15} = 90$

11. **a)** \$45 300 **b)** \$45 000
    c) mean: \$500, median: \$0

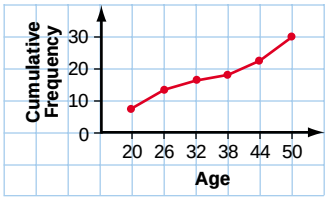
12. **a)** 34.5, 32.5, 48 **b)** mean or median
    c)

Age	Tally	Frequency
16.5–22.5	𝍸 III	8
22.5–28.5	𝍸	5
28.5–34.5	III	3
34.5–40.5	II	2
40.5–46.5	IIII	4
46.5–52.5	𝍸 III	8

   d)



   e)



   f) 34.1, 31.5, 19.5 and 49.5

13. **a)** 16.5–22.5 and 46.5–52.5
    b) No; there is more than one modal interval

14. **a)** none **b)** none

15. **a)** 67.4¢/L
    b) find the average of x when xy is a constant, for example finding the average speed for travelling a given distance

16. **a)** 2.38%
    b) finding the average of percents, ratios, indexes, or growth rates

Section 2.6, pp. 148–150

Practise

1. **a)** 7.1, 2.2, 5.0 **b)** \$27.42, \$10.88, 118.35

2. **a)** median: 6; Q1: 4.5; Q3: 10; interquartile range: 5.5; semi-interquartile range: 2.75
   b) median: 71.5; Q1: 60.5; Q3: 79.5; interquartile range: 19; semi-interquartile range: 9.5

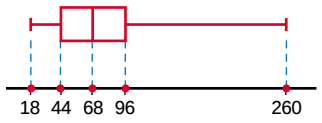
3. **a)** first quartile **b)** second quartile **c)** second quartile

4. **a)** 25th **b)** 50th **c)** 75th

5. –0.49, –1.23, 1.48, 0, 0.25 if data are a sample

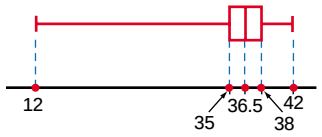
Apply, Solve, Communicate

6. **a)** 79.4, 57.53, 3309.7 **b)** 68, 52, 26
   c)

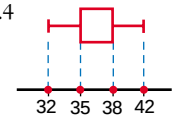


   d) 260

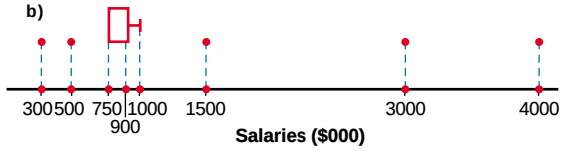
7. **a)** 35.8, 35.0, 5.9 if OR nurses are considered as a sample; 35.8, 33.6, 5.8 if they are considered as a population
   b) 36.5, 3, 1.5
   c)



8. **a)** The standard deviation will be smaller and the left-hand whisker will be shorter.
   b) new standard deviation: 3.4



9. **a)** 804 674, 6.475×10^{77}, 150 000, 75 000
   b)



   c) –0.87
   d) After calculating the new average salary and standard deviation, the z-score will be –0.037.

10. Chi-Yan seems to have better control over her drives (a lower standard deviation in their distances). If their putting abilities are essentially equal, then Chi-Yan is more likely to have a better score in a round of golf.

11. There must be either $4n + 2$ or $4n + 3$ (where $n \geq 1$) data points in the distribution.

12. a) 7, 8, 8, 8, 8, 9; these could be the sizes of the first six pairs of shoes sold in a day at a shoe store.

13. No, the standard deviation cannot be much less than half of the semi-interquartile range. For example, {0, 0, 0, 0, 5, 5, 5, 5, 5, 5, 10, 10, 10, 10} has $s = 3.8$ and a semi-interquartile range of 5.

14. a) i) 68 **ii)** 90

b) 12th to 14th **c)** 86.5, 78

16.
$$\Sigma(x - \bar{x}) = \Sigma x - \Sigma\bar{x}$$
$$\bar{x} = \frac{1}{n}\Sigma x, \text{ so } \Sigma\bar{x} = n\bar{x}$$
$$\Sigma x = n\bar{x}$$
$$\Sigma(x - \bar{x}) = \Sigma x - \Sigma\bar{x}$$
$$= n\bar{x} - n\bar{x}$$
$$= 0$$

17. a) By definition, $s = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n - 1}}$.

Then,
$$\Sigma(x - \bar{x})^2 = \Sigma(x^2 - 2x\bar{x} + (\bar{x})^2)$$
$$= \Sigma x^2 - 2\bar{x}\Sigma x + \Sigma(\bar{x})^2$$
$$= \Sigma x^2 - 2n(\bar{x})^2 + n(\bar{x})^2$$
$$= \Sigma x^2 - n(\bar{x})^2$$
$$= \Sigma x^2 - \frac{(n\bar{x})^2}{n}$$
$$= n(\Sigma x^2) - (\Sigma x)^2$$

b) The individual deviations do not need to be calculated, round-off error from calculation of mean is avoided, and the quantities in this formula are supplied by a graphing calculator.

18. a) midrange: 65.5; interquartile range: 27.5

b) Both measures are half of a certain range of data; the midrange is affected by outliers, whereas the interquartile range is not.

19. a) mean deviation: 17.6, standard deviation: 22.9

b) Both are measures of the deviations from the mean; the standard deviation emphasizes the larger differences by squaring.

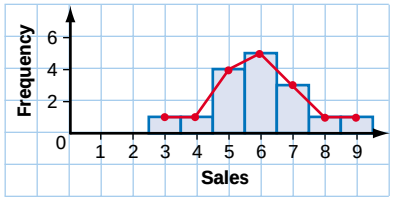
Review of Key Concepts, pp. 151–153

Section 2.1

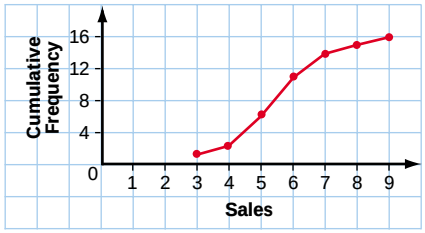
1. a)

Sales	Tally	Frequency
3	I	1
4	I	1
5	IIII	4
6	𝍸	5
7	III	3
8	I	1
9	I	1

b)



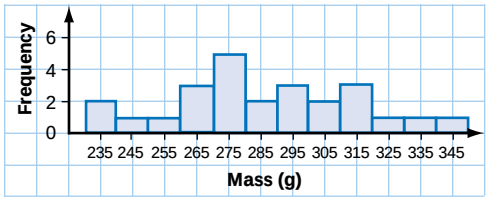
c)



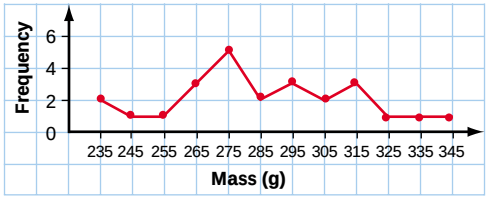
2. a)

Mass(g)	Tally	Frequency
230.5–240.5	II	2
240.5–250.5	I	1
250.5–260.5	I	1
260.5–270.5	III	3
270.5–280.5	𝍸	5
280.5–290.5	II	2
290.5–300.5	III	3
300.5–310.5	II	2
310.5–320.5	III	3
320.5–330.5	I	1
330.5–340.5	I	1
340.5–350.5	I	1

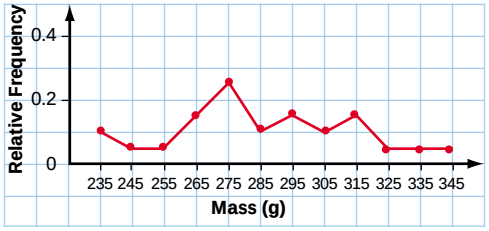
b)



c)

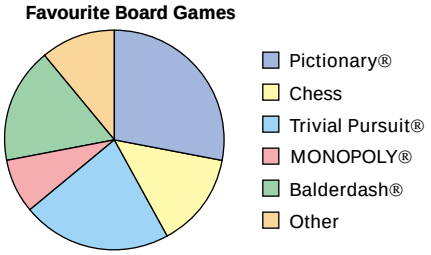


d)



3. a) categorical data

b)



Section 2.2

4. a) times-series/index **b)** fuel oil and other fuel
b) 1992 is the base year
d) i) coffee and tea **ii)** rent
e) Rent is controlled in some provinces.
Coffee and tea prices are weather dependent.

5. a) i) about $8.80 **ii)** about $7.05
b) about $620 **c)** about $1.74

Section 2.3

6. a) With a systematic sample, members are selected from the population at regular intervals whereas a stratified sample divides the population into groups according to characteristics and selects members from each group appropriately.
b) A convenience sample would be appropriate when respondents who are representative of the population are easily accessible—for example, sampling of classmates to determine a choice of field trip.
c) A voluntary-response sample is convenient and inexpensive since respondents submit information themselves. A disadvantage is the possibility of bias.

7. a) multi-stage sample
b) The classes from which students would be selected would be chosen at random; then, within these classes students could be chosen systematically from a class list.

8. Answers will vary.

Section 2.4

9. a) measurement bias; method of data collection (asking a leading question) is poor and the group of children may not be representative and so the sampling technique may also be poor.
b) response bias; method of data collection is poor.
c) response bias; method of data collection is poor.
d) measurement bias; the sample (convenience) is not necessarily representative of the population and the method of data collection (asking a leading question) is poor.

10. a) The sample of children should be representative of the population under consideration—a systematic sample of this population could be chosen. The question should be reworded to eliminate phrases such as "junk food."
b) The teacher could collect the data in such a way as to make respondents anonymous—this may remove the response bias.
c) The musician should play before a more random collection of people—perhaps at a coffee shop—to see how well the song is received.
d) A convenience sample may still be appropriate, but not necessarily at a public library. The potential respondents should first be asked if they are familiar with the work of Carol Shields and, if so, they may then be asked their opinion on her work. Questions with phrases such as "critically acclaimed" should not be used.

Section 2.5

11. a) 5.9, 6, 6
b) Each measure describes these data equally well.

12. a) mean: 285; median: 285
b) mean: 288.6; median: 285
c) The results in part a) are estimates.

13. a) median
b) No; the distribution of ages is skewed.

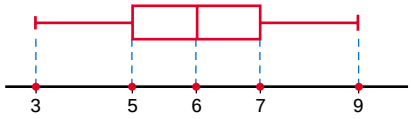
14. a) 90.2 **b)** 88.9
c) Students entering an engineering program must be proficient in sciences and mathematics courses which are given higher weightings.

15. Answers may vary. For example, the mode is useful when determining the "average" size such as for hats.

Section 2.6

16. a) standard deviation: 1.5; interquartile range: 2; semi-interquartile range: 1

b)



c) no

17. a) The interquartile range is not necessarily symmetric about the median.
b) Add these differences and divide by 2.

18. a) i) 267.5, 310 **ii)** 242.5, 267.5, 310, 332.5
b) The first quartile coincides with the 25th percentile and that the third quartile coincides with the 75th percentile. Rounding may produce slightly different values.

19. a) i) –1 **ii)** 2.33 **iii)** 0 **iv)** –2.53
b) i) 115 **ii)** 70 **iii)** 122.5 **iv)** 82

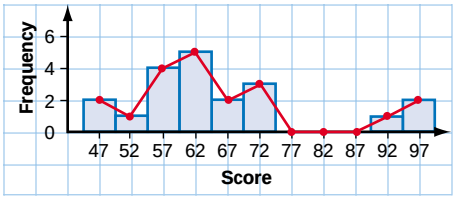
20. a) 68.1, 68; almost equal
b) standard deviation: 13.9; interquartile range: 30
c) standard deviation; the interquartile range is too large.

Chapter Test, pp. 154–155

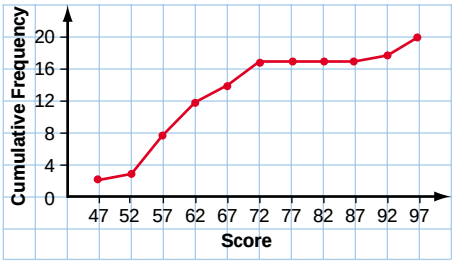
1. a)

Score	Tally	Frequency
44.5–49.5	II	2
49.5–54.5	I	1
54.5–59.5	IIII	4
59.5–64.5	~~IIII~~	5
64.5–69.5	II	2
69.5–74.5	III	3
74.5–79.5		0
79.5–84.5		0
84.5–89.5		0
89.5–94.5	I	1
94.5–99.5	II	2

b)

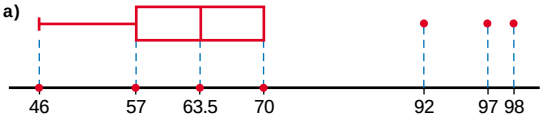


c)



2. a) mean: 66.06, median: 63.5, mode: 70
b) standard deviation: 14.52, variance: 210.68
c) 13, 6.5

3. a)



b) 92, 97, 98
c) There is no right whisker, so all the data above Q3 must be outliers.

4. The median is the best descriptor. The mean and mode are too high.

5. a) two **b)** 97, 98

6. Steven, with a weighted score of 4.

7. a) stratified sample
b) cluster sample
c) voluntary-response sample
d) simple random sample
e) convenience sample

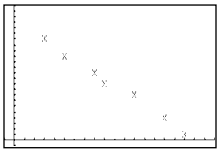
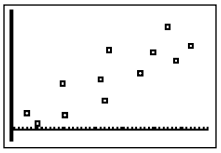
8. Obtain a list of the names of the 52 children. Randomly select one of the first 6 names on the list and then choose every 6th name from that point on.

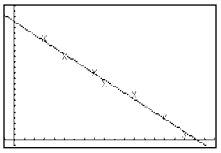
9. a) i) measurement bias—loaded question; may lead to many negative responses
ii) response bias—many false responses may be given
iii) sampling bias—the sample includes only business executives who are more likely to favour the channel
b) i) Reword the question and omit the references to "run around" and "get into trouble."
ii) Require the audience members to respond anonymously.
iii) A sample of the relevant population should be selected randomly.

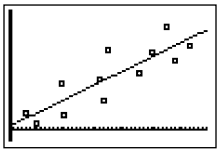
10. The company is stating that all of its funds have returns below the first quartile of comparable funds

CHAPTER 3

Review of Prerequisite Skills, p. 158

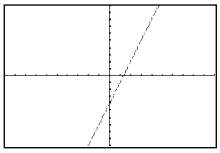
1. a)  **b)** 

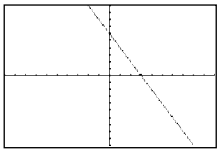
2. i) a)  $y = -1.16x + 21.17$

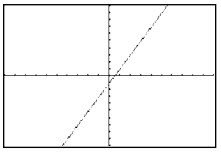
b)  $y = 0.73x + 1.16$

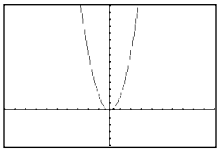
ii) a) x-intercept: 18.3, y-intercept: 21.17
b) x-intercept: −1.6, y-intercept: 1.16

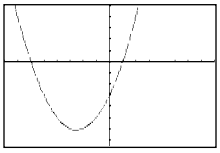
iii) a) 13.05 **b)** 6.3

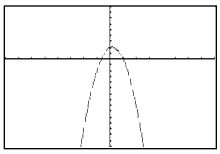
3. a) slope: 3, y-intercept: −4 

b) slope: −2, y-intercept: 6 

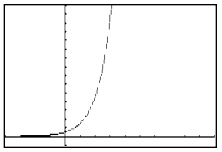
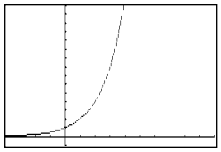
c) slope: 2, y-intercept: $-\frac{7}{6}$ 

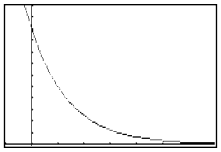
4. a)  y-intercept: 0, x-intercept: 0

b)  y-intercept: −6, x-intercepts: −6, 1

c)  y-intercept: 2, x-intercepts: −0.67, 1

5. a) i) base: 3, coefficient: 0.5
ii) base: 2, coefficient: 1
iii) base: 0.5, coefficient: 100

b) i)  **ii)** 

iii) 

c) i) x has a large negative value
ii) x has a large negative value
iii) x becomes very large

6. a) 36 **b)** 55

7. a) 6 **b)** 5

9. a) Every member of the population has an equal chance of being selected and the selection of any particular individual does not affect the chances of any other individual being chosen.
b) Scan the population sequentially and select members at regular intervals. The sampling interval size is determined by dividing the population size by the sample size.
c) a data point or observation that lies a long way from the main body of data

10. a) a measurement technique or sampling method that systematically decreases or increases the variable it is measuring
b) surveys that might include sensitive questions
c) surveys that ask leading questions
d) Answers may vary.

Section 3.1, pp. 168–170

Practise

1. Answers may vary.
a) strong positive
b) moderate positive
c) weak negative
d) zero
e) weak positive

2. a) independent: cholesterol level; dependent: heart disease
b) independent: practice; dependent: success rate
c) independent: amount of fertilizer; dependent: height
d) independent: level of education; dependent: income
e) independent: running speed; dependent: pulse rate

Apply, Solve, Communicate

3. a)  moderate positive

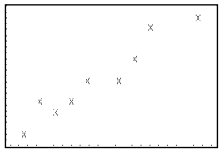
b)  moderate to strong negative

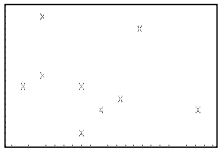
c) hours watching TV

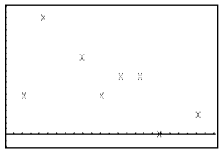
d) 0.755, –0.878; yes

4. a) Rogers method: strong positive linear correlation, 0.961
Laing System: weak negative linear correlation, –0.224

b) The scatter plot will be reflected in the line $y = x$; the correlation coefficient will remain unchanged.

c) Rogers method:  , $r = 0.961$

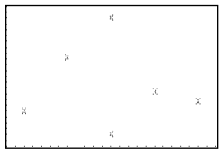
Laing System:  , $r = -0.224$

5. a)  moderate negative linear correlation

b) –0.61 **c)** yes

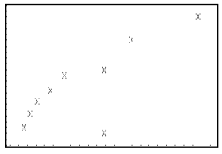
d) The point (1, 2) is an outlier.

e) Karrie may have worked only part of last year.

6. a) 

b) –0.050; weak negative linear correlation

c) no relationship between the two variables

7. a) 

b) moderate to strong positive linear correlation

c) 0.765

8. Answers may vary.

9. Answers may vary.

10. a) yes, for certain ranges of temperatures

11. Answers may vary.

12. a) Interchanging the order of multiplication in the formula for covariance does not affect the covariance; the variables you consider to be dependent and independent do not affect the spread in the values of these variables. Then, interchanging the independent and dependent variables, but retaining variable names, the correlation coefficient is calculated as $\frac{s_{YX}}{s_Y \times s_X}$, which is the same as $\frac{s_{XY}}{s_X \times s_Y}$ or r.

13. Answers may vary.

14. Answers may vary.

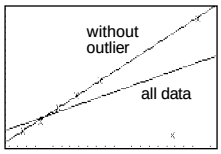
Section 3.2, pp. 180–183

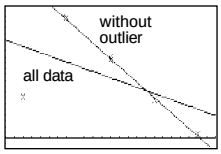
Practise

1. a) (92, 18) **b)** (4, 106)

2. a) a: $y = 0.61x + 17.2$ b: $y = -39.9x + 392.9$

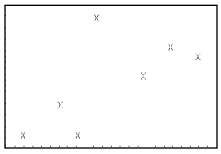
b) a: $y = 1.12x + 3.2$ b: $y = -98.7x + 794.4$

c) a: 

b: 

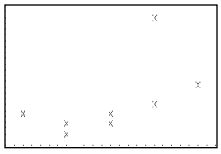
3. $y = 1.17x - 22.5$

Apply, Solve, Communicate

5. a)  moderate positive linear correlation

b) $y = 0.442x + 5.51$ **c)** 78.4 kg **d)** 166 cm

e) The correlation between height and weight is not perfect.

6. a)  moderate positive linear correlation

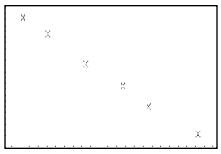
b) 0.568; $y = 1.67x - 20.42$ **c)** (17, 15)

d) 0.759; $y = 0.92x - 9.61$

e) Yes; the correlation is greater.

f) Answers will vary.

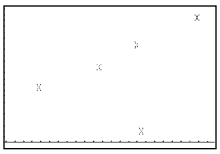
g) yes

7. a) 

b) $y = -162.92x + 4113.1$

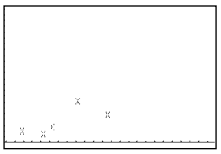
c) 1018

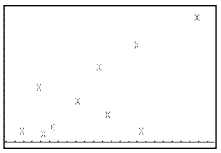
d) $16

8. a)  ; (4.2, 1.5) is an outlier.

b) 0.447; $y = 1.71x + 2.6$

c) 0.996; $y = 2.34x + 2.42$

d) a:  b: 0.700; $y = 1.55x - 0.97$

e) a:  b: 0.681; $y = 2.63x - 2.15$

f) Answers will vary.

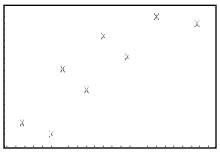
g) Answers will vary.

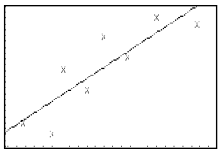
9. a) $y = 0.6x + 45.9$; 0.765 **b)** 196

c) (192, 140); possibly due to a recession

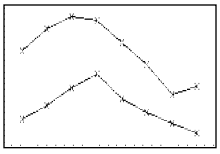
d) a: $y = 0.67x + 35.2$; 0.957 b: 203

e) Answers will vary.

10. a)  moderate to strong positive linear correlation

b) 0.865; $y = 1.28x + 19.31$ 

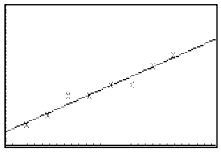
c) No; the correlation is not perfect.

d) 

e) It appears that the wolf population depends on the rabbit population.

11. b) $y = 0.627x + 29.6$

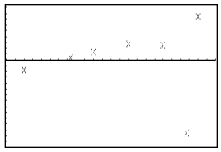
14. No; consider the points (1, 2), (2, 3), (3, 5), (4, 5), (5, 6), (6, 6), (7, 8) (8, 9). All points except (3, 5) and (6, 6) lie on the line $y = x + 1$. The point (3, 5) is one unit above the line and the point (6, 6) is one unit below the line. Performing a linear regression gives the line of best fit as $y = 0.93x + 1.32$ with correlation 0.976.



15. a) $a = \dfrac{n(\Sigma yx) - (\Sigma y)(\Sigma x)}{n(\Sigma y^2) - (\Sigma y)^2}$, $b = \bar{x} - a\bar{y}$

b) no

c) no

16. a) 

Section 3.3, pp. 191–194

Practise

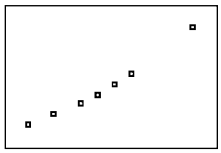
1. a) iii **b)** i **c)** iv **d)** ii

2. a) $y = -1.43x^2 - 2.46x + 4.64$; $r^2 = 0.966$

b) $y = 0.89x^3 + 3.68x^2 + 3.29x + 1.01$; $r^2 = 0.959$

c) $y = 1.60(1.77)^x$ or $y = 1.60(e^{0.57x})$; $r^2 = 0.979$

Apply, Solve, Communicate

3. a) 

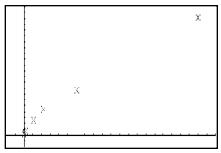
b) $r = 0.99$, $y = 6.48x - 6.53$

c) $r^2 = 0.9996$, $y = 1.475x^{2.018}$

d) Power regression fits the data a bit more closely.

e) i) 10.9 m^2 **ii)** 4.4 m

f) If the trees' outward growth is proportional to their upward growth, the areas will be proportional to the squares of the heights.

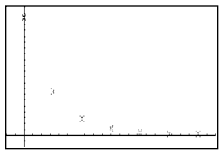
4. a) 

b) $y = 1.05x^{0.75}$

c) The curve fits these data well; $r^2 = 0.999$

d) i) 8 kJ/day **ii)** 314 kJ/day

5. a)



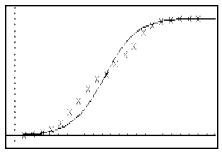
b) $y = 97.4(0.38)^x$ or $y = 97.4(e^{-0.98x})$
c) The curve fits these data well; $r^2 = 0.999$
d) 0.73 h or approximately 44 min

6. b) $y = 0.157x^2 - 1.10x + 28.3$; $r^2 = 0.83$

7. a) power level = $2.02 \times 10^{-3}d^{-2}$, where d is the distance to the transmitter in kilometres
b) i) 2.02 μW/m² **ii)** 126 μW/m² **iii)** 0.81 μW/m²

8. a) $y = \dfrac{1647}{1 + 46.9e^{-0.39x}}$

b)



d) The growth rate levels off due to environmental constraints.

9. a) $y = 99.8(1.3)^x$ billion or $y = 99.8e^{0.264x}$ where x = years since 1995
b) after 10.7 years
c) the average number of the crop-destroying insects that each arachnid will kill
d) population = $99.8(1.3)^x$ billion $- k \times (100 \text{ million} \times 2^x)$, where k is the number of insects destroyed by each arachnid

10. For linear regression, the coefficient of determination is equal to the square of the linear correlation constant.

11. a) i) A: 3, B: 4
ii) A: $y = 0.3125x^3 - 4.625x^2 + 20x - 19$; $r^2 = 1$
B: $y = -0.042x^4 + 1.153x^3 - 10.49x^2 + 35.19x - 30.98$; $r^2 = 1$

Section 3.4, pp. 199–201

Practise

1. a) cause-and-effect relationship: alcohol consumption impairs driving ability
b) common-cause factor: achievement in physics and calculus requires similar skills
c) reverse cause-and-effect relationship: better job performance leads to increases in pay
d) accidental relationship: no causal relationship between the variables
e) presumed relationship: seems logical that a student who has obtained a number of scholarships would be attractive as an employee, but there are many other qualities that employers seek
f) cause-and-effect relationship: coffee consumption keeps people awake
g) reverse cause-and-effect relationship: higher number of medals won at Olympic games encourages investors to fund athletic programs

2. a) overall athletic ability
b) overall disregard of safe driving practices
c) a large income

Apply, Solve, Communicate

3. Traffic accidents cause traffic congestion.

4. a) the teachers of the class
b) no
c) Conduct further trials giving teachers time to become comfortable with the new teaching method and examine the results of classes where the same teacher has used the different methods to minimize the effect of this possible extraneous variable.

5. The overall increase in computer use is likely to have caused parallel increases in the fortunes of the companies.

6. a) Accidental relationship is likely.

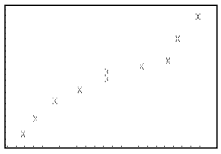
7. a) Accidental relationship is likely.
b) cause-and-effect relationship

8. Answers may vary.

9. In a double-blind study, neither the participants in a trial nor the investigators are aware of which intervention the participants are given. This method may prevent performance bias by the participants and may prevent detection bias by the investigators

10. a) no **b)** a decrease in the number of graduates hired

11. a)



b) $y = 1.06x - 46\,092$; 0.973
c) Answers may vary; for example, economy.
d) Answers may vary.

12. Answers may vary.

13. Hawthorne effect: an increase in worker productivity produced by the psychological stimulus of being singled out and made to feel important. Placebo effect: the measurable, observable, or felt improvement in health not attributable to treatment.

14. a) Create groups consisting entirely of members of the same gender.
b) Compare results of same-gender groups and mixed-gender groups.

Section 3.5, pp. 209–211

Apply, Solve, Communicate

1. Poor attendance could be a result of mathematics anxiety.

2. income

3. a) Yes, there is a strong negative correlation, $r = 0.9$.
b) The sample may not be representative, and is much too small in any event.
c) using a sufficiently large random sample

4. The results are skewed in favour of customer satisfaction since only satisfied customers return.

5. a) The line of best fit is $y = -2.1x + 84.9$; for values of x greater or equal to 17, the value of y is less than 50.

b) The outlier (20, 30) skews the regression due to the small sample size.

c) Investigate the possibility of removing the outlier; consider a larger random sample.

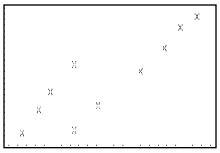
6. a) –$120 million **b)** no

c) The cubic model has no logical relationship to the situation and gives inaccurate predictions when extrapolated beyond the data.

d) Gina could obtain data for the years before 1992 to see if the trends are clearer over a longer period. Salaries that tend to have the same percent increase every year will fit an exponential model. Non-linear models may work better with the years elapsed since 1992 as the independent variable in place of the date.

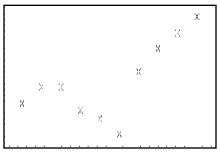
7. Answers may vary.

8. a)



b) 1995 **c)** Answers may vary.

d)



e) $y = 0.011x + 0.447$

f) in 1998, when productivity began increasing

9. Answers may vary.

10. Answers may vary.

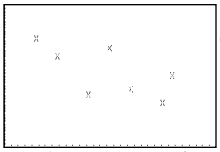
Review of Key Concepts, pp. 212–213

Section 3.1

1. a) **i)** moderate positive linear correlation
ii) strong negative linear correlation
iii) no relationship

b) **i)** approximately 0.66
ii) approximately –0.93
iii) approximately 0.03

2. a)



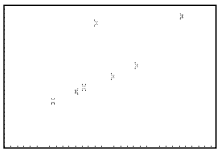
moderate negative linear correlation

b) –0.651 **c)** Answers may vary.

Section 3.2

3. $y = -1.20x + 87.9$

4. a)



b) 0.740; $y = 0.87x + 36.15$

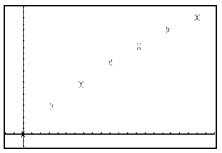
c) (177, 235)

d) 0.966; $y = 0.94x + 15.57$

e) 254, 251; 3 points

Section 3.3

5. a)



b) $y = -5.24x^2 + 10.1x + 0.095$; $r^2 = 0.9997$

c) 4.9 m **d)** 1.9 s

e) r^2 is very close to 1, but the data are only seven measurements with limited precision.

6. a) $y = \dfrac{450}{1 + 44e^{-0.29x}} - 10$

b) Answers may vary.

c) accelerated, drove at constant velocity, slowed to a stop

Section 3.4

7. a) An external variable causes two variables to change in the same way, for example, sunny weather could boost sales of both sunscreen and ice-cream cones.

b) The dependent and independent variables are reversed in the process of establishing causality, for example, the lower the golf score, the more hours a golfer will spend practising.

c) A variable distinct from the dependent and independent variables that influences either the dependent or independent variable, for example, in trying to find a correlation between traffic congestion and traffic accidents, the time of day is an extraneous variable.

8. a) Results from experimental and control groups are compared to minimize the effects of extraneous variables. The independent variable is varied for the experimental group but not for the control group.

b) A control group may be required to reduce the placebo effect.

9. a) In an accidental relationship, a correlation exists without any causal relationship whereas in a logical or presumed relationship, a correlation seems logical although no causal relationship is apparent.

b) accidental relationship: a correlation between an NFL team from a certain conference winning the super bowl and the state of the market; logical relationship: a correlation between musical and mathematical ability

10. common-cause factors

11. Answers may vary.

Section 3.5

12. Answers may vary.

13. a) A hidden variable is an extraneous variable that is not easily detected.

b) The presence of a hidden variable may be detected by analysing any disjoint clusters of data separately.

Chapter Test, pp. 214–215

1. a) A correlation with value −1; the dependent variable decreases at a constant rate as the independent variable increases.

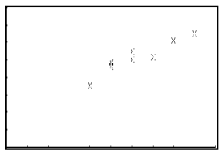
b) The design of experiments and analysis of data to establish the presence of relationships between variables.

c) A data point or observation that lies a long way from the main body of data.

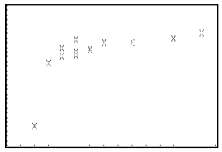
d) A variable distinct from the dependent and independent variables that influences either the dependent or independent variable.

e) An extraneous variable that is not easily detected.

2. a) −0.8 **b)** 1 **c)** 0.3 **d)** 0.6 **e)** −1

3. a)  strong positive linear correlation

b) 0.935 **c)** $y = 0.517x + 1.93$ **d)** 8.1

4. a) 

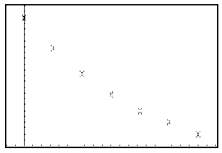
b) $y_1 = 1.79x + 57.9$; $r = 0.618$

c) (2, 38)

d) $y_2 = 0.894x + 66$; $r = 0.819$

5. y_2, assuming at most 9 h of study

6. Answers may vary.

7. a) 

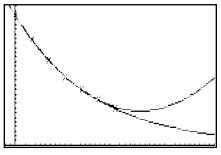
b) $y = 0.011x^2 - 0.167x + 0.855$; $r^2 = 0.9995$

c) $y = 0.854(0.812)^x$ or $y = 0.854e^{-0.208x}$; $r^2 = 0.9997$

d) Both fit the data very well.

e) quadratic: 0.286 lumens, exponential: 0.107 lumens

f) exponential; the quadratic curve begins to increase:



8. Use control groups.

9. The more money a person invests, the higher their income will be.

CUMULATIVE REVIEW: CHAPTERS 1 TO 3, pp. 218–220

1. a) $\begin{bmatrix} 2 & -6 \\ -10 & -8 \\ -8 & -2 \end{bmatrix}$ **b)** not possible

c) not possible **d)** $\begin{bmatrix} 59 & 12 \\ -60 & 11 \end{bmatrix}$

e) not possible **f)** $\begin{bmatrix} \frac{4}{37} & \frac{1}{37} \\ \frac{5}{37} & \frac{8}{37} \end{bmatrix}$

2. a) Place 1 in the centre. Place 2 one square to the right of 1. Place the numbers beginning with 3 in a counterclockwise circle around 1. When you reach 9 move one square to the right and place 10. Continue in a counterclockwise circle as before.

b)

37	36	35	34	33	32	31
38	17	16	15	14	13	30
39	18	5	4	3	12	29
40	19	6	1	2	11	28
41	20	7	8	9	10	27
42	21	22	23	24	25	26
43	44	45	46	47	48	49

3. a) no **b)** yes **c)** no **d)** maybe

4. a) voluntary response **b)** simple random sample

c) observational study

5. a) strong negative **b)** moderate positive

c) strong positive **b)** weak positive

6. Answers may vary.

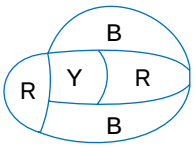
a) either a common-cause factor or a presumed relationship

b) common-cause factor **c)** accidental relationship

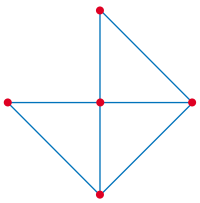
d) cause-and-effect relationship

7. Answers may vary.

a)



b)



8. a) **i)** no **ii)** yes **iii)** yes

b) **i)** yes **ii)** yes **iii)** no

9. Answers may vary.

10. a) Buffalo **b)** Buffalo

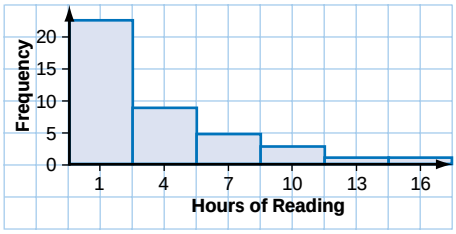
11. a) 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 5, 5, 6, 7, 7, 7, 8, 9, 10, 10, 14, 15

b) 3.4

c)

Class Interval	Tally	Frequency
–0.5–2.5	𝍸 𝍸 𝍸 𝍸 | | |	23
2.5–5.5	𝍸 | | | |	9
5.5–8.5	𝍸	5
8.5–11.5	| | |	3
11.5–14.5	|	1
14.5–17.5	|	1

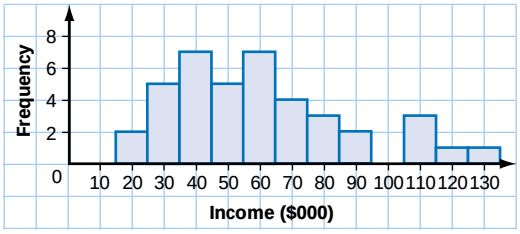
d)

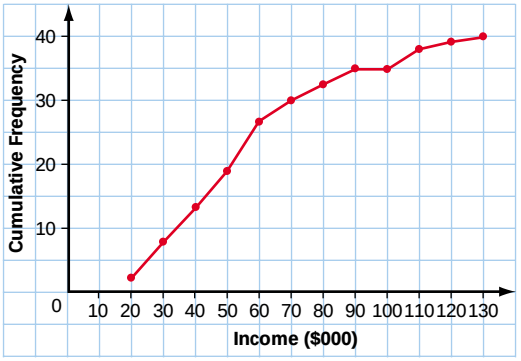


12. a)

Class Interval	Tally	Frequency
15.9–25.9	| |	2
25.9–35.9	𝍸	5
35.9–45.9	𝍸 | |	7
45.9–55.9	𝍸	5
55.9–65.9	𝍸 | |	7
65.9–75.9	| | | |	4
75.9–85.9	| | |	3
85.9–95.9	| |	2
95.9–105.9		0
105.9–115.9	| | |	3
115.9–125.9	|	1
125.9–135.9	|	1

b)



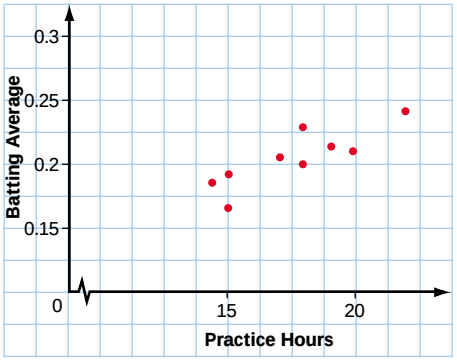


c) 60%

13. a) response bias b) response bias
c) sampling bias d) measurement bias

14. a) 198.6, 193.5, 260; the mean or median is the most descriptive; the mode is the least descriptive.
b) 40.4, 170, 222, 52
c) The standard deviation is a measure of the spread of the scores about the mean; 25% of the scores are below the 1st quartile, 170, and 75% are above; 75% of the scores are below the 3rd quartile, 222, and 25% are above; 50% of the scores fall within the interquartile range.
d) 193.5
e) No; this person is in the 75th percentile.

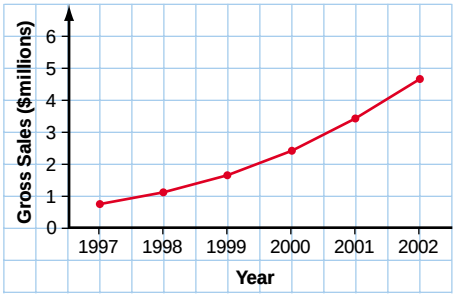
15. a) independent: practice hours; dependent: batting average
b)



strong positive correlation
c) 0.95; $y = 0.0098x + 0.0321$
d) i) 0.190 ii) 0.160 iii) 0.375

16. Answers will vary.

17. a)



b) predictions may vary; $6.3 million

c) factors may vary: the economy, health of the employee, change of employer, design of 2003 cars

d)

Year	Index
1997	100
1998	137.5
1999	200
2000	287.5
2001	437.5
2002	587.5

1997 = 100

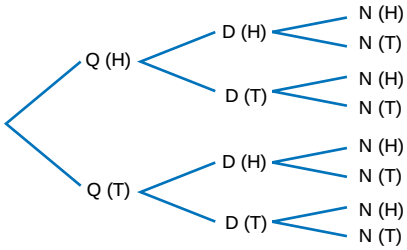
e) Answers may vary.

18. a) 100 **b)** 1980 **c)** 1980–1990 **d)** 5.2%

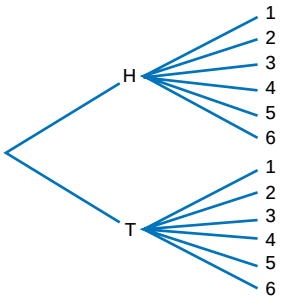
CHAPTER 4

Review of Prerequisite Skills, p. 224

1. 8 ways



2. a)



b) 12

3. a) 7 **b)** 28 **c)** 84

d) 1, 3, 6, 10, … ; $t_n = \frac{n(n+1)}{2}$

4. a) 15 **b)** 45 **c)** 90 **d)** 150

5. a) 96 **b)** 72 **c)** 8

6. a) 11 **b)** 209.1 **c)** 120 **d)** 2450 **e)** 605

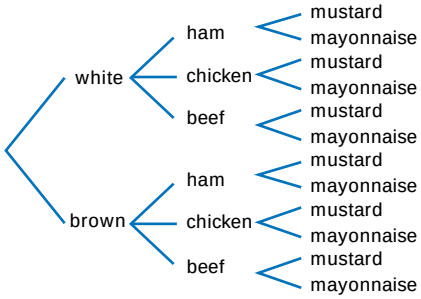
7. a) $\frac{x}{2} - \frac{y}{2} + 1$ **b)** $x^2 + 4x + 4$ **c)** $x^2 + 2$

d) $x^2 - 4x + 3$ **e)** $\frac{3y+2}{x}$

Section 4.1, pp. 229–231

Practise

1. 12



2. 5

3. 16

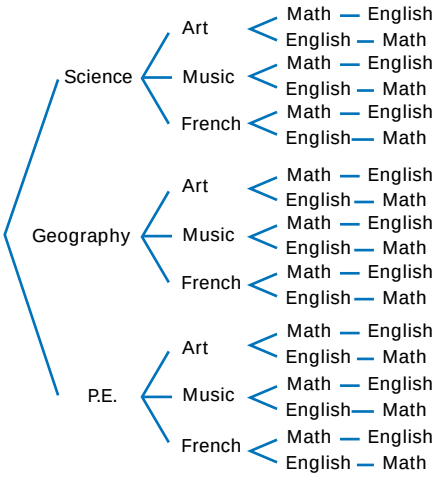
4. 8

5. a) 12 **b)** 6 **c)** 9

6. Use the product rule; for each of the 10 books there are 4 choices of a pen, and so there are $10 \times 4 = 40$ choices in total.

Apply, Solve, Communicate

7. a)



b) 18
8. 7776
9. 15
10. 64
11. 28
12. 32
13. a) Canada: 17 576 000; U.S.A.: 100 000 **b)** 17 476 000
14. 160
15. 12
16. a) 12 **b)** 26 **c)** 15 **d)** 14
e) Multiplicative counting principle in parts a) and b). Additive counting principle in parts c) and d).
17. a) 90 **b)** 56
18. a) 17 576 000 **b)** 456 976 000
19. 72
20. a) 30 **b)** 6
22. 1620
23. 23 816
24. a) 60 840
b) Answers may vary. The answer given in part a) assumes that any string of three numbers taken from the list 0, 1, …, 39 is a possible combination, provided no successive numbers are the same.
c) 59 319
d) If the first number can also be dialled counter clockwise from 0, the number of possible combinations increases to 118 638, assuming the remaining numbers are dialled clockwise.
e) 2 372 760, with assumptions similar to those in part b)
25. a) i) 2 **ii)** 9 **iii)** 22
b) No; the number of subsequent moves depends on the particular previous move.

Section 4.2, pp. 239–240

Practise

1. a) 6! **b)** 8! **c)** 3! **d)** 9!
2. a) 210 **b)** 110 **c)** 168 **d)** 5 405 400
e) 592 620 **f)** 30 270 240
3. a) ${}_6P_3$ **b)** ${}_9P_4$ **c)** ${}_{20}P_4$ **d)** ${}_{101}P_5$ **e)** ${}_{76}P_7$
4. a) 5040 **b)** 43 680 **c)** 20 **d)** 3024 **e)** 5040

Apply, Solve, Communicate

6. a) 720 **b)** 24 **c)** 11 880
7. a) 12! **b)** 9! **c)** $(n + 5)!$
8. The sequence $t_n = n!$ can be given by the recursion formula $t_1 = 1$, $t_n = n \cdot t_{n-1}$. A recursion formula is an example of an iterative process.
9. a) 5040 **b)** 720 **c)** 120 **d)** 1440
10. 479 001 599
11. 79 833 600
12. 201 600
13. 479 001 600
14. 6840
15. a) 311 875 200 **b)** 5.74×10^{16}
c) 7 893 600 **d)** 24
16. a) 9 **b)** 30 240
17. 479 001 600
18. a) 5040 **b)** 4320
19. No; there are 39 916 800 possible orders.
22. 120
23. 2^{97}
24. 3 628 800

Section 4.3, pp. 245–246

Practise

1. a) 2 of the letter *a*, 2 of the letter *m*, and 2 of the letter *t*.
b) the notebook of the same colour
c) the food items of the same kind
d) Thomas and Richard
2. a) 20 160 **b)** 907 200 **c)** 420 **d)** 9 081 072 000
3. 10
4. a) 720 **b)** 120 **c)** 180 **d)** 30

Apply, Solve, Communicate

5. 56
6. 210
7. 19 380
8. a) SUYFS: 60, YATTS: 60, SPEEXO: 360, HAREMM: 360
b) Answers may vary.
9. 2520
10. a) 720 **b)** 20 **c)** 90
11. 11 732 745 024
12. 1260
13. 15
15. 4032
16. 20
17. a) $\dfrac{165!}{72! \times 36! \times 57!}$ **b)** Estimates will vary.
18. 504
19. 190 590 400

Section 4.4, pp. 251–253

Practise

1.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1

2. a) $t_{8,3}$ **b)** $t_{52,41}$ **c)** $t_{17,11}$ **d)** $t_{n-1,r-1}$

3. a) 4096 **b)** 1 048 576 **c)** 33 554 432 **d)** 2^{n-1}

4. a) 8 **b)** 11 **c)** 14 **d)** 16

Apply, Solve, Communicate

5. a)

Row	Sum/Difference	Result
0	1	1
1	1 – 1	0
2	1 – 2 + 1	0
3	1 – 3 + 3 – 1	0
4	1 – 4 + 6 – 4 + 1	0
5	1 – 5 + 10 – 10 + 5 – 1	0
6	1 – 6 + 15 – 20 + 15 – 6 + 1	0
⋮	⋮	⋮

b) 0 **c)** 0

6. a) $t_{2n,n}$ **b)** n odd: 0, n even: $(-1)^{0.5n}t_{n,0.5n}$

7. a) The digits in 11^n are the same as the digits in row n of Pascal's triangle.

b) Power: $11^5 = 161\,051$. Add the digits in the 5th row of Pascal's triangle as follows:

1 5 10 10 5 1
1 6 1 0 5 1

c) Power: $11^6 = 1\,771\,561$. Add the digits in the 6th row of Pascal's triangle as follows:

1 6 15 20 15 6 1
1 7 7 1 5 6 1

Power: $11^7 = 19\,487\,171$. Add the digits in the 7th row of Pascal's triangle as follows:

1 7 21 35 35 21 7 1
1 9 4 8 7 1 7 1

8. a) 2, 5, 9 **b)** $t_{n-3,1} + t_{n-2,2}$ **c)** 14, 20

9. The terms of Pascal's triangle in prime-numbered rows are divisible by the row-number (with the exceptions of the leading and final 1).

10. a) 0, 1, 3, 7, 15, … Yes, the rows numbered $2^n - 1$ for $n \geq 0$ contain only odd numbers.

b) no **c)** odd

11. a) $t_{n+2,n-1}$ **b)** 364

12. a) $1 + 4 + \ldots + n^2 = t_{n+1,n-2} + t_{n+2,n-1}$

b) 385

13. a) $\frac{n^2 + n + 2}{2}$ **b)** 121

15. Answers will vary.

16. a) $3t_{n+1,n-1}$ **b)** 165

17. Answers may vary.

18. a)

1																		
1	1																	
1	2	1																
1	3	3	1															
1	4	6	4	1														
1	5	10	10	5	1													
1	6	15	20	15	6	1												
1	7	21	35	35	21	7	1											
1	8	28	56	70	56	28	8	1										
1	9	36	84	126	126	84	36	9	1									
1	10	45	120	210	252	210	120	45	10	1								
1	11	55	165	330	462	462	330	165	55	11	1							
1	12	66	220	495	792	924	792	495	220	66	12	1						
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1					
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1				
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1			
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1		
1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	2380	680	136	17	1	
1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	8568	3060	816	153	18	1
1	19	171	969	3876	11628	27132	50388	75582	92378	92378	75582	50388	27132	11628	3876	969	171	19	1

b)

1																		
1	1																	
1	2	1																
1	3	3	1															
1	4	6	4	1														
1	5	10	10	5	1													
1	6	15	20	15	6	1												
1	7	21	35	35	21	7	1											
1	8	28	56	70	56	28	8	1										
1	9	36	84	126	126	84	36	9	1									
1	10	45	120	210	252	210	120	45	10	1								
1	11	55	165	330	462	462	330	165	55	11	1							
1	12	66	220	495	792	924	792	495	220	66	12	1						
1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1					
1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1				
1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1			
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1		
1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	2380	680	136	17	1	
1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	8568	3060	816	153	18	1
1	19	171	969	3876	11628	27132	50388	75582	92378	92378	75582	50388	27132	11628	3876	969	171	19	1

c) Answers will vary.

19. a) The top term is 1, and the terms on the extreme left and right of row n are generated by increasing the denominator of the extreme left and right terms of row $n - 1$ by 1. Every other term is obtained by subtracting

the term to its immediate right from the term immediately above and to the right. Alternatively, each term is obtained by subtracting the term to its immediate left from the term immediately above and to the left. The values will be the same in each method.

b) $\frac{1}{7}, \frac{1}{42}, \frac{1}{105}, \frac{1}{140}, \frac{1}{105}, \frac{1}{42}, \frac{1}{7}; \frac{1}{8}, \frac{1}{56}, \frac{1}{168}, \frac{1}{280}, \frac{1}{280}, \frac{1}{168}, \frac{1}{56}, \frac{1}{8}$

c) Answers may vary.

Section 4.5, pp. 256–259

Practise

1. 924 792 495 330

1716 1287 825

3003 2112

5115

2. a) 128 **b)** 672 **c)** 80

3. 1, 15, 105, 455, 1365, 3003, 5005, 6435, 6435; 1, 17, 136, 680, 2380, 6188, 12 376, 19 448, 24 310

Apply, Solve, Communicate

4. a) 126 **b)** 270 **c)** 136

5. 56

6. No; there are only 126 different routes.

7. a) 32 **b)** 308

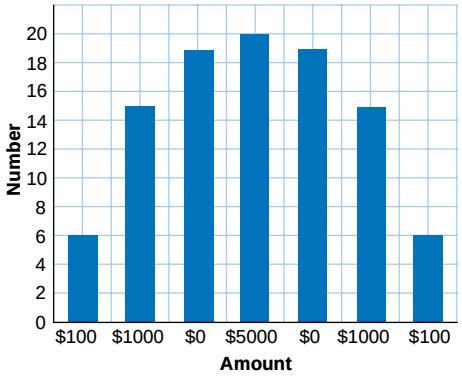
8. a) 20 **b)** the square labelled with 3: 54 routes

9. a) 4 **b)** 4 **c)** 12 **d)** 16 **e)** yes

10. 25

11. a) either 3 or 4; either 1 or 6

b)



12. a)

2

2 2

2 4 2

2 6 6 2

2 8 12 8 2

2 10 20 20 10 2

2 12 30 40 30 12 2

b) Conjectures will vary. **c)** Conjectures will vary.

13. $t_{n,r} = t_{n-3,r-3} + 3t_{n-3,r-2} + 3t_{n-3,r-1} + 3t_{n-3,r}$

15. $\frac{(m+n)!}{m!n!}$

16. 269

17. 30.8 L

18. yes

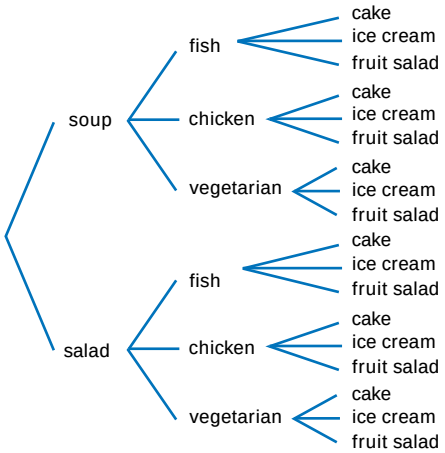
19. 320

20. Answers may vary.

Review of Key Concepts, p. 260

4.1 Organized Counting

1.



2. 336

3. 480

4. 48

4.2 Factorials and Permutations

5. 0, 1, 2, 3

6. 60

7. 1320

4.3 Permutations With Some Identical Items

8. 4200

9. a) 5040 **b)** 1260 **c)** 630

10. 900 900

4.4 Pascal's Triangle

11.

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

12. 128

13. Answers may vary.

4.5 Applying Pascal's Method

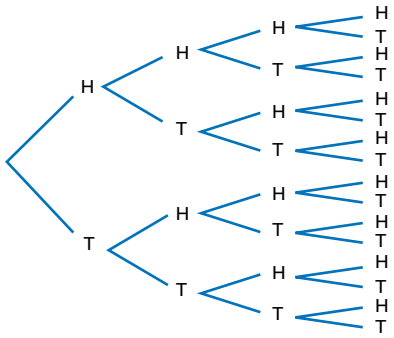
14. Pascal's method is iterative since previous terms are used to determine subsequent terms.

15. 204

Chapter Test, p. 261

1. a) 16

b)



2. a) $\frac{15!}{9!} = 3\ 603\ 600$ **b)** $\frac{6!}{4!} = 30$

c) $\frac{7!}{4!} = 210$ **d)** $\frac{9!}{0!} = 362\ 880$

e) $\frac{7!}{7!} = 1$

3. a) i) 27 **ii)** 39

b) For codes of the same length the multiplicative counting principle applies by multiplying the number of pulses possible for each position. The additive counting principle applies by adding the numbers of codes of various lengths since these are mutually exclusive cases.

4. a) 840 **b)** 480 **c)** 360

5. 6

6. a) 12 **b)** 226 800 **c)** 362 880

7. a) $t_{n,n-2}$ **b)** 15

CHAPTER 5

Review of Prerequisite Skills, p. 264

1. a) 40 320 **b)** 336 **c)** 552 **d)** 144

2. a) 120 **b)** 90 **c)** 12 **d)** 210

3. a) 30 240 **b)** 240 **c)** 3 628 800 **d)** 6 720

4. a) 1.96×10^{39} **b)** 2.69×10^{23}

c) 8.84×10^{30} **d)** 2.13×10^{35}

5. 35 152

6. a) 60 **b)** 226 800 **c)** 1320

7. a) 1 **b)** $-64x^3$ **c)** $576\ 240x^4y^2$ **d)** $\frac{21}{x^4}$

e) $432x^6y^5$ **f)** $\frac{3}{2}x^2y^3$ **g)** $-75x^5y^3$ **h)** $-8x^3y^3$

8. a) $x^2 - 10x + 25$ **b)** $25x^2 - 10xy + y^2$

c) $x^4 + 10x^2 + 25$ **d)** $x^3 - 7x^2 - 5x + 75$

e) $x^4 - 2x^2y + y^2$ **f)** $4x^2 + 12x + 9$

g) $x^3 - 10x^2 + 32x - 32$ **h)** $4x^4 + 12x^2y + 9y^2$

i) $4x^3 - 4x^2 - 7x - 2$ **j)** $x^3 - 3x^2y + 4y^3$

9. a) $\sum_{n=0}^{4} 2^n$ **b)** $\sum_{n=1}^{5} nx^n$ **c)** $\sum_{n=2}^{\infty} \frac{1}{n}$

10. a) $4 + 6 + 8 + 10$ **b)** $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

c) $2^1 + 1^2 + 2^2 + 2^2 + 2^3 + 3^2 + 2^4 + 4^2 + 2^5 + 5^2 = 117$

Section 5.1, pp. 270–272

Practise

1. a) i) apple **ii)** apple, orange, pear, banana

iii) apple, orange, pear, banana **iv)** apple, banana

iv) apple, orange, pear, banana

b) i) 5 **ii)** 4 **iii)** 4 **iv)** 4 **v)** 3

c) i) apple, pear; orange, pear; apple, orange

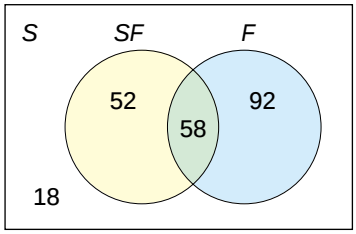
ii) apple, banana

iii) apple, orange; apple, pear; apple, banana; orange, pear; orange, banana; pear, banana

2. a) 82 **b)** 10 **c)** 56 **d)** 9 **e)** 6 **f)** 50

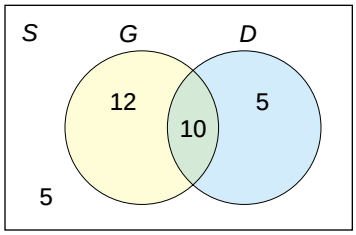
Apply, Solve, Communicate

3. 18,



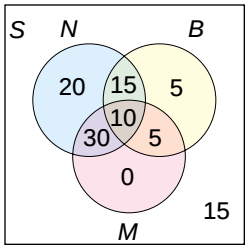
4. a) 95% **b)** 5% **c)** 45% **d)** 50% **e)** 35%

5. 5,



6. 10

7. a)



30

b) There are many consistent sets of information. If x represents the number of people that did not read books, newspapers, or magazines, then x can have any value between 0 and 15, and the number who read both newspapers and magazines is $25 + x$; the number who read newspapers only is $35 - x$, and the number who read magazines only is $15 - x$.

8. a) 5 **b)** 40

9. a) 22 **b)** 20 **c)** 19 **d)** 20

10. a) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

b) $n(A \cup B \cup C \cup D) = n(A) + n(B) + n(C) + n(D) - n(A \cap B) - n(B \cap C) - n(C \cap D) - n(D \cap A) - n(A \cap C) - n(B \cap D) + n(A \cap B \cap C) + n(B \cap C \cap D) + n(C \cap D \cap A) + n(D \cap A \cap B) - n(A \cap B \cap C \cap D)$

c) $(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{i=1}^{n} A_i - \sum_{\substack{i,j \\ i<j}}^{n} n(A_i \cap A_j) + \sum_{\substack{i,j,k \\ i<j<k}}^{n} n(A_i \cap A_j \cap A_k) - \ldots + (-1)^{n+1} n(A_1 \cap A_2 \cap \ldots \cap A_n)$

Section 5.2, pp. 279–281

Practise

1. a) 210 **b)** 435 **c)** 8568 **d)** 560 **e)** 3876 **f)** 53 130

2. a) 11, 11, equal **b)** 55, 55, equal **c)** 165, 165, equal

Apply, Solve, Communicate

3. 105

4. 270 725

5. 165

6. 74 613

7. 33 649

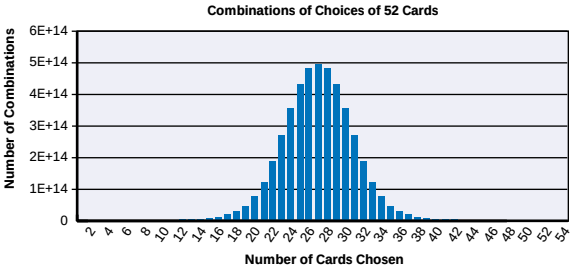
8. a) 2 220 075

9. a) 55 **b)** 165 **c)** 990

d) 6; the number of ways to order the three people in each combination of part b) is ${}_3P_3 = 3!$ or 6.

10.

Number of Cards Chosen	Combinations	Number of Cards Chosen	Combinations
0	1	27	4.77551E+14
1	52	28	4.26385E+14
2	1326	29	3.5287E+14
3	22100	30	2.70534E+14
4	270725	31	1.91992E+14
5	2598960	32	1.25995E+14
6	20358520	33	7.63604E+13
7	133784560	34	4.2672E+13
8	752538150	35	2.19456E+13
9	3679075400	36	1.03632E+13
10	15820024220	37	4.48138E+12
11	60403728840	38	1.76897E+12
12	2.06379E+11	39	6.35014E+11
13	6.35014E+11	40	2.06379E+11
14	1.76897E+12	41	60403728840
15	4.48138E+12	42	15820024220
16	1.03632E+13	43	3679075400
17	2.19456E+13	44	752538150
18	4.2672E+13	45	133784560
19	7.63604E+13	46	20358520
20	1.25995E+14	47	2598960
21	1.91992E+14	48	270725
22	2.70534E+14	49	22100
23	3.5287E+14	50	1326
24	4.26385E+14	51	52
25	4.77551E+14	52	1
26	4.95919E+14		



11. 26 460

12. 1 051 050

13. a) 65 780 **b)** 4950 **c)** 15 015
d) 25 025 **e)** 3465

14. a) 125 970 **b)** 9450 **c)** 203 490
d) 257 754

15. 42 504

16. a) 330 **b)** 35

17. 14 000

18. 190

20. a) 13!; the number of ways to arrange the 13 cards

b) ${}_{13}C_5 \times {}_{13}C_2 \times {}_{13}C_3 \times {}_{13}C_3$ **c)** ${}_{13}C_5 \times {}_{39}C_8$

d) 8 211 173 256, 79 181 063 676

21. a) 2 743 372 800 **b)** 2 472 422 400 **c)** 8 892 185 702 400

22. a) 330 **b)** 150 **c)** 5 **d)** 15

Section 5.3, pp. 286–288

Practise

1. 15

2. 11

3. a) 4 **b)** 16 **c)** 128

4. 255

5. a) combinations; ${}_{12}C_3$ **b)** permutations; ${}_{12}P_3$
c) combinations; ${}_3C_1 \times {}_4C_1 \times {}_2C_1$
d) permutations; ${}_4P_2$

Apply, Solve, Communicate

6. a) 3003 **b)** 2926

7. a) 287 **b)** 216

8. a) 593 775 **b)** 2925 **c)** 590 850

9. a) 792 **b)** 35 772 **c)** 21 252

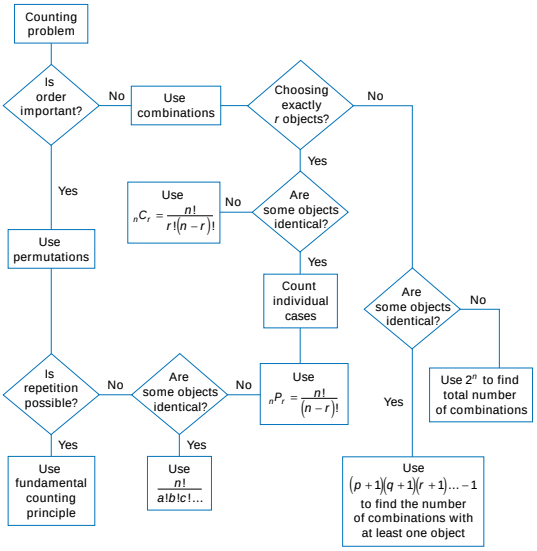
10. a) 28 561 **b)** 2 023 203 **c)** 844 272

11. a) 48 **b)** 36 **c)** 12

12. 252

13. 2047

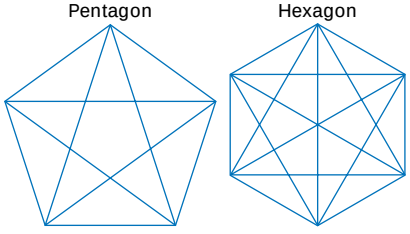
14.



15. a) 1365 **b)** 364 **c)** 1001

16. a) i) 5 **ii)** 9

b)



17. 19

18. 9240

19. a) 7.15×10^{18} **b)** 3 628 800 **c)** 256

21. a) DEVEL: 8, VEENT: 8, PAPNYS: 18, SIFOSY: 18
b) Because some choices could be identical.

22. 150

23. a) 6 **b)** 7 **c)** 6

24. 23

Section 5.4, pp. 293–295

Practise

1. a) ${}_{16}C_{10} + {}_{16}C_{11}$ **b)** ${}_{42}C_{35} + {}_{42}C_{36}$ **c)** ${}_nC_r + {}_nC_{r+1}$
d) ${}_{33}C_5$ **e)** ${}_{16}C_{10}$ **f)** ${}_{n+1}C_{r+1}$
g) ${}_{17}C_8$ **h)** ${}_{23}C_8$ **i)** ${}_{n-1}C_r$

2. a) 4 **b)** 2 **c)** 5

3. a) 13 **b)** 6 **c)** 21

4. a) ${}_{11}C_2$, 55 **b)** ${}_{11}C_0$, 1 **c)** ${}_{11}C_5$, 462

Apply, Solve, Communicate

5. a) 512 **b)** 0 **c)** 32 768 **d)** 2^n

6. 14

7. a) i) $2 \times {}_{2n-1}C_n$
ii) $(-1)^{\frac{n}{2}} \times 2 \times {}_{n-1}C_{\frac{n}{2}}$ for n even, 0 for n odd.
iii) ${}_{n-1}C_2 - 1$
b) i) 155 117 520 **ii)** 924 **iii)** 77

8. 9

9. a) $x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$
b) $64x^6 + 576x^5y + 2160x^4y^2 + 4320x^3y^3 + 4860x^2y^4 + 2916xy^5 + 729y^6$
c) $32x^5 - 400x^4y + 2000x^3y^3 - 5000x^2y^3 + 6250xy^4 - 3125y^5$
d) $x^8 + 20x^6 + 150x^4 + 500x^2 + 625$
e) $2187a^{14} + 20\,412a^{12}c + 81\,648a^{10}c^2 + 181\,440a^8c^3 + 241\,920a^6c^4 + 193\,536a^4c^5 + 86\,016a^2c^6 + 16\,384c^7$
f) $-38\,880c^{10} + 64\,800c^8p - 43\,200c^6p^2 + 14\,400c^4p^3 - 2400c^2p^4 + 160p^5$

10. a) $59\,049x^{10} + 196\,830x^9y + 295\,245x^8y^2 + 262\,440x^7y^3 + 153\,090x^6y^4$
b) $59\,049x^{10} - 196\,830x^9y + 295\,245x^8y^2 - 262\,440x^7y^3 + 153\,090x^6y^4$
c) The terms are identical, except the coefficients of the terms involving odd powers are negative in $(3x - y)^{10}$.

11. a) $x^{10} - 5x^7 + 10x^4 - 10x + 5x^{-2} - x^{-5}$
b) $16y^4 + 96y + 216y^{-2} + 216y^{-5} + 81y^{-8}$
c) $64x^{12} + 192x^{10.5} + 240x^9 + 160x^{7.5} + 60x^6 + 12x^{4.5} + x^3$
d) $5k^5m^{-2} + 10k^5m^{-4} + 10k^5m^{-6} + 5k^5m^{-8} + k^5m^{-10} + k^5$
e) $y^{3.5} - 14y^{2.5} + 84y^{1.5} - 280y^{0.5} + 560y^{-0.5} - 672y^{-1.5} + 448y^{-2.5} - 128y^{-3.5}$
f) $162m^8 - 432m^{5.5} + 432m^3 - 192m^{0.5} + 32m^{-2}$

12. a) $(x + y)^6$ **b)** $(y^3 + 2)^4$ **c)** $(3a - b)^5$

13. a) $0.5^5(1 + 1)^5 = 1$ **b)** $(0.7 + 0.3)^7 = 1$
c) $(7 - 1)^9 = 6^9$ or 10 077 696

14. a) $x^4 + 8x^2 + 32x^{-2} + 16x^{-4} + 24$
b) $\left(x + \frac{2}{x}\right)^4 = \left(\frac{1}{x}(x^2 + 2)\right)^4 = \frac{1}{x^4}(x^2 + 2)^4$

15. a) $15\,625x^6 + 56\,250x^5y + 84\,375x^4y^2 + 67\,500x^3y^3 + 30\,375x^2y^4 + 7290xy^5 + 729y^6$
b) $59\,049x^{10} - 131\,220x^8y^2 + 116\,640x^6y^4 - 51\,840x^4y^6 + 11\,520x^2y^8 - 1024y^{10}$

16. a) 5.153 92 after 6 terms **b)** 7

17. a) 210 **b)** 1023

19. a) $16x^5$, $208x^4$, $1080x^3$
b) $4096x^7$, $-11\,264x^6$, $8704x^5$

c) x^{36}, $-45x^{34}$, $12x^{33}$

20. a) $x^2 + 2xy + 2xz + y^2 + 2yz + z^2$

b) $x^3 + 3x^2y + 3x^2z + 3xy^2 + 6xyz + 3xz^2 + y^3 + 3y^2z + 3yz^2 + z^3$

c) $x^4 + 4x^3y + 4x^3z + 6x^2y^2 + 12x^2yz + 6x^2z^2 + 4xy^3 + 12xy^2z + 12xyz^2 + 4xz^3 + y^4 + 4y^3z + 6y^2z^2 + 4yz^3 + z^4$

d) $\sum_{\substack{i,j,k=0 \\ i+j+k=n}}^{n} \frac{n!}{i!j!k!} x^i y^j z^k$

e) $x^5 + 5x^4y + 5x^4z + 10x^3y^2 + 20x^3yz + 10x^3z^2 + 10x^2y^3 + 30x^2y^2z + 30x^2yz^2 + 10x^2z^3 + 5xy^4 + 20xy^3z + 30xy^2z^2 + 20xyz^3 + 5xz^4 + y^5 + 5y^4z + 10y^3z^2 + 10y^2z^3 + 5yz^4 + z^5$

21. a) $B^5 + 5B^4G + 10B^3G^2 + 10B^2G^3 + 5BG^4 + G^5$

b) The coefficient of G^k is the number of ways of having k girls in a family of 5 children.

c) 10 **d)** 5

22. a) 32 **b)** 10 **c)** 31

Review of Key Concepts, pp. 296–297

5.1 Organized Counting With Venn Diagrams

1. a) R2, R3, R5, R6, R7, R8 **b)** R7, R8

c) R5, R8 **d)** all regions

2. a) $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$

b) Because the elements counted by the last term have been counted twice in previous terms.

c) Answers will vary.

3. a) colour television, computers, and no dishwashers; colour television, dishwashers, and no computers; dishwashers, computers, and no colour televisions; colour television and no computers or no dishwashers; computers and no colour televisions or no dishwashers; dishwashers and no colour television or no computers; no colour televisions, no computers, and no dishwashers

b) 33%, 19%, 1%, 14%, 1%, 1%, 1%, respectively

c) Answers will vary.

5.2 Combinations

4. a) 95 548 245 **b)** 1 037 158 320 **c)** 1 081 575

d) 10 272 278 170 **e)** 45 **f)** 105

g) 5 **h)** 25 **i)** 1365

j) 53 130 **k)** 12 870 **l)** 27 405

5. 9900

6. 4 134 297 024

7. Order is important.

5.3 Problem Solving With Combinations

8. 512

9. 31

10. 210

11. 256

12. 756 756

13. 459

5.4 The Binomial Theorem

14. a) 6 **b)** 3

15. a) $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

b) $4096x^6 - 6144x^5y + 3840x^4y^2 - 1280x^3y^3 + 240x^2y^4 - 24xy^5 + y^6$

c) $16x^4 + 160x^3y + 600x^2y^2 + 1000xy^3 + 625y^4$

d) $16\,807x^5 - 36\,015x^4 + 30\,870x^3 - 13\,230x^2 + 2835x - 243$

16. a) $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

b) $1296x^4 - 4320x^3y + 5400x^2y^2 - 3000xy^3 + 625y^4$

c) $3125x^5 + 6250x^4y + 5000x^3y^2 + 2000x^2y^3 + 400xy^4 + 32y^5$

d) $729x^6 - 2916x^5 + 4860x^4 - 4320x^3 + 2160x^2 - 576x + 64$

17. a) $128x^7$, $2240x^6y$, $16\,800x^5y^2$

b) $4096x^6$, $-6144x^5y$, $3840x^4y^2$

18. The nth term in the expansion is given by ${}_6C_{n-1}(2x)^{6-(n-1)}(-3y)^{n-1}$. Determine the 1st through 7th terms and sum them.

19. $32x^5$

20. 43 750

21. $a = 1$, $x = 0.5$

22. $y^{24} - 24y^{20} + 240y^{16} - 1280y^{12} + 3840y^8 - 6144y^4 + 4096$

23. $(4x^2 - 3)^5$

Chapter Test, pp. 298–299

1. a) 1 **b)** 52 **c)** 220 **d)** 40 225 345 056

2. a) ${}_{11}C_8$ **b)** ${}_{22}C_{15}$

3. a) $81x^4 - 432x^3 + 864x^2 - 768x + 256$

b) $128x^7 + 1344x^6y + 6048x^5y^2 + 15\,120x^4y^3 + 22\,680x^3y^4 + 20\,412x^2y^5 + 10\,206xy^6 + 2187y^7$

4. a) $32\,768x^5 - 61\,440x^4 + 46\,080x^3 - 17\,280x^2 + 3240x - 243$

b) $64x^6 - 960x^5y + 6000x^4y^2 - 20\,000x^3y^3 + 37\,500x^2y^4 + 37\,500xy^5 + 15\,625y^6$

5. a) 1001 **b)** 35 **c)** 441 **d)** 966

6. a) 1140 **b)** 6840 **c)** no; order is important in part b).

7. a) 118 **b)** 1

8. a) 35 **b)** 34

9. a) 32 767 **b)** 216

10. $x = -0.1$, $n = 9$

11. $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

12. a) 66 **b)** 239 500 800

13. a) 756 **b)** 120 **c)** 176

d) 4 pizzas with 2 toppings, 2 with 1 topping; 1 pizza with 3 toppings, 2 with 2 toppings, and 3 with 1 topping; 2 pizzas with 3 toppings, 4 with 1 topping

e) 4725 ways; 12 600 ways; 2100 ways, respectively

CHAPTER 6

Review of Prerequisite Skills, pp. 302–303

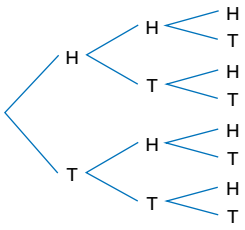
1. a) 35% **b)** 4% **c)** 95% **d)** 0.8% **e)** 8.5% **f)** 37.5%

2. a) 0.15 **b)** 0.03 **c)** 0.85 **d)** 0.065 **e)** 0.265 **f)** 0.752

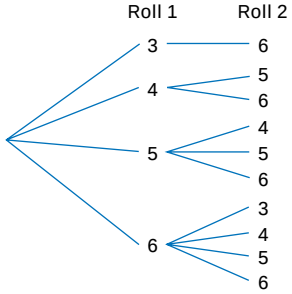
3. a) $\frac{3}{25}$ **b)** $\frac{7}{20}$ **c)** $\frac{67}{100}$ **d)** $\frac{1}{25}$ **e)** $\frac{1}{200}$ **f)** $\frac{49}{50}$

4. a) 25% **b)** 86.7% **c)** 78.6% **d)** 70% **e)** 44.4% **f)** 65%

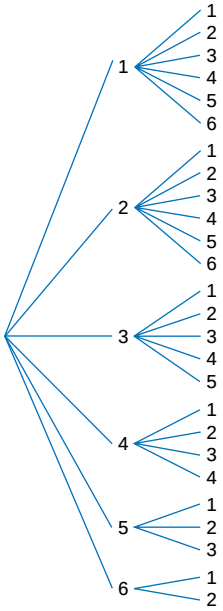
5.



6. a)



b)

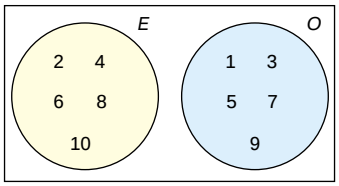


7. 12 ways, assuming that Benoit will wear an article of each type.

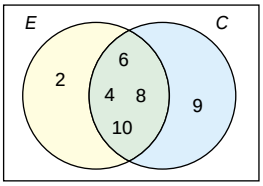
8. 6.33966×10^9

9. a) i) 2, 4, 6, 8, 10 **ii)** 1, 3, 5, 7, 9
iii) 4, 6, 8, 9, 10 **iv)** 1, 4, 9

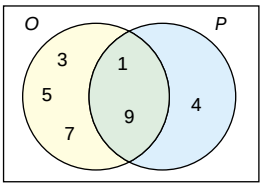
b) i)



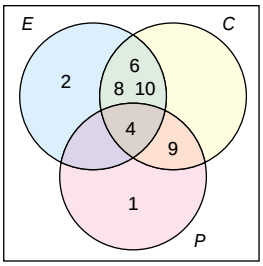
ii)



iii)



iv)



10. a) To count the elements of a set, each element must be counted exactly once. If an element is included in the count more than once, it must be excluded the appropriate number of times. For example, to count the elements of the set A or B, each element of A and B is counted (included) twice in the sum $n(A) + n(B)$ and so must be excluded once.
Thus, $n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$.

b) 22

11. a) 720 **b)** 1 **c)** 240 **d)** 220 **e)** 9900 **f)** 143

12. a) 60 **b)** 7 **c)** 30 **d)** 362 880 **e)** 100 **f)** 9900

13. 259 459 200

14. 800

15. a) 20 **b)** 4 **c)** 1 **d)** 1 **e)** 315 **f)** 100
g) 190 **h)** 190

16. 84

17. a) 6188 **c)** 2200

18. square matrices: a), d); row matrix: b); column matrix: f)

19. a) [0.505 0.495]
b) Not possible; the number of columns of B does not equal the number of rows of A.

c) $\begin{bmatrix} 0.49 & 0.51 \\ 0.4675 & 0.5325 \end{bmatrix}$ **d)** $\begin{bmatrix} 0.4765 & 0.5235 \\ 0.479875 & 0.520125 \end{bmatrix}$

e) Not possible; A is not a square matrix. **f)** [0.58]

Section 6.1, pp. 312–313

Practise

1. a) $\frac{1}{2}$ **b)** $\frac{1}{4}$ **c)** $\frac{7}{8}$ **d)** $\frac{2}{9}$ **e)** $\frac{29}{36}$ **f)** $\frac{3}{13}$

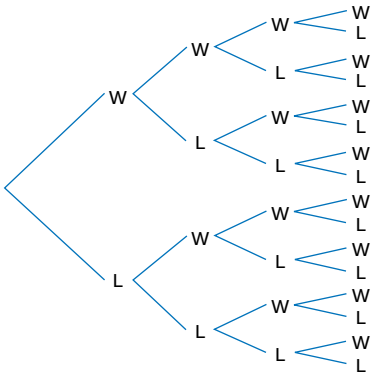
2. a) 100% **b)** 0% **c)** Answers may vary.
d) Answers may vary.

3. a) i) (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)
ii) no outcomes
iii) (2, 2)
b) Player A

4. a) 0.625 **b)** 0.281 25 **c)** 0.8

Apply, Solve, Communicate

5. a)



b) $\frac{3}{8}$ **c)** $\frac{1}{16}$

d) Assuming that a win or loss by either team is equally likely.

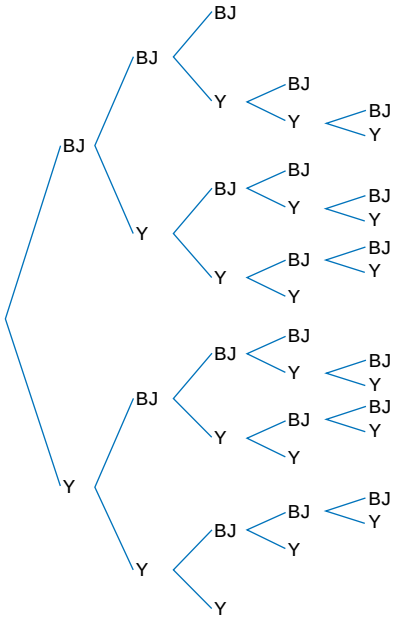
6. $\frac{2}{5}$

7. a) Player B has a winning probability of $\frac{11}{18}$ and so has the advantage.
b) Player B has a winning probability of $\frac{5}{9}$ and so has the advantage.
c) Player A wins if 1, 2, or 3 are thrown. Player B wins if 4, 5, or 6 are thrown.

8. a) $\frac{13}{24}$ **b)** 5

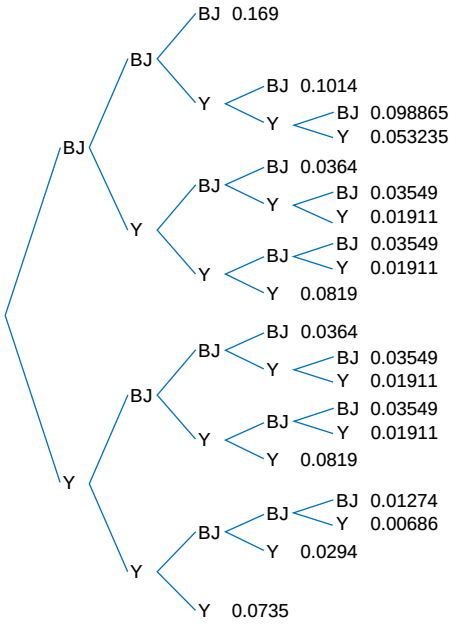
9. Answers will vary.

10. a)



b) 0.169

c)



d) by multiplying the branch probabilities

11. a) Trimble 36%, Yakamoto 23%, Audette 41%
b) Answers may vary.
c) In part a), votes for Jonsson are not counted. In part b), voters who intended to vote for Jonsson can now cast their votes for one of the remaining three candidates.

12. a) Answers may vary; for example, always choose B or C.
b) Answers may vary; for example, alternately choose B and C.

Section 6.2, pp. 318–319

Practise

1. a) 2:3 **b)** $\frac{3}{5}$

2. $\frac{1}{20}$

3. a) 1:35 **b)** 5:13 **c)** 5:7 **d)** 0

Apply, Solve, Communicate

4. a) 1:6 **b)** 1:7 **c)** 5:3 **d)** 1:25 **e)** 4:5

5. a) 6:5 **b)** 8:3

6. $\frac{7808}{19\,683}$

7. a) 7:41 **b)** 11:13

8. odds against $A = \frac{P(A')}{P(A)} = \frac{1}{\frac{P(A)}{P(A')}} = \frac{1}{\text{odds in favour of } A}$

9. 3:7

10. a) 10:3 **b)** 210:11

12. a) 3:7 **b)** 2:3 **c)** 1:1

13. No; the associated probabilities of a win, a loss, and a tie do not total 1.

14. approximately 43:7

15. $\frac{P(A)}{P(A')} = \frac{h}{k}$; $kP(A) = hP(A')$; $kP(A) = h(1 - P(A))$; $(h + k)P(A) = h$; $P(A) = \frac{h}{h+k}$

Section 6.3, pp. 324–326

Practise

1. $\frac{1}{3}$

2. 0.074

3. a) $\frac{5}{14}$ **b)** $\frac{25}{28}$

4. $\frac{2}{7}$

Apply, Solve, Communicate

5. a) $\frac{7}{102}$ **b)** $\frac{4}{17}$

6. $\frac{7}{30}$

7. a) $\frac{1}{28}$ **b)** $\frac{1}{56}$

c) Names are drawn at random.

8. a) $\frac{3}{10}$ **b)** $\frac{1}{120}$

c) Assuming that the friends arrive individually.

9. a) $\frac{14}{969}$ **b)** $\frac{1001}{4845}$

10. a) i) $\frac{1}{540\,000}$ **ii)** $\frac{1}{54\,000}$ **iii)** $\frac{1}{5400}$

b) 27 000 **c)** 270 000

11. $\frac{1}{5}$

12. a) i) $\frac{1}{209}$ **ii)** $\frac{170}{209}$

b) $\frac{1}{42}$

14. a) i) $\frac{3}{490}$ **ii)** $\frac{25}{784}$

b)–e) Answers may vary.

15. $\frac{35}{128}$

16. 7

Section 6.4, pp. 334–335

Practise

1. a) i) dependent **ii)** independent **iii)** independent **iv)** dependent **v)** independent **vi)** dependent

2. 0.06

3. $\frac{1}{1296}$

4. a) 84.55% **b)** 0.55% **c)** 99.45%

Apply, Solve, Communicate

5. a) Biff

6. 0.51

7. $\frac{121}{300}$

9. 0.2

10. a) 59.5% **b)** 9:191

c) More time spent studying one subject means less time spent studying the other.

11. Assuming independence, the probability is $\frac{1}{12^5}$.

12. $\frac{13}{80}$

13. a) $\frac{33}{182}$ **b)** Answers may vary. **c)** Answers may vary.

14. 1.3%

15. a) Each of the n tosses is independent and has a probability of $\frac{1}{2}$ of getting a head.

b) $\frac{127}{128}$

16. $\frac{64}{729}$

17. 0.48

18. 0.2187

Section 6.5, pp. 340–343

Practise

1. a) non-mutually exclusive **b)** non-mutually exclusive **c)** mutually exclusive **d)** mutually exclusive **e)** non-mutually exclusive **f)** non-mutually exclusive

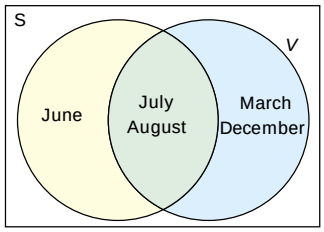
2. a) $\frac{1}{9}$ **b)** $\frac{1}{3}$ **c)** $\frac{4}{9}$

3. a) 7% **b)** 81% **c)** 74% **d)** 26%

Apply, Solve, Communicate

4. a) i) $\frac{124}{365}$ ii) $\frac{92}{365}$

b)



5. a) $\frac{13}{55}$ **b)** $\frac{4}{11}$ **c)** $\frac{1}{2}$ **d)** $\frac{3}{22}$

6. a) 0.55 **b)** 0.148 **c)** 0.948

7. a) $\frac{3}{28}$ **b)** $\frac{4}{7}$ **c)** $\frac{5}{14}$ **d)** $\frac{15}{28}$

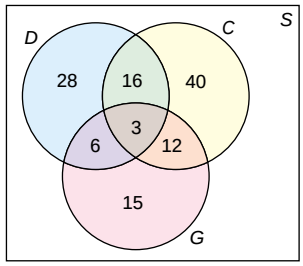
8. b) A and B are mutually exclusive.

9. Answers will vary.

10. $\frac{37}{120}$; mutually exclusive since bald patches relate to does only.

11. a) 0.232 **b)** 0.107

13. a)



b) $\frac{7}{8}$ **c)** $\frac{83}{120}$

14. 7:33

15. a) $\frac{3}{25}$ **b)** no

16. a) i) 0.418 ii) 0.058 iii) 0.040
iv) 0.960 v) 0.436
b) 0.090 **c)** 0.697 **d)** not independent

Section 6.6, pp. 353–356

Practise

1. a) Not a probability vector since the components do not sum to 1.
b) A probability vector.
c) Not a probability vector, since it is not a row vector.
d) Not a probability vector since there is a negative component.
e) A probability vector.

2. a) Not a transition matrix since the components of the 3rd row do not sum to 1.
b) A transition matrix.
c) Not a transition matrix since it is not square.

3. a) Initially, ZapShot has 67% of the market for digital cameras. E-pics has 33% of the market.
b) If the customer's initial purchase was with ZapShot, there is a 60% chance that the customer returns to ZapShot, and a 40% chance that the customer switches to E-pics. If the customer's initial purchase was with E-pics, there is a 50% chance that the customer returns to E-pics, and a 50% chance that the customer switches to ZapShot.

Apply, Solve, Communicate

4. a) Answers may vary.
b) $\begin{bmatrix} \frac{5}{9} & \frac{4}{9} \end{bmatrix}$
c) E-pics
d) Although ZapShot has a greater chance of drawing customers back than E-pics, the initial market share for ZapShot in question 3 was too high to sustain.

5. a) Regular since all are non-zero entries.
b) Not regular since all powers have entries equal to 0.
c) Regular since the second power has all non-zero entries.

6. a)
$$\begin{matrix} & \text{W} & \text{L} \\ \text{W} \\ \text{L} \end{matrix}\begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$
b) 0.52
c) $\begin{bmatrix} \frac{4}{7} & \frac{3}{7} \end{bmatrix}$; in the long run the team will win $\frac{4}{7}$ of their games.

7. a) [0.5 0.5]
b)
$$\begin{matrix} & \text{RP} & \text{BP} \\ \text{RP} \\ \text{BP} \end{matrix}\begin{bmatrix} 0.65 & 0.35 \\ 0.75 & 0.25 \end{bmatrix}$$
c) [0.7 0.3], [0.68 0.32]
d) approximately 0.68

8. a)
$$\begin{matrix} & \text{S} & \text{C} & \text{R} \\ \text{S} \\ \text{C} \\ \text{R} \end{matrix}\begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.25 & 0.35 & 0.4 \\ 0.2 & 0.35 & 0.45 \end{bmatrix}$$
b) 0.305 **c)** 0.310
d) Assuming that the transition matrix is seasonally invariant.

9. a) 0.25 **b)** 0.5415
c) Yes; approximately 67% of the time the value of the stock will rise or remain unchanged.

10. a) [0.7 0.3]
b)
$$\begin{matrix} & \text{D} & \text{B} \\ \text{D} \\ \text{B} \end{matrix}\begin{bmatrix} 0.8 & 0.2 \\ 0.35 & 0.65 \end{bmatrix}$$
c) 0.364

12. a) Answers may vary. **b)** $\begin{bmatrix} \frac{5}{9} & \frac{4}{9} \end{bmatrix}$

c) $P = \begin{matrix} & \text{C} & \text{S} \\ \text{C} \\ \text{S} \end{matrix}\begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$; $S^{(n)} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

13. a) not regular; steady state vector: $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$
b) regular; steady state vector: $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$
c) not regular; steady state vector: [1 0]

14. a) $\begin{bmatrix} 0.4711 & 0.3471 & 0.1818 \\ 0.4711 & 0.3471 & 0.1818 \\ 0.4711 & 0.3471 & 0.1818 \end{bmatrix}$

b) [0.4711 0.3471 0.1818]

c) A regular Markov chain will reach the same steady state regardless of the initial probability vector.

15. $m = 0.7$, $n = 0.3$

Review of Key Concepts, pp. 357–359

6.1 Basic Probability Concepts

1. a) $\frac{7}{20}$ **b)** $\frac{3}{4}$

2. a) $\frac{1}{4}$ **b)** $\frac{1}{6}$ **c)** chance variation

3. a) Answers may vary; normally low.

b) Answers may vary; normally high.

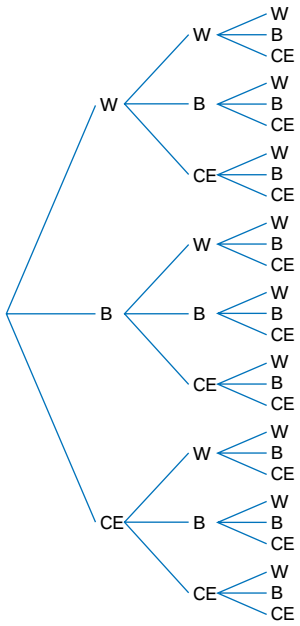
6.2 Odds

4. 1:7

5. a) 7:13 **b)** 1:1 **c)** 17:3

6.3 Probability Using Counting Techniques

6. a)



b) no **c)** $\frac{14}{285}$ **d)** $\frac{11}{57}$

7. $\frac{1}{72}$

8. a) $\frac{1}{12}$ **b)** $\frac{1}{120}$ **c)** $\frac{1}{720}$

6.4 Dependent and Independent Events

9. a) independent **b)** dependent

c) dependent **d)** independent

10. a) 0.3 **b)** subjective

11. a) $\frac{1}{2}$ **b)** $\frac{1}{3}$ **c)** $\frac{1}{2}$ **d)** 0.40

12. 0.025

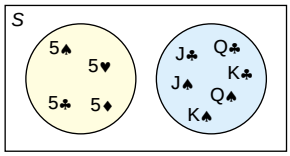
6.5 Mutually Exclusive Events

13. a) mutually exclusive **b)** mutually exclusive

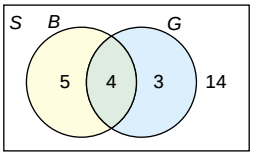
c) non-mutually exclusive **d)** non-mutually exclusive

14. a) $\frac{5}{26}$

b)



15. a)



b) $\frac{6}{13}$

16. a) 11:4 **b)** 2:13

17. a) 0.45 **b)** 0.87

6.6 Applying Matrices to Probability Problems

18. a) [1 0]

b) $\begin{matrix} & \text{S} & \text{N} \\ \end{matrix}$
$\begin{bmatrix} 0.5 & 0.5 \\ 0.35 & 0.65 \end{bmatrix}$ $\begin{matrix} \text{S} \\ \text{N} \end{matrix}$

c) 0.5, 0.425, 0.4118 **d)** 0.4118 **e)** 0.4118

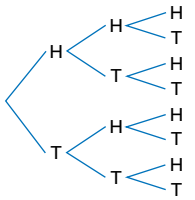
19. a) $\begin{matrix} & \text{BP} & \text{S} & \text{GW} \\ \end{matrix}$
$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.25 & 0.6 & 0.15 \\ 0.3 & 0.05 & 0.65 \end{bmatrix}$ $\begin{matrix} \text{BP} \\ \text{S} \\ \text{GW} \end{matrix}$

b) [0.4775 0.2703 0.2523]

c) approximately 25%

Chapter Test, pp. 360–361

1. a)



b) $\frac{1}{8}$ for all three tosses coming up heads; $\frac{1}{2}$ for a single toss coming up heads; $\frac{7}{8}$ for at least one toss in three coming up heads

2. a) $\frac{2}{3}$ **b)** 4:5

3. $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

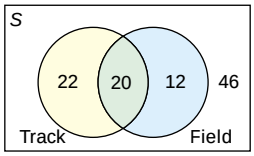
4. 0.05

5. a) i) $\frac{19}{30}$ **ii)** $\frac{11}{30}$

b) that nobody works at two jobs

6. 1:3

7. a)



b) i) 0.46 **ii)** 0.54

8. a) $\frac{1}{10}$ **b)** $\frac{1}{60}$

9. a) $\frac{7}{99}$ **b)** $\frac{35}{66}$ **c)** $\frac{13}{66}$ **d)** $\frac{2}{33}$

10. a) i) $[1 \quad 0]$

ii)
$$\begin{matrix} & \text{D} & \text{ND} \\ \begin{matrix}\text{D}\\ \text{ND}\end{matrix} & \begin{bmatrix} 0.07 & 0.93 \\ 0.004 & 0.996 \end{bmatrix} \end{matrix}$$

iii) 0.0049

b) Regular; all entries of the transition matrix are non-zero.

CUMULATIVE REVIEW: CHAPTERS 4 TO 6, p. 364

1. a) 5040 **b)** 7 **c)** 7 **d)** 5040
e) 21 **f)** 21

2. a) $243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

b) $2(a - b)^4$

3. a) 19 770 609 664 **b)** 19 461 693 888

4. 399 168 000

5. $\frac{7}{10}$

6. $\frac{4}{9}$

7. a) 24 **b)** 16

8. a) $\frac{1}{14}$ **b)** $\frac{3}{7}$

9. $\frac{1}{20}$

10. 0.073

11. a) 0.18 **b)** 0.0324

12. 0.64

13. a) Sasha: 40%; Pedro: 60%

b) 47.5%

c) Sasha: 53, Pedro: 47

CHAPTER 7

Review of Prerequisite Skills, p. 368

1. a) $\frac{2}{81}$ **b)** $\frac{9}{64}$ **c)** $\frac{36}{625}$ **d)** 6.28 **e)** 0.5904

2. a) $\sum_{k=1}^{12} t_k$ **b)** $\sum_{k=0}^{9} k_9C_k$ **c)** $\sum_{k=1}^{6} \frac{k+1}{k+2}$ **d)** $\frac{1}{6}\sum_{k=0}^{5} a_k$

3. a) $1 + 4 + 9 + 16 + 25 + 36 = 91$

b) $b_0 + b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7 + b_8 + b_9 + b_{10} + b_{11} + b_{12} + b_{13} + b_{14}$

c) ${}_7C_0 + {}_7C_1 + {}_7C_2 + {}_7C_3 + {}_7C_4 + {}_7C_5 + {}_7C_6 + {}_7C_7 = 128$

d) $0.7 + (0.3)(0.7) + (0.3)^2(0.7) + (0.3)^3(0.7) + (0.3)^4(0.7) + (0.3)^5(0.7) + (0.3)^6(0.7) + (0.3)^7(0.7) + (0.3)^8(0.7) = 1 - 0.3^9$

4. a) $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

b) $0.4^4 + 4(0.4)^3(0.6) + 6(0.4)^2(0.6)^2 + 4(0.4)(0.6)^3 + 0.6^4 = 1$

c) $\left(\frac{1}{3}\right)^5 + 5\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right) + 10\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2 + 10\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3 + 5\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4 + \left(\frac{2}{3}\right)^5 = 1$

d) $p^n + {}_nC_1p^{n-1}q + {}_nC_2p^{n-2}q^2 + \ldots + {}_nC_nq^n = \sum_{k=0}^{n} {}_nC_kp^{n-k}q^k$

5. a) $\frac{1}{6}$ **b)** $\frac{1}{18}$ **c)** $\frac{5}{9}$ **d)** $\frac{5}{36}$ **e)** $\frac{1}{6}$

6. $\frac{1}{16}$

7. a) $\frac{1}{676}$ **b)** $\frac{150}{169}$

Section 7.1, pp. 374–377

Practise

1. a) discrete **b)** continuous **c)** continuous
d) discrete **e)** continuous **f)** discrete

2. a) uniform
b) not uniform; not everyone has an equal probability of being selected.
c) uniform **d)** uniform
e) not uniform; some values will have a higher probability of being selected.
f) uniform
g) not uniform; some totals have a higher probability than other totals.

3. a) 10.75 **b)** 2 775 250 **c)** $\frac{29}{10}$

4. a) $\frac{1}{2}$ **b)** 4.5

Apply, Solve, Communicate

5. a) uniform **b)** $\frac{1}{10\,000}$ **c)** $\frac{1}{1000}$

6. a)

x	$P(x)$
2	$\frac{1}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{4}{36}$
6	$\frac{5}{36}$
7	$\frac{6}{36}$
8	$\frac{5}{36}$
9	$\frac{4}{36}$
10	$\frac{3}{36}$
11	$\frac{2}{36}$
12	$\frac{1}{36}$

b) 7 **c)**

x	$P(x)$
3	$\frac{1}{216}$
4	$\frac{3}{216}$
5	$\frac{6}{216}$
6	$\frac{10}{216}$
7	$\frac{15}{216}$
8	$\frac{21}{216}$
9	$\frac{25}{216}$
10	$\frac{27}{216}$
11	$\frac{27}{216}$
12	$\frac{25}{216}$
13	$\frac{21}{216}$
14	$\frac{15}{216}$
15	$\frac{10}{216}$
16	$\frac{6}{216}$
17	$\frac{3}{216}$
18	$\frac{1}{216}$

expected sum: 10.5

7. tetrahedron: 2.5, cube: 3.5, octahedron: 4.5, dodecahedron: 6.5, icosahedron: 10.5

8. a) $\frac{7}{2\,000\,000}$ **b)** \$1.48

9. a) −0.5 **b)** No; the expected value is not 0.

10. \$1.05

11.

x	$P(x)$
0	0.25
1	0.5
2	0.25

expected number: 1

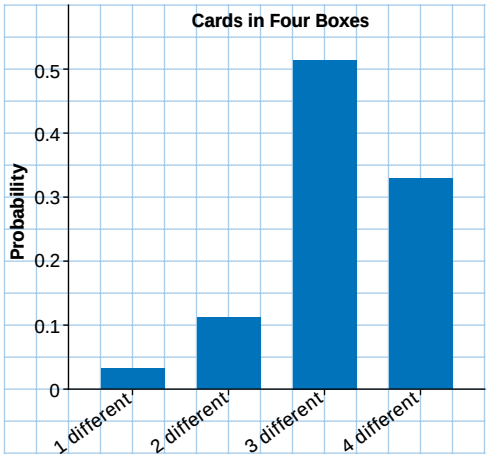
12. 26 cm²

13. a) Answers may vary. **b)** 2

14. a) Answers may vary.
b) Fatima has the greater probability of having two sons.

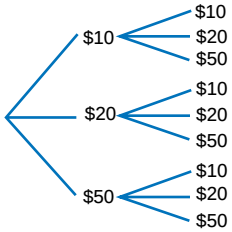
15. a) i) $\frac{1}{343}$ **ii)** $\frac{24}{343}$ **iii)** $\frac{180}{343}$ **iv)** $\frac{18}{343}$ **v)** $\frac{120}{343}$

b)



c) no

17. a)



b)

x	$P(x)$
0	$\frac{21}{66}$
1	$\frac{35}{66}$
2	$\frac{10}{66}$

c) $\frac{14}{99}$ **d)** 1.7

18. a) i) 5.3% **ii)** 5.3% **iii)** 5.3% **iv)** 5.3% **v)** 5.3%

b) Answers may vary.

c) The house advantage decreases.

19. inner region: $\frac{4}{25}$, middle region: $\frac{5}{25}$, outside region: $\frac{16}{25}$

20. Answers may vary.

a) expectation: 2.625 **b)** expectation: 1.75

21. equal

Section 7.2, pp. 385–387

Practise

1. a) binomial

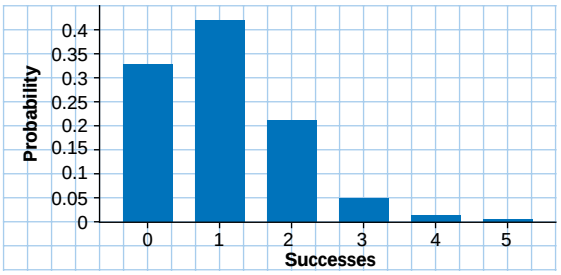
b) not binomial; there are different probabilities of a success.

c) not binomial; there is no fixed probability of success.

d) binomial

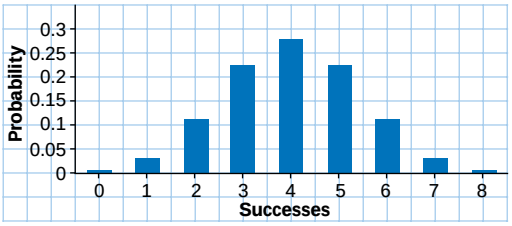
2. a)

x	$P(x)$
0	0.327 68
1	0.4096
2	0.2048
3	0.0512
4	0.0064
5	0.000 32



b)

x	$P(x)$
0	0.003 906 25
1	0.031 25
2	0.109 375
3	0.218 75
4	0.273 43 75
5	0.218 75
6	0.109 375
7	0.031 25
8	0.003 906 25



Apply, Solve, Communicate

3. a)

x	$P(x)$
0	0.735 09
1	0.232 13
2	0.030 54
3	0.002 14
4	8.4609×10^{-5}
5	1.7813×10^{-6}
6	1.5625×10^{-7}

b) 0.3

4. Answers may vary.

5. 0.2003

6. a) Answers may vary. **b)** 0.9358 **c)** 1.5

7. a) 0.8939 **b)** 5.2

8. a) 0.0319 **b)** 2 **c)** Answers may vary.

9. Answers may vary.

10. 0

11. $12\frac{2}{3}\%$

12. a) i) $(p+q)^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6$

ii) $(0.2 + 0.8)^5 = 0.2^5 + 5(0.2)^4(0.8) + 10(0.2)^3(0.8)^2 + 10(0.2)^2(0.8)^3 + 5(0.2)(0.8)^4 + (0.8)^5$

b) Each term is of the form of the probability of a certain number of successes for a binomial random variable.

14. a) Poisson approximation: 1.2276×10^{-8}, binomial distribution: 9.7936×10^{-9}

b) Poisson approximation: 5.3559×10^{-15}, binomial distribution: 4.4357×10^{-15}

c) Poisson approximation: 5.2337×10^{-7}, binomial distribution: 3.9805×10^{-7}

15. Answers may vary. Approximately 96% of the time the number of heads flipped by a fair coin in 20 flips will be between 6 and 14 inclusive.

16. a) $\frac{n!}{n_1!n_2!n_3!}p^{n_1}q^{n_2}r^{n_3}$; where $n_1 + n_2 + n_3 = n$

b) $\frac{10!}{2!4!4!}\left(\frac{1}{6}\right)^2\left(\frac{1}{6}\right)^4\left(\frac{4}{6}\right)^4 \doteq 0.0133$

17. There is a 79% chance that at least one model will be damaged.

Section 7.3, pp. 394–396

Practise

1. a) geometric

b) not geometric; the number of trials is fixed.

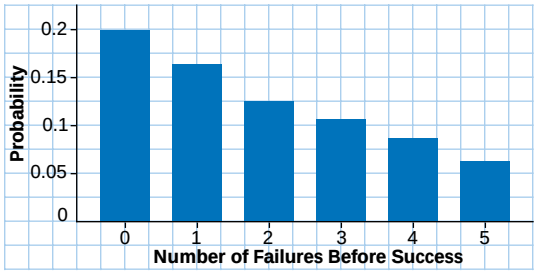
c) not geometric; the random variable for the waiting time is not discrete.

d) geometric

e) not geometric

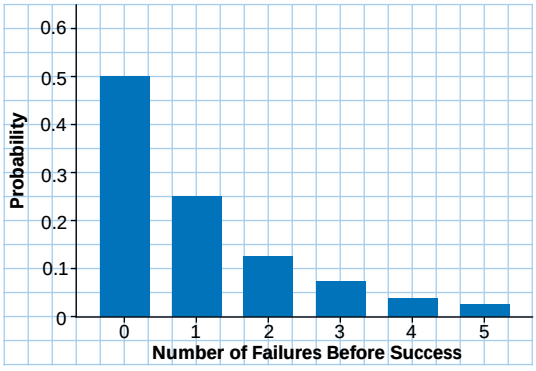
2. a)

x	$P(x)$
0	0.2
1	0.16
2	0.128
3	0.1024
4	0.081 92
5	0.065 536



b)

x	$P(x)$
0	0.5
1	0.25
2	0.125
3	0.625
4	0.031 25
5	0.015 625



Apply, Solve, Communicate

3. a) 0.0700 **b)** 11 rolls

4. a) 0.1001 **b)** 3 flights

5. No; the expected waiting time to roll a sum of 2 is 35 tries.

6. $\frac{16}{81}$

7. a) 0.0573 **b)** 0.5578 **c)** 11.5 tests

8. a) 0.0314 **b)** 9 hang-ups

9. a) 0.1296 **b)** 1.5 deliveries

10. a) 0.0426 **b)** 1.9 people

11. a) 0.0188 **b)** 0.0776 **c)** 49 cards
12. a) 0.0176 **b)** 13.3 shots
13. a) 0.0728 **b)** 65.7 chips
14. a) Answers may vary. **b)** 4.5
15. a) Answers may vary. **b)** 18.15
16. $P(x) = q^{x-1}p$, $E(x) = \frac{1}{p}$
17. The MTBF is the expected value of a continuous distribution that measures time before the failure of a component. The probability of the drive failing in any one-year period cannot be calculated knowing only the mean of the distribution—the distribution itself must be known. Probability of failure changes with time.
18. $4p^2q^3$
19. a) i) $\frac{1}{20}$ **ii)** $\frac{1}{20}$ **iii)** $\frac{1}{20}$ **iv)** 0.45
b) 95
c) The trials are not indpendent.

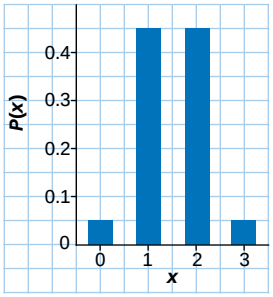
Section 7.4, pp. 404–405

Practise

1. a) hypergeometric
b) not hypergeometric; this is a geometric distribution.
c) not hypergeometric; this is a binomial distribution.
d) not hypergeometric; this is a binomial distribution.
e) hypergeometric **f)** hypergeometric
g) hypergeometric

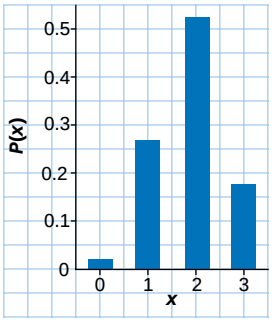
2. a)

x	$P(x)$
0	0.05
1	0.45
2	0.45
3	0.05



b)

x	$P(x)$
0	0.017 857 143
1	0.267 857 143
2	0.535 714 286
3	0.178 571 429



Apply, Solve, Communicate

3. a) $\frac{14}{33}$ **b)** $\frac{7}{6}$
4. a) 2167 **b)** chance variation in survey results
5. a) Answers may vary. **b)** 0.2173
6. a) Answers may vary. **b)** 8.3
7. a) 0.6339 **b)** 1.25
8. a) 0.3777 **b)** 1.6
9. a) 0.0707 **b)** 2.1
10. a) 0.2668 **b)** 0.9886 **c)** 5.3
11. a) 0.1758 **b)** 1.8
12. a) 0.0152 **b)** 0.3818 **c)** 0.1818
d) Red: 0.73, Black: 1.82, Green: 1.45
14. a) 4.34×10^{-8} **b)** 2.17×10^{-7} **c)** 0.842
15. If n is very large, non-replacement of successes will not change the ratio of successes to the population a great deal, and a binomial distribution will be a good approximation to the hypergeometric distribution.
16. $\frac{63}{256}$
17. 2.4 older scientists versus 2 younger scientists.

Review of Key Concepts, pp. 406–407

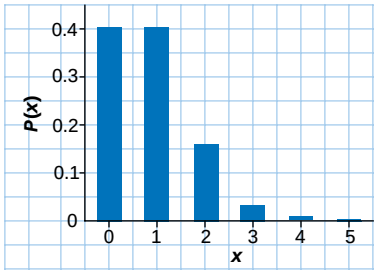
7.1 Probability Distributions

1. A uniform distribution is one in which each outcome has an equal probability. For example, when rolling a fair die, each of the six faces has an equal chance of facing upward.
2. a) forward 3.5 squares
b) Since the expected move is not 0, the game is not fair.
3. –$4.54
4. 57 m^2
5. 8-sided

7.2 Binomial Distributions

6. A binomial distribution is one in which there are a certain number of independent trials and two possible outcomes for each trial, with the probability of these outcomes being constant in each trial. For example, counting the number of times a 1 is rolled in 10 rolls of a fair die.
7. a) 0.0009 **b)** 2
8.

x	$P(x)$
0	0.401 877 572
1	0.401 877 572
2	0.160 751 029
3	0.032 150 206
4	0.003 215 021
5	0.000 128 601



9. a) 36.8% **b)** 6.2%

10. a) $252 \times \frac{19^5}{20^{10}}$ or 0.000 0609

b) 0.5

11. a)

x	$P(x)$	$xP(x)$
0	$\left(\frac{4}{5}\right)^{15}$	0
1	$15 \times \frac{4^{14}}{5^{15}}$	$15 \times \frac{4^{14}}{5^{15}}$
2	$105 \times \frac{4^{13}}{5^{15}}$	$210 \times \frac{4^{13}}{5^{15}}$
3	$455 \times \frac{4^{12}}{5^{15}}$	$1365 \times \frac{4^{12}}{5^{15}}$
4	$1365 \times \frac{4^{11}}{5^{15}}$	$5460 \times \frac{4^{12}}{5^{15}}$
5	$3003 \times \frac{4^{10}}{5^{15}}$	$15015 \times \frac{4^{10}}{5^{15}}$
6	$5005 \times \frac{4^9}{5^{15}}$	$30030 \times \frac{4^9}{5^{15}}$
7	$6435 \times \frac{4^8}{5^{15}}$	$45045 \times \frac{4^8}{5^{15}}$
8	$6435 \times \frac{4^7}{5^{15}}$	$51480 \times \frac{4^7}{5^{15}}$
9	$5005 \times \frac{4^6}{5^{15}}$	$45045 \times \frac{4^6}{5^{15}}$
10	$3003 \times \frac{4^5}{5^{15}}$	$30030 \times \frac{4^5}{5^{15}}$
11	$1365 \times \frac{4^4}{5^{15}}$	$15015 \times \frac{4^4}{5^{15}}$
12	$455 \times \frac{4^3}{5^{15}}$	$5460 \times \frac{4^3}{5^{15}}$
13	$105 \times \frac{4^2}{5^{15}}$	$1365 \times \frac{4^2}{5^{15}}$
14	$15 \times \frac{4}{5^{15}}$	$210 \times \frac{4}{5^{15}}$
15	$\frac{1}{5^{15}}$	$15 \times \frac{1}{5^{15}}$

7.3 Geometric Distributions

12. A geometric distribution is one in which there are independent trials that have two possible outcomes, with the probability of these outcomes being constant in each trial. The number of trials before a success is of interest. For example, counting the number of rolls of a fair die before a 1 is rolled.

13. 5

14. a) 0.0213 **b)** 0.1356 **c)** 40.7 boards

15. a) $\left(\frac{24}{25}\right)^5\left(\frac{1}{25}\right)$ **b)** 24 trials before a 10 appears

16.

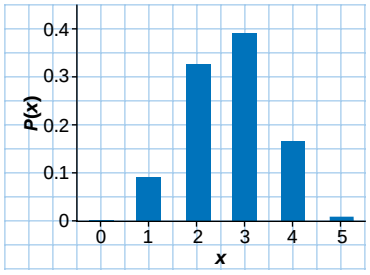
x	$P(x)$	$xP(x)$
1	$\frac{1}{36}$	$\frac{1}{36}$
2	$\frac{35}{36^2}$	$2 \times \frac{35}{36^2}$
3	$\frac{35^2}{36^3}$	$3 \times \frac{35^2}{36^3}$
4	$\frac{35^3}{36^4}$	$4 \times \frac{35^3}{36^4}$
5	$\frac{35^4}{36^5}$	$5 \times \frac{35^4}{36^5}$
6	$\frac{35^5}{36^6}$	$6 \times \frac{35^5}{36^6}$
7	$\frac{35^6}{36^7}$	$7 \times \frac{35^6}{36^7}$
8	$\frac{35^7}{36^8}$	$8 \times \frac{35^7}{36^8}$
9	$\frac{35^8}{36^9}$	$9 \times \frac{35^8}{36^9}$
10	$\frac{35^9}{36^{10}}$	$10 \times \frac{35^9}{36^{10}}$

7.4 Hypergeometric Distributions

17. A hypergeometric distribution is one in which there are a certain number of dependent trials, each having two possible outcomes. The number of successes is of interest. For example, counting the number of clubs dealt in a hand of 5 cards from a standard deck of 52 cards.

18. a)

x	$P(x)$
0	0.006 993 007
1	0.093 240 093
2	0.326 340 326
3	0.391 608 392
4	0.163 170 163
5	0.018 648 019



b) 2.7

19.

x	$P(x)$	$xP(x)$
0	$\frac{{}_{16}C_0 \times {}_{36}C_7}{{}_{52}C_7} = 0.062\ 40$	0
1	$\frac{{}_{16}C_1 \times {}_{36}C_6}{{}_{52}C_7} = 0.232\ 95$	0.239 25
2	$\frac{{}_{16}C_2 \times {}_{36}C_5}{{}_{52}C_7} = 0.338\ 15$	0.676 30
3	$\frac{{}_{16}C_3 \times {}_{36}C_4}{{}_{52}C_7} = 0.246\ 57$	0.739 70
4	$\frac{{}_{16}C_4 \times {}_{36}C_3}{{}_{52}C_7} = 0.097\ 13$	0.388 53
5	$\frac{{}_{16}C_5 \times {}_{36}C_2}{{}_{52}C_7} = 0.020\ 57$	0.102 85
6	$\frac{{}_{16}C_6 \times {}_{36}C_1}{{}_{52}C_7} = 0.002\ 15$	0.012 93
7	$\frac{{}_{16}C_7 \times {}_{36}C_0}{{}_{52}C_7} = 0.000\ 09$	0.000 60

20. 153

21. a) Simulations will vary. **b)** $P(x) = \frac{{}_{32}C_x \times {}_{50}C_{32-x}}{{}_{98}C_{32}}$

Chapter Test, pp. 408–409

1. a) hypergeometric **b)** geometric
c) hypergeometric **d)** binomial
e) uniform **f)** binomial
g) hypergeometric **h)** geometric

2. –$1.86

3. a) 0.2743 **b)** 0.2189 **c)** 0.96 **d)** 1.92

4. 34.56 cm^2

5. a)

Number of Prizes	Probability
0	0.430 467
1	0.382 638
2	0.148 803
3	0.033 067
4	0.004 593
5	0.000 408
6	2.27E -05
7	7.2E-07
8	1E-08

b) 50

6. a) 0.1314 **b)** 35 rolls

7. Answers may vary.

8. a) 0.469 **b)** 0.019 73 **c)** 2.5

9. a) $\frac{1}{6^4}$ **b)** 13.7

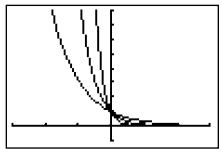
10. Answers may vary.

11. 43

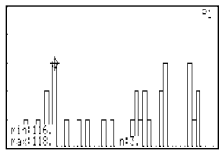
CHAPTER 8

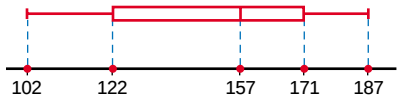
Note: *For some normal probability questions, answers calculated in a spreadsheet will differ slightly from those done on a graphing calculator.*

Review of Prerequisite Skills, pp. 412–413

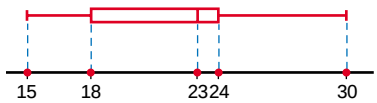
1. a)  **b)** 1

c) **i)** $y = 10^{-\frac{x}{2}}$ **ii)** $y = 10^{-2x}$

2. a)  **b)** 150.4, 27.2

c) 

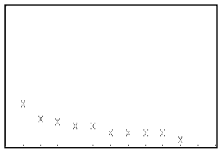
3. a) 21.9, 23, 24

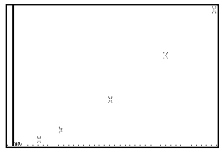
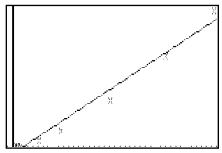
b) 

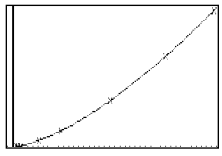
c) 4.2

4. mean: 24.9; median: 22; modes: 12, 14, 22; standard deviation: 14.8

5. mean: 77.3; median: 80; modes: 89, 90; standard deviation: 16.7

6. 

7. a)  **b)** 

c) 
The curve $y = \sqrt{x^3}$ is almost a perfect fit.

d) The data satisfies $T = \sqrt{R^3}$, where T is the time for one revolution and R is the mean distance from the sun.

8. a) –1.05 **b)** 2.49 **c)** 0.13
d) 25.59 to 51.01 **e)** 12.88 to 63.72 **f)** 21.78 to 54.82

9. Gavin's z-score is 2.28 while Patricia's is only 2.10.

10. a) 0.0683 **b)** 0.0922

c) 0.9761 **d)** 10.2

11. a) 0.047 **b)** 0.957 **c)** 2.75

Section 8.1, pp. 419–421

Practise

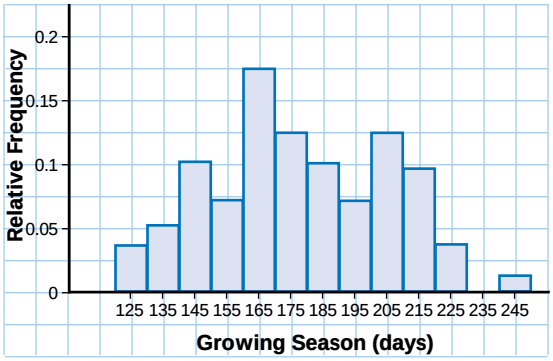
1. a) iii **b)** ii **c)** i

Apply, Solve, Communicate

2. a) about 76

b) Answers may vary.

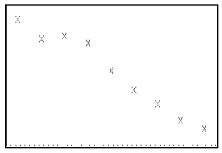
3. a)



c) Estimates may vary.

d) mean: 176.9; standard deviation: 27

4. a)



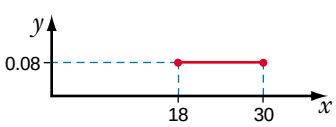
b) positively skewed

c) Lower proportion of older workers; lower accident rate among more experienced workers.

d) 420; average

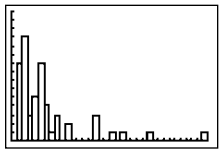
5. Answers will vary.

6. a)



b) $\frac{5}{6}$ **c)** $\frac{1}{2}$

7. a)



b) Yes; waiting times follow an exponential distribution.

c) 5.9 **d)** $y = 0.17e^{-0.17x}$

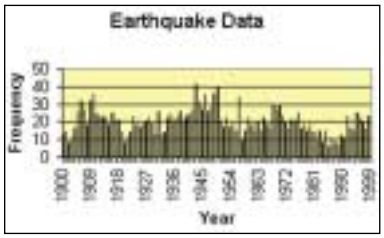
8. a) 1, ∞ **b)** 0.78, both estimates are low by about 10%.

c) The estimate from the simulation is low due to random chance. The slope of the exponential curve becomes less steep as x increases, so the midpoint of the intervals underestimates the area under the curve.

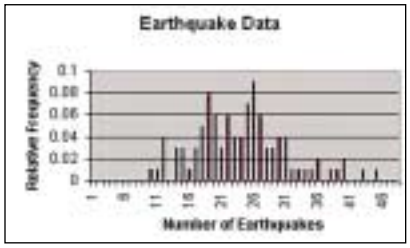
d) Increase the length of time being simulated to a week or more. Estimate the area under the exponential curve using narrower interval widths, such as tenths or hundredths of a minute.

9. a) 0.58 **b)** 0.24

10. a)

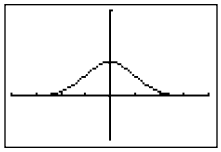


b)



c) 20.05, 7.23

11. a)



b) bell curve

c) so that the total area under the curve is 1

Section 8.2, pp. 430–431

Practise

1. 0.16, 0.16, 0.32, 0.18

Apply, Solve, Communicate

2. 0.8944

3. 13.33%

4. a) 88th percentile **b)** 6th percentile

c) 99th percentile

5. Ty Cobb: 4.15, Ted Williams: 4.26, George Brett: 4.07
Using z-scores, Williams is top ranked, followed by Cobb and then Brett.

6. 0.766%

7. 5.02%

8. a) 2.21% **b)** 45.77% **c)** 11.6%

9. a) 62.76% **b)** 14.60%

10. a) 0.0316 **b)** 145.5 cm to 168.5 cm

11. a) $y = \frac{10(x - 55)}{13} + 70$, where x is the old mark and y is the new mark.

b) 89 **c)** 58

12. between 20 cm and 43 cm

13. 4.35 min

Section 8.3, pp. 438–441

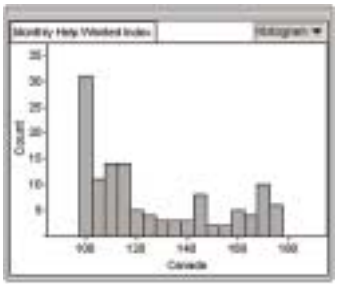
Apply, Solve, Communicate

1. a) 110.3, 10.7 **b)** 0.8315

2. a) $32 461.20, $3375.77 **b)** 0.26

3. a) $67.37, $17.00 **b)** 0.6677 **c)** 0.1534

d) With repeated sampling, the mean of the sample means will more closely approximate the true population mean and may yield more accurate results.

4. b)  ; no

c) Canada: 125.5, 25.9; Ontario: 128.3, 34.4

d) yes

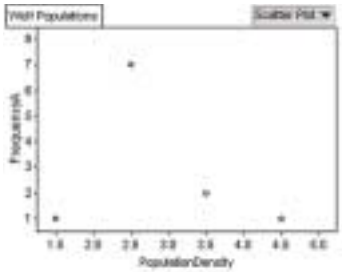
5. Answers depend on current data.

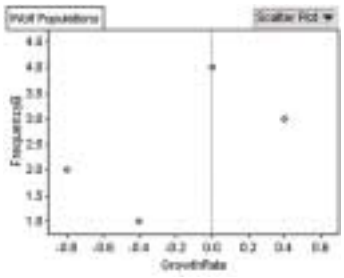
6. Answers depend on current data.

7. Answers depend on current data.

8. a) 43.9, 10.866 **b)** 0.408

9. 367

10. a)  ; no

b)  ; no

c) Yes, the data could be part of a larger set that is normally distributed.

11. 0.0742

13. a) Compound the gain from year to year.

b) 5-year mean 78.22%, standard deviation 26.52%. Ratio of standard deviation to return is markedly less for the 5-year returns: fluctuations in the annual returns tend to cancel out over longer periods.

c) 1-year mean 20.05, standard deviation 7.23; 5-year mean 101.08, standard deviation 27.29. Again, ratio of s to $\bar{x}$ is lower for the longer period.

Technology Extension, pp. 442–444

Normal Probability Plots

1. 20 min, 5.71 min

3. a) 0.5 **b)** 77%

Section 8.4, pp. 449–450

Practise

1. a) reasonable **b)** not reasonable since $np < 5$

c) reasonable **d)** reasonable

2.

Sample Size, n	Probablility of Success, p	μ	σ	Probablility
60	0.4	24	3.79	$P(X < 22) = 0.2550$
200	0.7	140	6.48	$P(X < 160) = 0.9999$
75	0.6	45	4.24	$P(X > 50 = 0.0974$
250	0.2	50	6.32	$P(X > 48) = 0.5937$
1000	0.8	800	12.65	$P(780 < X < 840) = 0.9375$
90	0.65	58.5	4.52	$P(52 < X < 62) = 0.6539$
100	0.36	36	4.80	$P(X = 40) = 0.0587$
3000	0.52	1560	27.36	$P(X = 1650) = 0.000\,0653$

Apply, Solve, Communicate

3. 0.150

4. 0.129

5. 0.69

6. Normal approx: 0.056; Binomial Dist.: 0.0541

7. Normal approx: 0.080; Binomial Dist.: 0.0775

8. a) i) 0.9273 **ii)** 0.8948 **b)** 6.7

9. a) i) 0.9507 **ii)** 0.0432

b) i) 0.951 **ii)** 0.0433

10. a) 0.0312 **b)** 0.0507

11. a) i) 0.209 **ii)** 0.209

b) i) 0.213 **ii)** 0.207

c) very closely

13. a) 32 **b)** 26.4, 37.6

Section 8.5, pp. 457–458

Practise

1. 20, 0.86; 12, 0.8; 5, 0.33; 40, 0.8; 8.4, 0.39; 17.6, 1.11; 73.9, 1.35

2. a) accept H_0 **b)** accept H_0 **c)** accept H_0

d) accept H_0 **e)** reject H_0

Apply, Solve, Communicate

3. $P(z > 11.79) < 10\%$; the machine needs to be adjusted.

4. No, $P(z < -1.23) > 10\%$; that only 31 agreed in the survey could have been due to chance variation.

5. $P(z \geq 0.35) > 0.01$; recommend acceptance—there is not enough evidence to reject the claim that the drug causes serious side effects in less than 0.01% of the population.
6. $\alpha = 5\%$: $P(z \leq -1.09) > 5\%$; there is not enough evidence to reject the claim that the product holds 28% of the market.
7. $P(z \leq -3.30) < 0.05\%$; the evidence refutes the company's claim even for $\alpha = 1\%$.
8. The standard deviation in loan amounts. A significance level of 1% would be appropriate due to the small sample size.
9. 50%
10. The advertising campaign was probably not a success.
11. Larger samples are likely to give more accurate statistics but will be more time-consuming and costly. Repeated sampling verifies the measurement process but will not detect bias.
13. Whether to accept your hypothesis for the value of μ.

Section 8.6, pp. 464–467

Practise

1. **a)** $14.2 < \mu < 15.8$ **b)** $27.7 < \mu < 32.3$
   c) $5.6 < \mu < 7.2$ **d)** $29.3 < \mu < 31.1$
   e) $40.4 < \mu < 43.2$ **f)** $3.9 < \mu < 4.6$
2. **a)** There is a 95% probability that between 39% and 45% of high school graduates expect to earn over $100 000 per year by the time they retire.
   b) There is a 90% probability that between 43% and 53% of decided voters will vote for the incumbent in the next election.
   c) There is a 73% probability that between 24% and 32% of teenagers will purchase the latest CD by the rock band.

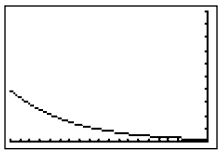
Apply, Solve, Communicate

3. $24.48 < \mu < 25.52$
4. $15.8 < \mu < 19.4$
5. $0.192 < p < 0.368$
6. $0.119 < p < 0.121$
7. $61.0 < \mu < 71.4$
8. $13.7\% < \mu < 18.3\%$
9. $18.3\% \pm 4.4\%$
10. 68
11. 44
12. 936
13. Answers may vary.
14. Answers may vary.
15. **a)** –0.111, 0.355 **b)** 0.62
    c) yes **d)** $-0.296 < \mu < 0.074$
16. 106
17. Answers may vary.
20. 1690

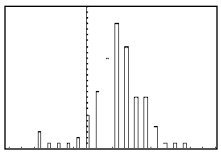
Review of Key Concepts, pp. 468–471

8.1 Continuous Probability Distributions

1. **a)** 0.6 **b)** 0.28
2. **a)** the exponential distribution



   b) 0.55
3. **a)**



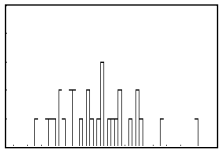
   b) approximately symmetric and bell-shaped
   c) mean: 8.8, standard deviation: 8.07

8.2 Properties of the Normal Distribution

4. **a)** 0.0856 **b)** 0.6486 **c)** 0.5695
5. **a)** 0.0228 **b)** 65.68% **c)** 35.3
6. **a)** 0.2119 **b)** 69.15% **c)** about 1158

8.3 Normal Sampling and Modelling

7. **a)** A histogram shows that these data could be normally distributed.



   b) 35.4, 10.5 **c)** using a normal distribution: 0.695
8. **a)** 2.304, 0.069 **b)** 21.67%
9. **a)** 75.7, 4.35
   b) The standard deviation of the class average scores equals the standard deviation of the individual scores divided by the square root of the class size.
   c) 31
10. 0.503

8.4 Normal Approximation to the Binomial Distribution

11. **a)** 0.176
    b) Yes; 110 is almost five standard deviations above expectation.
12. 0.8291

8.5 Repeated Sampling and Hypothesis Testing

13. $P(z \geq 5.3) < 0.10$; it takes longer than 0.9 h on average to paint a car.
14. The coach is correct at this significance level.
15. The evidence does not support the claim that the advertising campaign was a success at a 20% significance level.

8.6 Confidence Intervals

16. $5.2 < \mu < 5.6$
17. $0.304 < p < 0.416$
18. **a)** $1515 < \mu < 3185$
    b) The confidence interval includes less than half of the data because they are not normally distributed.
19. $45.5 < \mu < 394.8$ (in thousands)

20. 358
21. 210
22. a) $0.382 < p < 0.658$ **b)** Answers may vary.

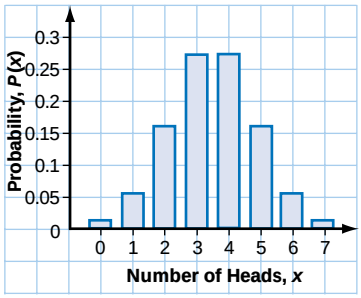
Chapter Test, pp. 472–473

1. Answers may vary.
a) test mark distribution for a large class
b) driving time between Toronto and Hamilton
c) waiting times for service at a bank machine
d) the distribution of shoe sizes of adult males and females
2. a) 0.588 **b)** 9.12% **c)** about 24
3. Currently about 18% of the cans contain the exceptional amounts. A lower standard deviation would be better.
4. a) 9.88, 3.48 **b)** 0.486
5. a) 0 **b)** 41%
c) 9% have less than 20% cashews.
d) "At least 19% cashews in every can" or "average 22% cashews."
6. 0.6808
7. a) 0.178 assuming defect rate is 0.03
b) If the defect rate is 3%, there is a 17.8% chance that 65 or more defective cars will be produced during a shift. If more samples yield similar numbers of defective cars, changes may be required. Otherwise, the high number of defects may have been just a chance variation.
8. $0.352 < p < 0.527$
9. a) 384
b) Yes, the poll indicates that the politician has a 78% chance of losing the election if he faces a single opponent.
c) The politician should try to convince some undecided voters to vote for him.

CUMULATIVE REVIEW: CHAPTERS 7 AND 8, p. 476

1. –68.8¢
2. a) 37% **b)** 15.9%
3.

Number of Heads, x	Probability, $P(x)$
0	0.007 8125
1	0.054 6875
2	0.164 0625
3	0.273 4375
4	0.273 4375
5	0.164 0625
6	0.054 6875
7	0.007 8125



4. a) 0.0158 **b)** 0.9810 **c)** 8.25
5. 0.38%
6. a) Answers will vary. **b)** 0.2616 **c)** 5.45
d) Statistical fluctuations could cause noticeable differences unless the number of simulated trials is quite large.
7. 504
8. a) $1.58 < \mu < 1.76$
b) Wolves are normally distributed.

COURSE REVIEW, pp. 477–479

1. a) 2:3 **b)** 3:2

2. Answers may vary.

3. a) E, F, G, and H

b) **i)** no **ii)** no **iii)** yes

4. 15

5.

Day of Month	Probability
1	$\frac{2}{61}$
2	$\frac{2}{61}$
3	$\frac{2}{61}$
4	$\frac{2}{61}$
5	$\frac{2}{61}$
6	$\frac{2}{61}$
7	$\frac{2}{61}$
8	$\frac{2}{61}$
9	$\frac{2}{61}$
10	$\frac{2}{61}$
11	$\frac{2}{61}$
12	$\frac{2}{61}$
13	$\frac{2}{61}$
14	$\frac{2}{61}$
15	$\frac{2}{61}$
16	$\frac{2}{61}$
17	$\frac{2}{61}$
18	$\frac{2}{61}$
19	$\frac{2}{61}$
20	$\frac{2}{61}$
21	$\frac{2}{61}$
22	$\frac{2}{61}$
23	$\frac{2}{61}$
24	$\frac{2}{61}$
25	$\frac{2}{61}$
26	$\frac{2}{61}$
27	$\frac{2}{61}$
28	$\frac{2}{61}$
29	$\frac{2}{61}$
30	$\frac{11}{366}$
31	$\frac{7}{366}$

6. a) Independent trials in which there are only two possible outcomes.

b) Unlike a hypergeometric distribution, trials in a geometric distribution are independent.

7. a) 3 **b)** 6

c) No, with $2 coins and quarters, there are only three choices.

d) 12

8. Answers may vary.

a) sampling students from each of the grades 9, 10, 11, and 12 in numbers according to the proportion each grade comprises of the total; ensures representation from all grades in the proper proportions

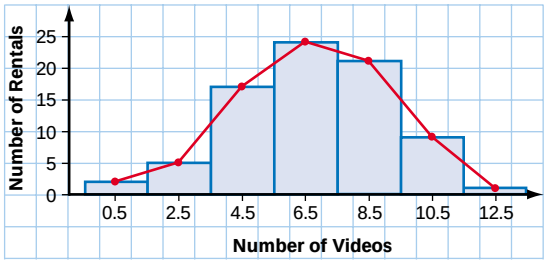
b) sampling persons from randomly-chosen households in Ontario for their provincial political preference; each person has an equal opportunity to be a part of the sample

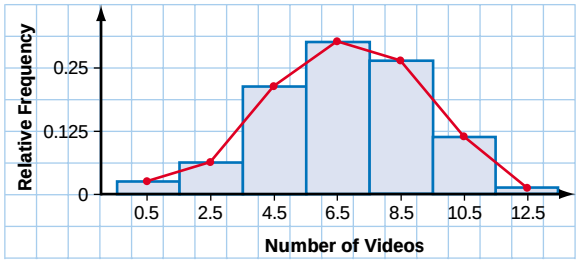
c) to sample 6 students from a class of 30, choose a random number from 1 to 5 and, from a class list, choose this student and every 6th student thereafter; easy to obtain the required percentage of respondents

9. a) 10 879 286 400 **b)** about 20 719.3 years

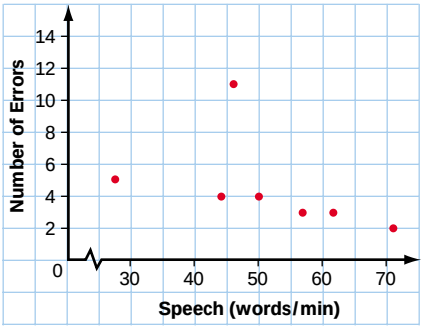
10. a) 6.7, 6.5

b)





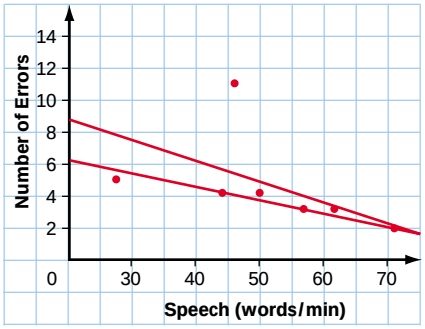
11. a)



weak negative correlation

b) -0.46; $y = -0.10x + 9.66$

c) (46, 11); -0.98; $y = -0.068x + 7.04$;



d) the faster the speed, the fewer the errors

12. No; both are dairy products and so have a common-cause relationship.

13. a) 600 **b)** random sample of all households in Ontario

14. a) 64 **b)** 729

15. 1 632 960

16. 256 851 595

17. a) 120 **b)** 14 **c)** 8 **d)** 51 **e)** 50

18. 17 153 136

19. a) $\frac{x^5}{243} + \frac{5x^4y}{27} + \frac{10x^3y^2}{3} + 30x^2y^3 + 135xy^4 + 243y^5$

b) $\binom{n+1}{2}$

20. $\frac{1}{5}$

21. a) 0.438 866 **b)** $3.\overline{3}$

22. $\frac{19}{24}$

23. a) yes **b)** no

24. a) 0.080 **b)** 0.594 **c)** 5.06

25. a) UltraChicken: 0.2807; Churrasqueira Champion: 0.5439; Mac's Chicken: 0.1754

b) that transition matrix does not change

26. a) 0.014 **b)** 0.88 **c)** 46.6

27. Leading questions offer information that may then be used by the respondent, when it otherwise would not have been. Loaded questions are intended to directly influence a response by supplying information in a certain way to obtain the desired result.

28. a) 0.069 **b)** 13.4

29. Answers will vary.

APPENDIX A

Evaluating expressions, p. 496

1. a) -31 **b)** 2 **c)** 271 **d)** -422 **e)** 48

f) -227 **g)** -42 **h)** -699 **i)** -36 **j)** $-\frac{14}{81}$

k) $\frac{1}{20}$ **l)** 4900

Exponent laws, p. 496

1. a) $1024x^{10}$ **b)** $9x^8y^{10}$ **c)** $1250x^9y^7$ **d)** $\frac{3}{400}$

e) a^7b **f)** $-9x^4y^4$ **g)** $\frac{1536y^3}{x^2}$ **h)** $-50x^{15}y^7$

Fractions, percents, decimals, p. 497

1. a) 75%, 0.75 **b)** 50%, 0.5 **c)** 840%, 8.4

d) $\frac{17}{50}$, 0.34 **e)** $\frac{3}{10\,000}$, 0.0003 **f)** $\frac{7}{125}$, 0.056

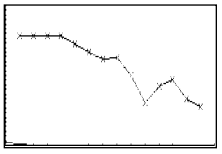
g) $\frac{9}{20}$, 45% **h)** $\frac{3}{100}$, 3% **i)** $2\frac{17}{25}$, 268%

2. 14.3%

3. \$23

Graphing data, p. 498

1. a)

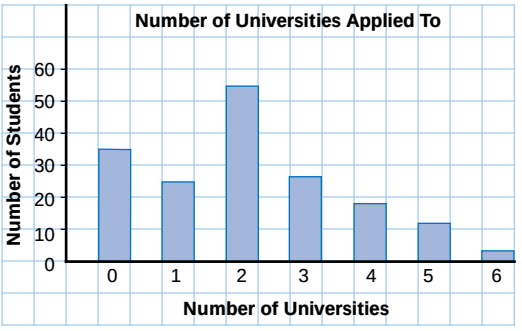


b) Initially sales are high, but then they begin to fall off as the number of appearances increases.

c) Graphs that do not exhibit changes over time would not be appropriate, for example, a circle graph.

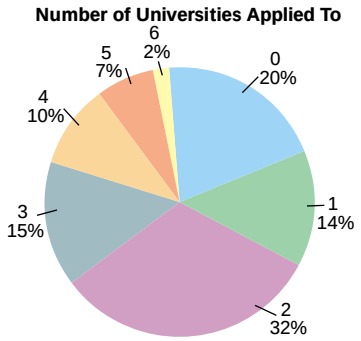
2. a) 176

b)



c) 0: 20%, 1: 14%, 2: 32%, 3: 15%, 4: 10%, 5: 7%, 6: 2%

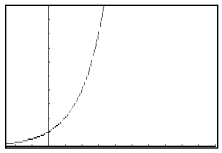
d)



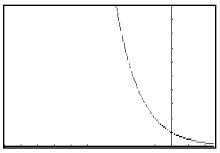
e) A line graph would not be appropriate.

Graphing exponential functions, p. 499

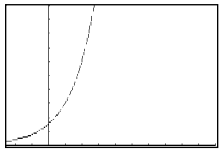
1. a)



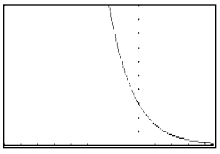
b)



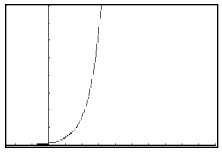
c)



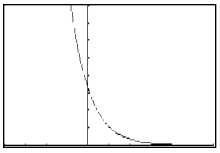
d)



e)

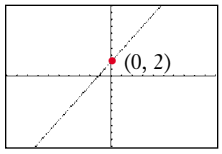


f)

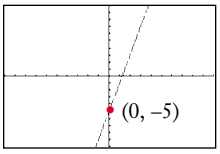


Graphing linear equations, p. 500

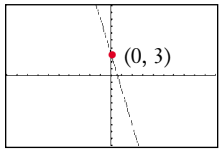
1. a)



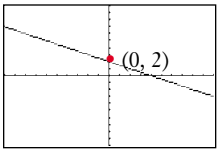
b)



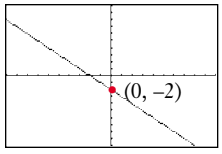
c)



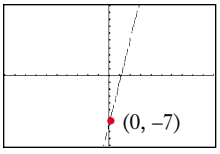
d)



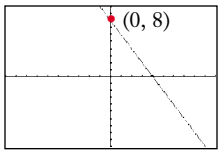
e)



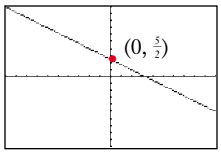
f)



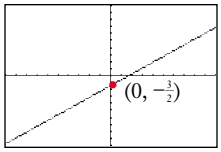
g)



h)

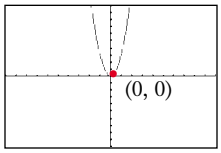


i)



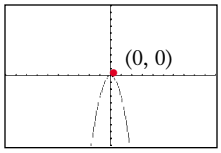
Graphing quadratic equations, p. 500

1. a)



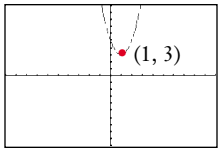
y-intercept: 0, *x*-intercept: 0

b)



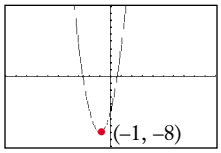
y-intercept: 0, *x*-intercept: 0

c)



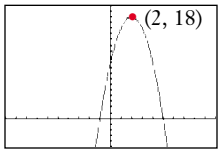
y-intercept: 5, *x*-intercepts: none

d)



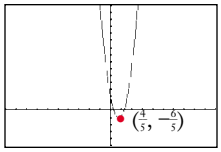
$y = 3(x + 1)^2 - 8$
y-intercept: –5,
x-intercepts: 0.6, –2.6

e)



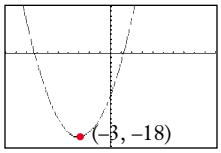
$y = -2(x - 2)^2 + 18$
y-intercept: 10, *x*-intercepts: –1, 5

f)

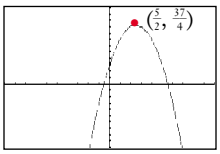


$y = 5(x - \frac{4}{5})^2 - \frac{6}{5}$
y-intercept: 2,
x-intercepts: 0.3, 1.3

g)

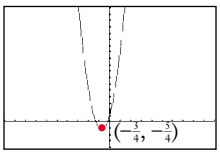


$y = (x + 3)^2 - 18$
y-intercept: –9,
x-intercepts: –7.2, 1.2

h) 

$y = -(x - \frac{5}{2})^2 + \frac{37}{4}$

y-intercept: 3,
x-intercepts: –0.5, 5.5

i) 

$y = 4(x + \frac{3}{4})^2 - \frac{5}{4}$

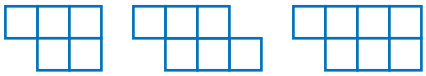
y-intercept: 1,
x-intercepts: –1.3, –0.2

Mean, median, mode, p. 501

1. a) 16.7, 17, 18 **b)** 82.1, 79, 79

Number patterns, p. 502

1. a) subtract 3; 6, 3, 0
b) multiply by 3: 108, 324, 972
c) perfect squares; 16, 25, 36
d) add 2; 8, 10, 12
e) divide by –3: $-\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$
f) divide by –2: –10, 5, $-\frac{5}{2}$
g) multiply by –1: 1, –1, 1
h) subtract 10: –15, –25, –35
i) powers of 2 or multiply by 2: 32, 64, 128
j) multiply by $\frac{2}{3}; \frac{2}{9}, \frac{4}{27}, \frac{8}{81}$
k) multiply p by q^x: $pqqqq$, $pqqqqq$, $pqqqqqq$
l) append twice the previous number of b's: $abbbbbbbb$, $abbbbbbbbbbbbbbb$, $abbbbbbbbbbbbbbbbbbbbbbbbbbbbbb$
m) add alternately to lower right and upper right;



n) add next larger square base

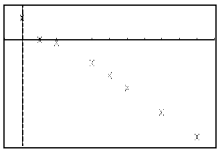
Order of operations, p. 503

1. a) 33 **b)** 195 **c)** 108 **d)** 64.4
e) 0.5 **f)** $1\frac{1}{2}$ **g)** $\frac{419}{270}$ **h)** $\frac{9}{8}$

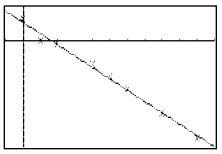
Ratio of areas, p. 503

1. a) 1:4 **b)** 1:9 **c)** 4:1 **d)** 125:1

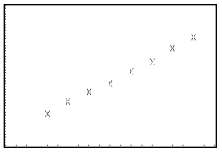
Scatter plots, p. 504

1. a) 

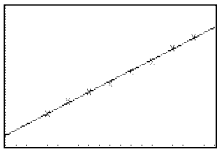
b) As altitude increases, temperature decreases.

c) 

d) –6.5; the rate of change of temperature with respect to altitude
e) $t = -6.5a + 11.25$ **f)** –66.8°

2. a) 

b) As floor space increases, heating cost increases.

c) 

d) 9.68; the rate of change of heating cost with respect to floor space
e) $c = 9.68s - 295.5$ **f)** $3092.50

Sigma notation, p. 505

1. a) $\sum_{i=1}^{6} 2^i$ **b)** $\sum_{i=3}^{15} i$ **c)** $\sum_{i=1}^{7} \frac{i}{i+1}$ **d)** $\sum_{i=5}^{11} ix$

2. a) 3 + 6 + 9 + 12 + 15 + 18 + 21 + 24 + 27 + 30
b) 5.2 + 5.4 + 5.8 + 5.16 + 5.32 + 5.64 + 5.128 + 5.256
c) 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29
d) $\frac{4}{9} + \frac{5}{16} + \frac{6}{25} + \frac{7}{36}$

Similar triangles, p. 505

1. $\triangle QPR \sim \triangle FGH$ because
QP:FG = QR:FH = PR:GH = 2:1

Simplifying expressions, p. 506

1. a) $x^2 + 4xy + 4y^2$ **b)** $4x^2 - 20x + 25$
c) $x^4 + 2x^2y + y^2$ **d)** $25x^2 - 20xy + 4y^2$
e) $12x^3 + 28x^2 + 19x + 4$ **f)** $75y^3 + 40y^2 - 68y + 16$
g) $k^3 - 2k^2 - 15k + 36$ **h)** $125m^3 - 15mn^2 + 2n^3$
i) $-64x^3 - 128x^2y - 20xy^2 + 50y^3$

Solving equations, p. 506

1. a) –4 **b)** –6 **c)** –3 **d)** 18 **e)** $\frac{15}{2}$ **f)** 1.575
g) –11 **h)** –4 **i)** ±6 **j)** ±12 **k)** 2 **l)** 6

Substituting into equations, p. 507

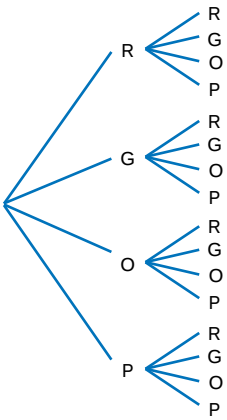
1. a) –3 **b)** 27 **c)** –19
d) $9x^4 - 36x^2 + 38$ **e)** 11 **f)** $3x^2 + 12x + 6$
g) 966 **h)** $6x^2 - 11$ **i)** $27x^4 - 54x^2 + 102$
j) 69 **k)** $x^4 + 4x^2 + 6$ **l)** 10 406

m) 7 **n)** 7797

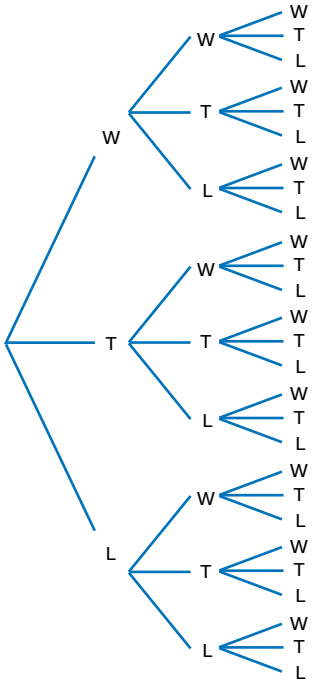
2. no

Tree diagrams, p. 507

1.



2.



3.