

Name: _____

$A\infty\Omega$
MATH@TD

FACTORIZING WORKBOOK

Unit Outline:

- 1) Polynomial Expansion
- 2) Common Factoring and Factor Pairs
- 3) Decomposition: $ax^2 + bx + c$
- 4) Special Case A: $x^2 + bx + c$
- 5) Special Cases B: Differences of Squares and C: Perfect Squares

1. Expanding Binomials

Find each product.

$$\begin{aligned} 1) (5k+1)(4k-7) \\ = 20k^2 - 35k + 4k - 7 \\ = 20k^2 - 29k - 7 \end{aligned}$$

$$\begin{aligned} 3) 2(3n+7)(3n+4) \\ = 2(9n^2 + 4n + 21n + 28) \\ = 2(9n^2 + 25n + 28) \\ = 18n^2 + 50n + 56 \end{aligned}$$

$$\begin{aligned} 5) (4x-7)(6x+4) \\ = 24x^2 + 16x - 42x - 28 \\ = 24x^2 - 26x - 28 \end{aligned}$$

$$\begin{aligned} 7) (6n-8)(6n+8) \\ = 36n^2 + 48n - 48n - 64 \\ = 36n^2 - 64 \end{aligned}$$

$$\begin{aligned} 9) 5(2m+3)^2 \\ = 5(2m+3)(2m+3) \\ = 5(4m^2 + 6m + 6m + 9) \\ = 5(4m^2 + 12m + 9) \\ = 20m^2 + 60m + 45 \end{aligned}$$

$$\begin{aligned} 2) (5n-3)(6n+4) \\ = 30n^2 + 20n - 18n - 12 \\ = 30n^2 + 2n - 12 \end{aligned}$$

$$\begin{aligned} 4) (3x+6)(4x-7) \\ = 12x^2 - 21x + 24x - 42 \\ = 12x^2 + 3x - 42 \end{aligned}$$

$$\begin{aligned} 6) (3r+6)(r+6) \\ = 3r^2 + 18r + 6r + 36 \\ = 3r^2 + 22r + 36 \end{aligned}$$

$$\begin{aligned} 8) (5k-3)^2 \\ = (5k-3)(5k-3) \\ = 25k^2 - 15k - 15k + 9 \\ = 25k^2 - 30k + 9 \end{aligned}$$

$$\begin{aligned} 10) (2n+3)(2n-3) \\ = 4n^2 - 6n + 6n - 9 \\ = 4n^2 - 9 \end{aligned}$$

$$11) (8x - 3)(12x + 6)$$

$$= 96x^2 + 48x - 36x - 18$$

$$= 96x^2 + 12x - 18$$

$$12) (8n - 3)(10n + 8)$$

$$= 80n^2 + 64n - 30n - 24$$

$$= 80n^2 + 34n - 24$$

$$13) (4n + 6)(10n + 1)$$

$$= 40n^2 + 4n + 60n + 6$$

$$= 40n^2 + 64n + 6$$

$$14) -3(3p - 3)(8p - 1)$$

$$= -3(24p^2 - 3p - 24p + 3)$$

$$= -3(24p^2 - 27p + 3)$$

$$= -72p^2 + 81p - 9$$

$$15) (n - 12)^2$$

$$= (n - 12)(n - 12)$$

$$= n^2 - 12n - 12n + 144$$

$$= n^2 - 24n + 144$$

$$16) (5n - 1)(5n + 12)$$

$$= 25n^2 + 60n - 5n - 12$$

$$= 25n^2 + 55n - 12$$

$$17) (4x + 6)(8x^2 + 5x + 2)$$

$$= 32x^3 + 20x^2 + 8x + 48x^2 + 30x + 12$$

$$= 32x^3 + 68x^2 + 38x + 12$$

$$18) (4m - 4)(2m^2 + 2m - 2)$$

$$= 8m^3 + 8m^2 - 8m - 8m^2 + 8m + 8$$

$$= 8m^3 - 8$$

$$19) (x - 1)(x + 4)(x - 8)$$

$$= (x - 1)(x^2 - 8x + 4x - 32)$$

$$= (x - 1)(x^2 - 4x - 32)$$

$$= x^3 - 4x^2 - 32x - 1x^2 + 4x + 32$$

$$= x^3 - 5x^2 - 28x + 32$$

$$20) (2x + 1)(3x - 5)(x + 6)$$

$$= (2x + 1)(3x^2 + 18x - 5x - 30)$$

$$= (2x + 1)(3x^2 + 13x - 30)$$

$$= 6x^3 + 26x^2 - 60x + 3x^2 + 13x - 30$$

$$= 6x^3 + 29x^2 - 47x - 30$$

2. Common Factoring and Factor Pairs

Factor the common factor out of each expression.

1) $10n - 4$

$$= 2(5n - 2)$$

2) $3x^2 + 12$

$$= 3(x^2 + 4)$$

3) $8k^3 + 2k^5$

$$= 2k^3(4 + k^2)$$

4) $5x^9 - 10x^3$

$$= 5x^3(x^6 - 2)$$

5) $15x + 25$

$$= 5(3x + 5)$$

6) $-35b^5 + 20b^4 - 50b^2$

$$= -5b^2(7b^3 - 4b^2 + 10)$$

7) $x^3 + 10x^2 - 5x$

$$= x(x^2 + 10x - 5)$$

8) $6x^6 - 2x^3 + 12x^2$

$$= 2x^2(3x^4 - x + 6)$$

9) $-30m^5 + 54m^3 + 54m^2$

$$= -6m^2(5m^3 - 9m - 9)$$

10) $18v^4 + 27v^3 - 54v^2$

$$= 9v^2(2v^2 + 3v - 6)$$

11) $5p^7 + 4p^4 + 6p^3$

$$= p^3(5p^4 + 4p + 6)$$

12) $14k^6 - 7k^5 + 7k$

$$= 7k(2k^5 - k^4 + 1)$$

13) $3x(x + 4) - 5(x + 4)$

$$= (x + 4)(3x - 5)$$

14) $4x^2(5x - 6y) + 3y^2(5x - 6y)$

$$= (5x - 6y)(4x^2 + 3y^2)$$

Find the factor pair satisfies the given conditions: (NOTE: M=multiplied to, A=adds to). One of these questions does not have a pair that works. You must show the entire list to verify that that is the case.

15) M: -40
A: -6

+1, -40
+2, -20
+4, -10

16) M: 68
A: 21

1, 68
2, 34
4, 17

17) M: 56
A: -15

-1, -56
-2, -28
-4, -14
-7, -8

18) M: 6
A: 7

1, 6

19) M: 72
A: -22

-1, -72
-2, -36
-3, -24
-4, -18
-6, -12
-8, -9

20) M: -6
A: 5

-1, +6

21) M: -100
A: 0

-1, +100
-2, +50
-4, +25
-5, +20
-10, +10

22) M: 169
A: 26

1, 169
13, 13

23) M: -12
A: 1

-1, +12
-2, +6
-3, +4

24) M: -756
A: -101

+1, -756
+2, -378
+3, -252
+4, -189
+6, -126
+7, -108

3. Factoring ax^2+bx+c by Decomposition

Factor each completely.

$$\begin{aligned}
 1) \quad & 3v^2 + 20v + 25 \\
 &= \underbrace{3v^2 + 5v}_{v} + \underbrace{15v + 25}_{+5} \\
 &= v(3v+5) + 5(3v+5) \\
 &= (3v+5)(v+5)
 \end{aligned}$$

M: 75 A: 20

1, 75
3, 25
5, 15

$$\begin{aligned}
 2) \quad & 2p^2 - p - 21 \\
 &= \underbrace{2p^2 + 6p}_{2p} - \underbrace{7p - 21}_{-7} \\
 &= 2p(p+3) - 7(p+3) \\
 &= (p+3)(2p-7)
 \end{aligned}$$

M: -42 A: -1

1, -42
2, -21
3, -14
6, -7

$$\begin{aligned}
 3) \quad & 7n^2 + 31n + 12 \\
 &= \underbrace{7n^2 + 3n}_n + \underbrace{28n + 12}_{+4} \\
 &= (n+4)(7n+3)
 \end{aligned}$$

M: 84 A: 3

1, 84
2, 42
3, 28

$$\begin{aligned}
 4) \quad & 4b^2 + 13b + 3 \\
 &= \underbrace{4b^2 + 1b}_b + \underbrace{12b + 3}_{+3} \\
 &= (b+3)(4b+1)
 \end{aligned}$$

M: 12 A: 13

1, 12

$$\begin{aligned}
 5) \quad & 6p^2 + 25p + 25 \\
 &= \underbrace{6p^2 + 10p}_{2p} + \underbrace{15p + 25}_{+5} \\
 &= (2p+5)(3p+5)
 \end{aligned}$$

M: 150 A: 25

1, 150
2, 75
3, 50
5, 30
6, 25
10, 15

$$\begin{aligned}
 6) \quad & 4n^2 - 31n + 21 \\
 &= \underbrace{4n^2 - 3n}_n - \underbrace{28n + 21}_{-7} \\
 &= (n-7)(4n-3)
 \end{aligned}$$

M: 84 A: -3

-1, -84
-2, -42
-3, -28

$$7) 3n^2 + 13n - 88$$

$$= \underbrace{3n^2 - 11n}_{+8} + \underbrace{24n - 88}_{+8}$$

$$= (n+8)(3n-11)$$

$$M: -264 \quad A: +13$$

$$\begin{array}{l} -1, +264 \\ -2, +132 \\ -3, +88 \\ -4, +66 \\ -6, +44 \\ -8, +33 \\ -11, +24 \end{array}$$

$$8) 11x^2 + 25x + 6$$

$$= \underbrace{11x^2 + 3x}_x + \underbrace{22x + 6}_{+3}$$

$$= (x+2)(11x+3)$$

$$M: 66 \quad A: 25$$

$$\begin{array}{l} 1, 66 \\ 2, 33 \\ 3, 22 \end{array}$$

$$9) 55k^2 + 415k - 200$$

$$= 5(11k^2 + 83k - 40)$$

$$= 5(\underbrace{11k^2 - 5k}_k + \underbrace{88k - 40}_{+8})$$

$$= 5(k+8)(11k-5)$$

$$M: -440 \quad A: +83$$

$$\begin{array}{l} -1, 440 \\ -2, 220 \\ -4, 110 \\ -5, 88 \end{array}$$

$$10) 25n^2 - 110n - 240$$

$$= 5(5n^2 - 22n - 48)$$

$$= 5(\underbrace{5n^2 + 8n}_n - \underbrace{30n - 48}_{-6})$$

$$= 5(n-6)(5n+8)$$

$$M: -240 \quad A: -22$$

$$\begin{array}{l} 1, -240 \\ 2, -120 \\ 3, -80 \\ 4, -60 \\ 5, -48 \\ 6, -40 \\ 8, -30 \end{array}$$

$$11) 5v^2 - 28v - 12$$

$$= \underbrace{5v^2 + 2v}_v - \underbrace{30v - 12}_{-6}$$

$$= (v-6)(5v+2)$$

$$M: -60 \quad A: -28$$

$$\begin{array}{l} 1, -60 \\ 2, -30 \end{array}$$

$$12) 33p^2 + 171p - 162$$

$$= 3(11p^2 + 57p - 54)$$

$$= 3(\underbrace{11p^2 - 9p}_p + \underbrace{66p - 54}_{+6})$$

$$= 3(p+6)(11p-9)$$

$$M: -594 \quad A: 57$$

$$\begin{array}{l} -1, +594 \\ -2, +297 \\ -3, +198 \\ -6, +99 \\ -9, +66 \end{array}$$

$$13) 5n^2 + 53n + 72$$

$$= \underbrace{5n^2 + 8n}_n + \underbrace{45n + 72}_{+9}$$

$$= (n+9)(5n+8)$$

$$M: 360 \quad A: 53$$

$$\begin{array}{l} 1, 360 \\ 2, 180 \\ 3, 120 \\ 4, 90 \\ 5, 72 \\ 6, 60 \\ 8, 45 \end{array}$$

$$14) 20v^2 - 22v - 12$$

$$= 2(10v^2 - 11v - 6)$$

$$= 2(\underbrace{10v^2 + 4v}_{2v} - \underbrace{15v - 6}_{-3})$$

$$= 2(2v-3)(5v+2)$$

$$M: -60 \quad A: -11$$

$$\begin{array}{l} 1, -60 \\ 2, -30 \\ 3, -20 \\ 4, -15 \end{array}$$

$$15) 42p^2 - 87p + 33$$

$$= 3(14p^2 - 29p + 11)$$

$$M: 154$$

$$A: -29$$

$$-1, -154$$

$$-2, -77$$

$$-7, -22$$

$$= 3(\underbrace{14p^2 - 7p}_{7p} - \underbrace{22p + 11}_{-11})$$

$$= 3(7p - 11)(2p - 1)$$

$$16) 10p^2 - 57p + 54$$

$$= \underbrace{10p^2 - 12p}_{2p} - \underbrace{45p + 54}_{-9}$$

$$= (2p - 9)(5p - 6)$$

$$M: 540$$

$$A: -57$$

$$-1, -540$$

$$-2, -270$$

$$-3, -180$$

$$-4, -135$$

$$-5, -108$$

$$-6, -90$$

$$-9, -60$$

$$-10, -54$$

$$\boxed{-12, -45}$$

$$17) 16x^2 - 160x + 396$$

$$= 4(4x^2 - 40x + 99)$$

$$= 4(\underbrace{4x^2 - 18x}_{2x} - \underbrace{22x + 99}_{-11})$$

$$= 4(2x - 11)(2x - 9)$$

$$M: 396$$

$$A: -40$$

$$-1, -396$$

$$-2, -198$$

$$-3, -132$$

$$-4, -99$$

$$-6, -66$$

$$-9, -44$$

$$-11, -36$$

$$-12, -33$$

$$\boxed{-18, -22}$$

$$18) 9x^2 - 7x - 91$$

Not Factorable!

$$M: -819$$

$$A: -7$$

$$1, -819$$

$$3, -273$$

$$7, -117$$

$$9, -91$$

$$\boxed{21, -39}$$

$$19) 10k^2 + 111k - 108$$

$$= \underbrace{10k^2 - 9k}_K + \underbrace{120k - 108}_{+12}$$

$$= (k + 12)(10k - 9)$$

$$M: -1080$$

$$A: +111$$

$$-1, 1080$$

$$-2, 540$$

$$-3, 360$$

$$-4, 270$$

$$-5, 216$$

$$-6, 180$$

$$-8, 135$$

$$\boxed{-9, 120}$$

$$20) 9x^2 - 50x - 24$$

$$= \underbrace{9x^2 + 4x}_x - \underbrace{54x - 24}_{-6}$$

$$= (x - 6)(9x + 4)$$

$$M: -216$$

$$A: -50$$

$$1, -216$$

$$2, -108$$

$$3, -72$$

$$\boxed{4, -54}$$

4. Special Case A: x^2+bx+c

Factor each completely.

$$1) \ n^2 - n - 12$$

$M: -12 \ A: -1$
 $3, -4$

$$= (n+3)(n-4)$$

$$2) \ 4x^2 - 28x + 40$$

$$= 4(x^2 - 7x + 10)$$

$$= 4(x-2)(x-5)$$

$M: 10 \ A: -7$
 $-2, -5$

$$3) \ 6p^2 + 54p - 60$$

$$= 6(p^2 + 9p - 10)$$

$$= 6(p-1)(p+10)$$

$M: -10$
 $A: 9$
 $-1, +10$

$$4) \ x^2 + 9x + 8$$

$$= (x+1)(x+8)$$

$M: 8 \ A: 9$
 $1, 8$

$$5) \ a^2 + 4a - 32$$

$$= (a-4)(a+8)$$

$M: -32$
 $A: 4$
 $-1, 32$
 $-2, 16$
 $-4, 8$

$$6) \ 4r^2 + 20r - 144$$

$$= 4(r^2 + 5r - 36)$$

$$= 4(r-4)(r+9)$$

$M: -36 \ A: +5$
 $-1, 36$
 $-2, 18$
 $-3, 12$
 $-4, 9$

$$7) \ n^2 + 2n - 8$$

$$= (n-2)(n+4)$$

$M: -8 \ A: 2$
 $-2, 4$

$$8) \ 2v^2 - 20v + 48$$

$$= 2(v^2 - 10v + 24)$$

$$= 2(v-4)(v-6)$$

$M: 24 \ A: -10$
 $-4, -6$

$$9) \ x^2 + x - 20$$

$$= (x-4)(x+5)$$

$M: -20 \ A: 1$
 $-4, +5$

$$10) \ r^2 - 3r - 18$$

$$= (r+3)(r-6)$$

$M: -18 \ A: -3$
 $+3, -6$

$$\begin{aligned}
 11) \quad & 3x^2 + 27x + 54 \\
 & = 3(x^2 + 9x + 18) \\
 & = 3(x+3)(x+6)
 \end{aligned}$$

$M: 18$
 $A: 9$
 $3, 6$

$$\begin{aligned}
 12) \quad & 4n^2 - 20n - 56 \\
 & = 4(n^2 - 5n - 14) \\
 & = 4(n+2)(n-7)
 \end{aligned}$$

$M: -14$ $A: -5$
 $2, -7$

$$\begin{aligned}
 13) \quad & v^2 - 11v + 30 \\
 & = (v-5)(v-6)
 \end{aligned}$$

$M: 30$
 $A: -11$
 $-5, -6$

$$\begin{aligned}
 14) \quad & x^2 - 9x - 112 \\
 & = (x+7)(x-16)
 \end{aligned}$$

$M: -112$
 $A: -9$
 $1, -112$
 $2, -56$
 $4, -28$
 $7, -16$

$$\begin{aligned}
 15) \quad & k^2 + 4k + 3 \\
 & = (k+1)(k+3)
 \end{aligned}$$

$$\begin{aligned}
 16) \quad & 3b^2 - 63b + 270 \\
 & = 3(b^2 - 21b + 90) \\
 & = 3(b-6)(b-15)
 \end{aligned}$$

$M: 90$
 $A: -21$
 $-6, -15$

$$\begin{aligned}
 17) \quad & v^2 - 11v + 24 \\
 & = (v-3)(v-8)
 \end{aligned}$$

$$\begin{aligned}
 18) \quad & x^2 - 8x - 9 \\
 & = (x+1)(x-9)
 \end{aligned}$$

$$\begin{aligned}
 19) \quad & 2x^2 - 42x + 180 \\
 & = 2(x^2 - 21x + 90) \\
 & = 2(x-6)(x-15)
 \end{aligned}$$

AGAIN?!

$$\begin{aligned}
 20) \quad & v^2 - 21v + 90 \\
 & = (v-6)(v-15)
 \end{aligned}$$

5. Special Cases B (Differences of Squares) and C (Perfect Square)

Factor the Difference of Squares (watch out for GCFs)

1) $49r^2 - 81$

$$(7r - 9)(7r + 9)$$

2) $81x^2 - 49$

$$(9x + 7)(9x - 7)$$

3) $16m^2 + 49$

not a DoS

4) $100n^2 - 1$

$$= (10n - 1)(10n + 1)$$

5) $1156b^2 - 1521$

$$(34b - 39)(34b + 39)$$

6) $529x^2 - 1444$

$$= (23x - 38)(23x + 38)$$

7) $1600n^2 - 841$

$$= (40n - 29)(40n + 29)$$

8) $v^2 - 676$

$$= (v - 26)(v + 26)$$

9) $5n^2 - 80$

$$= 5(n^2 - 16)$$

$$= 5(n - 4)(n + 4)$$

10) $343x^2 - 252$

$$= 7(49x^2 - 36)$$

$$= 7(7x - 6)(7x + 6)$$

11) $k^2 - 9$

$$= (k - 3)(k + 3)$$

12) $49n^2 - 15$

not a perfect square.

13) $175m^2 - 112$

$$= 7(25m^2 - 16)$$

$$= 7(5m - 4)(5m + 4)$$

14) $64p^2 - 49$

$$= (8p - 7)(8p + 7)$$

15) $600x^2 - 486$

$$= 6(100x^2 - 81)$$

$$= 6(10x - 9)(10x + 9)$$

16) $16m^2 - 81$

$$= (4m - 9)(4m + 9)$$

17) $300x^2 - 243$

$$= 3(100x^2 - 81)$$

$$= 3(10x - 9)(10x + 9)$$

18) $490x^2 - 40$

$$= 10(49x^2 - 4)$$

$$= 10(7x - 2)(7x + 2)$$

19) $36n^2 + 25$

not a DoS.

20) $49a^2 - 9$

$$= (7a - 3)(7a + 3)$$

Check to see if it is a perfect square. If so, factor it. If not, state why not.

$$21) 16a^2 + 8a + 1$$

$$(4a+1)^2$$

$$22) 25n^2 + 70n + 49$$

$$(5n+7)^2$$

$$23) 9b^2 + 42b + 49$$

$$= (3b+7)^2$$

$$24) 16x^2 - 40x + 27$$

not a square #

$$25) x^2 + 6x + 9$$

$$(x+3)^2$$

$$26) 81n^2 - 126n + 49$$

must be positive
∴ not a P.S.

$$27) 100v^2 - 20v + 1$$

$$(10v-1)^2$$

$$28) 64x^2 - 144x + 81$$

$$= (8x-9)^2$$

$$29) 25n^2 + 34n + 9$$

not a P.S.

$$30) 100k^2 - 60k + 9$$

$$= (10k-3)^2$$

$$31) 7m^2 + 126m + 567$$

$$= 7(m^2 + 18m + 81)$$

$$= 7(m+9)^2$$

$$32) 245p^2 + 350p + 125$$

$$= 5(49p^2 + 70p + 25)$$

$$= 5(7p+5)^2$$

$$33) 18x^2 + 8x + 1$$

$2 \times 9 \times 1 = 18$, not 8.
∴ not P.S.

$$34) 40n^2 + 200n + 250$$

$$= 10(4n^2 + 20n + 25)$$

$$= 10(2n+5)^2$$

$$35) 405b^2 - 720b + 320$$

$$= 5(81b^2 - 144b + 64)$$

$$= 5(9b-8)^2$$

$$36) x^2 + 4x + 4$$

$$= (x+2)^2$$

$$37) 49m^2 + 112m + 64$$

$$(7m+8)^2$$

$$38) 4n^2 - 12n + 9$$

$$= (2n-3)^2$$

$$39) 500r^2 + 100r + 5$$

$$= 5(100r^2 + 20r + 1)$$

$$= 5(10r+1)^2$$

$$40) 81v^2 - 90v + 25$$

$$= (9v-5)^2$$