Mathematics 10D

Properties of Quadratics

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Line: $y = mx + b$

~~Standard: $Ax + By + C = 0$~~

- Quadratics are the equations.
- Parabolas are the graphs.

There are three equations of quadratics.

1. Standard Form / Expanded Form

   $y = ax^2 + bx + c$ → y-intercept

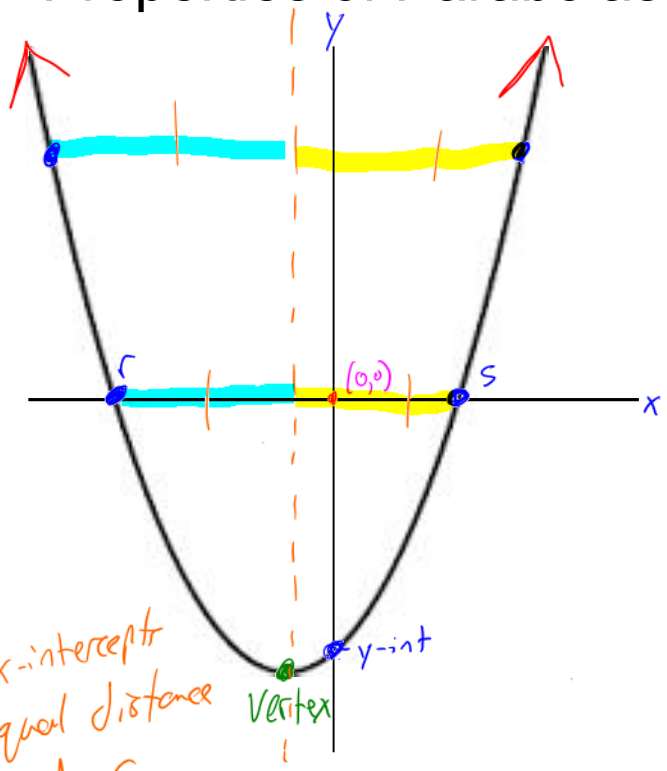
2. Zeros/Factored Form

   $y = a(x - r)(x - s)$ → x-intercept/zeros.

3. Vertex Form

   $y = a(x - h)^2 + k$ → (h, k) = vertex

Properties of Parabolas



The x-intercepts have equal distance to the AoS

① y-int: $y = c$ or $(0, c)$

② x-int: $x = r$ and $x = s$

Zeros: $(r, 0)$ and $(s, 0)$

③ Vertex: (h, k) max/min value

④ Axis of Symmetry

AoS: $x = h$

$h = \frac{r+s}{2}$ The average of the zeros/x-ints.

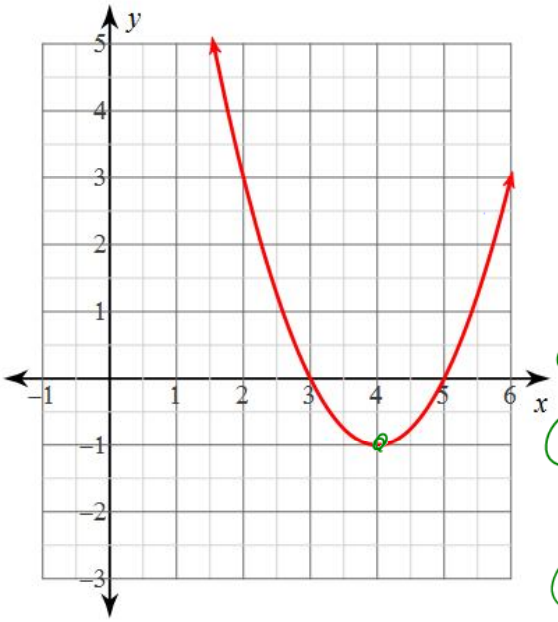
⑤ If $a > 0$, then the parabola opens up ∴ minimum of k.

If $a < 0$, the parabola opens down ∴ maximum of k.

∪ = happy = +

∩ = sad = −

1) $y = x^2 - 8x + 15$ (c)



State all the properties:

① y-int: $y = 15$

② x-int: $x = 3$ and $x = 5$

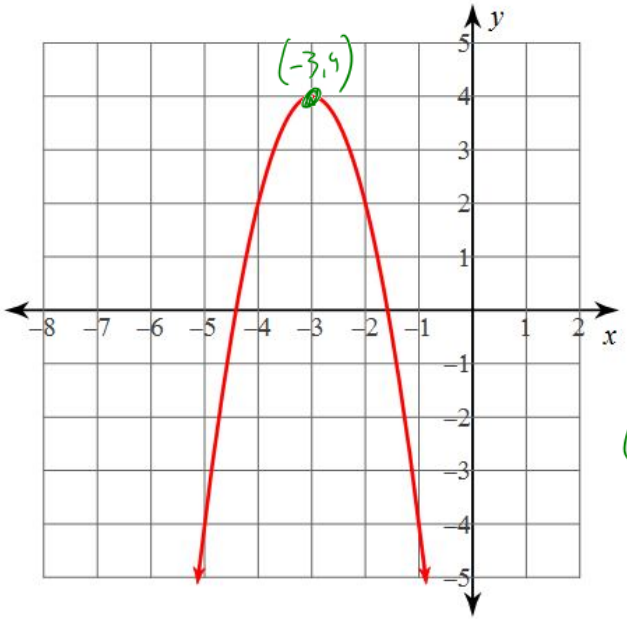
$(3, 0)$ $(5, 0)$

③ Vertex: $(4, -1)$

④ AoS: $x = 4$ $h = \frac{3+5}{2} = \frac{8}{2} = 4$!!

⑤ Minimum of $y = -1$

2) $y = -2x^2 - 12x - 14$



① y-int: $y = -14$

② x-int: $x = -1.5$ and $x = -4.5$

③ vertex: $(-3, 4)$

④ AoS: $x = -3$

⑤ Max of $y = 4$

} vertex

Mathematics 10D **Q.02 – Zeros/Factored Form**

Standard: $y = ax^2 + bx + c$ → y-int

Factor

Zeros/Factored: $y = a(x - r)(x - s)$

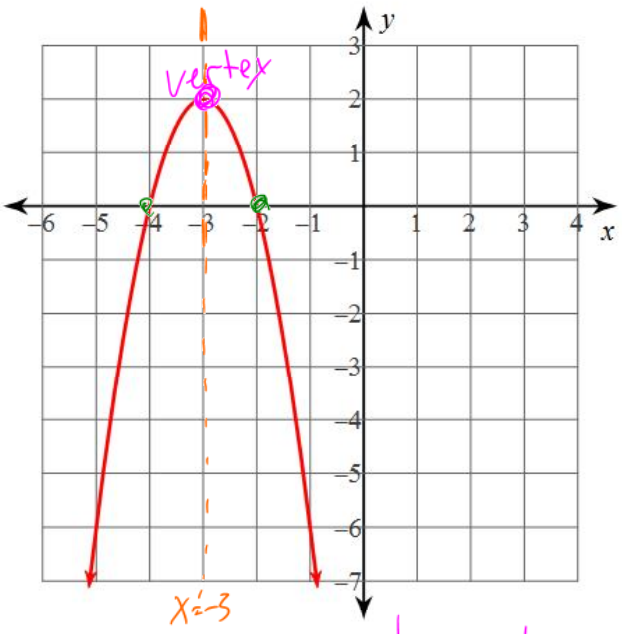
x-int/zeros/solutions/roots

$y = -2x^2 - 12x - 16$ factor $y = -2(x^2 + 6x + 8)$

m: 8
A: 6
2, 4

$y = -2(x+2)(x+4)$

$x+2=0$ $x=-2$ $x+4=0$ $x=-4$



The vertex is $(-3, 2)$

Zeros are $x=-4$ and $x=-2$

AoS: $h = \frac{-4+-2}{2} = \frac{-6}{2} = -3$

Vertex is (h, k) (x, y)

Vertex: $(-3, ?)$ $x=-3$

$k \Rightarrow y = -2(-3)^2 - 12(-3) - 16$

$y = -18 + 36 - 16$

$y = 2 \Rightarrow k$

Given the standard form, find the zeros then the vertex.

Factor AoS and plug it in.

$y = 3x^2 + 15x - 18$

M: -6 -1, +6
A: 5 2, 3

$y = 3(x^2 + 5x - 6)$

$y = 3(x-1)(x+6)$

Zeros are $x = 1$ and $x = -6$

AoS: $h = \frac{1+-6}{2} = \frac{-5}{2} = -2.5$

$k: y = 3(-2.5-1)(-2.5+6)$

$y = 3(-3.5)(3.5)$

$y = -36.75 = k$

Vertex is $(-2.5, -36.75)$

"a" is $3 > 0$ ∴ min

Given the standard form, find the zeros then the vertex.

$y = 5x^2 + 8x - 4$

M: -20 Factor
A: 8
-2, 10

AoS, plug it in

$y = \frac{5x^2 + 10x}{5x} \quad \frac{-2x - 4}{-2}$

$y = (5x - 2)(x + 2)$

$5x - 2 = 0$ | $x = -2$

$5x = 2$

$x = \frac{2}{5} = 0.4$

The vertex is $(-0.8, -7.2)$ and this is a minimum.

AoS: $h = \frac{0.4 + -2}{2}$

$h = \frac{-1.6}{2}$

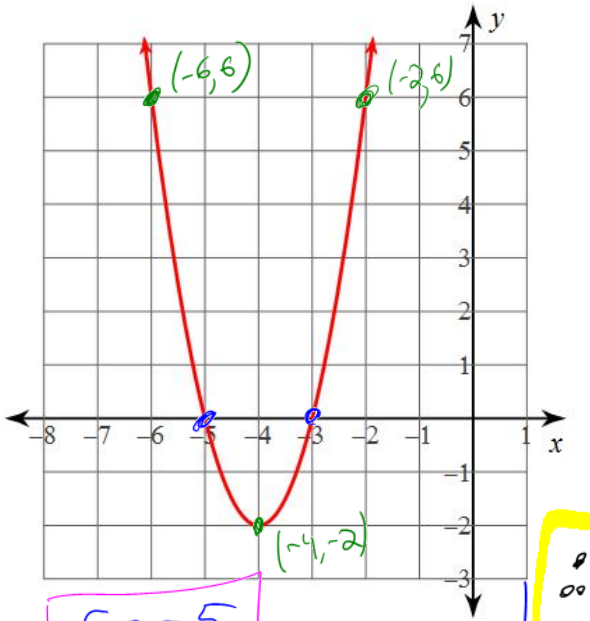
$h = -0.8$

k: $y = 5(-0.8)^2 + 8(-0.8) - 4$

$y = 3.2 - 6.4 - 4$

$y = -7.2 = k$

Given the graph, state the equation of the parabola in both zeros form and standard form, then state the y-intercept.



$y = a(x - r)(x - s)$

$6 = a(-2 + 5)(-2 + 3)$

$6 = a(3)(1)$

$\frac{6}{3} = \frac{3a}{3}$

$2 = a$

$y = 2(x + 5)(x + 3)$ zeros form

FOIL

$y = 2(x^2 + 3x + 5x + 15)$

$y = 2(x^2 + 8x + 15)$

$y = 2x^2 + 16x + 30$ standard form.

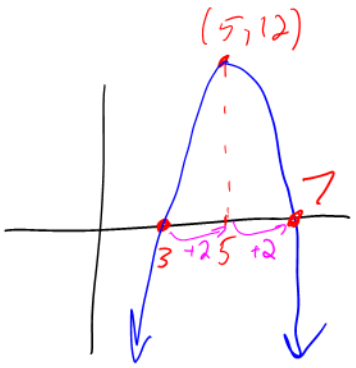
↳ y-int

Zeros $r = -5$, $s = -3$

Any other point (x, y)

$(-2, 6)$

A parabola has a zero at (3,0) and a vertex at (5,12). State the equation of the parabola in both zeros and standard form.



$r = 3$ $\quad x = 5$

$s = 7$ $\quad y = 12$

$y = a(x-r)(x-s)$

$12 = a(5-3)(5-7)$

$12 = a(2)(-2)$

$12 = -4a$

$\frac{12}{-4} = \frac{-4a}{-4}$

$-3 = a$

$y = -3(x-3)(x-7)$ zeros.

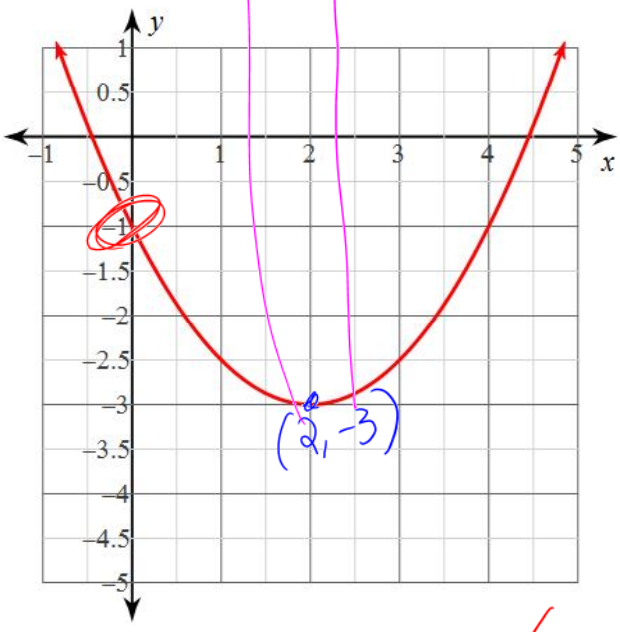
$y = -3(x^2 - 10x + 21)$

$y = -3x^2 + 30x - 63$ standard.

y-int.

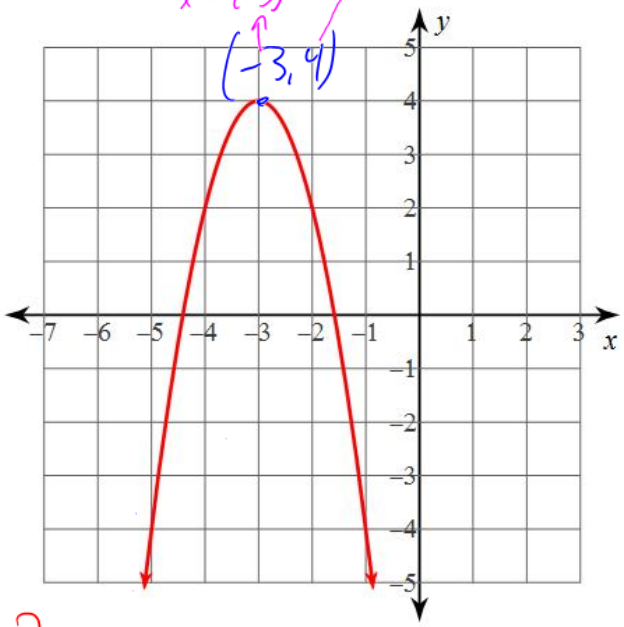
Mathematics 10D Q.03 – Vertex Form

$y = \frac{1}{2}(x-2)^2 - 3$



$y = -2(x+3)^2 + 4$

$x - (-3)$



$y = a(x-h)^2 + k$

Convert from Vertex form to Standard form Just do algebra.

$y = \frac{1}{2}(x-2)^2 - 3$ expand vertex

$y = \frac{1}{2}(x-2)(x-2) - 3$

$y = \frac{1}{2}(x^2 - 4x + 4) - 3$

$y = \frac{1}{2}x^2 - 2x + 2 - 3$

$y = \frac{1}{2}x^2 - 2x - 1$ standard. y-int.

$y = -2(x+3)^2 + 4$

$y = -2(x+3)(x+3) + 4$

$y = -2(x^2 + 6x + 9) + 4$

$y = -2x^2 - 12x - 18 + 4$

$y = -2x^2 - 12x - 14$ y-int.

Convert from Standard form to Vertex form via the zeros form.

$y = 4x^2 + 24x - 64$ standard Factor, get zeros, get AoS, plug it in.

$y = 4(x^2 + 6x - 16)$ M: -16, A: 6 -1, 16 -2, 8

$y = 4(x-2)(x+8)$ zeros

$x = 2 \quad x = -8$

AoS: $h = \frac{2 + -8}{2} = -3$

$k = y = 4(-3)^2 + 24(-3) - 64$

$y = 36 - 72 - 64$

$y = -100 = k$

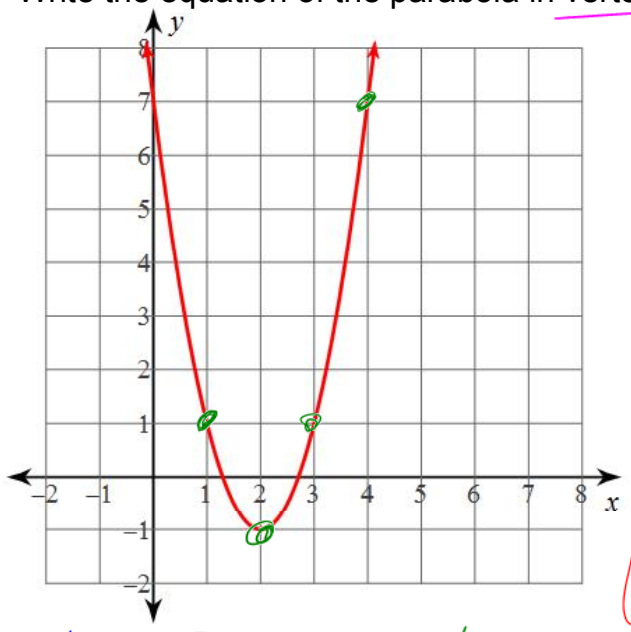
$\therefore$ The vertex is $(\underset{h}{-3}, \underset{k}{-100})$

$y = a(x-h)^2 + k$

$y = 4(x+3)^2 - 100$

no zeros so this way won't work :?

Write the equation of the parabola in vertex form



vertex $h = 2$, $k = -1$

$x = 4$, $y = 7$ > any other point

$y = a(x-h)^2 + k$

$7 = a(4-2)^2 - 1$

$7 = a(4) - 1$

$7 = 4a - 1$

$+1 \quad +1$

$\frac{8}{4} = \frac{4a}{4}$

$2 = a$

$\therefore y = 2(x-2)^2 - 1$

A parabola has a zero at (3,0) and a vertex at (5,12). State the equation of the parabola in both vertex and standard form.

$h = 5$, $k = 12$

$x = 3$, $y = 0$

$y = a(x-h)^2 + k$

$0 = a(3-5)^2 + 12$

$-12 \quad -12$

$\frac{-12}{4} = \frac{4a}{4}$

$-3 = a$

$\therefore y = -3(x-5)^2 + 12$ vertex Form

$y = -3(x-5)(x-5) + 12$

$y = -3(x^2 - 10x + 25) + 12$

$y = -3x^2 + 30x - 75 + 12$

$y = -3x^2 + 30x - 63$ y-int.

Mathematics 10D Q.04 – Graphing Quadratics From the Vertex Form

$$y = a(x-h)^2 + k.$$

Base/Parent graph: $y = x^2$

We will graph using transformations.

shape [a: vertical stretch / compression. $\Rightarrow a \neq 0$
- stretch: if $a > 1$ or $a < -1$ — skinny
- compression: between 0 and 1 or between -1 and 0 — wider.

position
- h: horizontal shift — pushes the graph horizontally
- k: vertical shift — pushes the graph vertically

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a: vertical (y's) stretch (multiply)

h: horizontal (x's) shift (add)

k: vertical (y's) shift (add)

$y = x^2$

x	y	
−2	4	left
−1	1	
0	0	vertex
1	1	right
2	4	

$y = a(x-h)^2 + k$

x + h	ay + k
−2 + h	4a + k
−1 + h	1a + k
0 + h	0a + k
1 + h	1a + k
2 + h	4a + k

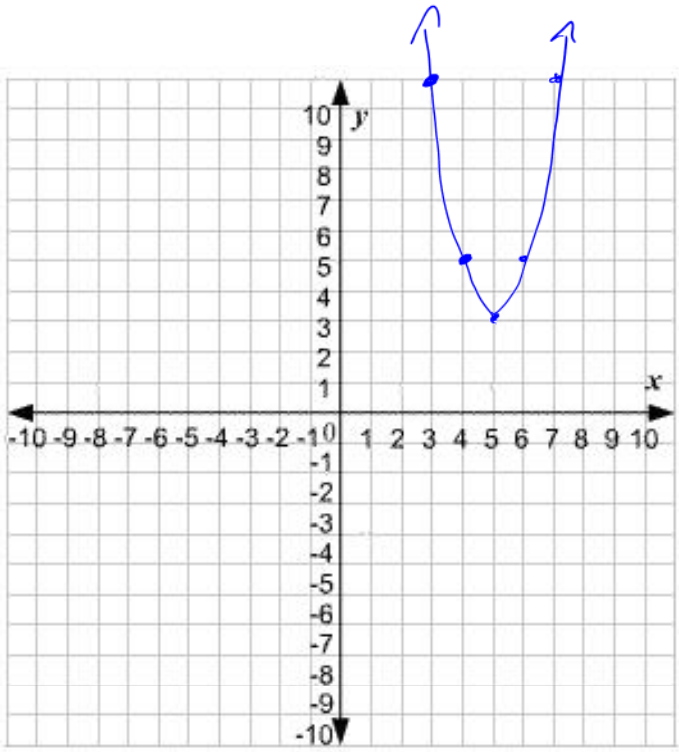
$$y = \overset{a}{2}(x - \overset{h}{5})^2 + \overset{k}{3}$$

1. Vertical Stretch of 2
2. Horizontal Shift of +5 right
3. Vertical Shift of +3 up

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

x + 5	2y + 3
3	11
4	5
5	3
6	5
7	11

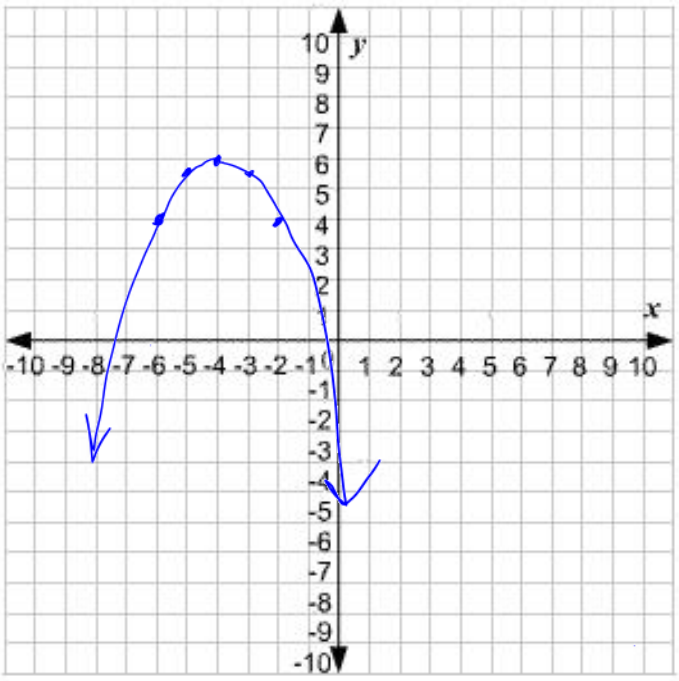


$$y = \frac{-1}{2}(x \overset{\text{opposite}}{+} 4)^2 + 6$$

1. V. Str. of $-\frac{1}{2}$
2. H. Shift of -4
3. V. Shift of $+6$

x	y
-2	4
-1	1
0	0
1	1
2	4

x − 4	$-\frac{1}{2}y + 6$
-6	4
-5	5.5
-4	6
-3	5.5
-2	4



Mathematics 10D

Q.05 – Completing the Square → converting standard form to vertex form

$(x+4)^2 = (x+4)(x+4) = x^2+8x+16$

$(x-1)^2 = (x-1)(x-1) = x^2-2x+1$

$y = a(x-h)^2 + k$ (perfect square)

$(x\underline{-7})^2 = x^2 \underset{\div 2}{-14}x + \underline{49}$

$(x\underline{+5})^2 = x^2 + 10x + \underline{25}$

$(x\underline{-174})^2 = x^2 - 348x + \underline{29584}$ $\quad (x+3)^2 = x^2+6x+9$

$(x\underline{-16.5})^2 = x^2 - 33x + \underline{272.25}$

$(x\underline{-1.42})^2 = x^2 - 2.84x + \underline{2.0164}$

Convert from Standard form to Vertex form by completing the square, then state the vertex.

$y = 2x^2 + 12x - 5$

$y = 2(x^2 + 6x + 0) - 5$ (b = 6, c = 0)

$y = 2(x^2 + 6x + 9 - 9) - 5$ (multiply)

Perfect Square; not

$y = 2(x^2 + 6x + 9) - 5 - 18$ ($\div 2$)

$y = 2(x+3)^2 - 23$ (opposite)

Vertex: $(-3, -23)$ (h, k)

steps:

① Factor the "a" from the first two terms.

② Find the missing # to complete the square $\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$

③ multiply "a" with the -9.

④ Factor the P.S. and simplify

Let's do another!

$y = -5x^2 - 40x + 71$

$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$ (4: end, 16: now)

$y = -5(x^2 + 8x + 0) + 71$

$y = -5(x^2 + 8x + 16 - 16) + 71$

P.S.

$y = -5(x^2 + 8x + 16) + 71 + 80$

$y = -5(x+4)^2 + 151$

Vertex: $(-4, 151)$

What happens when *a* is a fraction?

$y = \frac{1}{2}x^2 - 5x + 14$

$5 \div \frac{1}{2} = 5 \times \frac{2}{1} = 10$

$\frac{1}{2}$

$y = \frac{1}{2}(x^2 - 10x + 0) + 14$

$\left(\frac{10}{2}\right)^2 = (-5)^2 = 25$ (-5: End, 25: now)

$y = \frac{1}{2}(x^2 - 10x + 25 - 25) + 14$

$y = \frac{1}{2}(x^2 - 10x + 25) + 14 - 12.5$

$y = \frac{1}{2}(x-5)^2 + 1.5$

Vertex: $(5, 1.5)$

Uh oh, we have decimals! No worries!

$$y = 2.84x^2 - 8.23x + 5.4$$

2.84

$$\left(\frac{-2.9}{2}\right)^2 = (-1.45)^2$$

end

$$= 2.1025$$

now

$$y = 2.84(x^2 - 2.9x + 0) + 5.4$$

$$y = 2.84(x^2 - 2.9x + 2.1025 - 2.1025) + 5.4$$

$$y = 2.84(x^2 - 2.9x + 2.1025) + 5.4 - 5.97$$

$$y = 2.84(x - 1.45)^2 - 0.57$$

Vertex: $(1.45, -0.57)$

AoS

min value

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