Mathematics 10D

Unit 2 – Analytic Geometry

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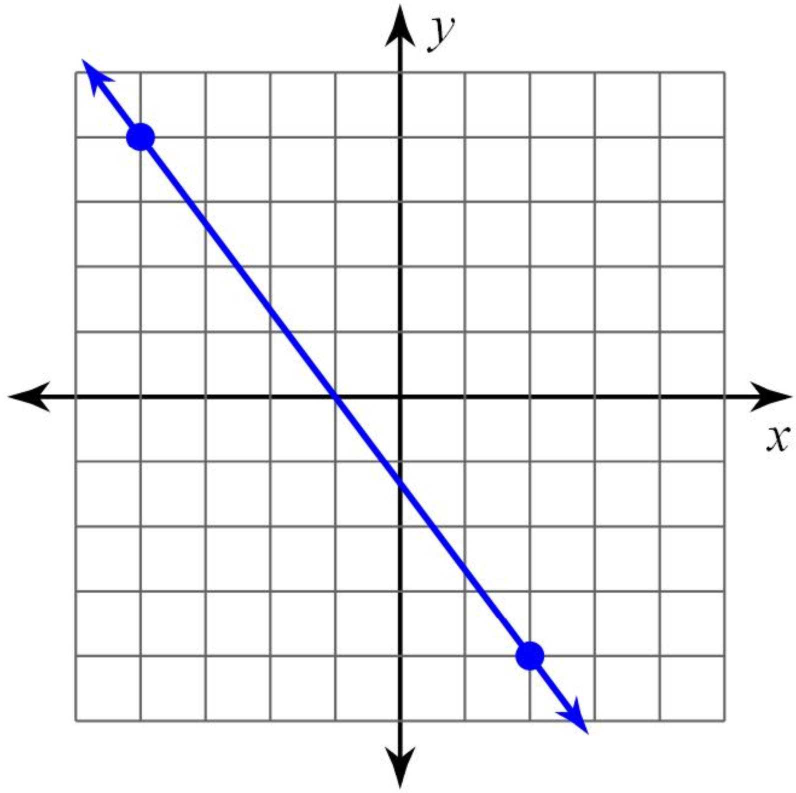
Mathematics 10D 2.1 – Midpoint of a Line Segment

A line segment is a line that connects two points. A midpoint, then, is the point that represents the middle of that line segment.

Question: If you scored a 70% on a test and then an 82% on the next test, what is the average of those tests?

Find the midpoint of the line segment below:



The coordinate of the midpoint is:

$$M_{AB}\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

Find the midpoint of the points:

$(6, -9),\ (-2, -4)$

$(-655, 848),\ (117, 976)$

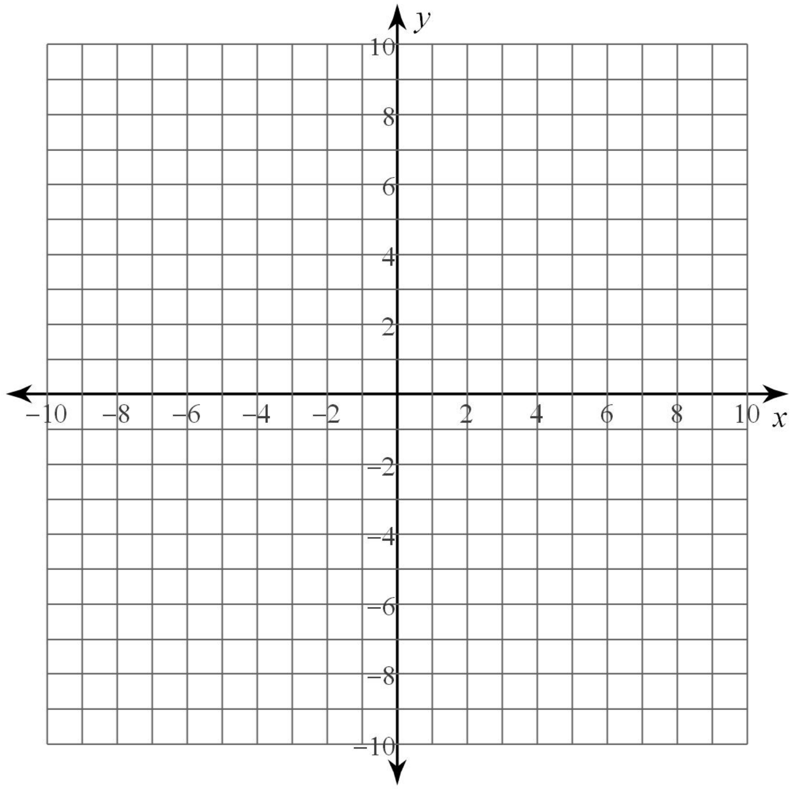
Given one endpoint and the midpoint, find the second endpoint.

Endpoint: $(-8, -6)$, midpoint: $(2, 1)$

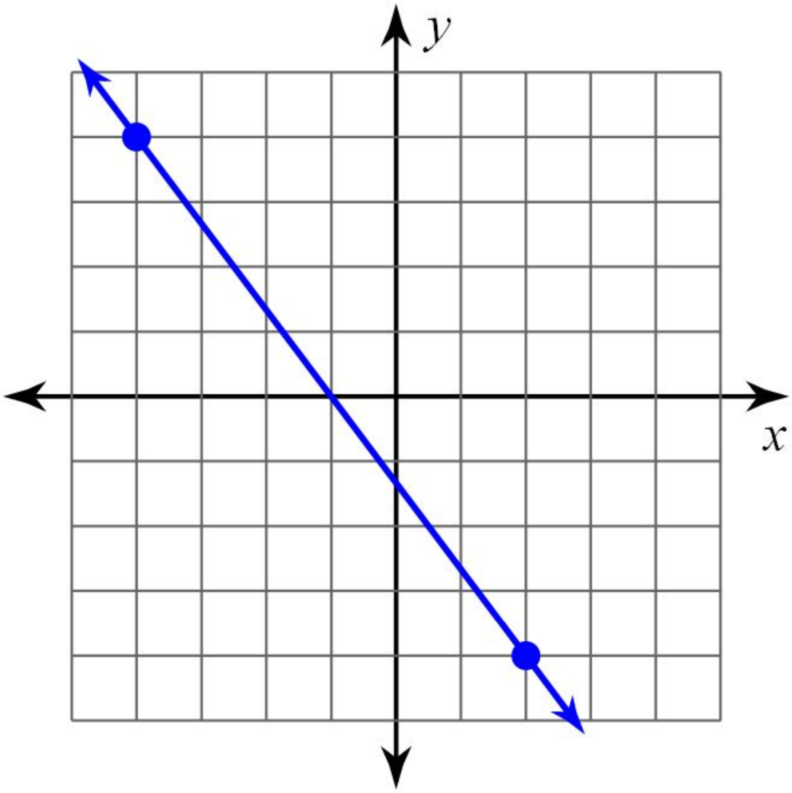
Endpoint: $(5, 4)$, midpoint: $(27, 40)$

The Big Question: A triangle has vertices at A(-3,1), B(3,5) and C(7,-3). Determine the equation of the median from vertex A.

A median is a line segment that joins a vertex of a triangle to the midpoint of the opposite side.



Find the distance between the two points:



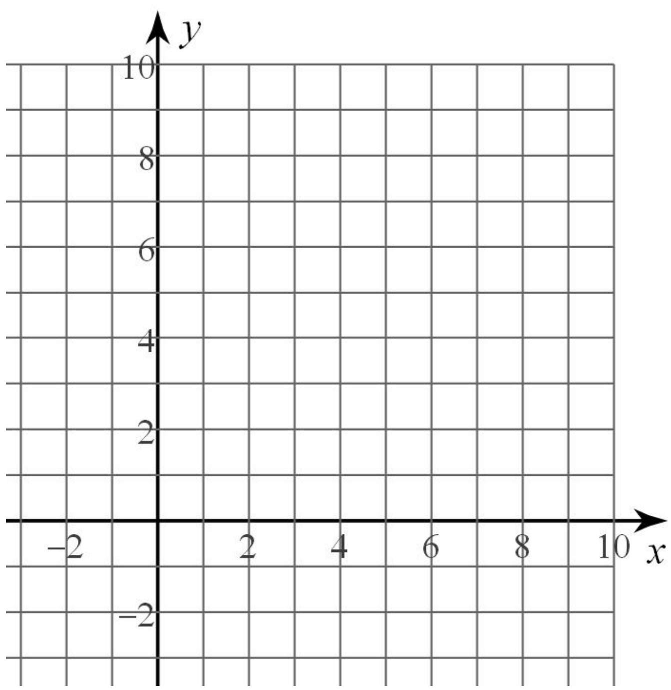
Let's derive the Distance Formula:

Find the distance between the two given points:

$(8, 6), \ (-2, -7)$

$(18, -2), \ (-17, 4)$

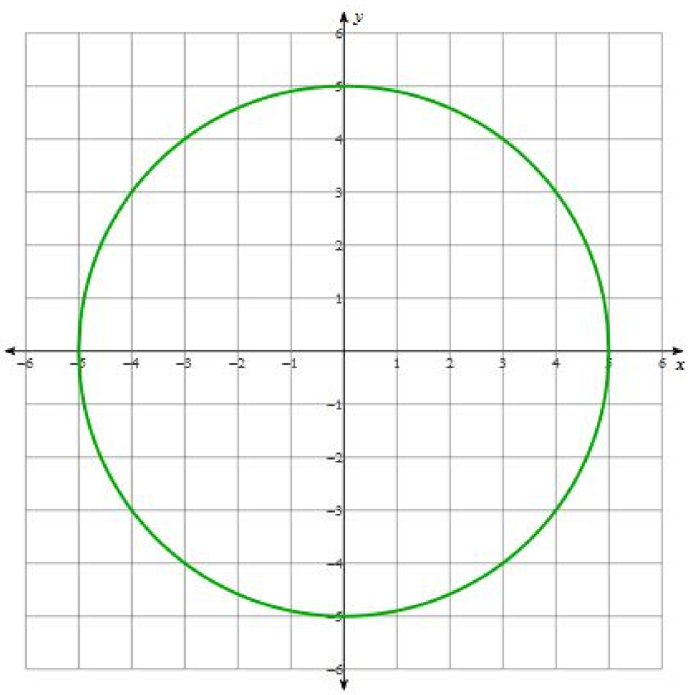
The Big Question: Calculate the shortest distance from point A(6,5) and the line y=2x+3.



Let's Do Another!: Calculate the shortest distance from point B(-2,-3) and the line $y = -\frac{2}{5}\mathrm{x} + 6$

Mathematics 10D 2.3 – Circles

A Circle centred around origin at (0,0):



Write the equation of the circle given:

$r = 8$ (-3,7)

Given the circle $x^2+y^2=80$, determine if the following points are inside, on or outside the circle:

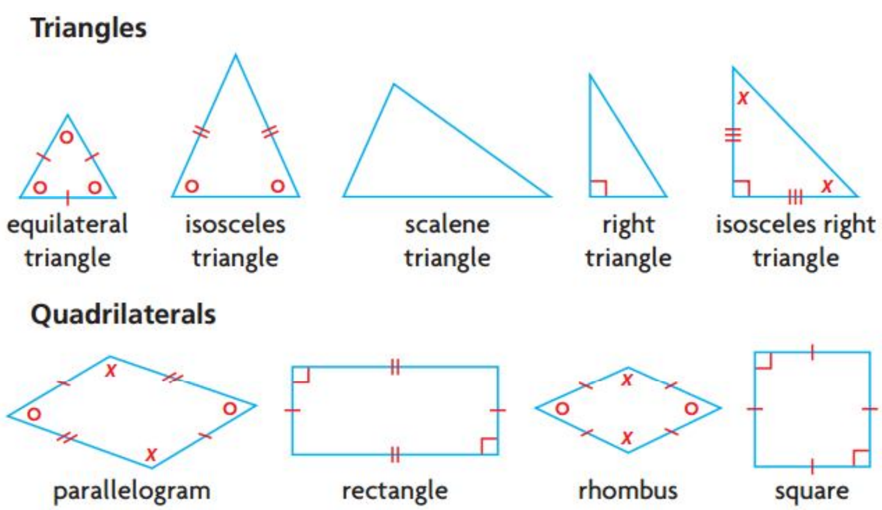
A(2,4) B(7,-5) C(5,-6)

Last Question: A stone is dropped into a pond, creating a circular ripple. The radius of the ripple increases by 4cm/s. Determine an equation that models the circular ripple after 10 seconds.

Mathematics 10D 2.4 – Classifying Figures on a Coordinate Grid

Things you must know and understand for the rest of this unit:

1. Midpoint
2. Distance
3. Slope – understand parallel and perpendicular
4. Write linear equations
5. Find point of intersection
6. Terms – median, vertice, line segment, midsegment, etc…
7. "Using analytic geometry" – geometry that uses an xy grid, algebra and equations to describe relations and solve problems related to geometric figures
8. Shapes



Example 1: Verify what type of quadrilateral is formed by the points P(-5,-5), Q(-30,10), R(-5,25) and S(20,10).

Example 2: A triangle has vertices at A(-1,-1), B(2,0) and C(1,3). Using analytic geometry, determine what type of triangle it is.

Example 3: Tony is constructing a patterned concrete patio that is in the shape of an isosceles triangle, as requested by his client. On his plan, the vertices of the triangle are at P(2,1), Q(5,7) and R(8,4). Each unit represents 1m.

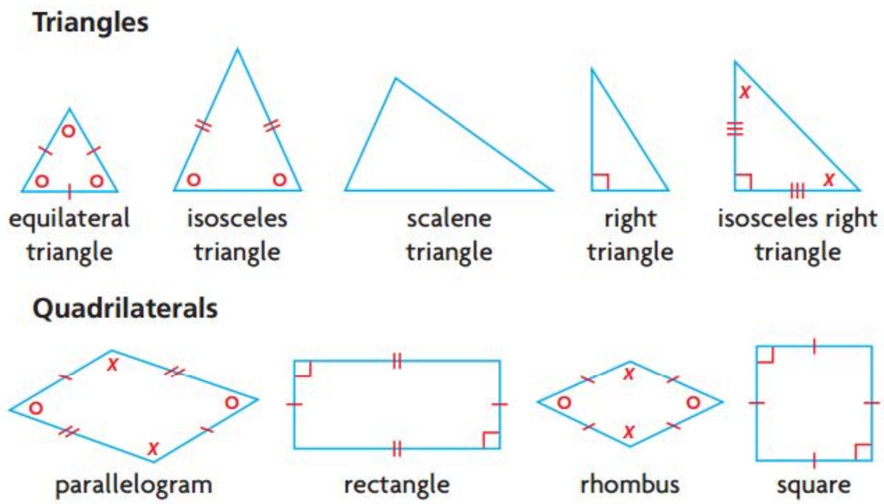
a) Confirm that the plan shows an isosceles triangle.

b) Calculate the area of the patio.

Mathematics 10D 2.5 – Verifying Properties of Geometric Figures

Things you must know and understand for the rest of this unit:

1. Midpoint
2. Distance
3. Slope – understand parallel and perpendicular
4. Write linear equations
5. Find point of intersection
6. Terms – median, vertice, line segment, midsegment, etc…
7. "Using analytic geometry" – geometry that uses an xy grid, algebra and equations to describe relations and solve problems related to geometric figures
8. Shapes



Example 1: Show that the midsegments of the quadrilateral, with vertices at P(-7,9), Q(9,11), R(9,-1) and S(1,-11), form a parallelogram.

Example 2: A triangle has vertices at A(-2,2), B(1,3) and C(4,-1). Show that the midsegment joining the midpoints of AB and AC is parallel to BC and half its length.

Example 3: Show that points A(10,5) and B(2,-11) lie on the circle with equation $x^2 + y^2 = 125$. Also show that the perpendicular bisector of chord AB passes through the centre of the circle.

Mathematics 10D 2.6 – Exploring Properties: Centroid and Circumcenter

Centroid: Find the centroid of the triangle with vertices at A(-2,6), B(-4,-1) and C(9,2).

The centroid is the intersection of the medians. It is the centre of mass of a triangle.

Steps to finding the centroid:
1. Make a sketch of the triangle.
2. Calculate a midpoint. Label it on your sketch.
3. Calculate the slope from the above midpoint to the opposite corner/vertice.
4. Find the equation (y=mx+b) of that line (the median)
5. Repeat steps 2-4 starting with a different midpoint
6. Find the point of intersection using the two equations. That is your centroid.

Centroid: Find the centroid of the triangle with vertices at A(-2,6), B(-4,-1) and C(9,2).

Circumcentre: Find the circumcentre of the triangle with vertices at X(4,8), Y(-2,6) and Z(1,1).

The circumcentre is the intersection of the perpendicular bisectors. It is the centre of the circle formed by the corners of the triangle.

Steps to finding the circumcentre:

1. Make a sketch of the triangle.
2. Calculate a midpoint. Label it on your sketch.
3. Calculate the slope of the same line that contains the above midpoint.
4. Perpendicularize the slope from 3 (the negative reciprocal).
5. Find the equation (y=mx+b) of that line (the perpendicular bisector)
6. Repeat steps 2-5 starting with a different midpoint
7. Find the point of intersection using the two equations. That is your circumcentre.

Circumcentre: Find the circumcentre of the triangle with vertices at X(4,8), Y(-2,6) and Z(1,1).