

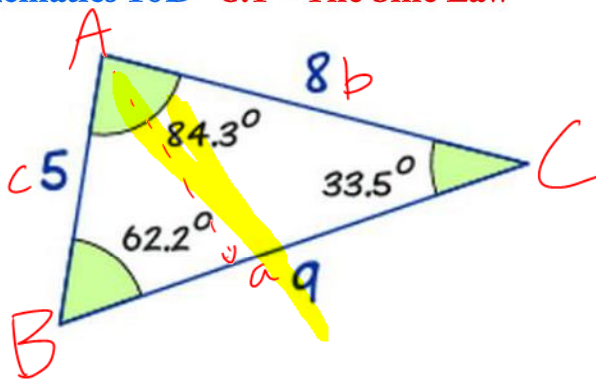
Mathematics 10D

Unit 8 – Acute Triangle Trigonometry

Mr. D. Hagen

Triangles with no right angle.

Mathematics 10D 8.1 – The Sine Law



The Three Bears Theorem

- The largest angle is paired with the largest side.

- The smallest angle is paired with the smallest side.

$$\frac{\sin A}{a} = \frac{\sin 84.3}{9} = 0.11056$$

$$\frac{\sin B}{b} = \frac{\sin 62.2}{8} = 0.11057$$

$$\frac{\sin C}{c} = \frac{\sin 33.5}{5} = 0.11039$$

essentially equal.

Also true if all fractions are flipped

Sine Law

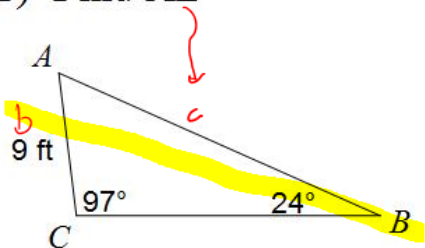
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- To use sine law, you need one pair and one other angle/side.

- Always put what you are looking for at the top.

1) Find AB



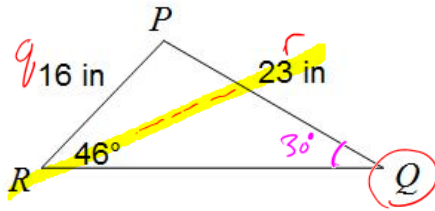
looks for this

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$
$$\frac{c}{(\sin 97^\circ)} = \frac{9 \sin 97^\circ}{\sin 24^\circ}$$

$$c = \frac{9 \sin(97^\circ)}{\sin(24^\circ)}$$

$$c = 22 \text{ ft}$$

2) Find $m\angle Q$



$$\frac{\sin Q}{9} = \frac{\sin R}{r}$$

$$\frac{\sin Q}{16} = \frac{16 \sin 46^\circ}{23}$$

$$\sin Q = \frac{16 \sin 46^\circ}{23}$$

$$\sin Q = 0.5004$$

$$Q = \sin^{-1}(0.5004)$$

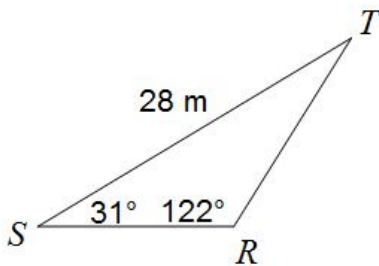
$$Q = 30^\circ$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \frac{\text{small}}{\text{big}}$$

therefore always smaller than 1

Must be less than one.

3) Solve $\triangle RST$



Solve means to find all missing pieces.

$$\begin{aligned} \angle R &= 122^\circ & r &= 28 \text{ m} \\ \angle S &= 31^\circ & s &= 17 \text{ m} \\ \angle T &= 27^\circ & t &= 15 \text{ m} \end{aligned}$$

All three angles add up to 180°

$$\begin{aligned} \textcircled{1} \angle T &= 180^\circ - \angle S - \angle R \\ &= 180 - 122 - 31 \\ &= 27^\circ \end{aligned}$$

$$\textcircled{2} \frac{s}{\sin S} = \frac{r}{\sin R}$$

$$\frac{s}{\sin 31^\circ} = \frac{28 \sin 31^\circ}{\sin 122^\circ}$$

$$s = \frac{28 \sin 31^\circ}{\sin 122^\circ}$$

$$s = 17$$

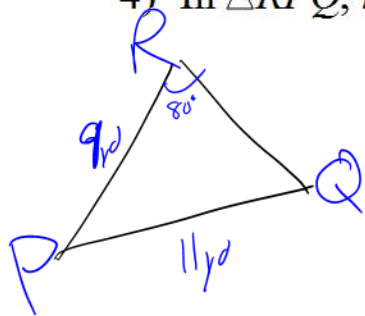
$$\textcircled{3} \frac{t}{\sin T} = \frac{r}{\sin R}$$

$$\frac{t}{\sin 27^\circ} = \frac{28 \sin 27^\circ}{\sin 122^\circ}$$

$$t = \frac{28 \sin 27^\circ}{\sin 122^\circ}$$

$$t = 15$$

4) In $\triangle RPQ$, $m\angle R = 80^\circ$, $q = 9$ yd, $r = 11$ yd



$$\begin{array}{ll} \angle R = 80^\circ & r = 11 \text{ yd} \\ \textcircled{2} \angle P = 46^\circ & \textcircled{3} p = 8 \text{ yd} \\ \textcircled{1} \angle Q = 54^\circ & q = 9 \text{ yd} \end{array}$$

$$\textcircled{1} \frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin Q}{9} = \frac{\sin 80}{11}$$

$$\sin Q = \frac{9 \sin 80}{11}$$

$$\sin Q = 0.80575$$

$$Q = \sin^{-1}(0.80575)$$

$$Q = 54^\circ$$

$$\begin{array}{l} \textcircled{2} \angle P = 180 - \angle R - \angle Q \\ \angle P = 180 - 80 - 54 \\ \angle P = 46^\circ \end{array}$$

$$\textcircled{3} \frac{p}{\sin P} = \frac{r}{\sin R}$$

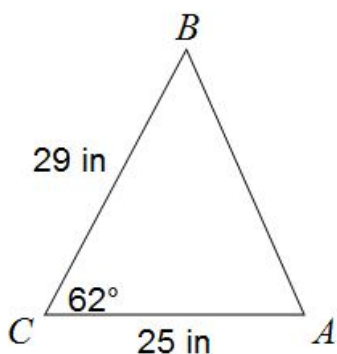
$$\frac{p}{\sin 46} = \frac{11}{\sin 80}$$

$$p = \frac{11 \sin 46}{\sin 80}$$

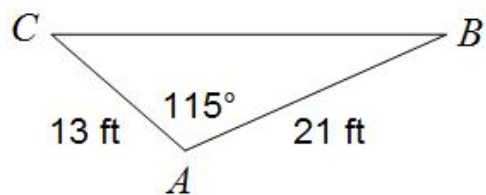
$$p = 8$$

Mathematics 10D 8.4 – Cosine Law

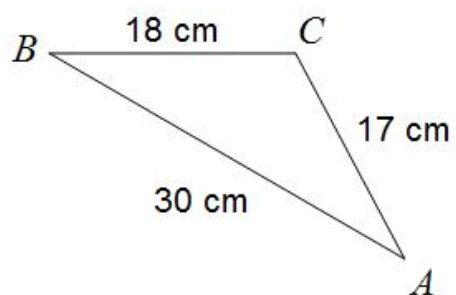
1) Find AB



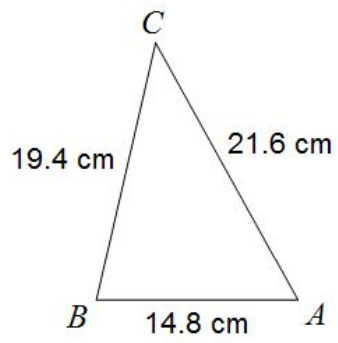
2) Find BC



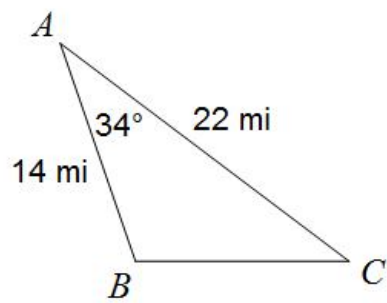
3) Find $m\angle B$



4) Find $m\angle C$



5)



Mathematics 10D 8.5 – Solving Acute Triangle Problems

The roof of a new house must be built to exact specifications so that solar panels can be installed. The long rafters at the front of the house must be inclined at an angle of 26° to the horizontal beam. The short rafters at the back of the house must be inclined at an angle of 66° . The house is 15.3 m wide. Determine the length of the long rafters.

A weather balloon is directly between two tracking stations. The angles of elevation from the two tracking stations are 55° and 68° . If the tracking stations are 20 km apart, determine the altitude of the weather balloon.

The captain of a boat leaves a marina and heads due west for 25 km. Then the captain adjusts the course of his boat and heads $N30^\circ E$ for 20 km. How far is the boat from the marina?