Name: Mr Hagen

Math 10
Unit 1

Systems of Linear Equations

Learning Goals. We are learning to:

- create the graphs which represent given linear equations;
- determine a solution to a linear system graphically;
- explain what the solution to a linear system means;
- determine a solution to a linear system algebraically by substitution;
- determine a solution to a linear system algebraically by elimination; and
- create and solve a linear system which models a given situation (word problem)

What do you remember from previous Math classes?

- Algebra → binomials → trinomials → polynomials
  - → Distributive Property
    $3(x+2)$
    $= 3x+6$
  - → Expanding
    ex: $(x-3)(x+5)$
  - → Factoring $x^2+2x-15$

- Slope = $\frac{\text{Rise}}{\text{Run}}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

- parallel and perpendicular
- Linear Equations
  - $y = mx + b$ → slope intercept form
  - $Ax + By + C = 0$ → Standard Form

Exponent Rules:

$4^2 \times 4^3 = 4^5$ Product rule: add the exponents

$4^6 \div 4^2 = 4^4$ Quotient rule: subtract the exponents

$(3^2)^5 = 3^{10}$ Power rule: multiply exponents

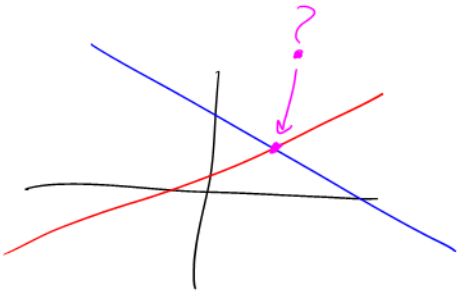
$(65)^0 = 1$ Zero rule: anything to the power of zero is one.

Solving Equations

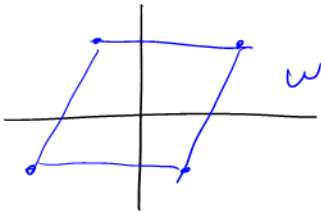
Fractions

In Grade 10:

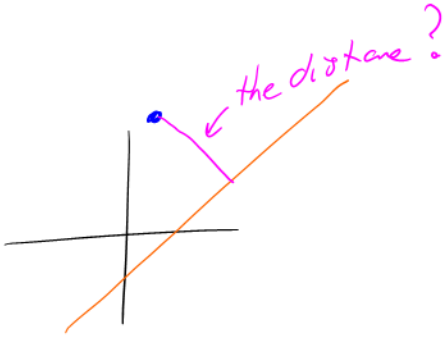
① Solving Linear Systems



② Coordinate Geometry



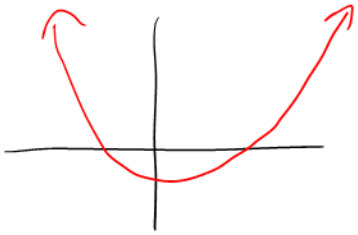
what shape is this ?



③ Trigonometry → the study of triangles.

sin cos tan need a scientific calculator.

④ Quadratics



Algebra: Solving for the Unknown

Consider this to be "undoing" whatever has been done to the unknown, often *x*. Solve for *x* in the following equations.

$-23 = n + (-6)$

$+6 \quad +6$

$-17 = n$

$8v + 4 = 96$

$-4 \quad -4$

$\frac{8v}{8} = \frac{92}{8}$

$v = 11.5$

$3(6a - 8) = -132$

$18a - 24 = -132$

$+24 \quad +24$

$\frac{18a}{18} = \frac{-108}{18} \quad a = -6$

$-4(1 + 8x) - 8x = 196$

$-4 - 32x - 8x = 196$

$-4 \quad -40x = 196$

$+4 \quad +4$

$\frac{-40x}{-40} = \frac{200}{-40} \quad x = -5$

Fractions:

$\left(\frac{x}{1}\right)\left(\frac{y}{z}\right) = \frac{xy}{z}$	$\left(\frac{m}{n}\right)\left(\frac{o}{p}\right) = \frac{mo}{np}$
$\frac{\left(\frac{a}{b}\right)}{\frac{c}{1}} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{bc}$	$\left(\frac{m+n}{q}\right) = \frac{m}{q} + \frac{n}{q}$
$\frac{\left(\frac{a}{b}\right)}{\left(\frac{m}{n}\right)} = \left(\frac{a}{b}\right)\left(\frac{n}{m}\right) = \frac{an}{bm}$	common denominators $\left(\frac{v}{x}\right) + \left(\frac{y}{z}\right) = \frac{vz}{xz} + \frac{xy}{xz} = \frac{vz + xy}{xz}$

$\frac{7}{5} = \frac{n}{2}$

$\frac{14}{10} = \frac{5n}{10}$

$14 = 5n$

$\frac{14}{5} = \frac{5n}{5}$

$2.8 = n$

$\frac{4}{x} = \frac{7}{10}$

$\left(\frac{x}{2}\right)\left(\frac{3}{5}\right) - \frac{x}{3} = 5$

$\frac{3x}{10} - \frac{x}{3} = \frac{5}{1}$

$9x - 10x = 150$

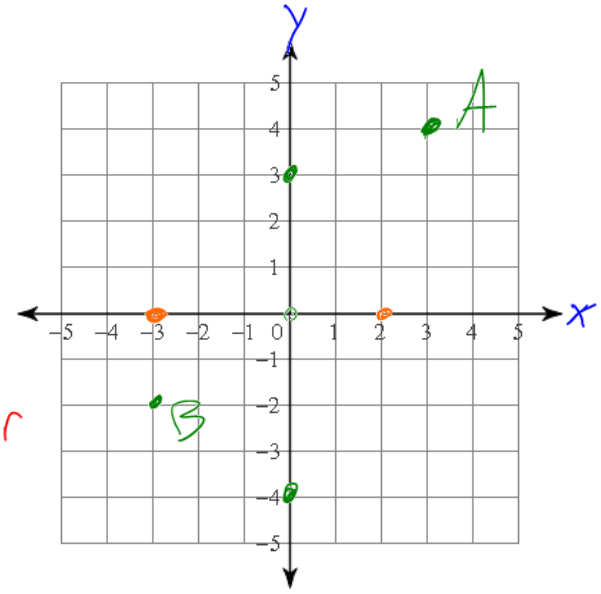
$-1x = 150$

$x = -150$

Unit 1: Systems of Linear Equations

Cartesian Plane: a graph with two number lines called the x-axis and the y-axis

Coordinates: Points on the Cartesian plane.
Ex. Plot the points *A* (3,4) and B (-3,-2)
(x, y)



Intercepts: when the line crosses the x or y axis.
→ y-intercept, (0, #)
→ x-intercept, (#, 0)

Independent Variable
x-value
domain

Dependent Variable
y-value
range

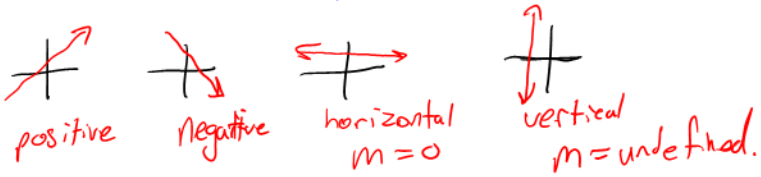
Table of Values: used to organize points.

x	y
2	-3
1	-5
0	-7

Slope: measure of the steepness of a line

Slope Formula: $m = \frac{\text{Rise}}{\text{run}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

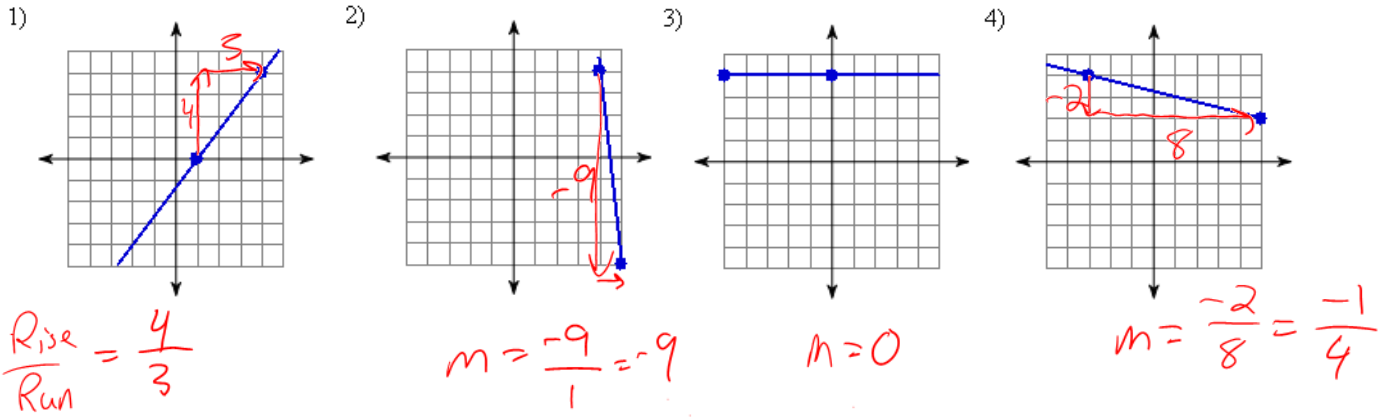


Ex. What is the slope of a line that passes through (-2,5) and (-6, 2)?

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{-6-(-2)} = \frac{-3}{-4} = \frac{3}{4}$

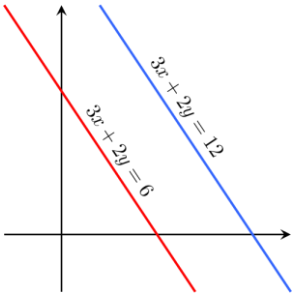
$m = \frac{-3}{-4} = \frac{3}{4}$

Ex. Find the slope of the following lines



$m = \frac{Rise}{Run} = \frac{4}{3}$ $\quad m = \frac{-9}{1} = -9$ $\quad m = 0$ $\quad m = \frac{-2}{8} = \frac{-1}{4}$

What are the slopes of the two lines shown in the graph?

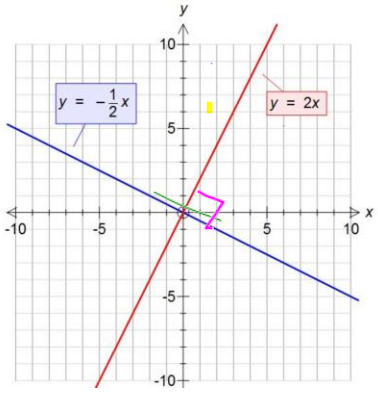


$3x + 2y = 12$ convert to $y =$ $\quad 2y = -3x + 12$ $\quad y = -\frac{3}{2}x + 6$

Parallel Lines have the Same slope

The slope of the line parallel to $y = \left(\frac{2}{3}\right)x + 5$ is $m = \frac{2}{3}$

Perpendicular Lines have negative reciprocal slopes.



Ie. $m_1 = -\frac{1}{2}$ $\quad m_2 = +\frac{2}{1}$ $\quad m = \frac{a}{b}, \; m_1 = \frac{-b}{a}$

The slope of the line perpendicular to $y = \left(\frac{7}{3}\right)x + 4$ is _____

$m = \frac{7}{3}$ $\quad m_1 = \frac{-3}{7}$

Equation of a Line:

Slope-intercept form	*Standard Form*	~~*Point Slope Form*~~
$y = mx + b$ (m: Slope, b: y-int)	$Ax + By + C = 0$ A, B, and C are integers (whole #)	

Steps to Graphing a Linear Equation:

1. Put the equation in $y = mx + b$ form
2. Plot the y-intercept b on the graph
3. Plot the slope m from the b and draw the line

(Other options: Table of Values, x & y intercepts, Standard form x-int = C/A, y-int C/B)

State the slope and y-intercept, then graph the following lines given their equation:

1. $y = 2x - 3$ $m = \frac{2}{1}$ $b = -3$

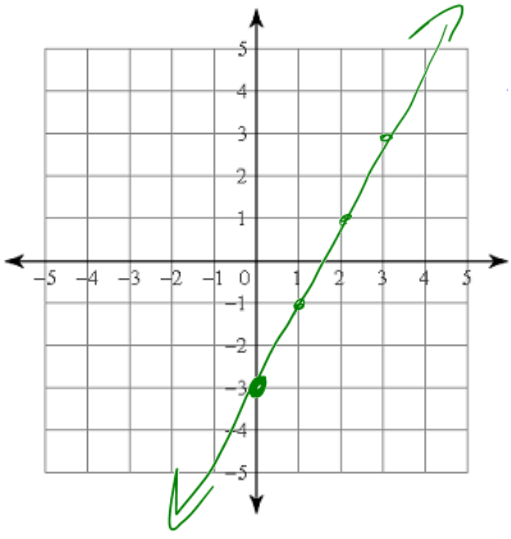
2. $y = -\frac{4}{3}x + 4$ $m = \frac{-4}{3}$ $b = 4$

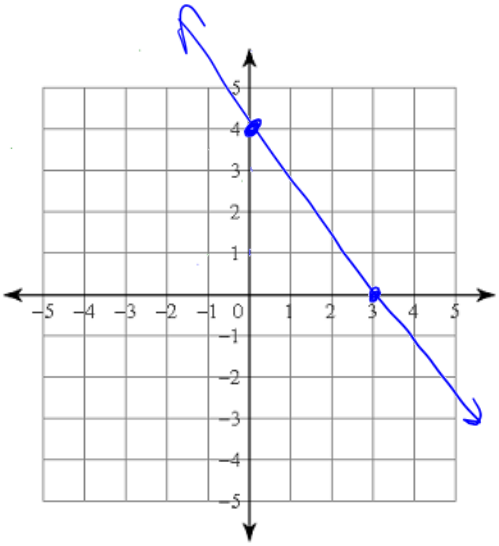
3. $2x - 3y = 6$

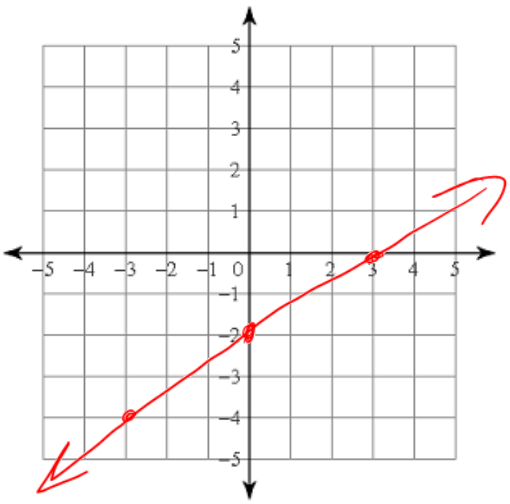
$-3y = -2x + 6$

$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{6}{-3}$

$y = \frac{2}{3}x - 2$ $m = \frac{2}{3}$ $b = -2$



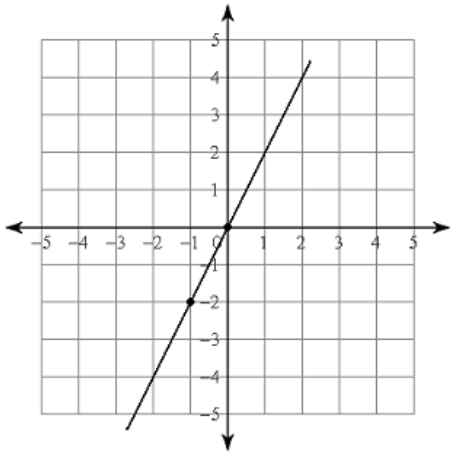


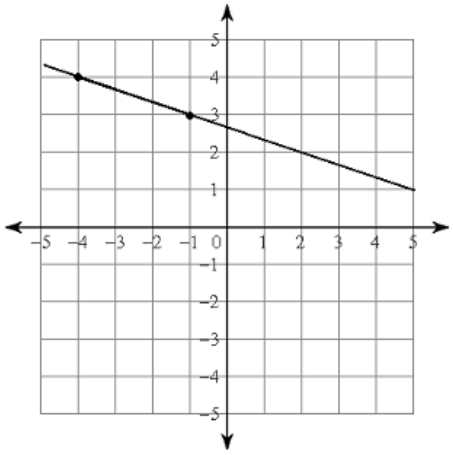


Writing the Equation of a Line

Method 1: Given a graph:

Find the equation of the following lines:



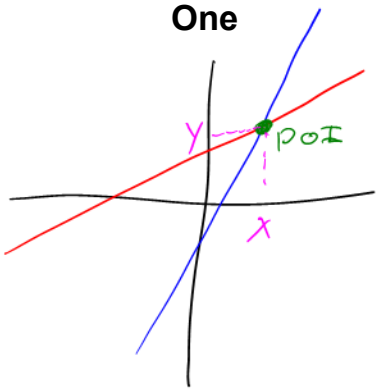
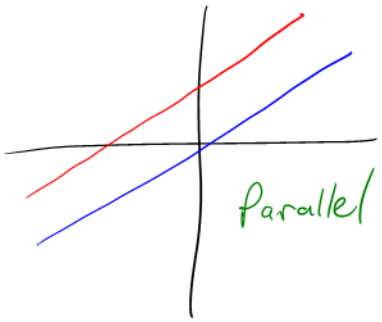
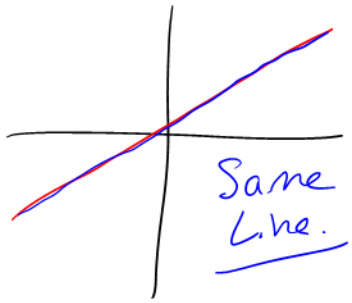


Method 2: When you have 2 points

What is the equation of a line that passes through (-2,5) and (-6, 2)?

Solving Linear Systems

"Solving" a Linear System means finding the solution which is the point at which all the lines intersect. A solution will always satisfy each equation of the Linear System. A Linear System can intersect in 3 ways:

One	None	Infinite
		

We will refer to the solution as the Point of Intersection POI = (x, y)

Method 1: Solve by Graphing:

Steps:

1. **Graph** the 2 linear equations
2. **State the POI** (Point of Intersection) by stating "POI = (x , y)"

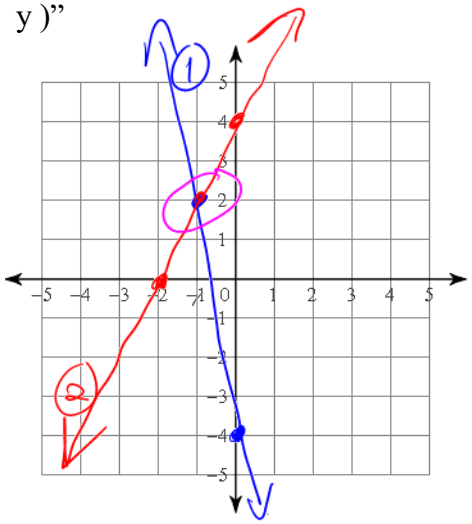
Examples

① $y=-6x-4$ $m=\frac{-6}{1}$ $b=-4$

② $y-2x=4$

$y=2x+4$ $m=\frac{2}{1}$ $b=4$

∴ the POI is $(-1,2)$



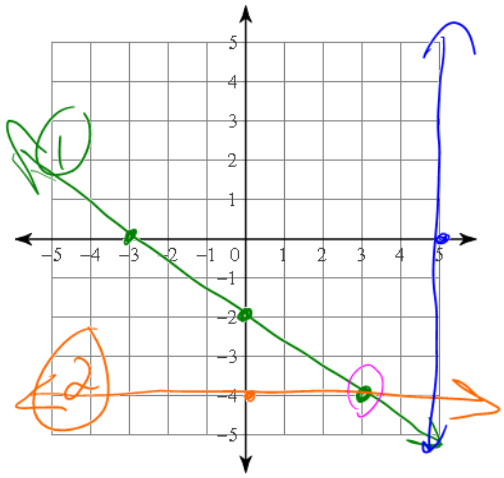
① $y=-\frac{2}{3}x-2$ $m=\frac{-2}{3}$ $b=-2$

② $y=-4$ → Horizontal Line

$m=0$

$b=-4$

∴ the POI is $(3,-4)$



$x=5$

Vertical

Method 2: Solve by Substitution:

Steps:

1. Isolate a variable in one equation. Look for a variable with no coefficient. Let's call this equation one.
2. Substitute the first equation into the varible of the second equation
3. Solve for the remaining variable, then use that to solve for the second variable.
4. State the POI

Examples

① $x = -6y - 11$
② $x = y + 3$

Sub $-6y-11$ into the x of equation 2

$-6y - 11 = y + 3$
$+11 \quad +11$
$-y \quad -y$

$-7y = 14$
$\frac{-7y}{-7} = \frac{14}{-7}$
$y = -2$

$x = -2 + 3$
$x = 1$

∴ the POI is $(1, -2)$ (x, y)

① $2x + y = -15$
② $3x - y = -15$

$\Rightarrow$ ① $y = -15 - 2x$ or $y = -2x - 15$

$3x - (-15 - 2x) = -15$
$3x + 15 + 2x = -15$
$5x + 15 = -15$
$-15 \quad -15$
$5x = -30$
$\frac{5x}{5} = \frac{-30}{5}$
$x = -6$

$y = -15 - 2(-6)$
$y = -15 + 12$
$y = -3$

∴ POI is $(-6, -3)$

① $y - 3x = 5$
② $y + x = 3$

$\Rightarrow y = 5 + 3x$ ①

$(5 + 3x) + x = 3$
$5 + 4x = 3$
$4x = -2$
$\frac{4x}{4} = \frac{-2}{4}$
$x = -\frac{1}{2}$

$y = 5 + 3\left(-\frac{1}{2}\right)$
$y = \frac{5}{1} - \frac{3}{2}$
$y = \frac{10}{2} - \frac{3}{2}$
$y = \frac{7}{2}$

∴ The POI is $\left(-\frac{1}{2}, \frac{7}{2}\right)$

① $4x + y = 11$
② $x + 2y = 8$

$y = 11 - 4x$

$x + 2(11 - 4x) = 8$
$x + 22 - 8x = 8$
$-7x + 22 = 8$
$-22 \quad -22$
$-7x = -14$
$\frac{-7x}{-7} = \frac{-14}{-7}$
$x = 2$

$y = 11 - 4(2)$
$y = 11 - 8$
$y = 3$

∴ POI is $(2, 3)$

Method 3: Solve by Elimination:

Steps:

1. Arrange the equations into $Ax + By = C$ form.
2. Same. Use multiplication to make the coefficients on the x or y the same.
3. Eliminate Add or subtract the equations together, thus eliminating the variable that you made the same.
4. Solve
5. State the POI

$3+(-3)=0$ $\quad 3-(-3)=6$ $\quad 3-3=0$

Examples

$4x + 3y = -15$
$+\ 1x - 3y = 0$
$5x = -15$
$\frac{5x}{5} = \frac{-15}{5}$
$x = -3$

$-3 - 3y = 0$
$-3y = 3$
$\frac{-3y}{-3} = \frac{3}{-3}$
$y = -1$

∴ PoI is $(-3, -1)$

$-5x + 3y = -1$
$-(-7x + 3y = -11)$
$2x = 10$
$\frac{2x}{2} = \frac{10}{2}$
$x = 5$

$-5(5) + 3y = -1$
$-25 + 3y = -1$
$+25 \quad +25$
$\frac{3y}{3} = \frac{24}{3}$
$y = 8$

POI is $(5, 8)$

$5(3x+4y=-6)$
$3(5x+6y=-8)$
$15x + 20y = -30$
$-(15x + 18y = -24)$
$2y = -6$
$\frac{2y}{2} = \frac{-6}{2}$
$y = -3$

$3x + 4(-3) = -6$
$3x - 12 = -6$
$\frac{3x}{3} = \frac{6}{3}$
$x = 2$

∴ POI is $(2, -3)$

$4x = -2y + 24$
$-x - 3y = -21$

① $4x + 2y = 24$
② $4(-x - 3y = -21)$

① $4x + 2y = 24$
② $+ -4x - 12y = -84$
$-10y = -60$
$\frac{-10y}{-10} = \frac{-60}{-10}$
$y = 6$

$4x = -2(6) + 24$
$4x = -12 + 24$
$\frac{4x}{4} = \frac{12}{4}$
$x = 3$

The POI is $(3, 6)$

Unit 1 - Linear Systems: Success Criteria

1. I can determine the best method to use to solve the system.

a) I can solve by Graphing: Usually used for applications comparing two scenarios.
To be successful:

1. use an appropriate scale
2. use a ruler label: axes, equations scale, pencil .
3. Be able to interpret the graph to state the solution.

b) I can solve By Substitution:
To be successful:

1. Rewrite one equation to Isolate "x" or "y".
2. Substitute the new equation into the other equation.
3. Solve for one variable,
4. Substitute found value in #3 and solve for the other.

c) I can solve By Elimination:
To be successful:

1) Set up the equations so one of the variables have opposite coefficients.
2) Add the equations together to eliminate one variable and solve for the other.
3) Substitute found value in #2 and solve for the other variable.

2. I can clearly state the solution POI = (,).
3. I can check the solution.

To create a linear system: (Application Word problems)

1. I can Interpret the question to Identify the unknown variables (x and y)
2. I can Interpret the question to create 2 equations to represent the information in the question in one of 3 formats:
   a. y=mx+b
   b. x + y = total (sum)
   c. _____ x + _____ y = _____ total where _____ is a #, $ or %
      i. $4x + $2y = $ 250
      ii. 13% x + 35% y =25% total
3. I can Solve the system using graphing, substitution or elimination (see above criteria)
4. I can state the solution, ensuring that it answers the question.
5. I can check the solution.

If you are struggling one area, check the video list posted on Edsby to review that concept.