Math 10D – Unit 2: Analytic Geometry **Name:**_____

Homework 2.4 – Verifying Figures **Date:**_____

1. Show that the line segment joining points $P(1,4)$ and $Q(5,5)$ is parallel to the line segment joining points $R(3,-4)$ and $S(7,-3)$.

same slope

$m_{PQ} = \frac{5-4}{5-1} = \frac{1}{4}$ $m_{RS} = \frac{-3-(-4)}{7-3} = \frac{1}{4}$

$m_{PQ} = m_{RS}$ ∴ parallel

2. Show that TU, $T(-1,7)$ and $U(3,5)$, is perpendicular to VW, $V(-4,1)$ and $W(-1,7)$.

$m_{TU} = \frac{5-7}{3-(-1)} = \frac{-2}{4} = \frac{-1}{2}$

$m_{VW} = \frac{7-1}{-1-(-4)} = \frac{6}{3} = \frac{2}{1}$

these are the negative reciprocals of each other AND $\frac{-1}{2} \times \frac{2}{1} = \frac{-2}{2} = -1$!!

∴ TU and VW are perpendicular.

3. Prove what type of triangle is formed by $J(2,5)$, $K(5,-2)$, and $L(-1,-2)$.

First sides

$d_{JK} = \sqrt{58} = 7.62$

$d_{KL} = \sqrt{36} = 6$

$d_{LJ} = \sqrt{58} = 7.62$

2 sides equal,

∴ Isosceles.

Second Angles (slopes)

$m_{JK} = \frac{-7}{3}$

$m_{KL} = 0$

$m_{LJ} = \frac{7}{3}$

no sides perpendicular

∴ not a right isosceles.

4. Show that triangle ABC is a right scalene triangle with points $A(-2,2)$, $B(-1,-2)$, and $C(7,0)$.

$d_{AB} = \sqrt{17}$

$d_{BC} = \sqrt{68}$

$d_{CA} = \sqrt{85}$

All sides different,

$\therefore$ scalene

$m_{AB} = \frac{-2}{9}$

$m_{BC} = \frac{1}{4}$

$m_{CA} = \frac{-4}{1}$

Negative reciprocals!

$\therefore$ perpendicular.

OR

$\sqrt{17}^2 + \sqrt{68}^2 = \sqrt{85}^2$

$17 + 68 = 85$

$85 = 85$

works in Pythagorean Theorem!!

Therefore this is a right scalene triangle

5. A polygon is defined by points $A(-5,1)$, $B(5,3)$, $C(2,-1)$, and $D(-8,-3)$. Show that the polygon is a parallelogram.

First Sides

$d_{AB} = \sqrt{104}$

$d_{BC} = \sqrt{25}$

$d_{CD} = \sqrt{104}$

$d_{DA} = \sqrt{25}$

opposite sides equal, so this is a rectangle or parallelogram

Second slopes for "angles"

$m_{AB} = \frac{1}{5}$

$m_{BC} = \frac{4}{3}$

$m_{CD} = \frac{1}{5}$

$m_{DA} = \frac{4}{3}$

opposite sides parallel and consecutive slopes are not perpendicular. (no 90°)

∴ a parallelogram

6. Determine the type of quadrilateral described by $P(-5,1)$, $Q(3,3)$, $R(4,-1)$, and $S(-4,-3)$.

First Sides:

$d_{PQ} = \sqrt{68}$

$d_{QR} = \sqrt{17}$

$d_{RS} = \sqrt{68}$

$d_{SP} = \sqrt{17}$

opposite sides equal, therefore rectangle or parallelogram

Second angles (slope)

$m_{PQ} = \frac{1}{4}$

$m_{QR} = \frac{-4}{1}$

> perpendicular, or 90° angle.

$m_{RS} = \frac{1}{4}$

$m_{SP} = \frac{-1}{4}$

∴ PQRS is a rectangle!