A(-6,5), B(-1,3), C(1,-8)

Centroid:

1. Midpoint of AB and BC (you can also do AC)

$$M_{AB} = (\frac{-6+-1}{2}, \frac{5+3}{2}) = (\frac{-7}{2}, 4) \qquad M_{BC} = (\frac{-1+1}{2}, \frac{3+-8}{2}) = (0, \frac{-5}{2})$$

2. Slope from C to M_{AB} and A to M_{BC}

$$\begin{aligned} m_{CM_{AB}} &= \frac{-8-4}{1+\frac{7}{2}} \\ &= \frac{-12}{\frac{9}{2}} \\ &= \frac{-8}{3} \end{aligned} \qquad \begin{aligned} m_{AM_{BC}} &= \frac{5+\frac{5}{2}}{-6+0} \\ &= \frac{\frac{15}{2}}{-6} \\ &= \frac{-5}{4} \end{aligned}$$

3. Equation of the Medians using the above slopes and their opposite vertex.

Use C(1,-8)

$$\begin{aligned} y &= mx+b \\ -8 &= \frac{-8}{3}(1)+b \\ -8+\frac{8}{3} &= b \\ \frac{-16}{3} &= b \\ \therefore y &= \frac{-8}{3}x - \frac{16}{3} \end{aligned}$$

Use A(-6,5)

$$\begin{aligned} y &= mx+b \\ 5 &= \frac{-5}{4}(-6)+b \\ 5-\frac{15}{2} &= b \\ \frac{-5}{2} &= b \\ \therefore y &= \frac{-5}{4}x - \frac{5}{2} \end{aligned}$$

4. Find PoI via substitution

$$\begin{aligned} \frac{-8}{3}x - \frac{16}{3} &= \frac{-5}{4}x - \frac{5}{2} \qquad \text{Multiply by 12} \\ -32x - 64 &= -15x - 30 \\ -17x &= 34 \\ x &= -2 \\ y &= \frac{-5}{4}(-2) - \frac{5}{2} \\ y &= \frac{-5}{2} - -\frac{5}{2} \\ y &= 0 \end{aligned}$$

$\therefore$ the centroid is (-2,0).