

Mathematics 10D

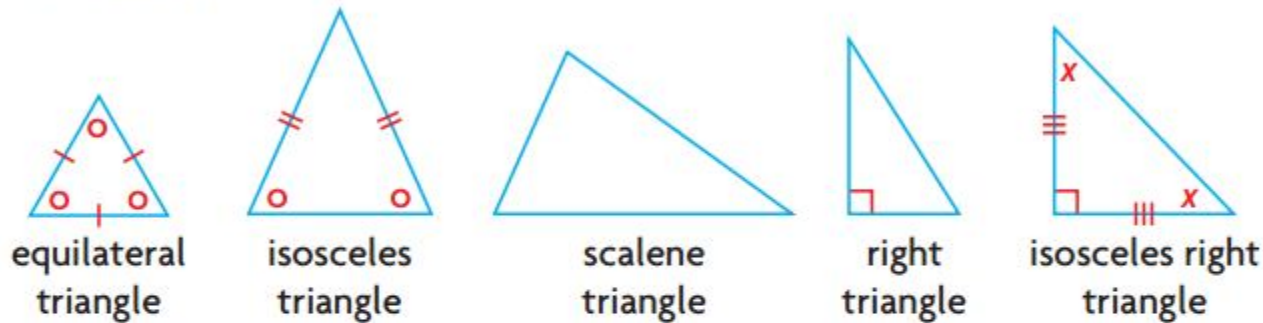
2.5 – Verifying Properties of Geometric Figures

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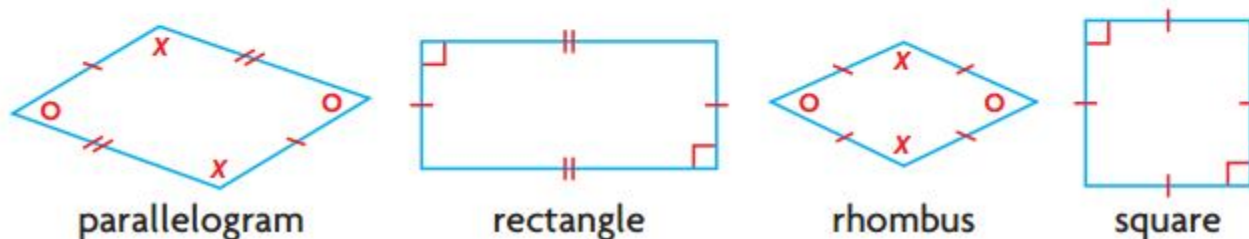
Things you must know and understand for the rest of this unit:

1. Midpoint
2. Distance
3. Slope – understand parallel and perpendicular
4. Write linear equations
5. Find point of intersection
6. Terms – median, vertex, line segment, midsegment, etc...
7. “Using **analytic geometry**” – geometry that uses an xy grid, algebra and equations to describe relations and solve problems related to geometric figures
8. Shapes

Triangles



Quadrilaterals



Example 1: Show that the midsegments of the quadrilateral, with vertices at P(-7,9), Q(9,11), R(9,-1) and S(1,-11), form a parallelogram.

① Calculate the 4 midpoints

$$M_{PQ} = A\left(\frac{-7+9}{2}, \frac{9+11}{2}\right) \\ = A(1, 10)$$

$$M_{QR} = B(9, 5)$$

$$M_{RS} = C(5, -6)$$

$$M_{SP} = D(-3, -1)$$

② calculate the 4 distances

$$d_{AB} = \sqrt{(1-9)^2 + (10-5)^2} \\ = \sqrt{(-8)^2 + (5)^2} \\ = \sqrt{89} = 9.43$$

$$d_{BC} = 11.7$$

$$d_{CD} = 9.43$$

$$d_{DA} = 11.7$$

③ calculate 4 slopes

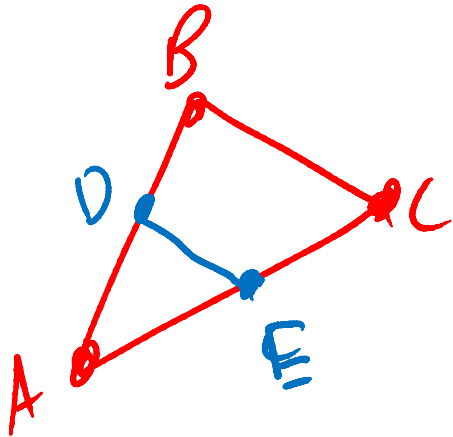
$$m_{AB} = \frac{10-5}{1-9} = \frac{5}{-8} = -0.63$$

$$m_{BC} = 2.75 = \frac{11}{4}$$

$$m_{CD} = \frac{-5}{8} = -0.63$$

$$m_{DA} = 2.75 = \frac{11}{4}$$

Example 2: A triangle has vertices at A(-2,2), B(1,3) and C(4,-1). Show that the midsegment joining the midpoints of AB and AC is parallel to BC and half its length.



① Find midpoints

$$M_{AB} = D(-0.5, 2.5)$$

$$M_{AC} = E(1, 0.5)$$

② Calculate distances

$$\begin{aligned} d_{DE} &= \sqrt{(1 - -0.5)^2 + (0.5 - 2.5)^2} \\ &= \sqrt{2.25 + 4} \\ &= \sqrt{6.25} = 2.5 \end{aligned}$$

$$d_{BC} = 5$$

③ calculate slopes

$$\begin{aligned} m_{DE} &= \frac{0.5 - 2.5}{1 - -0.5} = \frac{-2}{1.5} \\ &= -1.33 \\ &= -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} m_{BC} &= -1.33 \\ &= -\frac{4}{3} \end{aligned}$$

Example 3: Show that points A(10,5) and B(2,-11) lie on the circle with equation $x^2 + y^2 = 125$. Also show that the perpendicular bisector of **chord** AB passes through the centre of the circle.

① $x^2 + y^2 = 125$

$$10^2 + 5^2 = 125$$

$$100 + 25 = 125$$

$$125 = 125$$

✓

$$2^2 + (-11)^2 = 125$$

$$4 + 121 = 125$$

$$125 = 125$$

✓

② Equation of the perpendicular bisector

$$1. m_{AB} = \frac{-11-5}{2-10} = \frac{-16}{-8} = 2$$

$$m_{\perp} = -\frac{1}{2}$$

$$2. M_{AB} \left(\frac{10+2}{2}, \frac{5+(-11)}{2} \right)$$

$$M_{AB} (6, -3)$$

$$3. y = m_{\perp}x + b$$

$$-3 = -\frac{1}{2}(6) + b$$

$$-3 = -3 + b$$

$$0 = b$$

$$\text{origin} = (0,0)$$

$$y = -\frac{1}{2}(0)$$

$$y = 0$$

$\therefore y = -\frac{1}{2}x$, thus showing that the y-intercept is 0