Mathematics 10D

Q.05 – Completing the Square

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Binomial Perfect Square:

$(x + 3)^2 = (x+3)(x+3) = x^2 + 6x + 9$

$(x + 5)^2 = x^2 + 10x + 25$

$(x + \underline{7})^2 = \boxed{x^2 + 14x + \underline{49}}$

$\dfrac{14}{2} = 7$

$(x - \underline{4})^2 = x^2 - 8x + \underline{16}$

$\dfrac{8}{2} = 4$

$(x + \underline{17.5})^2 = x^2 + 35x + \underline{306.25}$

$\dfrac{35}{2}$

Convert from Standard form to Vertex form by completing the square, then state the vertex.

$$y = 2x^2 + 12x - 5$$

① Factor a from first two terms.

$$y = 2(x^2 + 6x + 0) - 5$$

a b c

② $\left(\frac{b}{2}\right)^2 = c$

$$\left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

$$y = 2(x^2 + 6x + 9 - 9) - 5$$

Perfect Square
$$(x + \underline{})^2$$

③ multiply a with -9

④ Factor the P.S. and Simplify

$$y = 2(x^2 + 6x + 9) - 5 - 18$$

$$y = 2(x + 3)^2 - 23$$

$$\therefore (-3, -23)$$

Let's do another!

$$y = -5x^2 - 40x + 71$$

-5

$$y = -5(x^2 + 8x + 0) + 71$$

$$\left(\frac{8}{2}\right)^2 = (4)^2 = 16$$

$$y = -5(x^2 + 8x + 16 - 16) + 71$$

$$y = -5(x^2 + 8x + 16) + 71 + 81$$

$$y = -5(x + 4)^2 + 151$$

$$\therefore (-4, 151) \text{ is the vertex}$$

What happens when *a* is a fraction?

$$y = \frac{1}{2}x^2 - 5x + 14$$

$$y = \frac{1}{2}(x^2 - 10x + 0) + 14$$

$$\left(\frac{10}{2}\right)^2 = (5)^2 = 25$$

$$y = \frac{1}{2}(x^2 - 10x + 25 - 25) + 14$$

$$y = \frac{1}{2}(x - 5)^2 + 1.5$$

∴ vertex is $(5, 1.5)$

$$\left(5, \frac{3}{2}\right)$$

Uh oh, we have decimals! No worries!

$$y = \underline{2.84x^2 - 8.23x} + 5.4$$
$$2.84$$

$$y = 2.84(x^2 - 2.9x + 0) + 5.4 \qquad \left(\frac{2.9}{2}\right)^2 = (1.45)^2$$
$$= 2.1025$$

$$y = 2.84(x^2 - 2.9x + 2.1025(-2.1025)) + 5.4 \underline{-5.97}$$

$$y = 2.84(x - 1.45)^2 - 0.57$$

$$\therefore \text{ vertex is } \left(1.45, -0.57\right)$$