

Mathematics 10D

Q.09 – Quadratic Word Problems

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Students really dislike word problems, no matter what type they are. Quadratic word problems come with the same disdain. There are two types of word problems: equations given and equations not given. However, no matter the situation, once you have an equation, there are 4 things you can do:

1. You are given an x value. Plug it in and work it out.
2. You are asked, in some way, to find the zeros. Solve it by whichever method works.
3. You are given a y value. Plug it into the y , then bring it over to the other side, then solve it using whichever method works.
4. Find the maximum or minimum. Complete the Square, or if you have the zeros, find the axis of symmetry, then plug that into the original equation.

None of this is new! You just need to apply the skills you have acquired over the past few weeks!

Let's do 4 examples. Two with equations given, 2 without.

1. An automated hose on a tower sprays water on a forest fire. The height of the water, h , in metres, can be modelled by the relation $h = -2.25x^2 + 4.5x + 6.75$, where x is the horizontal distance of the water from the hose, in metres.

a) What is the maximum height of the water?

$$h = -2.25(x^2 - 2x + 0) + 6.75$$

$$h = -2.25(x^2 - 2x + 1 - 1) + 6.75$$

$$h = -2.25(x^2 - 2x + 1) + 6.75 + 2.25$$

$$h = -2.25(x-1)^2 + 9$$

$\therefore$ the max height is 9m after 1 metre;
(1, 9)

b) ~~When will the water hit the ground?~~

How far will the water be from the tower when it hits the ground?

$$0 = -2.25(x-1)^2 + 9$$

$$-9 = -2.25(x-1)^2$$

$$\sqrt{4 = (x-1)^2}$$

$$\pm 2 = x - 1$$

$$\pm 2 + 1 = x$$

$$\textcircled{+} 2 + 1 = x$$

$$3 = x$$

$$\textcircled{+} -2 + 1 = x$$

$$-1 = x$$

$\therefore$ the water hits the ground 3m from the tower.

2. A person throws a ball from a roof of a building. The relation $h = -5t^2 + 20t + 12$ models the height of the ball, in metres, and the time, in seconds.

a) What is the height of the building?

Before ball is thrown, $t=0$. $\therefore h = -5(0)^2 + 20(0) + 12$

$$h = 12$$

the y-int!

b) How high will the ball be after one second?

$$h = -5(1)^2 + 20(1) + 12$$

$$h = -5 + 20 + 12 = 27$$

= the ball is 27m after one second.

c) When will the ball hit the ground?

$$0 = -5t^2 + 20t + 12$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-5)(12)}}{2(-5)}$$

$$t = \frac{-20 \pm \sqrt{640}}{-10}$$

$$t = \frac{-20 \pm 25.3}{-10}$$

$$[h=0, t=??]$$

$$\oplus t = \frac{-20 + 25.3}{-10} = -0.53$$

$$\ominus t = \frac{-20 - 25.3}{-10} = 4.53$$

$\therefore$ The ball hits the ground after 4.53 seconds

2. A person throws a ball from a roof of a building. The relation $h = -5t^2 + 20t + 12$ models the height of the ball, in metres, and the time, in seconds.

d) What is maximum height of the ball?

① Complete the Square

$$AoS = \frac{-0.53 + 4.53}{2} = \frac{4}{2} = 2$$

② Use the zeros

$$h = -5(2)^2 + 20(2) + 12$$

$$h = -20 + 40 + 12 = 32$$

∴ the max height of the ball is 32m.

e) How long will the ball be above 20 metres?

$$20 = -5t^2 + 20t + 12$$

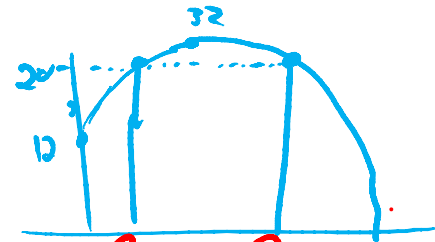
$$0 = -5t^2 + 20t - 8$$

$$t = \frac{-20 \pm \sqrt{20^2 - 4(-5)(-8)}}{2(-5)}$$

$$t = \frac{-20 \pm \sqrt{240}}{-10}$$

$$t = \frac{-20 \pm 15.5}{-10}$$

$$h = 20, t = ?$$



$$+ t = \frac{-20 + 15.5}{-10} = 0.45$$

$$- t = \frac{-20 - 15.5}{-10} = 3.55$$

The duration is $3.55 - 0.45 = 3.1$ seconds

3. The length of a rectangle is one more than two times the width. If the area of the rectangle is 136, what are the dimensions of the rectangle?

$$\boxed{\begin{matrix} 2x+1 \\ A=136 \end{matrix}} x$$

$$A = lw$$

$$136 = (2x+1)(x)$$

$$136 = 2x^2 + x$$

$$0 = \underset{a}{2}x^2 + \underset{b}{1}x - \underset{c}{136}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-136)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{1089}}{4}$$

$$x = \frac{-1 \pm 33}{4}$$

$$\oplus x = \frac{-1+33}{4} = \frac{32}{4} = 8$$

$$\ominus x = \frac{-1-33}{4} = \frac{-34}{4}$$

$\therefore$ width is
8m,
length is
17m.

4. The ~~sum~~ product of two consecutive numbers is 156. Find the numbers (without guess and check!)

Consecutive = in a row.

Let $x = \text{Num 1}$

$x+1 = \text{Num 2}$

$$x(x+1) = 156$$

$$x^2 + x = 156$$

$$x^2 + x - 156 = 0$$

$$(x+13)(x-12) = 0$$

$$\therefore x = -13 \text{ and } x = 12$$

$\therefore$ the two consecutive numbers are
12, 13
-13, -12