

3.4

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$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

$$7^2 = 49$$

$$8^2 = 64$$

$$9^2 = 81$$

$$10^2 = 100$$

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$$11^2 = 121$$

$$12^2 = 144$$

$$13^2 = 169$$

$$14^2 = 196$$

$$15^2 = 225$$

$$16^2 = 256$$

$$17^2 = 289$$

$$18^2 = 324$$

$$19^2 = 361$$

$$20^2 = 400$$

Simplifying Radicals:

1. $2\sqrt{48}$

$$\begin{aligned} &= 2\sqrt{16 \times 3} \\ &= 2\sqrt{16}\sqrt{3} \\ &= 2(4)\sqrt{3} = 8\sqrt{3} \end{aligned}$$

3. $\frac{1}{8}\sqrt{320}$

$$\begin{aligned} &= \frac{1}{8}\sqrt{64}\sqrt{5} \\ &= \frac{1}{8}(8)\sqrt{5} \\ &= \sqrt{5} \end{aligned}$$

2. $-\sqrt{20}$

$$\begin{aligned} &= -\sqrt{4}\sqrt{5} \\ &= -2\sqrt{5} \end{aligned}$$

4. $-3\sqrt{513}$

$$\begin{aligned} &= -3\sqrt{9}\sqrt{57} \\ &= -3(3)\sqrt{57} \\ &= -9\sqrt{57} \end{aligned}$$

$$\frac{513}{9} = 57$$

Simplifying Radicals with variables:

$$\begin{aligned} 1. \quad & 7\sqrt{288b^4} \\ &= 7\sqrt{144b^4} \sqrt{2} \\ &= 7(12b^2)\sqrt{2} \\ &= 84b^2\sqrt{2} \end{aligned}$$

$$\begin{aligned} 2. \quad & -5\sqrt{45n^3} \\ &= -5\sqrt{9n^2} \sqrt{5n} \\ &= -5(3n)\sqrt{5n} \\ &= -15n\sqrt{5n} \end{aligned}$$

$$\begin{aligned} & \sqrt{x^{23}} \\ &= \sqrt{x^{22} x^1} \\ &= x^{11} \sqrt{x} \end{aligned}$$

$$\begin{aligned} & \sqrt{x^3} \\ &= \sqrt{x^2 x^1} \\ &= x\sqrt{x} \end{aligned}$$

$$\begin{aligned} & \sqrt{x^2} = x \\ & \sqrt{x^4} = x^2 \\ & \sqrt{x^{100}} = x^{50} \end{aligned}$$

Adding and Subtracting Radicals:

① Simplify

$$\begin{aligned} 1. & -2\sqrt{12} - 3\sqrt{8} + 3\sqrt{32} + 2\sqrt{27} \\ & = -2\sqrt[2]{4}\sqrt{3} - 3\sqrt[2]{4}\sqrt{2} + 3\sqrt[2]{16}\sqrt{2} + 2\sqrt[3]{9}\sqrt{3} \\ & = \underline{-4\sqrt{3}} - \underline{6\sqrt{2}} + \underline{12\sqrt{2}} + \underline{6\sqrt{3}} \\ & = 2\sqrt{3} + 6\sqrt{2} \end{aligned}$$

$$\boxed{2x + 3x = 5x}$$

$$\begin{aligned} 2. & 2\sqrt{45} - \sqrt{8} - 2\sqrt{32} - 2\sqrt{18} \\ & = 2\sqrt{9}\sqrt{5} - \sqrt{4}\sqrt{2} - 2\sqrt{16}\sqrt{2} - 2\sqrt{9}\sqrt{2} \\ & = 6\sqrt{5} - \underline{2\sqrt{2} - 8\sqrt{2} - 6\sqrt{2}} \\ & = 6\sqrt{5} - 16\sqrt{2} \end{aligned}$$

Multiplying Radicals:

$$1. \quad 2\sqrt{6} \times 4\sqrt{5}$$

(Handwritten: blue arrow from 2 to 4, red arrow from 6 to 5)

$$= 8\sqrt{30}$$

$$2. \quad -3\sqrt{10} \times 6\sqrt{2}$$

(Handwritten: blue arrow from -3 to 6, blue arrow from 10 to 2)

$$= -18\sqrt{20}$$
$$= -18\sqrt{4}\sqrt{5} = -36\sqrt{5}$$

$$3. \quad (\sqrt{2} + \sqrt{3})(2\sqrt{2} - 5\sqrt{3})$$

(Handwritten: blue arrows from 1 to 2 and 1 to 5; red numbers 2 and 3 above 4 and 9 respectively)

$$= 2\sqrt{4} - 5\sqrt{6} + 2\sqrt{6} - 5\sqrt{9}$$
$$= 4 - 3\sqrt{6} - 15$$
$$= -11 - 3\sqrt{6}$$

Rationalizing the Denominator:

$$1. \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$2. \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{24}}{3}$$

$$= \frac{\sqrt{4} \sqrt{6}}{3} \\ = \frac{2\sqrt{6}}{3}$$

$$3. \frac{9\sqrt{63}}{7\sqrt{54}}$$

$$= \frac{9\cancel{\sqrt{9}}\sqrt{7}}{7\cancel{\sqrt{9}}\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$$

$$= \frac{9\sqrt{42}}{7(6)} = \frac{9\sqrt{42}}{42} = \frac{3\sqrt{42}}{14}$$

$$4. \frac{7+\sqrt{2}}{5\sqrt{40}}$$