Recap 4.2 and 4.3

4.2 → Properties of Exponents.

Bases must be the same!!

① Product Rule

$$4^2 \times 4^5 = 4^{2+5} = 4^7$$

$$a^m \times a^n = a^{m+n}$$

② Quotient Rule.

$$\frac{4^5}{4^2} = 4^{5-2} = 4^3$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$\frac{4^2}{4^5} = \frac{1}{4^3}$$

There are more 4's in the denominator!

③ Power Rule.

$$(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4^6$$

$$(a^m)^n = a^{mn}$$

$$(4^{-3})^{-5} = 4^{15}$$

④ Zero Rule:

$4^0 = 1$ $\quad 1{,}000{,}000^0 = 1$ $\quad \text{Elephant}^0 = 1$ $\quad 0^0 = 1$ or undefined.

$$\left(\frac{w^4-8}{3x+4}\right)^0 = 1$$

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⑤ Negative Exponents

- a negative exponent means the reciprocal (or inverse)

ex: $4^{-2} = \frac{1}{4^2}$ | $\frac{1}{3^{-3}} = 3^3$ | $\frac{2}{3^{-3}} = 2 \times 3^3$

$\frac{2^{-2}}{3^{-3}} = \frac{3^3}{2^2}$ | $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2}$

↳ $\frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2}$

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ex: $\frac{3^2 \times 3^{-4}}{3^5} = \frac{3^{-2}}{3^5} = 3^{-2-5} = 3^{-7} = \frac{1}{3^7}$

$$\left(\frac{(2^3)^4(2^{-1})^0}{(2^2)(2^{-3})}\right)^{-2} = \left(\frac{2^{12}}{2^{-1}}\right)^{-2} = \left(2^{13}\right)^{-2}$$

$12 - -1$

$$= 2^{-26}$$

$$= \frac{1}{2^{26}}$$

$$\left(\frac{(3^{-1})^{-4}(3^2)^3}{(3^2)(3^1)^4}\right)^2 = \left(\frac{3^4 \times 3^6}{3^6}\right)^2 = \left(\frac{3^{10}}{3^6}\right)^2 = (3^4)^2$$

$$= 3^8$$

4.3 Rational Exponents

ex: $\dfrac{4^{0.2} \times 4^{0.3}}{4^{1.5}} = \dfrac{4^{0.5}}{4^{1.5}} = 4^{-1} = \dfrac{1}{4}$

$$\frac{4^{1/5} \times 4^{3/10}}{4^{3/2}} = \frac{4^{2/10} \times 4^{3/10}}{4^{15/10}} = \frac{4^{5/10}}{4^{15/10}} = 4^{-10/10} = 4^{-1} = \frac{1}{4}$$

$$9^{1/2} = 3 \qquad \sqrt{9} = 3 \qquad \sqrt{9} = 9^{1/2}$$

why? solve

$$x^2 = 9$$

$$(x^2)^{1/2} = 9^{1/2}$$

$$x = 3$$

$$\sqrt[3]{64} = 64^{1/3} = 4$$

$\sqrt[3]{\ } = \_\_ \times \_\_ \times \_\_$

on calc: $64 \wedge (1/3)$

$\sqrt[4]{16} = 16^{1/4} = 2$

$\sqrt[m]{a} = a^{\frac{1}{m}}$

$32^{1/5} = \sqrt[5]{32}$

$\sqrt[3]{9^6} = 9^{6/3} = 9^2 = 81$

$(\sqrt[3]{9})^6$ ↗

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ex: $\dfrac{9^{-1/5}}{9^{2/3}} = \dfrac{9^{-3/15}}{9^{10/15}} = 9^{-13/15} = \dfrac{1}{9^{13/15}}$

ex: evaluate:

$81^{1/2} + \sqrt[3]{8} - 32^{4/5} + 16^{3/4}$

$\sqrt{81}$ $\quad$ $8^{1/3}$

$= 9 + 2 - 16 + 8$

$= 3$