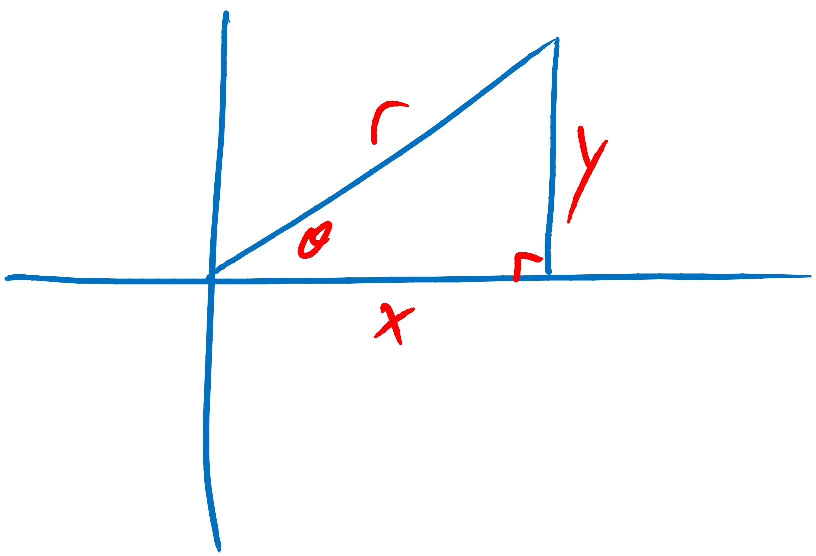
5.5 – Trigonometric Identities.

A Trigonometric Identity is an equation involving trigonometric ratios which is true for any angle.

Recall:



$$x^2 + y^2 = r^2$$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$

Our first identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Memorize

R.S. $\dfrac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{\cancel{r}} \times \frac{\cancel{r}}{x}$

$= \frac{y}{x}$

$= \tan\theta$

$\therefore$ L.S. = R.S.

$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Our second identity:

$$\sin^2\theta + \cos^2\theta = 1$$

L.S. $\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{\cancel{y^2 + x^2}\ r^2}{r^2}$

$= \frac{r^2}{r^2} = 1$

$\therefore$ L.S. = R.S

Pythagorean Identity

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$= (1 - \sin\theta)(1 + \sin\theta)$

Prove:

$$\cos\theta\tan\theta = \sin\theta$$

L.S $\quad \dfrac{\cancel{\cos\theta}}{1}\left(\dfrac{\sin\theta}{\cancel{\cos\theta}}\right)$

$= \sin\theta$

$\therefore$ L.S = R.S

Prove:

$$1 + \tan^2\theta = \sec^2\theta$$

L.S. $\dfrac{1}{1} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$ $\quad\therefore$ L.S. = R.S

Prove:

$$\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$$

L.S. $\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$

$= \dfrac{1}{\sin^2\theta\cos^2\theta} = \csc^2\theta\sec^2\theta$

$\therefore$ L.S. = R.S.

$\dfrac{1}{2} + \dfrac{1}{3}$

$\dfrac{3}{6} + \dfrac{2}{6} = \dfrac{5}{6}$

$\dfrac{1}{x} + \dfrac{1}{y}$

$\dfrac{y}{xy} + \dfrac{x}{xy}$

Second last question to prove:

$$\cos\theta + \cos\theta\tan^2\theta = \sec\theta$$

L.S. $\cos\theta(1 + \tan^2\theta)$

$= \cos\theta(\sec^2\theta)$

$= \frac{\cancel{\cos\theta}}{1}\left(\frac{1}{\cos^{\cancel{2}}\theta}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta \quad \therefore LS = R.S.$

$\frac{x}{x^2} = \frac{1}{x}$

$\frac{3}{3^2} = \frac{1}{3}$

Last question to prove:

$$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$$

$x^2 - y^2 = (x+y)(x-y)$

$x^4 - y^4 = (x^2+y^2)(x^2-y^2)$

L.S. $(\underbrace{\cancel{\cos^2\theta + \sin^2\theta}}_{1})(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

$= \underline{1 - \sin^2\theta} - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$\therefore$ L.S = R.S.