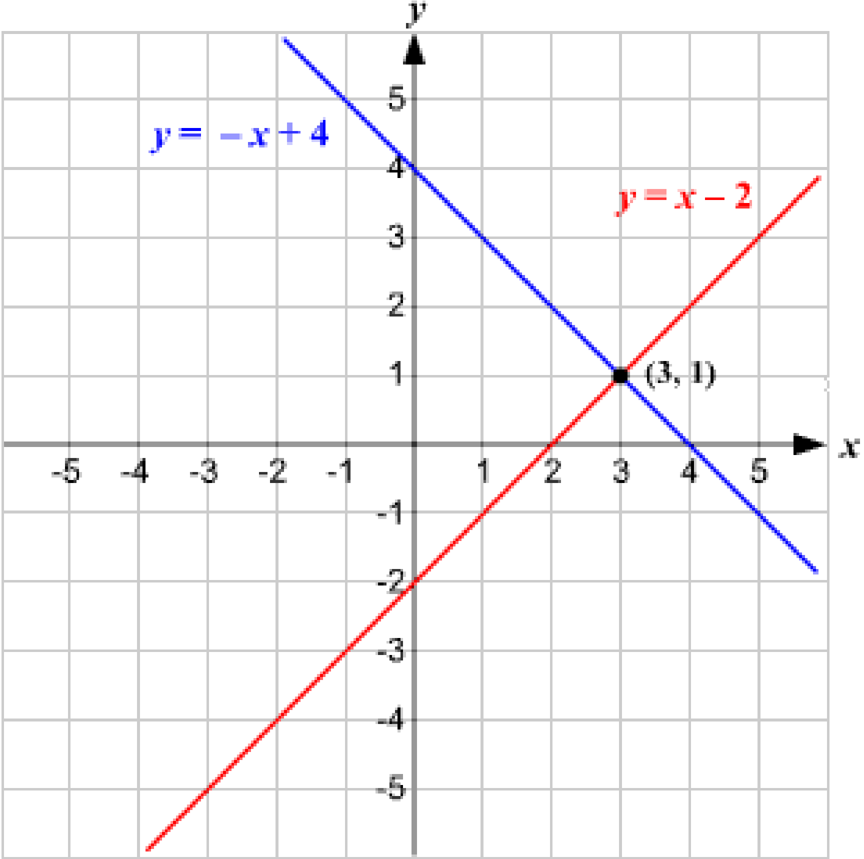
Mathematics 11

3.8 – Linear Quadratic Systems

Mr. D. Hagen

Systems of Linear Equations:

Recall in Grade 10, we looked at how to solve for the point of intersection between two lines. We looked at three ways to solve. The method I want to talk about is substitution, as we will use this method to solve for when a line and a parabola intersect.



Let’s have a quick refresher on this method!

Find the Point of Intersection between the lines:

$y = \boxed{4x - 5}$ and $y = -3x + 9$.

=

$\overset{+3x}{4x} - \underset{+5}{5} = -\overset{+3x}{3x} + \underset{+5}{9}$

$\frac{7x}{7} = \frac{14}{7}$

$x = 2$

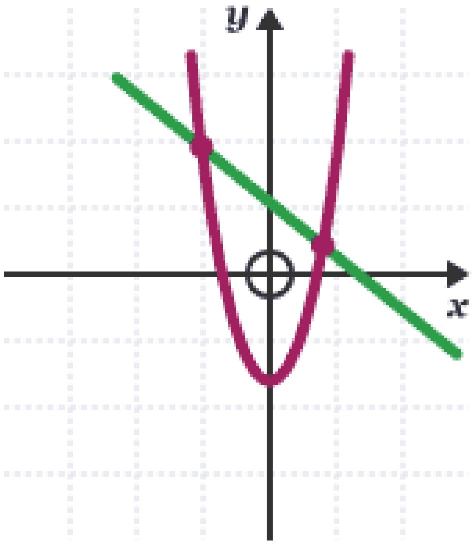
$y = 4(2) - 5$

$y = 8 - 5$

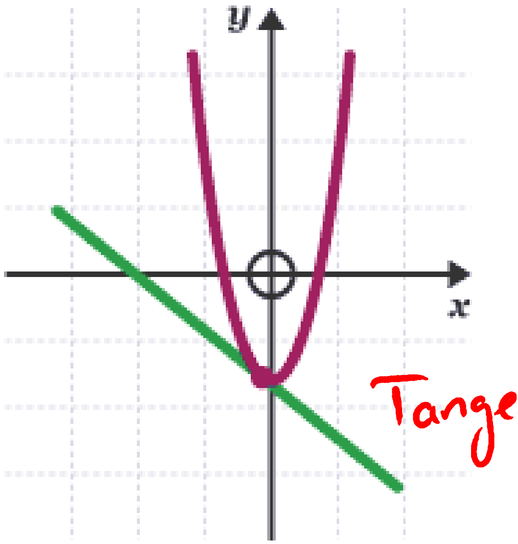
$y = 3$

$\therefore$ the PoI is $(2, 3)$

How do lines interact with parabolas?

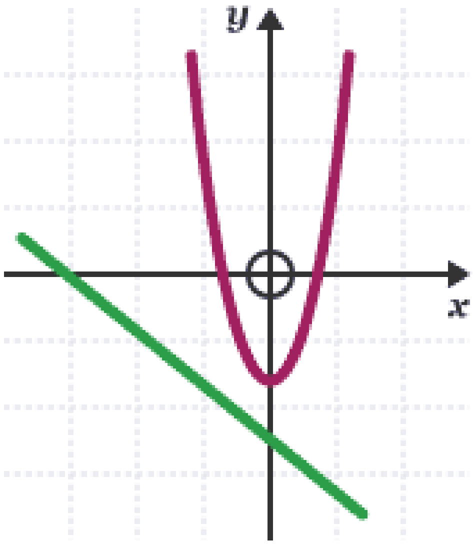


2 points of intersection



Tangent

1 point of intersection



No points of intersection

When working with Quadratic Linear Systems, there are two types of questions we can ask:

1. How many points of intersections are there?
2. What are the points of intersection? (solve for…)

Let’s do two examples of each.

Example 1 $f(\cancel{x}) = -2x+8 \quad g(\cancel{x}) = 4x^2+12x-7$

How many points of intersection?

$f(x) = g(x)$ at the PoI

$$\overset{+2x}{-2x} + \overset{-8}{8} = 4x^2 + \overset{+2x}{12x} \overset{-8}{-7}$$

$$0 = \underset{a}{4}x^2 + \underset{b}{14}x \underset{c}{-15}$$

"how many zeros or solutions?"

$$b^2-4ac \Rightarrow 14^2 - 4(4)(-15)$$

→ use the discriminant.

$$= 196 + 240$$

$$= 436$$

∴ there are two Points of Intersection

Example 2 $f(x) = 3x + 4 \quad g(x) = -2x^2 + 5x - 3$

How many points of intersection?

$$\overset{-3x}{3x} + \overset{-4}{4} = -2x^2 + \overset{-3x}{5x} - \overset{-4}{3}$$

$$0 = \underset{a}{-2}x^2 + \underset{b}{2}x \underset{c}{-7}$$

$$b^2 - 4ac \Rightarrow 2^2 - 4(-2)(-7)$$

$$= 4 - 56$$

$$= -52 < 0$$

$\therefore$ No points of intersection.

Example 3 $f(x) = 2x + 6$ $g(x) = x^2 + 4x + 3$

Find the point(s) of intersection.

$$\overset{-2x}{2x} + \overset{-6}{6} = x^2 + \overset{-2x}{4x} + \overset{-6}{3}$$

$$0 = \underset{a}{1}x^2 + \underset{b}{2}x - \underset{c}{3}$$

- solve by factoring or using the quadratic formula.

- this only gets the x-coordinates.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{16}}{2}$$

$$\rightarrow x = \frac{-2 \pm 4}{2}$$

$\oplus$ $x = \frac{-2+4}{2} = \frac{2}{2} = 1$

$\ominus$ $x = \frac{-2-4}{2} = \frac{-6}{2} = -3$

(1, __) (-3, __)

$(1, \_)$ $(-3, \_\_)$

$f(x) = 2x + 6$

$f(1) = 2(1) + 6$

$f(1) = 8$

$f(-3) = 2(-3) + 6$

$f(-3) = -6 + 6$

$f(-3) = 0$

$\therefore$ the points of intersection are $(1, 8)$ and $(-3, 0)$

Example 3 $f(x) = 2x - 6 \quad g(x) = 4x^2 + 18x + 10$

Find the point(s) of intersection.

$$\overset{-2x}{2x} - \overset{+6}{6} = 4x^2 + \overset{-2x}{18x} + \overset{+6}{10}$$

$$0 = \underset{a}{4x^2} + \underset{b}{16x} + \underset{c}{16}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(4)(16)}}{2(4)}$$

$$x = \frac{-16 \pm \sqrt{\overset{0}{\cancel{256 - 256}}}}{8}$$

$$\rightarrow x = \frac{-16 + 0}{8} = -2$$

$(-2, \_\_)$

$f(-2) = 2(-2) - 6$

$f(-2) = -4 - 6$

$f(-2) = -10.$

∴ the point of intersection is $(-2, -10)$