Mathematics 11U

4.3 – Working With Rational Exponents

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Three questions…

What exponent on 9 is equivalent to $\sqrt{9}$?

$$9^? = 3 \qquad \boxed{9^{1/2} = 3} \qquad 4 = 3$$

Why does $\sqrt{x^6} = x^3$?

$$\left(x^3\right)^2 = x^6$$

1) Does $\sqrt{}$ divide exponents by 2?

YES!!

How can you evaluate $4^{\frac{3}{2}}$?

Solve/work out

$$4^{3/2} = \left(4^3\right)^{1/2} = \sqrt{4^3} = \sqrt{64} = 8$$

Do $4^{1.5}$ on Calculator to verify.

$$= \left(4^{1/2}\right)^3 = \left(\sqrt{4}\right)^3 = (2)^3 = 8$$

Note:

A square root is $\sqrt{x}$ or $\sqrt[2]{x}$, but we don't typically put the 2 in the "hook" because it is the lowest radical.

A cubic root is $\sqrt[3]{x}$, meaning "what number do you multiply by itself 3 times to get x". Ex: $\sqrt[3]{64} = 4$ because $4^3 = 64$.

A 4th root is $\sqrt[4]{x}$. Ex: $\sqrt[4]{16} = 2$ because $2^4 = 16$. NOT $4\sqrt{16}$
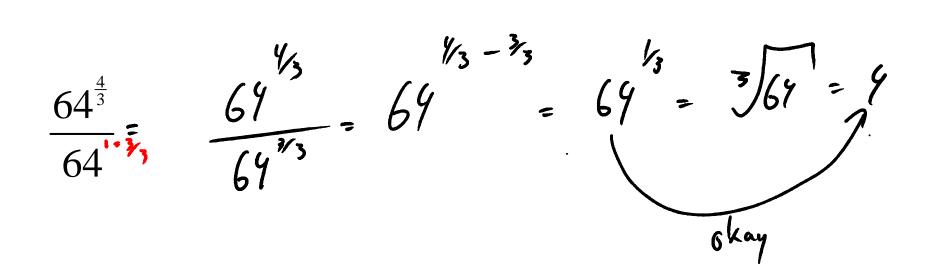
IT IS $\sqrt[4]{16}$

Let's make a mathematical leap of logic:

If $\sqrt{x} = x^{\frac{1}{2}}$, then does $\sqrt[3]{x} = x^{\frac{1}{3}}$ and $\sqrt[4]{x} = x^{\frac{1}{4}}$? YES!!

The "rule":

$\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}} = x^{\frac{m}{n}}$
n^{th} root

denominator = root.

ex: $\sqrt[5]{6^2} = 6^{\frac{2}{5}}$

Examples: Simplify, then evaluate.

$$81^{0.25} = 81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

$$\underline{3} \times \underline{3} \times \underline{3} \times \underline{3} = 81$$

$$\left(3^{\frac{2}{3}}\right)\left(3^{\frac{1}{3}}\right) = 3^{\frac{2}{3}+\frac{1}{3}} = 3^{\frac{3}{3}} = 3^{1} = 3$$

$$\frac{64^{\frac{4}{3}}}{64^{1 \cdot \frac{2}{3}}} = \frac{64^{\frac{4}{3}}}{64^{\frac{2}{3}}} = 64^{\frac{4}{3}-\frac{2}{3}} = 64^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

okay

Examples: Simplify, then evaluate.

$$\left[\frac{\sqrt[4]{5^8}}{\sqrt[3]{25^6}}\right]^{-2}$$

(5^2)

$$= \left[\frac{5^{8/4}}{5^{12/3}}\right]^{-2}$$

$$= \left(\frac{5^2}{5^4}\right)^{-2}$$

$$= \left(5^{-2}\right)^{-2} = 5^4 = 625$$

Evaluate

$$4^{-2} + \sqrt[3]{27}^{-1} - 8^0$$

3

$$= \frac{1}{4^2} + 3^{-1} - 1$$

$$= \frac{1}{16} + \frac{1}{3} - 1$$

$$= \frac{3}{48} + \frac{16}{48} - \frac{48}{48}$$

$$= \frac{-29}{48}$$