This chapter deals with Sinusoidal Functions, which are just a type (a subset) of Periodic Functions. The two sinusoidal functions we will work with are: $f(\theta) = \sin\theta$ and $f(x) = \cos x$

Periodic Function:

- a graph which repeats itself. The pattern must be exactly the same each time.

Period:

- one section of the graph that is repeated
- the length on the x-axis of one cycle

one period

Peak:

- the maximum

Trough:

- the minimum

Equation of Axis:

- the middle of the graph on the y-axis
- horizontal "line"

$y = \frac{\text{peak} + \text{trough}}{2}$

Amplitude:

- the distance from the middle to the peak or trough

always positive

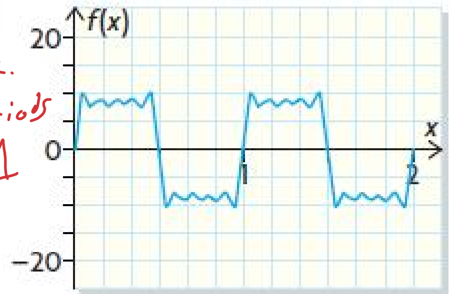
amp = peak − middle

amp = middle − trough

or amp = $\frac{\text{peak} - \text{trough}}{2}$

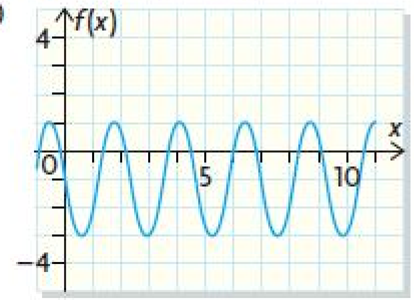
1. Which of the following graphs are periodic? Explain why or why not.

a)



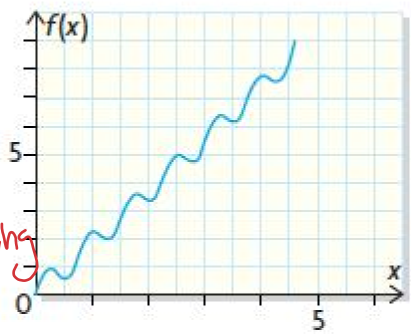
- periodic.
- the periods are 1

c)



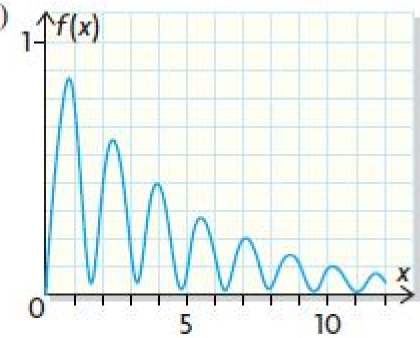
Is periodic
- same peaks and troughs
- equally spaced out

b)



- not periodic
- keep increasing

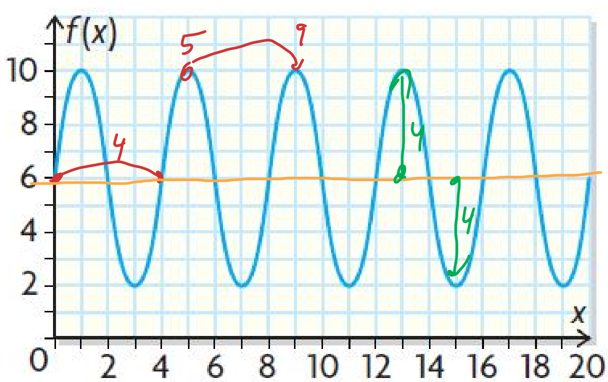
d)



- not periodic
- the peaks are always changing

the y-axis

2. Determine the range, period, equation of the axis, and amplitude of the function shown.



peak = 10 trough = 2

range: $\{f(x) \in \mathbb{R} \mid 2 \le f(x) \le 10\}$

period: 4

EoA: $y = \frac{10+2}{2} = \frac{12}{2} = 6$

↳ middle $y = 6$

Amp: $\overset{p}{10} - \overset{m}{6} = 4$

or $\overset{m}{6} - \overset{t}{2} = 4$

3. The motion of an automated device for attaching bolts to a household appliance on an assembly line can be modelled by the graph shown at the left.

a) What is the period of one complete cycle? one second.

b) What is the maximum distance between the device and the appliance? 1.5 cm

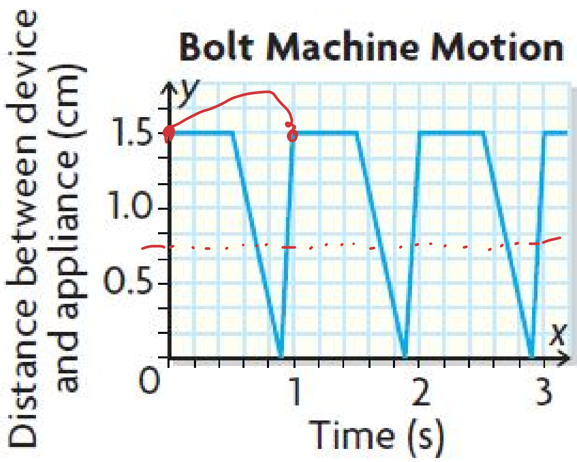
c) What is the range of this function? $\{y \in \mathbb{R} \mid 0 \le y \le 1.5\}$

d) If the device can run for five complete cycles only before it must be turned off, determine the domain of the function. $\{t \in \mathbb{R} \mid 0 \le t \le 5\}$

e) Determine the equation of the axis.

f) Determine the amplitude.

g) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "attaching the bolt."



e) $EoA = \dfrac{1.5+0}{2}$

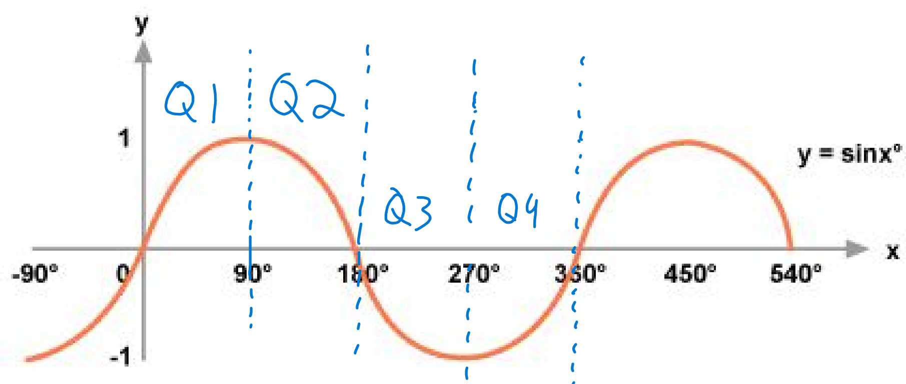
$y = 0.75$

f) $Amp = 0.75$

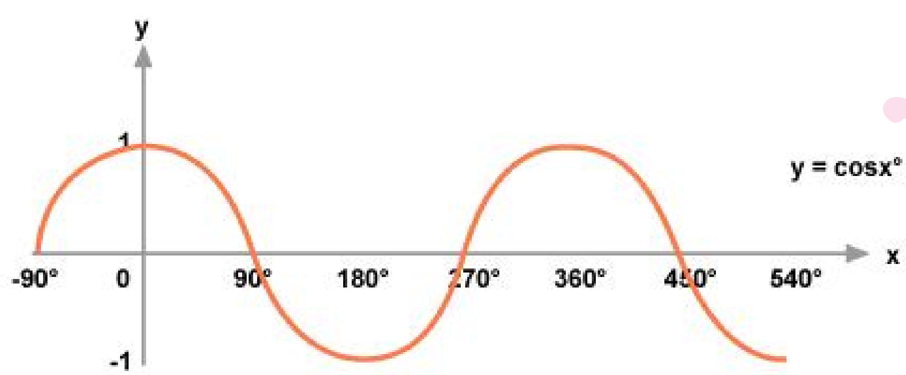
6.2 – Sinusoidal Functions

Homework: To be handed out.

What do the graphs of: $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$ look like?



x	y=sinx
0	0
90	1
180	0
270	-1
360	0



x	y=cosx
0	1
90	0
180	-1
270	0
360	1

$f(x) = a\sin(k(x-d)) + c$

Function	$f(x) = 3\sin(\underbrace{2x - 180}_{\text{factor}}) + 4$
Proper Function	$f(x) = \overset{a}{3}\sin(\overset{k}{2}(x\ \overset{d}{-90})) + \overset{c}{4}$
Amplitude $\lvert a \rvert$	3
Period $\frac{360°}{\lvert k \rvert}$	$\frac{360}{2} = 180°$
Phase Shift d Horizontal Shift	$90°$
Equation of Axis $y = c$	$y = 4$
Domain (2 cycles) always start @ $x=0$	$\{x \in \mathbb{R} \mid 0° \le x \le 360°\}$
Range Trough → Peak	peak = amp + mid $= 3 + 4 = 7$ trough = mid − amp $= 4 - 3 = 1$

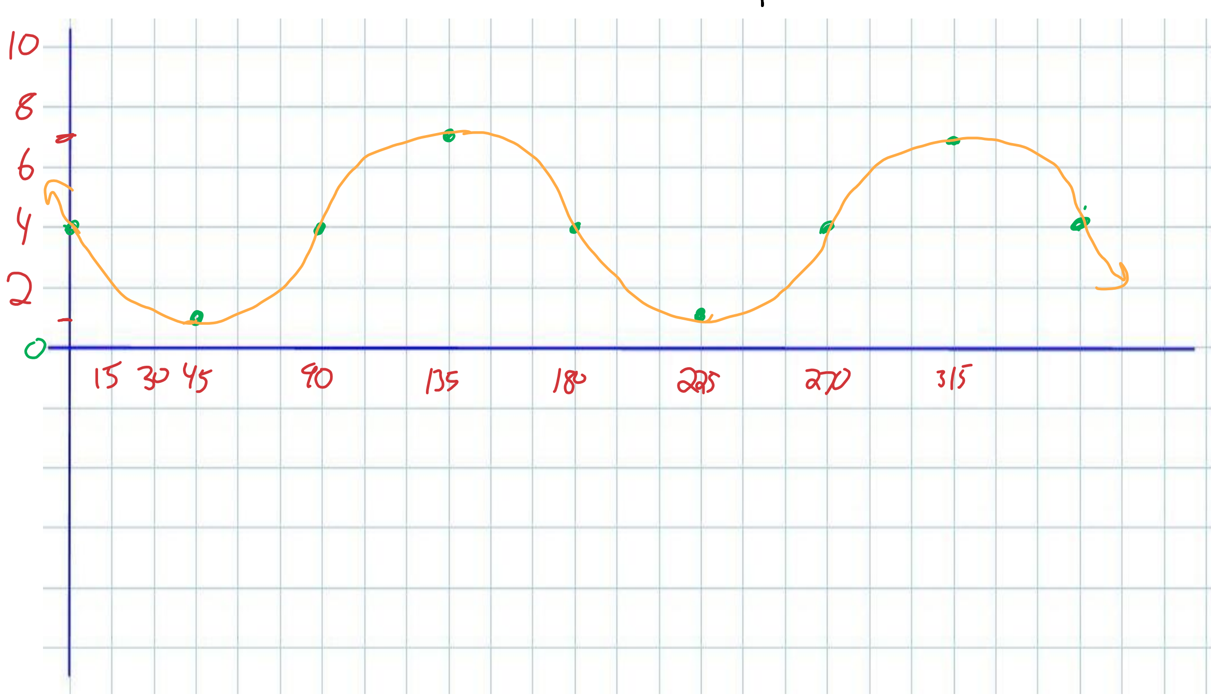
$\{y \in \mathbb{R} \mid 1 \le y \le 7\}$

x's → $\frac{1}{k}x + d$

y's → $ay + c$

x	y
0	0
90	1
180	0
270	-1
360	0

$\frac{1}{2}x + 90$	$3y + 4$
90	4
135	7
180	4
225	1
270	4



Function	$f(x) = -2\cos\left(\frac{1}{2}x - 60\right) - 3$
Proper Function	$f(x) = -\overset{a}{2}\cos\left(\overset{k}{\frac{1}{2}}(x - \overset{d}{120})\right) \overset{c}{-3}$
Amplitude $\lvert a \rvert$	$\lvert -2 \rvert = 2$
Period $\frac{360}{k}$	$360 \div \frac{1}{2} = 720^\circ$
Phase Shift d	120°
Equation of Axis $y = c$	$y = -3$
Domain (2 cycles)	$\{x \in \mathbb{R} \mid 0^\circ \le x \le 1440^\circ\}$
Range $\{f(x) \in \mathbb{R} \mid \overset{T}{-5} \le f(x) \le \overset{P}{-1}\}$	P = mid + amp $= -3 + 2 = -1$; T = mid − amp $= -3 - 2 = -5$

$\frac{1}{k}x + d$

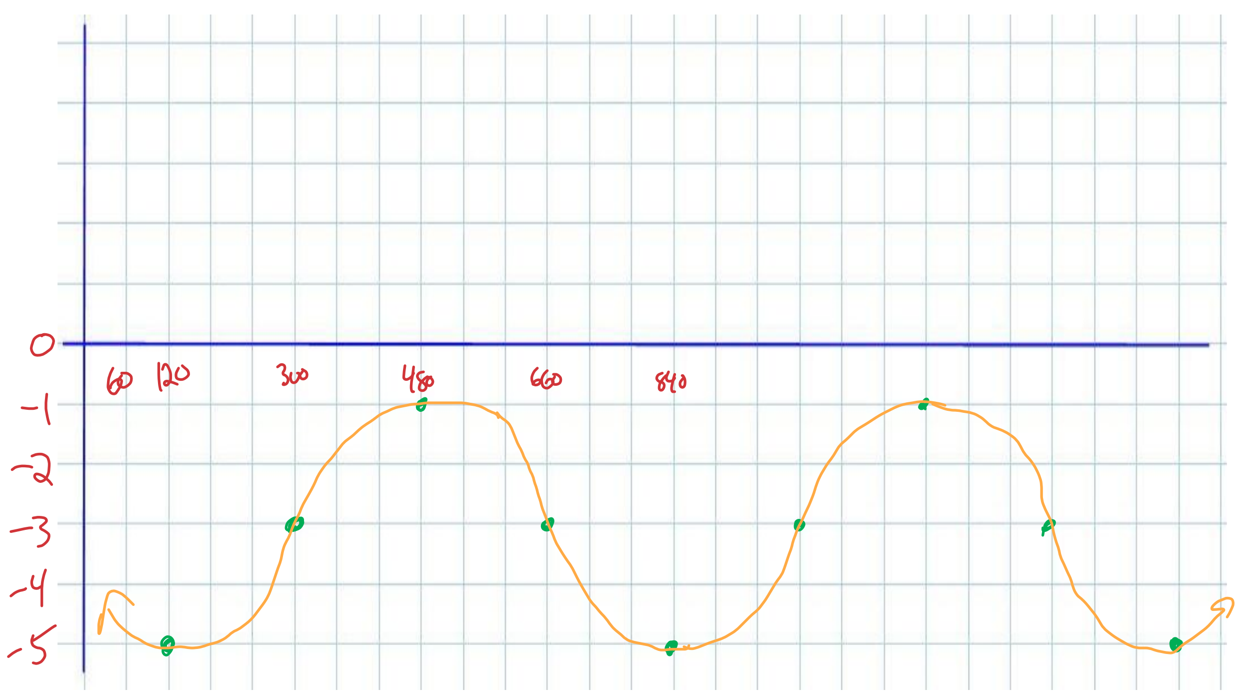
$\frac{1}{\frac{1}{2}}x + 120$

$2x + 120$

720×2

x	$y = \cos x$
0	1
90	0
180	−1
270	0
360	1

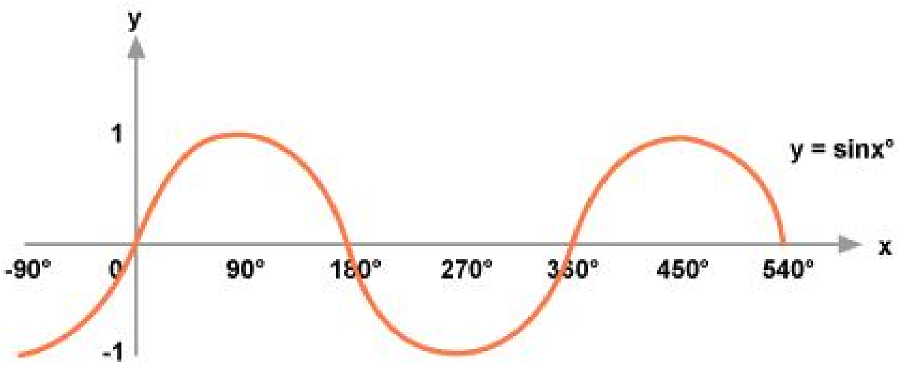
$2x + 120$	$-2y - 3$
120	−5
300	−3
480	−1
660	−3
840	−5

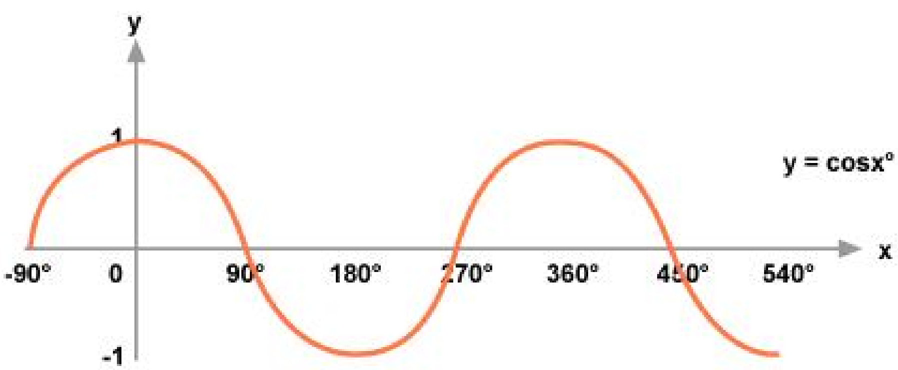


6.6 – Models of Sinusoidal Functions

Homework: pg 391 #4,5,6,7,8,11
pg 398 # 1,2,3,5,6

A reminder of our sinusoidal functions:





The key to creating equations:

$$f(x) = a\sin\big(k(x-d)\big) + c$$

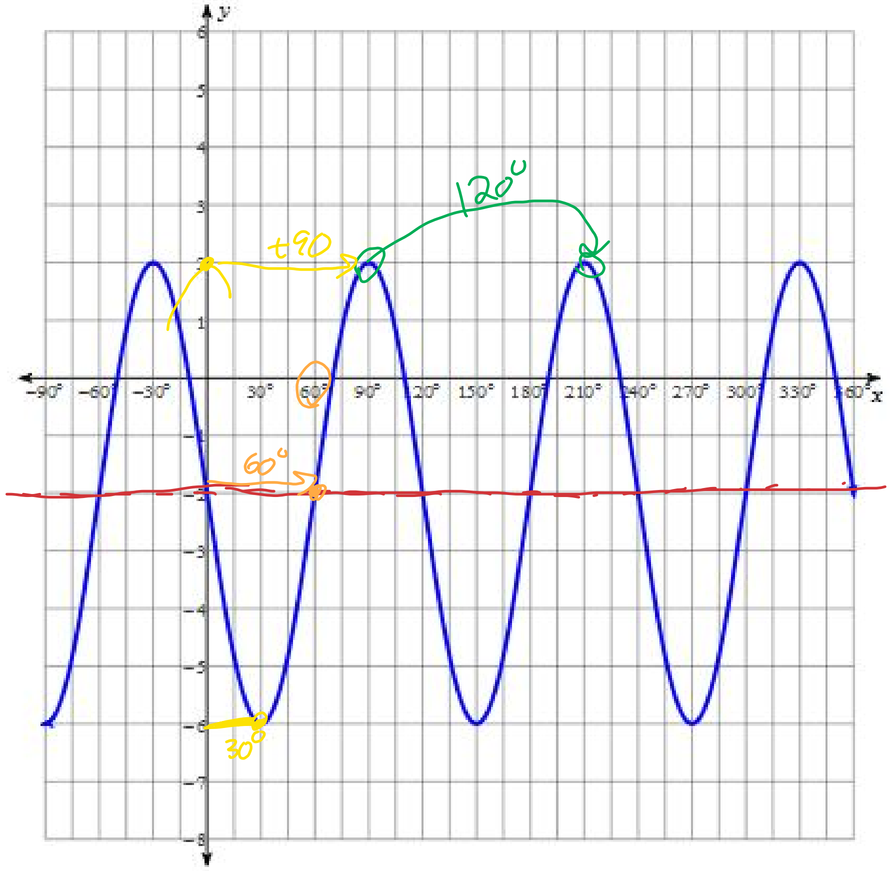
Amplitude = a, found by *peak-EoA*

Period = $\frac{360°}{k}$ therefore $k = \frac{360}{Period}$

Phase Shift = d – this is your "starting point" – must be peak, EoA or trough

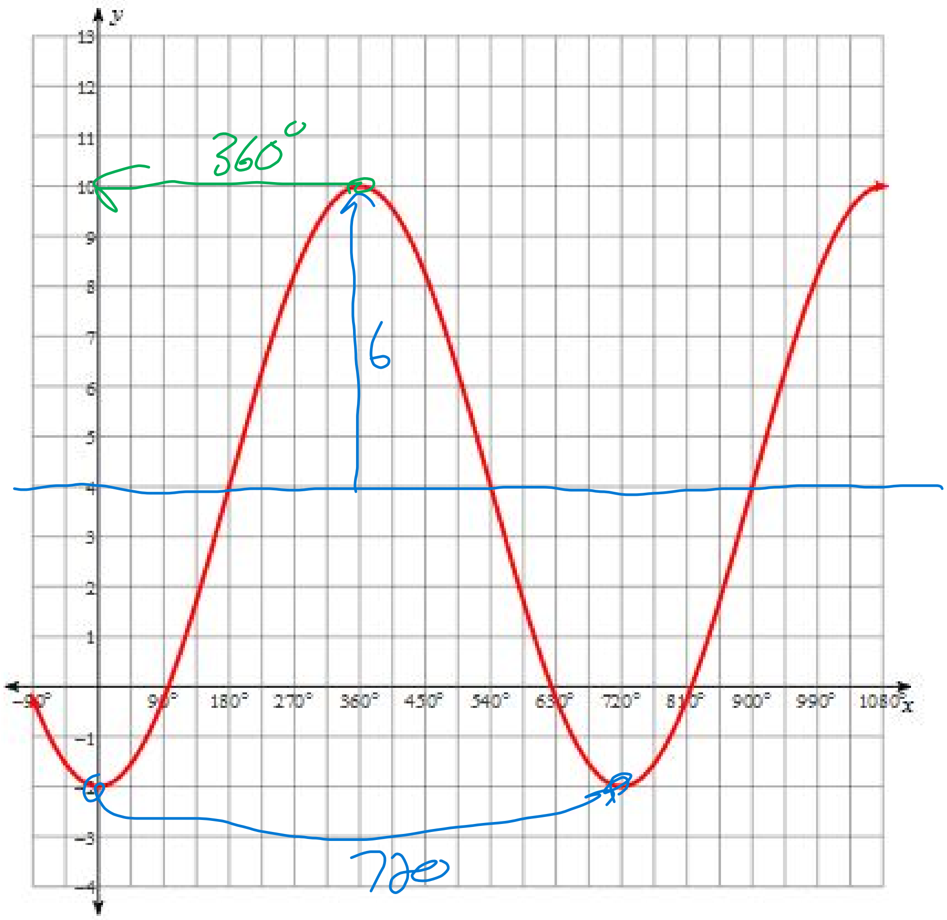
Equation of Axis = c, found by $\frac{peak+trough}{2}$

	Starting at…
+sin	Equation of axis, then heads to peak
-sin	Equation of axis, then heads to trough
+cos	Peak
-cos	Trough



Peak and Trough	$P = 2 \quad T = -6$
Equation of Axis	$y = \frac{2 + -6}{2} = -2 \to c$
Amplitude	$amp = 4 \longrightarrow a$
Period and *k* $Period = \frac{360}{k}$	$Period = 120^\circ \qquad k = \frac{360}{120} = 3$
Phase Shift for sine	d=60, then a is pos. d=0, then a is negative
Phase Shift for cosine	d=90, then a is positive (peak) d=30, then a is negative (trough)
Functions	$f(x) = 4\sin(3(x-60)) - 2$ $f(x) = -4\sin(3(x-0)) - 2$ $f(x) = 4\cos(3(x-90)) - 2 \longrightarrow$ easiest.

$f(x) = -4\cos(3(x-30)) - 2$



Peak and Trough	$P = 10 \quad T = -2$
Equation of Axis	$y = \frac{10 + -2}{2} = 4 \rightarrow c$
Amplitude	amp $= 6 \rightarrow a$
Period and *k*	period $= 720 \quad k = \frac{360}{720} = \frac{1}{2}$
Phase Shift for sine	
Phase Shift for cosine	$d = 360^\circ$
Functions	$f(x) = 6\cos\left(\frac{1}{2}(x - \underbrace{360}_{\text{peak}})\right) + 4$

x	0°	45°	90°	135°	180°	225°	270°
y	9	7	5	7	9	7	5

Peak = 9

Trough: 5

EoA: 7 → c

Amp = 2 → a

Period = 180°

$k = \frac{360}{180} = 2$

d = 0 or 180 = +cos

$\therefore f(x) = 2\cos(\underbrace{2x}_{2(x-0)}) + 7$

A sinusoidal function has an amplitude of 4 units, a period of 120°, and a maximum at (0,9). Determine the equation of the function.

amp = 4 → a

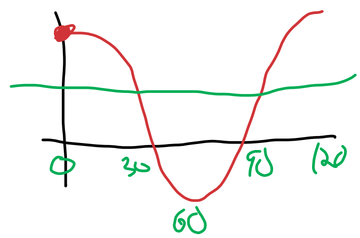
period = 120

$k = \frac{360}{120} = 3$

peak = 9

EoA = 5 → c

→ d = 0

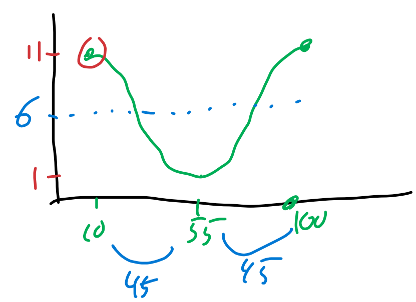


$\therefore f(x) = 4\cos(3x) + 5$

A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches a maximum height of 11m at 10s and then reaches a minimum height of 1m at 55s. How high is John after 2 minutes?

Max $(\overset{d}{10}, 11)$

Min $(55, 1)$



EoA = 6 c

Amp: 5 a

Period = 90 $\therefore k = \frac{360}{90} = 4$ k

$f(x) = 5\cos(4(x-10)) + 6$

2min: 120 seconds

$f(120) = 5\cos(4(120-10)) + 6$

$f(120) = 6.9$ metres