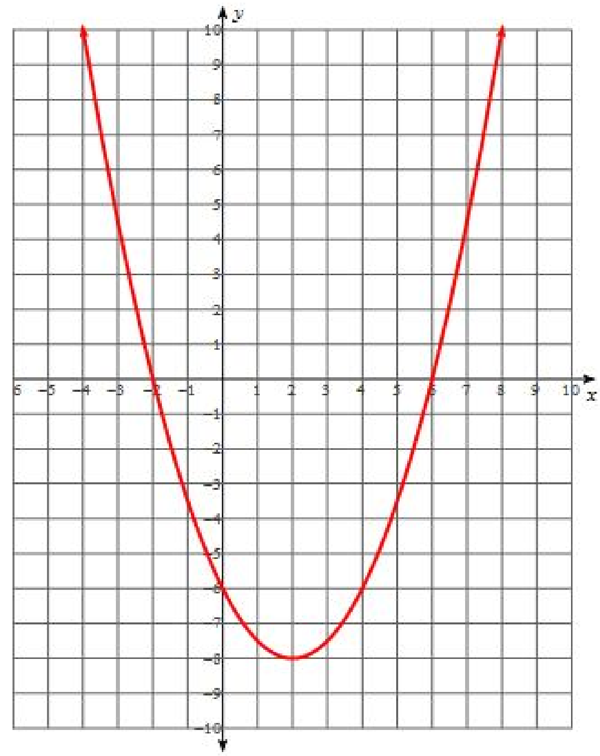
Our dear friend from Grade 10 is back. What are quadratics?



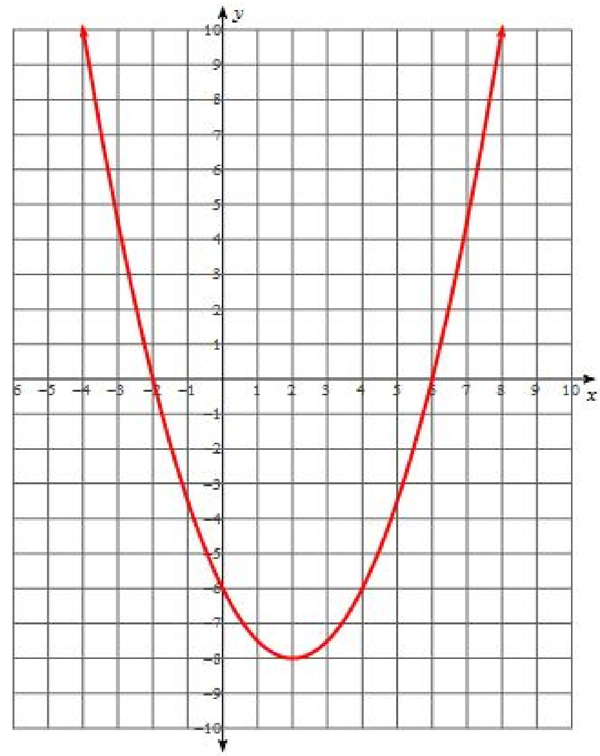
Three forms (equations) of Quadratics:

$$f(x) = a(x-h)^2 + k$$

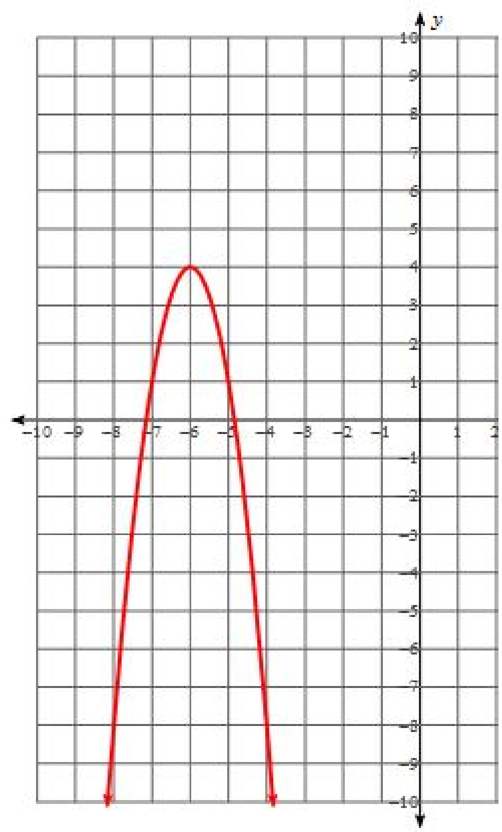
$$f(x) = ax^2 + bx + c$$

$$f(x) = a(x-r)(x-s)$$

Find the equation of the parabola:



Write the standard form of the parabola:



Convert to Standard Form:

$$f(x) = -2(x-4)(x+7)$$

$$g(x) = \frac{1}{2}(x+4)^2 - 6$$

State the direction of opening, the equation of axis and the vertex:

$$f(x) = 3(x+6)(x-2)$$

Determine the equation of axis:

$$(4,3),(12,3)$$

3.2 – Determining Maximums and Minimums of Quadratic Functions

The maximum/minimum is the y-coordinate of the vertex, or the *k*. A maximum occurs when $a<0$ and a minimum occurs when $a>0$.

Given vertex form:

$$f(x) = a(x-h)^2 + k$$

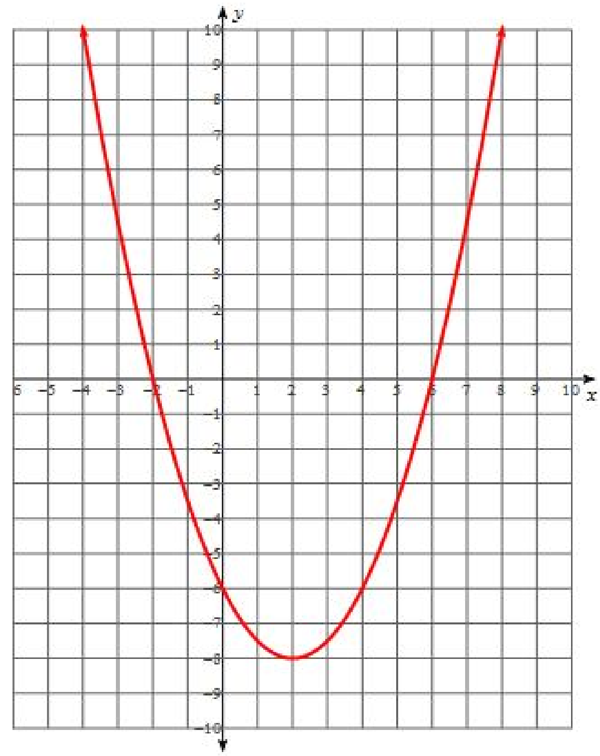
$$f(x) = -2(x+5)^2 - 8$$

Given Zeros Form:

$$f(x) = a(x-r)(x-s)$$

$$f(x) = 3(x+2)(x-8)$$

Given Standard Form: $f(x) = ax^2 + bx + c$



$$f(x) = 2x^2 + 10x - 5$$

$$f(x) = 3x^2 + 48x - 19$$

$$f(x) = \frac{1}{2}x^2 - 5x + 8$$

$$f(x) = -2.2x^2 + 8.4x + 5.76$$

3.4 – Operations with Radicals

$1^2 =$	$11^2 =$
$2^2 =$	$12^2 =$
$3^2 =$	$13^2 =$
$4^2 =$	$14^2 =$
$5^2 =$	$15^2 =$
$6^2 =$	$16^2 =$
$7^2 =$	$17^2 =$
$8^2 =$	$18^2 =$
$9^2 =$	$19^2 =$
$10^2 =$	$20^2 =$

Simplifying Radicals:

1. $2\sqrt{48}$

2. $-\sqrt{20}$

3. $\frac{1}{8}\sqrt{320}$

4. $-3\sqrt{513}$

Simplifying Radicals with variables:

1. $7\sqrt{288b^4}$

2. $-5\sqrt{45n^3}$

Adding and Subtracting Radicals:

1. $-2\sqrt{12} - 3\sqrt{8} + 3\sqrt{32} + 2\sqrt{27}$

2. $2\sqrt{45} - \sqrt{8} - 2\sqrt{32} - 2\sqrt{18}$

Multiplying Radicals:

1. $2\sqrt{6} \times 4\sqrt{5}$

2. $-3\sqrt{10} \times 6\sqrt{2}$

3. $(\sqrt{2} + \sqrt{3})\,(2\sqrt{2} - 5\sqrt{3})$

Rationalizing the Denominator:

1. $\frac{10}{\sqrt{2}}$

2. $\frac{\sqrt{8}}{\sqrt{3}}$

3. $\frac{9\sqrt{63}}{7\sqrt{54}}$

4. $\frac{7+\sqrt{2}}{5\sqrt{40}}$

3.5 – Solving Quadratics

What is solving? It is simple to understand when the question itself is simple:

$$5x - 1 = 4$$
$$5x = 5$$
$$x = 1$$

$$3x^2 - 12 = 0$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

However, when we need to solve:
$x^2 - 6x + 8 = 0$ or $2x^2 + 5x - 5 = 7$,
it gets more complicated.

My definition

Solving for *x* means to find the value(s) of *x* to satisfy (make true) the equation/function. With a linear function, there is one main method to solve. A quadratic has 4 methods!!! The use of each method depends on the problem and how comfortable you are with the methods.

1. Graphing Calculator/Technology.

This is my least favourite method. While it works every time, you aren't "learning" or practicing your math. This method is great to check your work or to use on those pesky large questions. Standard Form is not required.

2. Solving from the Vertex Form (Completing the Square)

I very rarely use this method straight from the Standard Form. However, if your question is already in Vertex Form, this is a powerful method.

3. Factoring

This is not partial factoring. You need to be able to FULLY factor the quadratic. This is a very quick method if you can see that it is factorable. The solutions are then just in the factored form.

4. Quadratic Formula

This method works every time! It can even tell you if there are no solutions or just one solution. If you can master this formula, you will never sweat while solving quadratics! Standard Form is required.

Note

Standard Form is required in most cases. This may require you to move all the components to one side, leaving the other side equal to zero. Hence why this is called solving for the zeros....

Examples of each method (expect technology):

2. Solving from the Vertex Form (Completing the Square)

$$f(x) = -3(x-4)^2 + 27$$

2. Solving from the Vertex Form (Completing the Square)

$$g(x) = 4(x+5)^2 - 32$$

3. Factoring

$$f(x) = x^2 + 18x + 80$$

$$g(x) = 2x^2 + 5x - 5$$
$$g(x) = 7$$

4. Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$f(x) = -3x^2 - 8x + 14$$

4. Quadratic Formula

$$ax^2 + bx + c = 0$$

$$5x(x+3.4) = 2x^2 + 9x - 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3.6 – Zeros of a Quadratic Function

Zero, One, or Two?

Three methods to determine the number of zeros.

1. Graph it.
2. Vertex Form

$$f(x) = -3(x-4)^2 + 5$$

$$g(x) = -5(x-6)^2 - 10$$

$$h(x) = 2(x+1)^2 - 8$$

3. Algebra!

Recall: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

To find the number of zeros, just look at the radical, or more commonly known as the discriminant.

If,

$b^2 - 4ac > 0$, then there are 2 zeros.

If,

$b^2 - 4ac = 0$, then there is 1 zero.

If,

$b^2 - 4ac < 0$, then there are 0 zeros.

Example:

$$f(x) = -2x^2 + 6x - 9$$

The curveball…

$$f(x) = -2x^2 + 3x + k$$

Solve for k so that $f(x)$ has only one solution.

How do lines interact with parabolas?

Example 1 $f(x) = -2x + 8 \quad g(x) = 4x^2 + 12x - 7$

How many points of intersection?

Example 2 $f(x) = 3x + 5$ $g(x) = 2x^2 - 6x - 4$

Find the point(s) of intersection.

Example 3 $f(x) = 2x + k \quad g(x) = 3x^2 + 5x - 2$

Find the value of *k* so that *f(x)* and *g(x)* intersect only once.