Laws of Exponents: (when bases are the same)

1. Product Rule (multiplying)

$4^3 \times 4^2 = (4\times4\times4)\times(4\times4) = 4^5$

$4^3 \times 5^2$
$= 64 \times 25$

$a^m \times a^n = a^{m+n}$

Law: When multiplying with the same base, add the exponents

Laws of Exponents: (when bases are the same)

2. Quotient Rule (dividing)

$$\frac{5^6}{5^2} = \frac{5\times5\times5\times5\times5\times5}{5\times5} = 5^4$$

$6 - 2 = 4$

$$\frac{a^m}{a^n} = a^{m-n}$$

Law: When dividing with the same base, subtract the exponents.

Laws of Exponents: (when bases are the same)

3. Power Law

$(3^2)^4 = (3\times3)\times(3\times3)\times(3\times3)\times(3\times3) = 3^8$

$2 \times 4 = 8$

$(a^m)^n = a^{mn}$

Law: when raising the base to a power, multiply the exponents.

$(3^1)^2 = 3^2$

Laws of Exponents: (when bases are the same)

4. Zero Law

$7^0 = 1$

$\frac{7^2}{7^2} = \frac{49}{49} = 1$

Quotient

$\frac{7^2}{7^2} = 7^{2-2} = 7^0 = 1$

$(2000^{1001})^0 = 1$

Law: Anything to the power of 0 is equal to 1.

$0^2 = 0$ $0^{100} = 0$

$0^0 = ?$

Laws of Exponents: (when bases are the same)

5. Negative Exponent Law

Law: A negative exponent means the reciprocal.

$\frac{2}{3}$, $\frac{3}{2}$ is the reciprocal

Exponent	-3	-2	-1	0	1	2	3	4	5
Base	4^{-3}	4^{-2}	4^{-1}	4^{0}	4^{1}	4^{2}	4^{3}	4^{4}	4^{5}
Result	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64	256	1024

(×4 moving right; ÷4 moving left)

$4^{-5} = \frac{1}{1024}$

$\frac{1}{4} \div 4 = \frac{1}{4} \times \frac{1}{4}$

$$4^{-4} = \frac{1}{4^4} = \frac{1}{256} \qquad \frac{1}{4^{-2}} = 4^2 = 16 \qquad \frac{3^{-3}}{4^{-2}} = \frac{4^2}{3^3} = \frac{16}{27}$$

Examples: Simplify, then evaluate. Answers need positive exponents

$$\frac{(2^3)(2^4)}{2^2} = \frac{2^7}{2^2} = 2^5 = 32$$

(added; subtract)

$$\frac{(3^{-1})^2}{3^{-3}} = \frac{3^{-2}}{3^{-3}} = 3^{-2-(-3)} = 3^1 = 3$$

$$\left[\frac{(4^6)(4^3)}{(4^2)(4^7)}\right]^{-2} = \left[\frac{4^9}{4^9}\right]^{-2} = \left[4^0\right]^{-2} = 4^0 = 1$$

$(4^{-3})^{-2} = 4^6$

$(4^3)^{-2} = \frac{4^{-6}}{1} = \frac{1}{4^6}$

4.3 – Working With Rational Exponents

Three questions…

What exponent on 9 is equivalent to $\sqrt{9}$?

$9^{\frac{1}{2}} = \sqrt{9}$

$\sqrt{9} = 3$

$(9^x)^2 = (3)^2$

$9^{2x} = 9^1$

$2x = 1$

$x = \frac{1}{2}$

$9^{\frac{1}{2}} = 3$

Why does $\sqrt{x^6} = x^3$?

$(x^6)^{\frac{1}{2}} = x^3$

How can you evaluate $4^{\frac{3}{2}}$?

$\frac{3}{2} = 3 \times \frac{1}{2}$

$(4^3)^{\frac{1}{2}} = 64^{\frac{1}{2}} = \sqrt{64} = 8$

or $(4^{\frac{1}{2}})^3 = (\sqrt{4})^3 = 2^3 = 8$

$16 \times 4 = 64$

Note:

A square root is $\sqrt{x}$ or $\sqrt[2]{x}$, but we don't typically put the 2 in the "hook" because it is the lowest radical.

A cubic root is $\sqrt[3]{x}$, meaning "what number do you multiply by itself 3 times to get x". Ex: $\sqrt[3]{64}$ =4 because $4^3 = 64$.

A 4th root is $\sqrt[4]{x}$. Ex: $\sqrt[4]{16} = 2$ because $2^4 = 16$.

Let's make a mathematical leap of logic:

If $\sqrt{x} = x^{\frac{1}{2}}$, then does $\sqrt[3]{x} = x^{\frac{1}{3}}$ and $\sqrt[4]{x} = x^{\frac{1}{4}}$?

The "rule":

$\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}} = x^{\frac{m}{n}}$

ex: $\sqrt[5]{6^2} = 6^{\frac{2}{5}}$

Examples: Simplify, then evaluate.

$81^{0.25} = 81^{\frac{1}{4}} = \sqrt[4]{81^1} = 3$

$\underline{3} \times \underline{3} \times \underline{3} \times \underline{3}$

$\left(3^{\frac{2}{3}}\right)\left(3^{\frac{1}{3}}\right) = 3^{\frac{2}{3}+\frac{1}{3}} = 3^1 = 3$

$\frac{64^{\frac{4}{3}}}{64^{\frac{1}{?}}} = \frac{64^{\frac{4}{3}}}{64^{3/3}} = 64^{\frac{4}{3}-\frac{3}{3}} = 64^{\frac{1}{3}} = \sqrt[3]{64} = 4$

Examples: Simplify, then evaluate.

$\left[\frac{\sqrt[4]{5^8}}{\sqrt[3]{25^6}}\right]^{-2}$

$25 = 5^2$

$= \left[\frac{\sqrt[4]{5^8}}{\sqrt[3]{(5^2)^6}}\right]^{-2}$

$= \left[\frac{5^{\frac{8}{4}}}{5^{\frac{12}{3}}}\right]^{-2}$

$= \left[\frac{5^2}{5^4}\right]^{-2}$

$= \left[5^{-2}\right]^{-2}$

$= 5^4$

$= 625$

$4^{-2} + \sqrt[3]{27^{-1}} - 8^0$

$= \frac{1}{4^2} + 27^{\frac{-1}{3}} - 1$

$= \frac{1}{16} + \frac{1}{27^{1/3}} - 1$ ($27^{1/3} = 3$)

$= \frac{1^{\times 3}}{16_{\times 3}} + \frac{1^{\times 16}}{3_{\times 16}} - \frac{1^{\times 48}}{1_{\times 48}}$

$= \frac{3}{48} + \frac{16}{48} - \frac{48}{48}$

$= \frac{-29}{48}$

4.4 – Algebraic Expressions

$$\frac{(n^{-4})(n^{-6})}{(n^{-2})^7}$$

$$= \frac{n^{-10}}{n^{-14}}$$

$$= n^{-10-(-14)}$$

$$= n^4$$

$$\frac{(-2x^5)^3}{8x^{10}}$$

$(-2x^5)(-2x^5)(-2x^5)$

$$= \frac{-8x^{15}}{8x^{10}}$$

$15-10$

$$= -1x^5$$

coefficients = regular math

variables = exponent math

$$\frac{(4r^{-6})(-2r^2)^5}{(-2r)^4}$$

$$= \frac{(4r^{-6})(-32r^{10})}{16r^4}$$

$$= \frac{-128r^4}{16r^4}$$

$$= -8r^0$$

$$= -8$$

$(-2)^4 = 16$

$(-2)^4 \neq -2^4$

16   -16

$$\frac{\sqrt[6]{(8x^6)^2}}{\sqrt[4]{625x^8}}$$

$$= \frac{\sqrt[6]{64x^{12}}}{\sqrt[4]{625x^8}}$$

$$= \frac{\sqrt[6]{64}\, x^{\frac{12}{6}}}{\sqrt[4]{625}\, x^{\frac{8}{4}}}$$

$$= \frac{2x^2}{5x^2}$$

$$= \frac{2}{5}$$

$2^6 = 64$

$5^4 = 625$

$$\frac{(mn^3)^{-\frac{1}{2}}}{m^{\frac{1}{2}}n^{-\frac{5}{2}}}$$

$$= \frac{m^{-\frac{1}{2}}n^{-\frac{3}{2}}}{m^{\frac{1}{2}}n^{-\frac{5}{2}}} = m^{-\frac{1}{2}-\frac{1}{2}}\, n^{-\frac{3}{2}-\left(-\frac{5}{2}\right)=\frac{2}{2}}$$

$$= \frac{m^{-1}n^{1}}{1}$$

$$= \frac{n^1}{m^1}$$

$$\frac{1}{m} \times \frac{n}{1} = \frac{n}{m}$$

$$\sqrt{\frac{9b^3(ab)^2}{(a^2b^3)^3}}$$

$$= \sqrt{\frac{(9b^3)(a^2b^2)}{a^6b^9}}$$

$$= \sqrt{\frac{9a^2b^5}{a^6b^9}}$$

$$= \sqrt{9a^{-4}b^{-4}}$$

$$= \sqrt{9}\,a^{\frac{-4}{2}}b^{\frac{-4}{2}}$$

$$= 3a^{-2}b^{-2}$$

$$= \frac{3}{a^2b^2}$$

4.7 – Applications Involving Exponential Functions

The exponential function equation is:

$$f(x) = ab^x$$

- $f(x)$: final value
- a: initial/beginning value
- x: periods/time
- growth: $b = 1 + r$ (r = rate in decimal form)
- decay: $b = 1 - r$

bacteria grows at a rate of 5% $\frac{5}{100}$

$r = 0.05$

$b = 1 + 0.05$

$b = 1.05$

bacteria decays at a rate of 5%

$r = 0.05$

$b = 1 - 0.05$

$b = 0.95$

Special b's

- doubles, $b = 2$
- triples, $b = 3$
- half-life, $b = \frac{1}{2}$

A population of 320 frogs grows at a rate of 4.5% per year. How many frogs will there be in 15 years?

$r = 0.045$

rate time and the periods time must match

$f(x)=$
$a=$
$b=1+\_$
$x=$

$f(x) = ab^x$ $\qquad b = 1+0.045$

$f(15) = 320(1.045)^{15}$

$f(15) = 619.29$

∴ In 15 years, there will be 619 frogs.

A new car depreciates at a rate of 20% per year. Steve bought a new car for $26,000.
a) How much will Steve's car be worth in 3 years?

decay

$r = 0.2 \therefore b = 1 - .2 = 0.8$

$f(3) = 26000(0.8)^3$

$f(3) = \$13,312$

b) When will Steve's car be worth $4000?

time, x = ? $\qquad$ f(x)

$\frac{4000}{26000} = \frac{26000(0.8)^x}{26000}$

$0.153846 = 0.8^x$

try $x=7$, $0.8^7 = 0.2097$ $\qquad x=8$, 0.1677
$x=12$, $0.8^{12} = 0.0687$ $\qquad x=9$, 0.134
$x=10$, 0.8^{10} = too small

Steve's car will be worth $4000 between 8 and 9 years

Unfortunately, its not always that simple….

c) What will steve's car be worth in 30 months?

Turn months into years: $\frac{30}{12} = 2.5$

$f(2.5) = 26000(0.8)^{2.5}$

$= \$14,883.27$

Sometimes the rate is not a single unit of time

↳ daily, month, year

A non single unit of time is: every 3 years, exponent is $\frac{x}{3}$

every 5th day, exp: $\frac{x}{5}$

A 200g (a) sample of radio-active material has a half-life of 138 days. How much will be left in 5 years?

$b = \frac{1}{2}$

$a = 200$

$5 \times 365 = 1825$ days

$f(x) = 200\left(\frac{1}{2}\right)^{\frac{x}{138}}$

$f(1825) = 200\left(\frac{1}{2}\right)^{\frac{1825}{138}}$

$f(1825) = 200\left(\frac{1}{2}\right)^{13.22}$

$f(1825) = 0.02$ grams