There are two special triangles: Exact value = no decimal

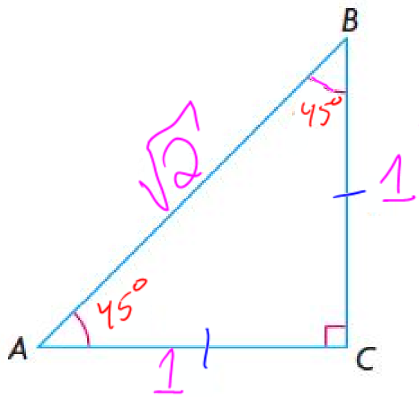
Isosceles Δ

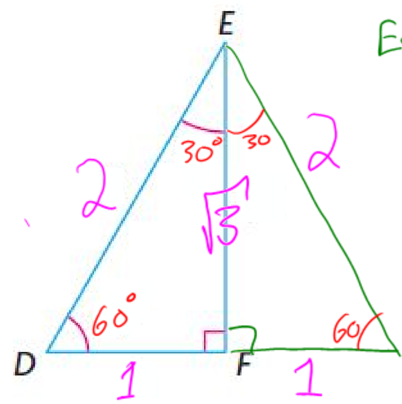
$a^2+b^2=c^2$

$1^2+1^2=c^2$

$2=c^2$

$\sqrt{2}=c$





Equilateral Δ.

$a^2+b^2=c^2$

$1^2+b^2=2^2$

$1+b^2=4$

$b^2=3$

$b=\sqrt{3}$

$\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

θ	$\sin\theta$ (o/h)	$\cos\theta$ (a/h)	$\tan\theta$ (o/a)
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{1}=1$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}=\sqrt{3}$

$\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{\sqrt{3}}{3}$

Determine the exact values of the following: → $(\cos 45)^2$

1. $\sin 30+\boxed{\cos^2 45}-\tan 45$

$=\frac{1}{2}+\left(\frac{\sqrt{2}}{2}\right)^2-\frac{1}{1}$

$=\frac{1}{2}+\boxed{\frac{2}{4}}-\frac{1}{1}$

$=\frac{1}{2}+\frac{1}{2}-\frac{1}{1}$

$=1-1=0!$

2. $\tan 30\times\overset{\to s.h}{\csc}60-\overset{\to \cos}{\sec}60$

$=\frac{\cancel{\sqrt{3}}}{3}\times\frac{2}{\cancel{\sqrt{3}}}-\frac{2}{1}$

$=\frac{2}{3}-\frac{2^{\times 3}}{1_{\times 3}}$

$=\frac{2}{3}-\frac{6}{3}=\frac{-4}{3}$

Determine the exact values of the following:

3. $\sin^2 60 + \cos^2 60$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4} = 1$

Determine the following angle:

4. $\frac{\cancel{\sqrt{2}}\sin\theta}{\cancel{\sqrt{2}}} = \frac{1}{\sqrt{2}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\theta = 45^\circ$

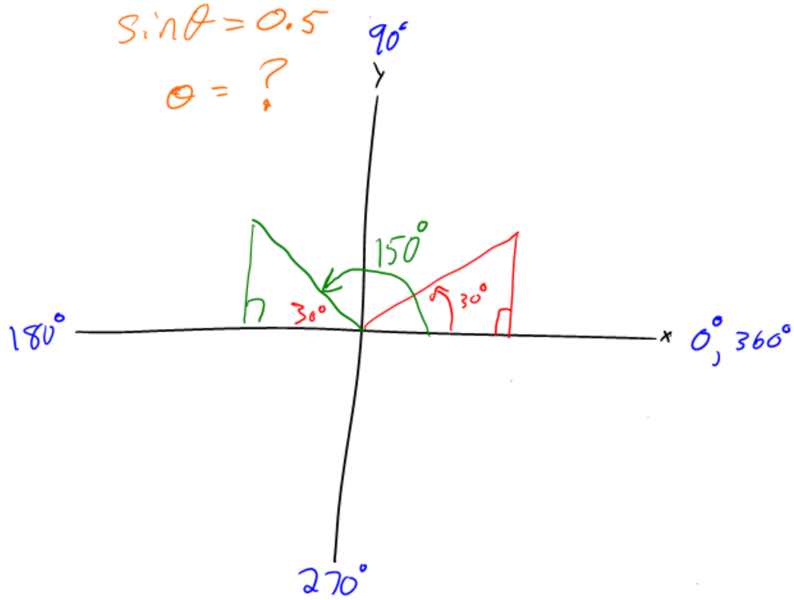
5.3 & 5.4 – Trigonometric Ratios for Angles over 90°

The Big Idea: Every ratio exists twice between 0° and 360°

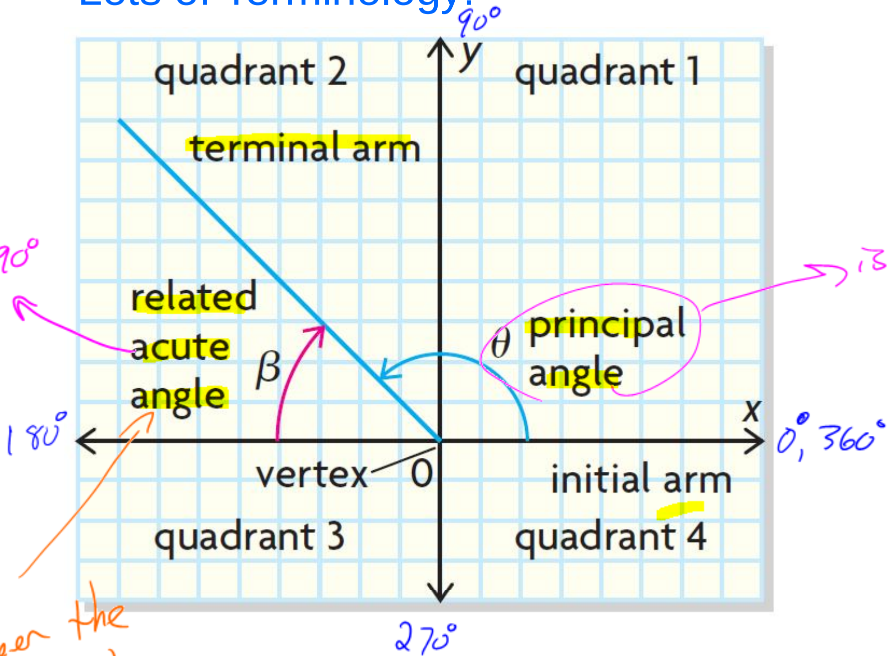
$\sin 30 = 0.5$ $\sin 150 = 0.5$

$\sin\theta = 0.5$

$\theta = ?$



Lots of Terminology:



0° – 90°

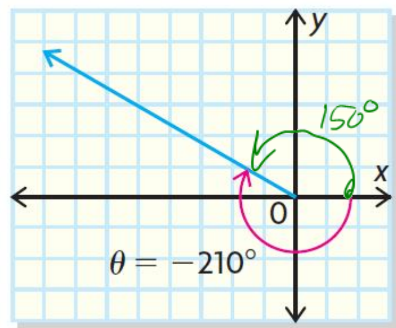
is between 0° and 360°

$0° \le \theta \le 360°$

always between the x-axis and the terminal arm

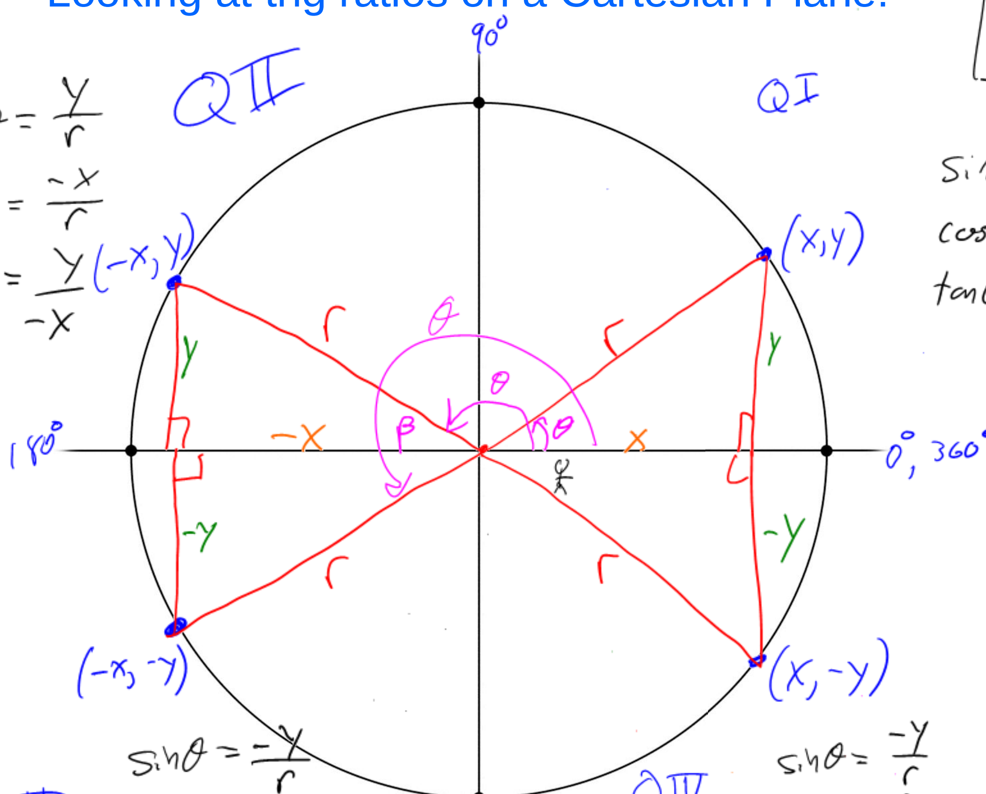
Positive Angle = counter clockwise

Negative Angle = clockwise



Looking at trig ratios on a Cartesian Plane:

$$\boxed{x^2+y^2=r^2}$$



QII:

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{y}{-x}$

QI:

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

QIII:

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

QIV:

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{-y}{x}$

$\sin\theta = \sin(180 - \theta)$

$\cos\theta = \cos(360 - \theta)$

$\tan\theta = \tan(180 + \theta)$

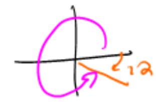
CAST Rule

S	A
T	C

Find the second equivalent trig ratio:

$0^\circ \le \theta \le 360^\circ$

1. $\sin \boxed{20} = \sin(180-20) = \sin 160$

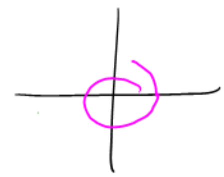


2. $\cos 280 = \cos(360-280) = \cos 80$

3. $\tan 110 = \tan(180+110) = \tan 290$

4. $\csc 192 = \csc(180-192) = \csc(-12) = \csc 348$ "sin" $360-12$

5. $\sec 18 = \sec(360-18) = \sec 342$ "cos"



6. $\cot 215 = \cot(180+215) = \cot 395$ "tan" -360

$= \cot 35$

Each point lies on the terminal arm of angle θ in standard position. ← always starts at 0°

i) Draw a sketch of each angle θ.

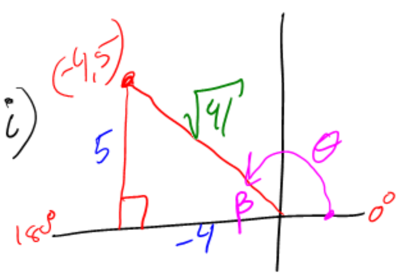
ii) Determine the value of r to the nearest tenth.

iii) Determine the primary trigonometric ratios for angle θ.

iv) Calculate the value of θ to the nearest degree.

$\boxed{P(\overset{x}{-4}, \overset{y}{5})}$

i)



ii)
$r^2 = x^2 + y^2$
$r^2 = (-4)^2 + 5^2$
$r^2 = 16 + 25$
$r^2 = 41$
$r = \sqrt{41} = 6.4$

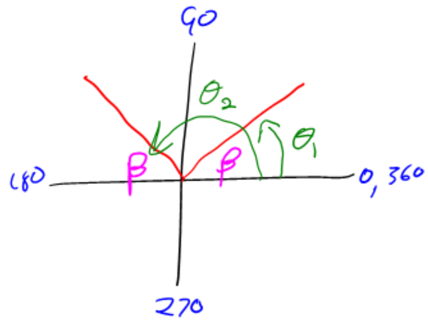
iii)
$\sin\theta = \frac{5}{\sqrt{41}}$
$\cos\theta = \frac{-4}{\sqrt{41}}$
$\tan\theta = \frac{5}{-4}$

iv) First calculate β, which uses positive ratios
$\tan\beta = \frac{5}{4}$
$\beta = \tan^{-1}\left(\frac{5}{4}\right)$
$\beta = 51^\circ$
$\theta = 180 - 51$
$\theta = 129^\circ$

$\sin\theta = \frac{o}{h}$ ✓ $\frac{\text{small}}{\text{big}} = 0.\#$

Use each trigonometric ratio to determine BOTH values of θ between 0º and 360º.

1. $\sin\theta = \overset{+}{0}.8942$



$\sin\beta = 0.8942$

$\beta = \sin^{-1}(0.8942)$

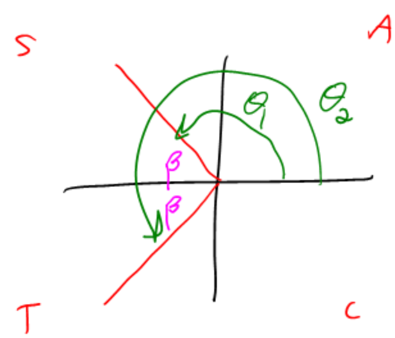
$\beta = 63^\circ$

$\theta_1 = 63^\circ$ $\quad$ $\theta_2 = 180 - 63$

$\theta_2 = 117^\circ$

Use each trigonometric ratio to determine BOTH values of θ between 0º and 360º.

2. $\cos\theta = -0.8931$



$\cos\beta = 0.8931$

$\beta = \cos^{-1}(0.8931)$

$\beta = 27^\circ$

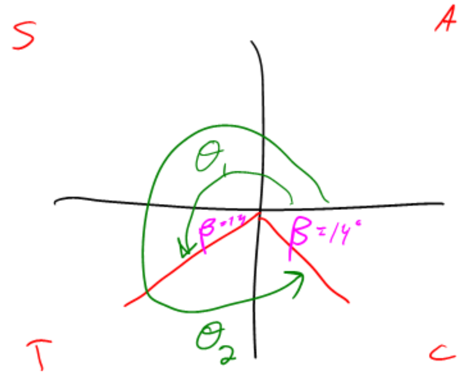
$\theta_1 = 180 - 27$ $\quad$ $\theta_2 = 180 + 27$

$\theta_1 = 153^\circ$ $\quad$ $\theta_2 = 207^\circ$

Use each trigonometric ratio to determine BOTH values of θ between 0º and 360º.

"sin"

3. $\csc\theta = -4.2013$



$\csc\beta = 4.2013$

$\sin\beta = \frac{1}{4.2013}$

$\beta = \sin^{-1}\left(\frac{1}{4.2013}\right)$

$\beta = 14°$

$\theta_1 = 180 + 14$

$\theta_1 = 194°$

$\theta_2 = 360 - 14$

$\theta_2 = 346°$

5.5 – Trigonometric Identities Mr. Hagen's Favourite!!!

Here's an example question:

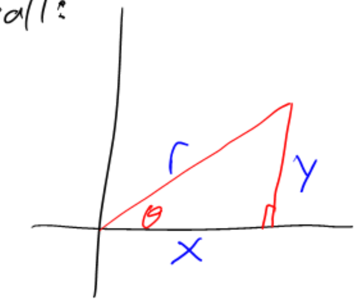
$\sin\theta + \cos\theta\cot\theta = \csc\theta$

Our first identity:

Memorize

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

recall:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

R.S. $\dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\frac{y}{r}}{\frac{x}{r}} \div$

$= \dfrac{y}{\cancel{r}} \times \dfrac{\cancel{r}}{x}$

$= \dfrac{y}{x}$

$= \tan\theta \ \square$

$\therefore$ L.S. = R.S.

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

Our second identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Memorize

L.S. $\sin^2\theta + \cos^2\theta$

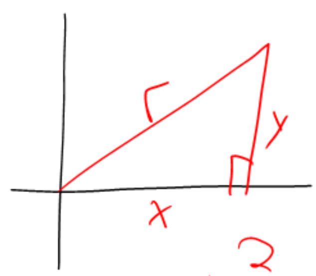
$= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \dfrac{y^2}{r^2} + \dfrac{x^2}{r^2}$

$= \dfrac{y^2 + x^2}{r^2}$ $= r^2$

$= \dfrac{r^2}{r^2}$

$= 1$ $\therefore$ L.S. = R.S.



$x^2 + y^2 = r^2$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

Prove:

$$\cos\theta \tan\theta = \sin\theta$$

Technique:
- turn all tan's/cot's into sines and cosines.

L.S. $\cos\theta \tan\theta$

$= \frac{\cancel{\cos\theta}}{1}\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)$

$= \sin\theta$ $\therefore$ L.S = R.S

Prove:

$$1 + \tan^2\theta = \sec^2\theta$$ Is a cosine

L.S. $1 + \boxed{\tan^2\theta}$

$= \frac{1}{1}\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\overset{1}{\cancel{\cos^2\theta + \sin^2\theta}}}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} = \sec^2\theta$ $\therefore$ L.S. = R.S.

Prove:

$$\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$$

Write everything as sines and cosines

L.S. $\sec^2\theta + \csc^2\theta$

$= \frac{1}{\cos^2\theta}\cdot\frac{\sin^2\theta}{\sin^2\theta} + \frac{1}{\sin^2\theta}\cdot\frac{\cos^2\theta}{\cos^2\theta}$

$= \frac{\cancel{\sin^2\theta + \cos^2\theta}\ 1}{\cos^2\theta\sin^2\theta}$

$= \frac{1}{\cos^2\theta\sin^2\theta}$

$= \sec^2\theta\csc^2\theta$ $\therefore$ R.S. = L.S.

$\frac{1}{2}^{\times 3} + \frac{1}{3}^{\times 2}$ $\quad \frac{1}{x} + \frac{1}{y}$

$= \frac{3+2}{(2)(3)}$

Second last question to prove:

$$\cos\theta + \cos\theta\tan^2\theta = \sec\theta$$

L.S. $\cos\theta + \cos\theta\tan^2\theta$

$= \cos\theta(1 + \tan^2\theta)$

$= \cos\theta\sec^2\theta$

$= \frac{\cancel{\cos\theta}}{1}\left(\frac{1}{\cos^{\cancel{2}}\theta}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta$ $\therefore$ L.S. = R.S.

$x + xy^2$

$x(1+y^2)$

$\frac{x}{1}\left(\frac{1}{x^2}\right) = \frac{x}{x^2} = \frac{1}{x}$

$\frac{3}{9} = \frac{1}{3}$

$\frac{4}{16} = \frac{1}{4}$

Last question to prove:

$$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$$

L.S. $\cos^4\theta - \sin^4\theta$

$= (\cos^2\theta - \sin^2\theta)(\cancelto{1}{\cos^2\theta + \sin^2\theta})$

$= \boxed{\cos^2\theta} - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$ $\therefore$ L.S. = R.S.

Difference of Squares

$x^2 - y^2 = (x-y)(x+y)$

$x^4 - y^4 = (x^2 - y^2)(x^2 + y^2)$

$x^6 - y^6 = (x^3 - y^3)(x^3 + y^3)$

$\sin^2\theta + \cos^2\theta = 1 -$

$\cos^2\theta = 1 - \sin^2\theta$