This chapter deals with Sinusoidal Functions, which are just a type (a subset) of Periodic Functions. The two sinusoidal functions we will work with are: $f(x) = \sin x$ and $f(\theta) = \cos\theta$

Periodic Function:

- a graph which repeats itself. The pattern must be exactly the same each time.

Period:

- one section of the graph that is repeated
- the length on the x-axis of one $\underbrace{\text{cycle}}_{\text{one period}}$

Peak:

- the maximum

Trough:

- the minimum

$$\frac{3}{7} = 0.\overline{428571}4285714285\ldots$$

Period = 428571 Length of Period = 6

Equation of Axis:

- the "middle" of the graph on the y-axis.

$$y = \frac{\text{peak} + \text{trough}}{2}$$

Amplitude:

- the distance from the middle to the peak or trough

always positive.

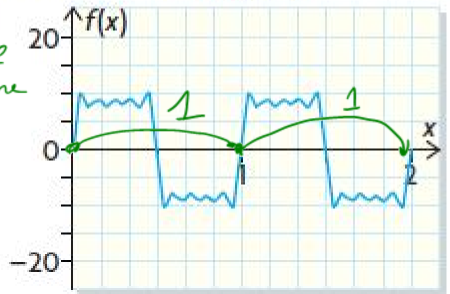
amp = peak − middle

amp = middle − trough

$$\text{amp} = \frac{\text{peak} - \text{trough}}{2}$$

1. Which of the following graphs are periodic? Explain why or why not.

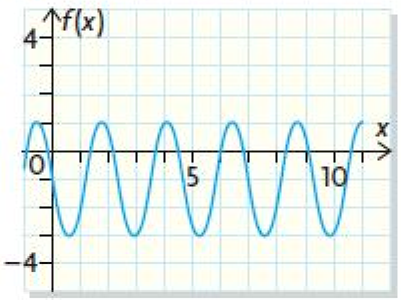
a)



period is one and the same each time.

YES!

c)



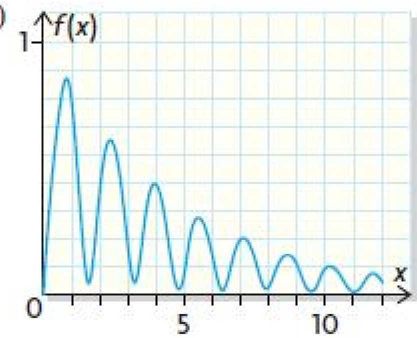
same peaks, troughs, periods ∴ Periodic.

b)

- doesn't repeat
- keeps increasing

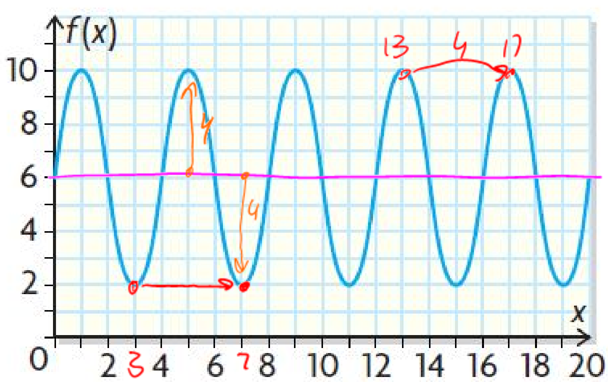
Not periodic

d)



doesn't repeat not periodic.

2. Determine the range, period, equation of the axis, and amplitude of the function shown.



1. Range: Peak = 10 Trough = 2

$\{f(x) \in \mathbb{R} \mid \underbrace{2 \le f(x) \le 10}_{\text{y is inbetween 2 and 10.}}\}$

2. Period: $7 - 3 = 4$

3. Equation of Axis:

$y = \frac{10+2}{2}$

$y = 6$

4. Amplitude:

amp = 4

$amp = \frac{10-2}{2} = 4$

3. The motion of an automated device for attaching bolts to a household appliance on an assembly line can be modelled by the graph shown at the left.

a) What is the period of one complete cycle? one second

b) What is the maximum distance between the device and the appliance? 1.5 cm

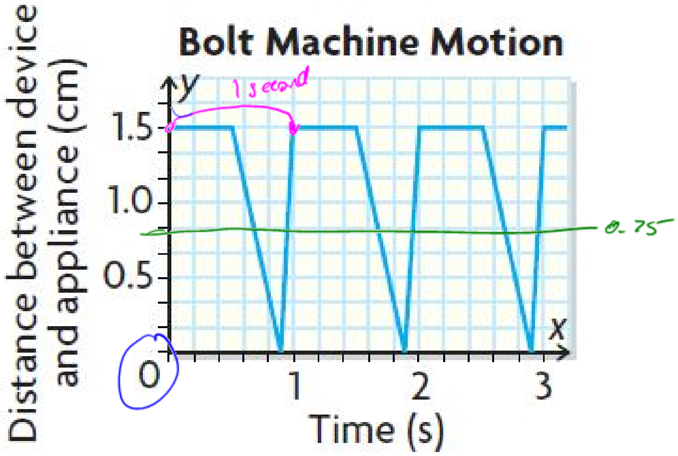
c) What is the range of this function? $\{y \in \mathbb{R} \mid 0 \le y \le 1.5\}$

d) If the device can run for five complete cycles only before it must be turned off, determine the domain of the function. 5 periods = 5 seconds $\{x \in \mathbb{R} \mid 0 \le x \le 5\}$

e) Determine the equation of the axis. $y = \frac{1.5+0}{2} = 0.75$

f) Determine the amplitude. $y = \frac{1.5-0}{2} = 0.75$; peak − middle = $1.5 - 0.75 = 0.75$

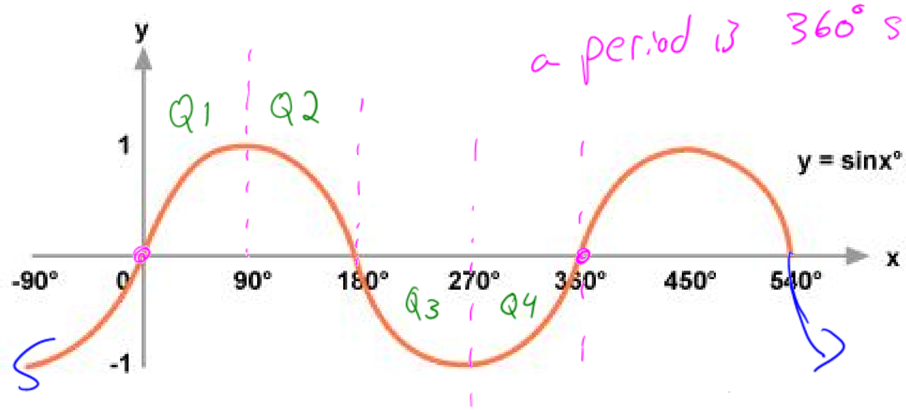
g) There are several parts to each complete cycle of the graph. Explain what each part could mean in the context of "attaching the bolt."



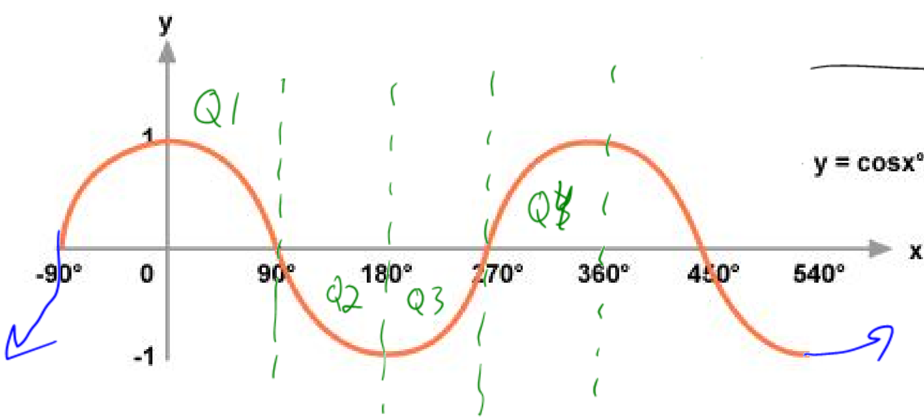
6.2 – Sinusoidal Functions

What do the graphs of: $f(\theta) = \sin\theta$ and $f(\theta) = \cos\theta$ look like?

a period is 360° starting at 0°



x	y
0	0
90	1
180	0
270	−1
360	0



x	y
0	1
90	0
180	−1
270	0
360	1

$f(x) = a\sin(k(x-d)) + c$

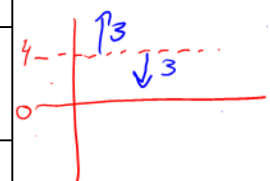
Function	$f(x) = 3\sin(\underbrace{2x - 180}_{\text{factored}}) + 4$
Proper Function	$f(x) = \overset{a}{3}\sin(\overset{k}{2}(x - \overset{d}{\boxed{90}})) \overset{c}{+4}$
Amplitude = $\lvert a \rvert$, must be positive.	3 vertical Stretch
Period $\frac{360°}{k}$	$\frac{360°}{2} = 180°$ one cycle; $\frac{1}{2}$ = H. Stretch
Phase Shift d	90° horiz. shift
Equation of Axis $y = c$	$y = 4$ vert. shift
Domain (2 cycles) start at 0°	$\{x \in \mathbb{R} \mid 0 \le x \le \underset{180 \times 2}{360}\}$
Range Trough to Peak	Peak = $c + a = 4 + 3 = 7$; Trough = $c - a = 4 - 3 = 1$; $1 \le f(x) \le 7$

a = vertical stretch

$\frac{1}{k}$ = Horizontal stretch

d = Horizontal Shift

c = Vertical shift.



Start with $y = \sin x$

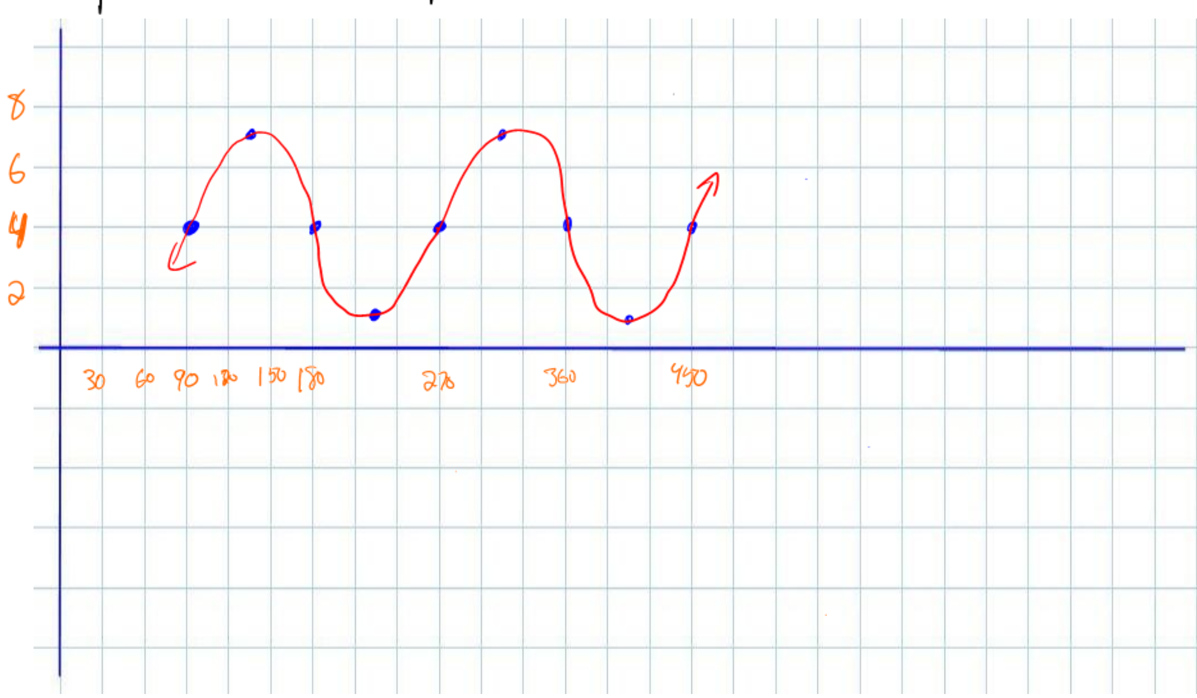
x	y
0	0
90	1
180	0
270	−1
360	0

$\frac{1}{2}x + 90$	$3y + 4$
90	4
135	7
180	4
225	1
270	4

180°

reminder:

$\frac{1}{k}x + d$	$ay + c$



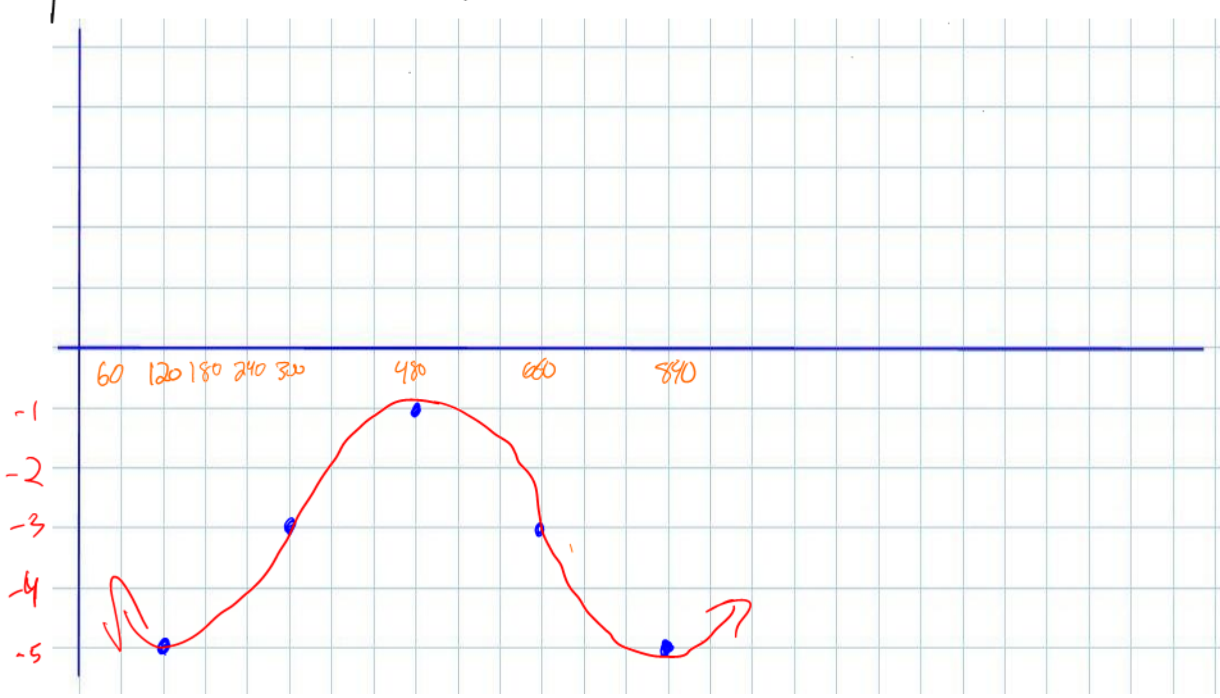
Function	$f(x) = -2\cos\left(\frac{1}{2}x - 60\right) - 3$	
Proper Function	$f(x) = \overset{a}{-2}\cos\left(\overset{k}{\tfrac{1}{2}}\left(x - \overset{d}{\boxed{120}}\right)\right) \overset{c}{-3}$	
Amplitude $\lvert a \rvert$	2	$a = -2$
Period	$\frac{360}{1/2} = 720$	$\frac{1}{k} = \frac{2}{1} = 2$
Phase Shift	120	$d = 120$
Equation of Axis	$y = -3$	$c = -3$
Domain (2 cycles)	$\{x \in \mathbb{R} \mid 0 \le x \le 1440\}$	
Range	Peak = C + amp = −3+2 = −1 | Trough = C − amp = −3 − 2 = −5	$-5 \le y \le -1$

Transformations

start with cosine

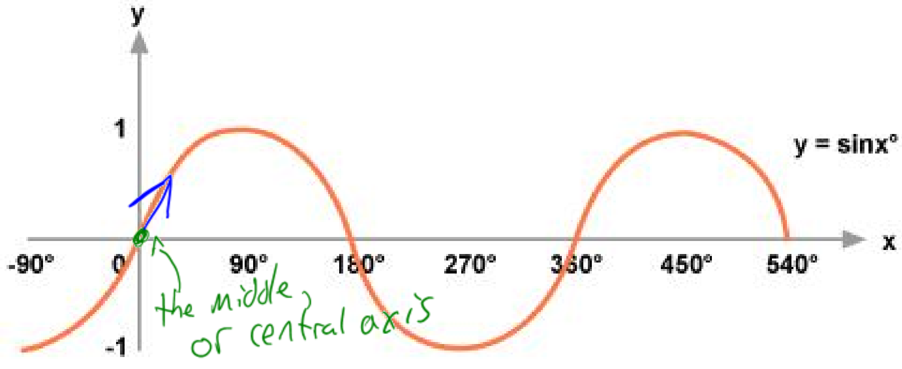
x	y
0	1
90	0
180	−1
270	0
360	1

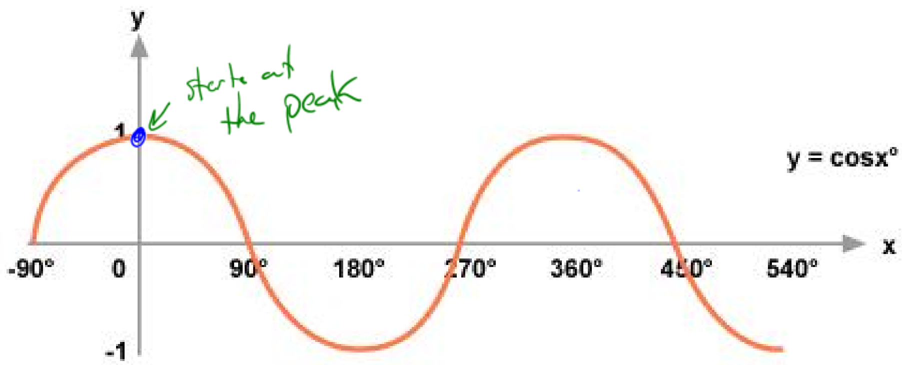
$2x + 120$	$-2y - 3$
120	−5
300	−3
480	−1
660	−3
840	−5



6.6 – Models of Sinusoidal Functions

A reminder of our sinusoidal functions:





The key to creating equations:

cos

$$f(x) = a\sin\big(k(x-d)\big) + c$$

middle

Amplitude = a, found by *peak-EoA* or $amp = \dfrac{peak - trough}{2}$

Period = $\dfrac{360^\circ}{k}$ therefore $k = \dfrac{360}{Period}$

→ x-coordinate

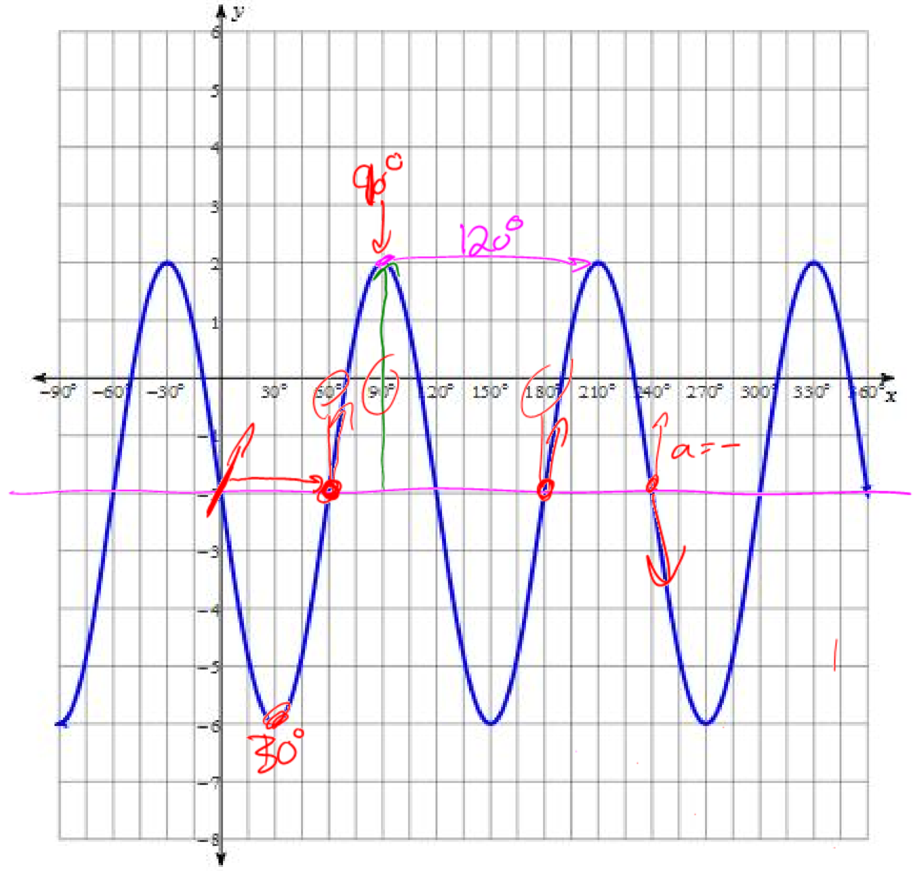
middle

Phase Shift = d – this is your "starting point" – must be peak, EoA or trough

Equation of Axis = c, found by $\dfrac{peak+trough}{2}$

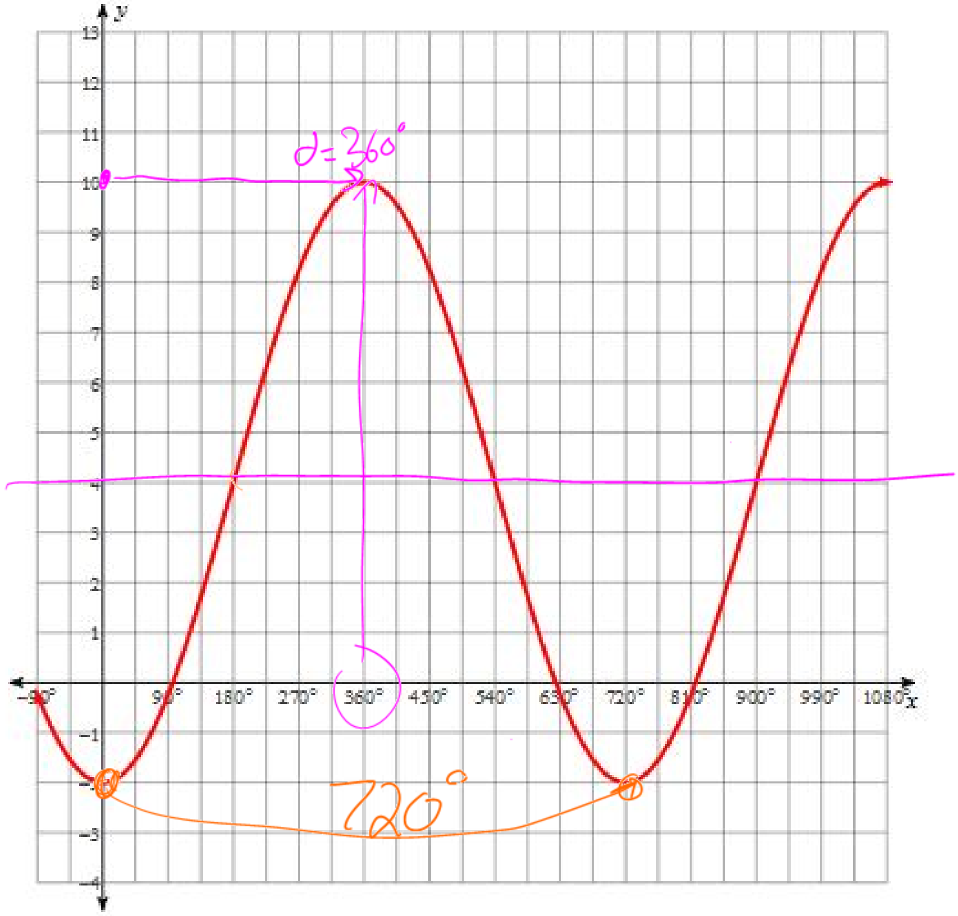
	Starting at…
+sin	Equation of axis, then heads to peak
-sin	Equation of axis, then heads to trough
+cos	Peak
-cos	Trough

depends on the phase shift or "d"



Peak and Trough	$P=2 \quad T=-6$
Equation of Axis	$y=\frac{2+-6}{2}=-2$ (c)
Amplitude	amp $=4$ | $\frac{2--6}{2}=4$ (a)
Period and *k*	period $=120^\circ$ $\quad k=\frac{360}{120}=3$
Phase Shift for sine	$d=60$ || $d=240$ $f(x)=-4\sin(3(x-240))-2$
Phase Shift for cosine	x-value for peak $=90^\circ$ a=+ | x-value for a trough $=30^\circ$ a=−
Functions	$f(x)=4\sin(3(x-60))-2$ $f(x)=4\cos(3(x-90))-2$

$f(x)=-4\cos(3(x-30))-2$



Peak and Trough	P=10 T= −2
Equation of Axis	$y = \frac{10 + -2}{2} = 4$ c
Amplitude	amp = 6 a
Period and *k*	period = 720° $k = \frac{360}{720} = \frac{1}{2}$ k
Phase Shift for sine	blah
Phase Shift for cosine	d = 360° peak a +
Functions	$f(x) = 6\cos\left(\frac{1}{2}(x - 360)\right) + 4$

x	0°	45°	90°	135°	180°	225°	270°
y	9	7	5	7	9	7	5

Peak = 9

Trough: 5

Middle ⇒ $y = 7$ (c)

Amp = $\frac{9-5}{2} = 2$ (a)

Period = 180°

$k = \frac{360}{180} = 2$

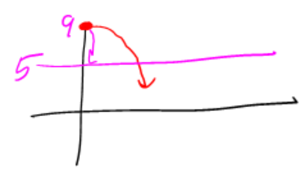
Zach said use 90° which is a trough ∴ −cosine.

$f(x) = -2\cos(2(x-90)) + 7$

A sinusoidal function has an amplitude of 4 units, a period of 120°, and a maximum at (0,9). Determine the equation of the function.

amp = 4 (a)

x-coordinate of the peak ∴ $d = 0$



period = 120°

$k = \frac{360}{120} = 3$ (k)

$\therefore f(x) = 4\cos(3(x-0)) + 5$

Peak at 9.

$f(x) = 4\cos(3x) + 5$

Middle = Peak − amp

$= 9 - 4 = 5$ (c)

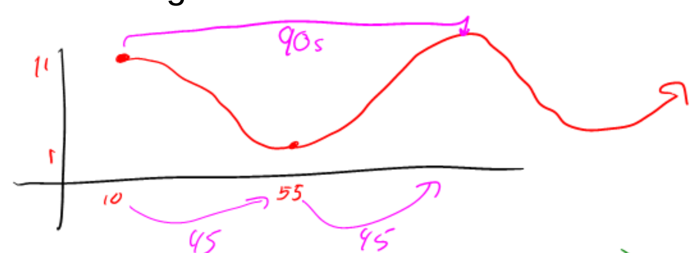
A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches a maximum height of 11m at 10s and then reaches a minimum height of 1m at 55s. How high is John after 2 minutes?

time = x

height = y

Max: (10, 11)

min: (55, 1)



Peak = 11

Trough = 1

Middle: $y = \frac{1+11}{2} = 6$ (c)

Amp: $11 - 6 = 5$ (a)

Period: 90° $k = \frac{360}{90} = 4$ (k)

$10 = d$

$f(x) = 5\cos(4(x-10)) + 6$

2 minutes = 120 seconds

$f(120) = 5\cos(4(120-10)) + 6$

$f(120) = 6.9$ metres