Mathematics 11U

3.2 – Determining Maximums and Minimums of Quadratic Functions

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The maximum/minimum is the y-coordinate of the vertex, or the *k*. A maximum occurs when *a*<0 and a minimum occurs when *a*>0.

Given vertex form:

$$f(x) = a(x - h)^2 + k$$

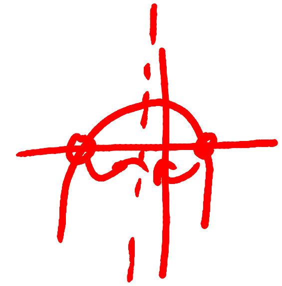
$$f(x) = -2(x + 5)^2 - 8$$

max of -8

(-5, -8)

Given Zeros Form:

$$f(x) = a(x-r)(x-s)$$



$x, f(x)$

vertex (h, k)

$h = \frac{r+s}{2}$

$f(h) = k$

$$f(x) = 3(x+2)(x-8)$$

$r = -2 \quad s = 8$

$h = \frac{-2+8}{2} = 3$

min of -75

$f(3) = 3(3+2)(3-8)$

$= 3(5)(-5) = -75$

Given Standard Form: $f(x) = ax^2 + bx + \boxed{c}$

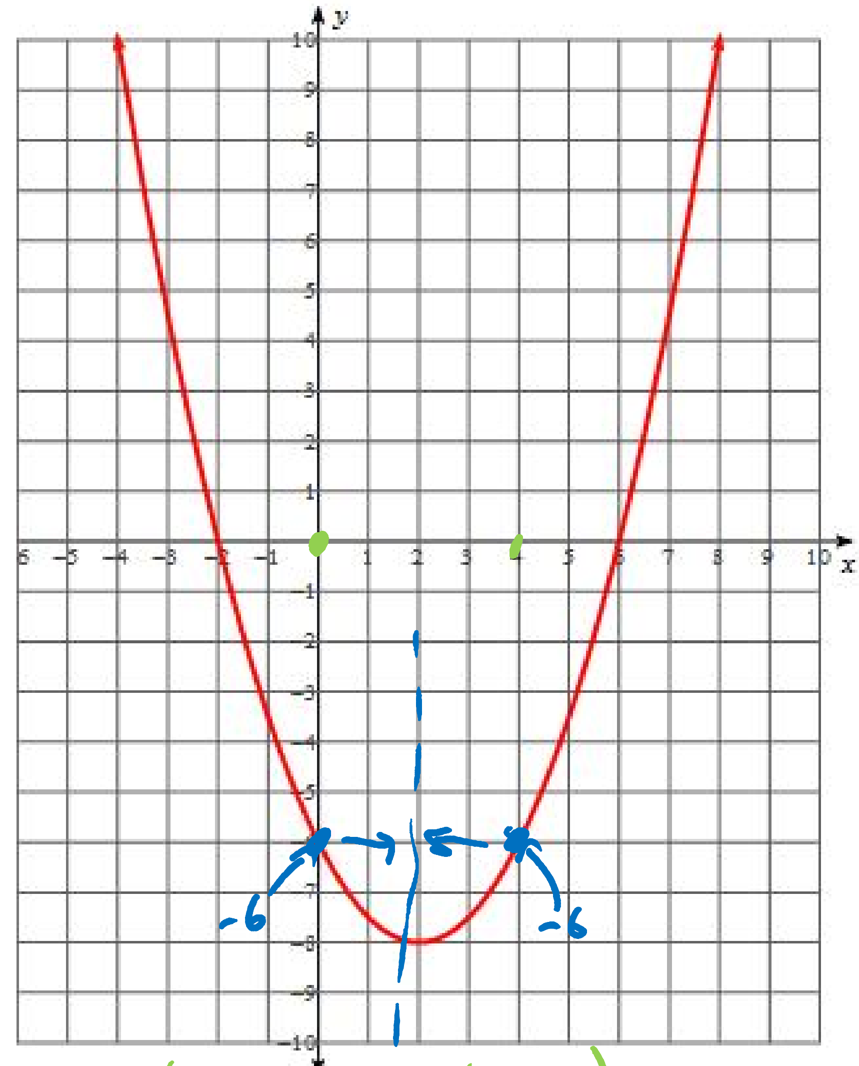
Partial Factoring

- All quadratics have y-ints.

we want the other "y-int" coordinate

$(0, c)$

$(?, c)$



$(0, -6)$ $(4, -6)$

ex: $f(x) = \underbrace{2x^2 + 10x}_{2x} - 5$

$f(0) = -5$

$f(?) = -5$

$f(x) = 2x(x+5) - 5$

$0 \quad -5$

$(0, -5)$

$(-5, -5)$

$h = \frac{0-5}{2} = -\frac{5}{2} = -2.5$

$f(-2.5) = 2(-2.5)^2 + 10(-2.5) - 5$

$= 12.5 - 25 - 5$

$= -17.5$

$\therefore$ minimum of -17.5

ex2. $f(x) = \dfrac{3x^2 + 48x}{3x} - 19$

$f(x) = 3x(x+16) - 19$

$\uparrow$ 0 $\quad$ $\uparrow$ -16

$h = \dfrac{0 - 16}{2} = -8$

$f(-8) = 3(-8)^2 + 48(-8) - 19$

$= 192 - 384 - 19$

$= -211$

$\therefore$ min at -211

$$f(x) = \underbrace{\frac{1}{2}x^2 - 5x}_{\frac{1}{2}x} + 8$$

$$f(x) = \frac{1}{2}\underset{0}{\underset{\uparrow}{x}}(\underset{10}{\underset{\uparrow}{x}} - 10) + 8$$

$$h = \frac{0+10}{2} = 5$$

$$f(5) = \frac{1}{2}(5)^2 - 5(5) + 8$$

$$f(5) = 12.5 - 25 + 8$$

$$f(5) = -4.5$$

$\therefore$ m.h of -4.5

$$f(x) = \underbrace{-2.2x^2 + 8.4x}_{-2.2x} + 5.76$$

$$f(x) = \underset{\overset{\uparrow}{0}}{-2.2x}\left(\underset{\overset{\uparrow}{3.82}}{x} - 3.82\right) + 5.76$$

$$h = \frac{0 + 3.82}{2} = 1.91$$

$$f(1.91) = -2.2(1.91)^2 + 8.4(1.91) + 5.76$$

$$= -8.03 + 16.04 + 5.76$$

$$= 13.77$$

$$\boxed{\therefore \text{Max of } 13.77}$$