

Mathematics 11U

3.4 – Operations with Radicals

Mr. D. Hagen

$$\sqrt{5} \quad \sqrt{4} = 2$$

$$\begin{aligned} 1^2 &= 1 \\ 2^2 &= 4 \\ 3^2 &= 9 \\ 4^2 &= 16 \\ 5^2 &= 25 \\ 6^2 &= 36 \\ 7^2 &= 49 \\ 8^2 &= 64 \\ 9^2 &= 81 \\ 10^2 &= 100 \end{aligned}$$

$$\begin{aligned} 11^2 &= 121 \\ 12^2 &= 144 \\ 13^2 &= 169 \\ 14^2 &= 196 \\ 15^2 &= 225 \\ 16^2 &= 256 \\ 17^2 &= 289 \\ 18^2 &= 324 \\ 19^2 &= 361 \\ 20^2 &= 400 \end{aligned}$$

Simplifying Radicals: $\sqrt{2} \times \sqrt{3} = \sqrt{6}$

$$\begin{aligned} 1. \quad & 2\sqrt{48} \\ &= 2\sqrt{16}\sqrt{3} \\ &= 2(4)\sqrt{3} \\ &= 8\sqrt{3} \end{aligned}$$

$$\begin{aligned} 2. \quad & -\sqrt{20} \\ &= -\sqrt{4}\sqrt{5} \\ &= -2\sqrt{5} \end{aligned}$$

$$\begin{aligned} 3. \quad & \frac{1}{8}\sqrt{320} \\ &= \frac{1}{8}\sqrt{64}\sqrt{5} \\ &= \sqrt{5} \end{aligned}$$

$$\begin{aligned} 4. \quad & -3\sqrt{513} \\ &= -3\sqrt{81}\sqrt{57} \\ &= -9\sqrt{57} \end{aligned}$$

Simplifying Radicals with variables:

$$1. \ 7\sqrt{288b^4}$$

$$= 7\sqrt{144b^4}\sqrt{2}$$

$$= 84b^2\sqrt{2}$$

$$2. \ -5\sqrt{45n^3}$$

$$= -5\sqrt{9n^2}\sqrt{5n}$$

$$= -15n\sqrt{5n}$$

Adding and Subtracting Radicals: *Same radicals*

$$1. -2\sqrt{12} - 3\sqrt{8} + 3\sqrt{32} + 2\sqrt{27}$$

$$= -2\sqrt{4}\sqrt{3} - 3\sqrt{4}\sqrt{2} + 3\sqrt{16}\sqrt{2} + 2\sqrt{9}\sqrt{3}$$

$$= \underline{-4\sqrt{3}} - \underline{6\sqrt{2}} + \underline{12\sqrt{2}} + \underline{6\sqrt{3}}$$

$$= 2\sqrt{3} + 6\sqrt{2}$$

$$2. 2\sqrt{45} - \sqrt{8} - 2\sqrt{32} - 2\sqrt{18}$$

$$= 2\sqrt{9}\sqrt{5} - \sqrt{4}\sqrt{2} - 2\sqrt{16}\sqrt{2} - 2\sqrt{9}\sqrt{2}$$

$$= 6\sqrt{5} - 2\sqrt{2} - 8\sqrt{2} - 6\sqrt{2}$$

$$= 6\sqrt{5} - 16\sqrt{2}$$

Multiplying Radicals:

$$1. \quad 2\sqrt{6} \times 4\sqrt{5}$$
$$= 8\sqrt{30}$$

$$2. \quad -3\sqrt{10} \times 6\sqrt{2}$$
$$= -18\sqrt{20}$$
$$= -18\sqrt{4}\sqrt{5}$$
$$= -36\sqrt{5}$$

$$3. \quad (\sqrt{2} + \sqrt{3})(2\sqrt{2} - 5\sqrt{3})$$
$$= 2\sqrt{4} - 5\sqrt{6} + 2\sqrt{6} - 5\sqrt{9}$$
$$= 4 - 3\sqrt{6} - 15$$
$$= -11 - 3\sqrt{6}$$

Rationalizing the Denominator:

$$1. \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

$$2. \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{24}}{3} = \frac{\sqrt{4}\sqrt{6}}{3} = \frac{2\sqrt{6}}{3}$$

$$\begin{aligned} 3. \frac{9\sqrt{63}}{7\sqrt{54}} &\rightarrow \frac{9\sqrt{7}}{7\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} \\ &= \frac{9\cancel{\sqrt{9}}\sqrt{7}}{7\cancel{\sqrt{9}}\sqrt{6}} = \frac{9\sqrt{42}}{42} \\ &= \frac{9\sqrt{7}}{7\sqrt{6}} = \frac{3\sqrt{42}}{14} \end{aligned}$$

$$\begin{aligned} 4. \frac{(7+\sqrt{2})}{5\sqrt{40}} \times \frac{\sqrt{40}}{\sqrt{40}} &= \frac{7\sqrt{40} + \sqrt{80}}{200} \\ &= \frac{7\sqrt{4}\sqrt{10} + \sqrt{16}\sqrt{5}}{200} \\ &= \frac{14\sqrt{10} + 4\sqrt{5}}{200} = \frac{7\sqrt{10} + 2\sqrt{5}}{100} \end{aligned}$$