Mathematics 11U

5.5 – Trigonometric Identities

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Here's an example question:

$$\sin\theta + \cos\theta \cot\theta = \csc\theta$$

$$L.S \quad \sin\theta + \cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$= \sin\theta + \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

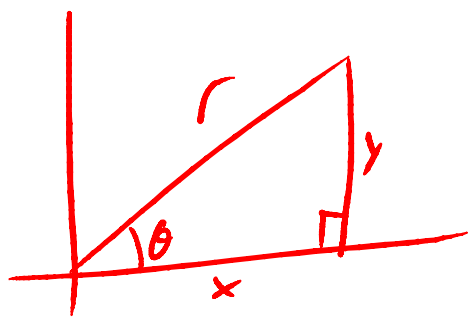
$$= \frac{1}{\sin\theta}$$

$$= \csc\theta \quad \therefore L.S = R.S.$$

Our first identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Recall:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{r}$

R.S. $= \dfrac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{r} \times \frac{r}{x}$

$= \frac{y}{x}$

$= \tan\theta \quad \therefore L.S. = R.S.$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\tan^2\theta = \dfrac{\sin^2\theta}{\cos^2\theta}$

Our second identity:

$$\sin^2\theta + \cos^2\theta = 1$$

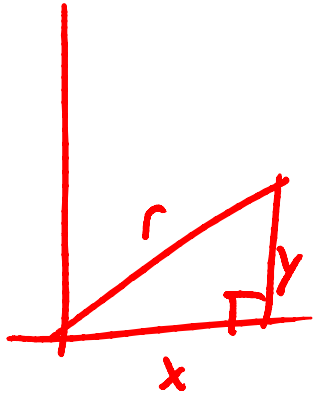
L.S. $\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2$

$= \frac{y^2}{r^2} + \frac{x^2}{r^2}$

$= \frac{x^2+y^2}{r^2}$

$= \frac{r^2}{r^2}$

$= 1 \quad \therefore LS = RS$



$x^2+y^2=r^2$

$\cos^2\theta = 1 - \sin^2\theta$

$= (1+\sin\theta)(1-\sin\theta)$

$\sin^2\theta = 1 - \cos^2\theta$

$= (1-\cos\theta)(1+\cos\theta)$

Prove:

$\cos\theta \tan\theta = \sin\theta$

L.S. $\frac{\cos\theta}{1}\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \sin\theta$

$\therefore$ L.S. = R.S.

Prove:

$1 + \tan^2\theta = \sec^2\theta$

L.S. $1 + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta}$

$= \sec^2\theta$

$\therefore$ L.S. = R.S.

Prove:

$$\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta$$

L.S. $\dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \dfrac{1}{\cos^2\theta \sin^2\theta}$

$= \sec^2\theta \csc^2\theta$ $\therefore$ L.S. = R.S.

Second last question to prove:

$\cos\theta + \cos\theta \tan^2\theta = \sec\theta$

L.S. $\cos\theta(1 + \tan^2\theta)$

$= \cos\theta(\sec^2\theta)$

$= \cos\theta\left(\frac{1}{\cos^2\theta}\right)$

$= \frac{1}{\cos\theta}$

$= \sec\theta$ $\therefore$ L.S. = R.S.

$\frac{x}{x^2} = \frac{1}{x}$

Last question to prove:

$$\cos^4\theta - \sin^4\theta = 1 - 2\sin^2\theta$$

L.S. $(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)$

$1 \cdot (\cos^2\theta - \sin^2\theta)$

$1 - \sin^2\theta - \sin^2\theta$

$1 - 2\sin^2\theta$

$\therefore$ L.S. = R.S

$x^2 - y^2 = (x-y)(x+y)$

$x^4 - y^4 = (x^2-y^2)(x^2+y^2)$

$x^6 - y^6 = (x^3-y^3)(x^3+y^3)$

$\cos^2\theta + \sin^2\theta = 1$