Mathematics 11U

6.6 – Models of Sinusoidal Functions

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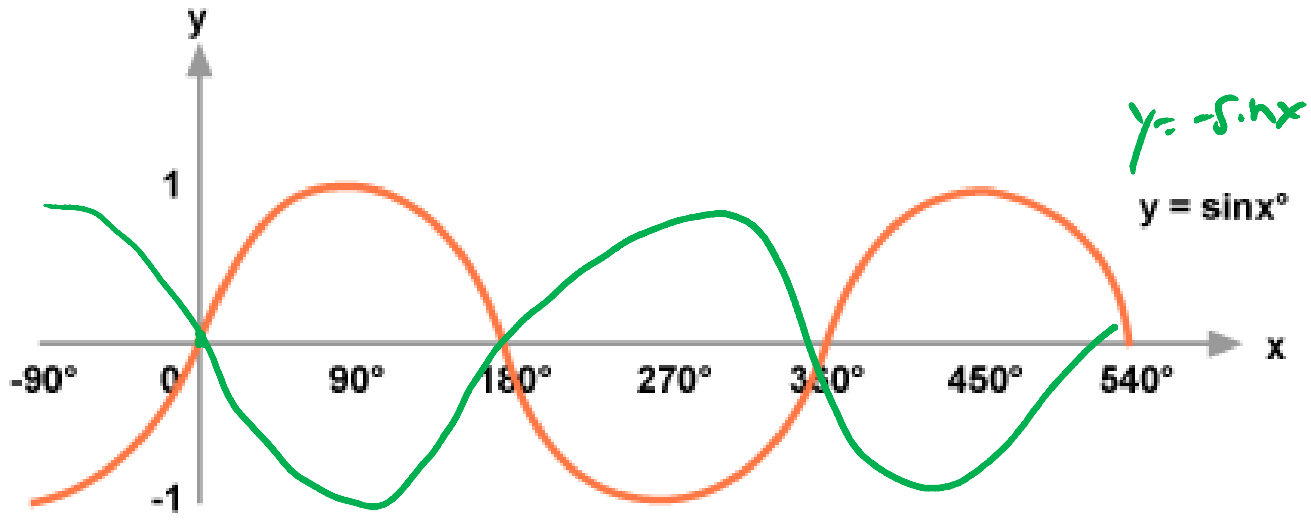
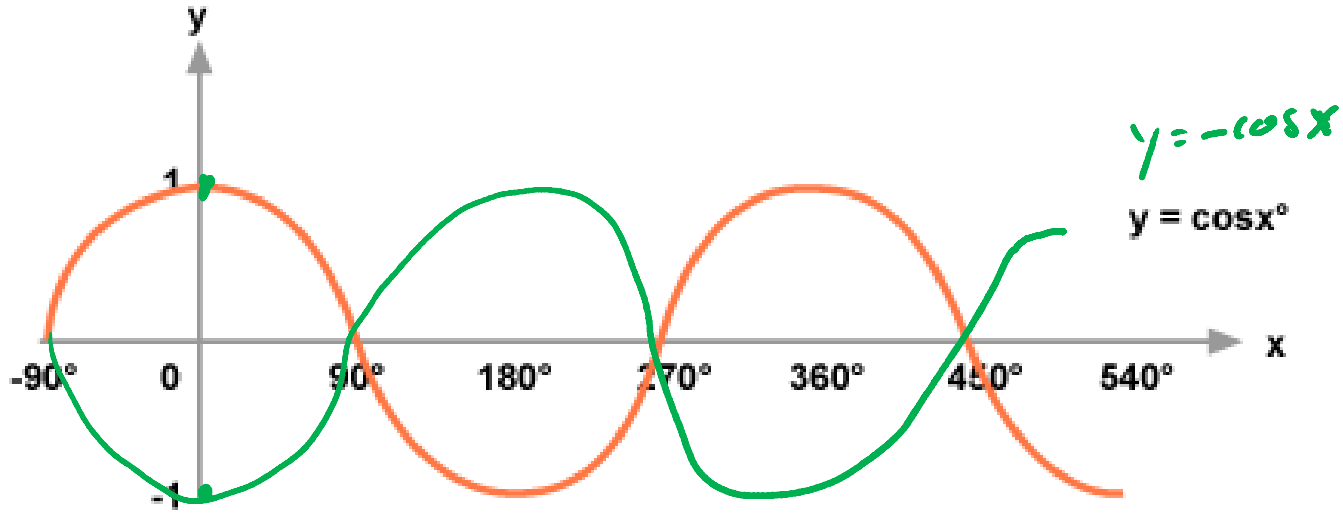
A reminder of our sinusoidal functions:





The key to creating equations:

$f(x) = a\sin\left(k(x - d)\right) + c$

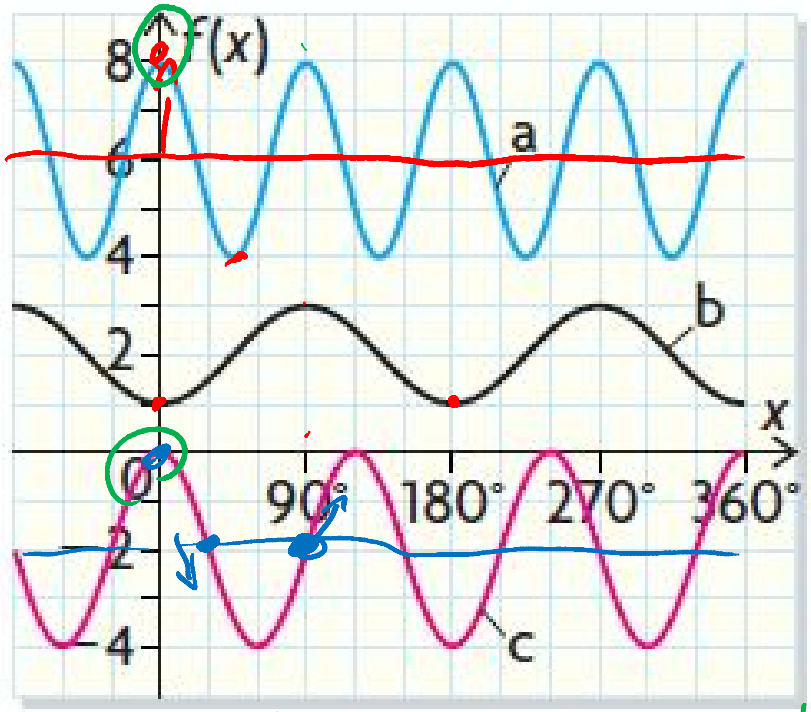
Amplitude = a, found by *peak-EoA*

Period = $\frac{360°}{k}$ therefore $k = \frac{360}{Period}$

Phase Shift = d – this is your "starting point" – must be peak, EoA or trough

Equation of Axis = c, found by $\frac{peak+trough}{2}$

	Starting at…
+sin	Equation of axis, then heads to peak
-sin	Equation of axis, then heads to trough
+cos	Peak
-cos	Trough



a) Amp: 2 a

EoA: y=6 c

P: 8 $\frac{8}{2}+4=\frac{12}{2}=6$

T: 4

Period: 90° ⇒ $k=\frac{360}{90}=4$

Peak = + cosine

$f(x)=2\cos(4(x-0))+6$

$f(x)=2\cos(4(x-90))+6$

P: 3 T: 1 ∴ EoA = 2

Amp: 1 Period = 180 ∴ $k=\frac{360}{180}$

$k=2$

$f(x)=-1\cos(2x)+2$

$f(x)=-1\cos(2(x-180))+2$

$f(x)=1\cos(2(x-90))+2$

P: 0 T: -4 ∴ EoA = -2

Amp: 2 Period = 120 ∴ $k=\frac{360}{120}=3$

$f(x)=2\sin(3(x-90))-2$

$f(x)=-2\sin(3(x-30))-2$

$f(x)=2\cos(3(x-240))-2$

x	0°	45°	90°	135°	180°	225°	270°
y	9	7	5	7	9	7	5

Peak = 9

Trough = 5

EoA = 7

Amp = 2

Period = 180 $\therefore k = \frac{360}{180} = 2$

$f(x) = 2\cos(2x) + 7$

$f(x) = -2\cos(2(x-270)) + 7$

$f(x) = 2\sin(2(x-135)) + 7$

A sinusoidal function has an amplitude of 4 units, a period of 120°, and a maximum at (0,9). Determine the equation of the function.

Amp = 4 ↑ starting point

Period = 120, $\therefore k = \frac{360}{120} = 3$

$f(x) = 4\cos(3x) + 5$

Peak: 9

EoA = 9 − 4 = 5

A group of students is tracking a friend, John, who is riding a Ferris wheel. They know that John reaches a maximum height of 11m at 10s and then reaches a minimum height of 1m at 55s. How high is John after 2 minutes?

(time, height)

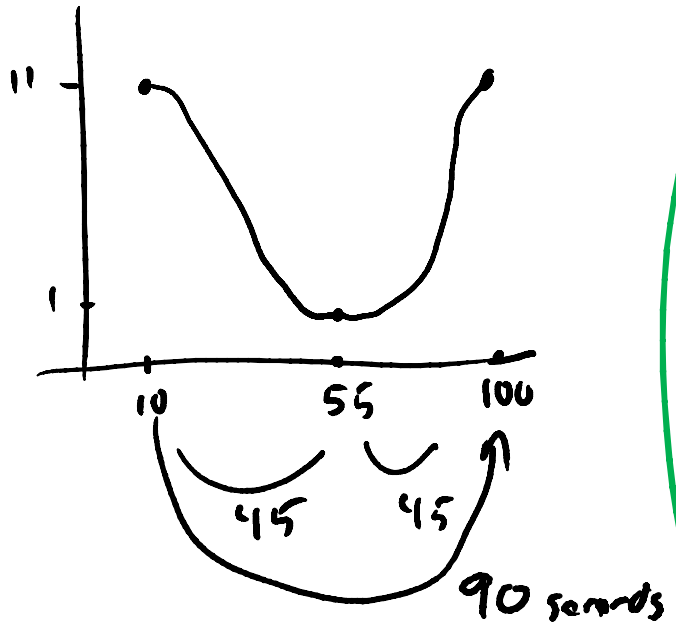
Max (10, 11) S.P.

Min (55, 1)

Peak = 11, Trough = 1, ∴ EoA = $\frac{11+1}{2} = 6$

Amp = 11 − 6 = 5

Period = 90, ∴ $k = \frac{360}{90} = 4$



$f(x) = 5\cos(4(x-10)) + 6$

2 min = 120 seconds

$f(120) = 5\cos(4(120-10)) + 6$

$f(120) = 5\cos(440) + 6$

$f(120) = 6.87$ m.