

Mathematics 11U

7.1 – Arithmetic Sequences

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First, some terminology:

Sequence:

- an ordered list of numbers
- ex: -4, 1, 4, 9, 10, 12, 100

Term:

- a number in a sequence
- subscripts are used to identify the position of the term
- $t_2 = 1$ $t_6 = 12$

Arithmetic Sequence:

- a sequence that has the same difference between consecutive terms, known as the **common difference**, or **d**

- ex: 4, 9, 14, 19, 24... *adding 5, d = 5*

- ex: 10, 4, -2, -8 ... *~~subtracting 6~~
adding -6 $\therefore d = -6$*

Geometric Sequence:

- a sequence that has the same ratio between consecutive terms, known as the **common ratio**, or r

- ex: 4, 8, 16, 32, 64... $\frac{8}{4} = 2 \quad \therefore r = 2$

- ex: 10, 4, $\frac{8}{5}$, $\frac{16}{25}$... $\frac{4}{10} = \frac{2}{5} \quad \therefore r = \frac{2}{5}$

Recursive Sequence:

- a sequence for which one or more terms is given and each successive term is determined from the previous term(s)

- ex: 1, 1, 2, 3, 5, 8, 13, 21, 34,

Fibonacci Sequence.

$$1 + 1 = 2$$

$$1 + 2 = 3$$

$$2 + 3 = 5$$

$$3 + 5 = 8$$

$$5 + 8 = 13$$

General Term:

- a formula, labeled t_n , that expresses each term of the sequence as a function of its position.
- ex: If $t_n = 2n + 1$, then $t_{12} = 2(12) + 1 = 25$.

The formula: (known as the General Term formula):

$$t_n = a + (n-1)d$$

$\downarrow$ First term or t_1

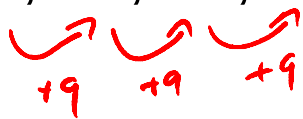
$\swarrow$ term #

$\searrow$ common difference

Example 1

Determine if the following sequence is arithmetic. If it is, state the General Term, then find the 40th term in the sequence.

3, 12, 21, 30, ...


+9 +9 +9

$$d = +9$$

$$a = 3$$

$$t_n = a + (n-1)d$$

$$t_n = 3 + (n-1)(9)$$

$$t_n = 3 + 9n - 9$$

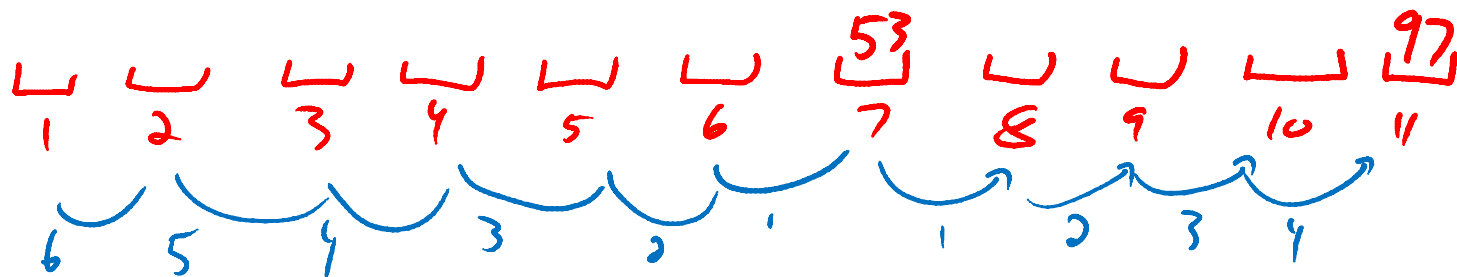
$$t_n = 9n - 6$$

$$t_{40} = 9(40) - 6$$

$$t_{40} = 354$$

Example 2

The 7th term in an arithmetic sequence is 53 and the 11th is 97. Find the common difference, the first term, the General Term, then the 18th term.



$$\begin{aligned} t_7 &= a + (7-1)d = 53 \\ t_{11} &= a + (11-1)d = 97 \\ \hline -4d &= -44 \\ d &= 11 \end{aligned}$$

$$\begin{aligned} 53 + 4d &= 97 \\ 4d &= 44 \\ d &= 11 \\ a &= 53 - 6(11) \\ a &= -13 \end{aligned} \quad \begin{aligned} t_n &= -13 + (n-1)(11) \\ t_n &= -13 + 11n - 11 \\ t_n &= 11n - 24 \\ \hline t_{18} &= 11(18) - 24 \\ t_{18} &= 174. \end{aligned}$$

Example 3

How many terms in the finite arithmetic sequence?

$$t_1 = a$$

18, 11, 4, -3, ..., -129

$$\begin{array}{cc} \curvearrowright & \curvearrowright \\ -7 & -7 \end{array}$$

Last term.

$$d = -7$$

$$t_n = a + (n-1)d$$

$$\overset{-18}{-129} = \overset{-18}{18} + (n-1)(-7)$$

$$\frac{-147}{-7} = \frac{(n-1)(-7)}{-7}$$

$$21 = n - 1$$

$$22 = n$$

$\therefore$ 22 terms
in this list.
