Functions 11

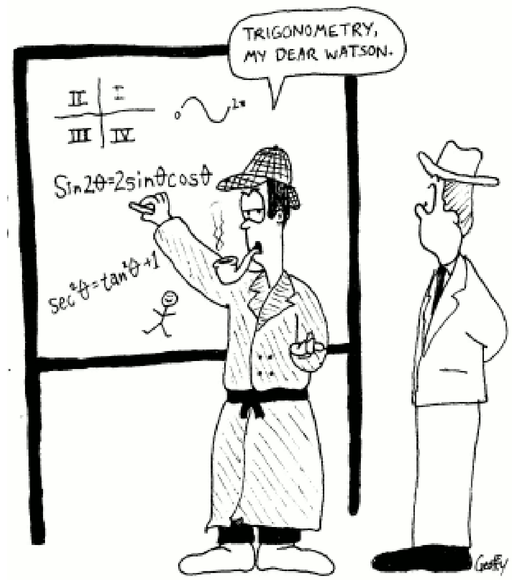
Course Notes

Chapter 5 – Trigonometric Ratios

TRIGONOMETRY IS MORE THAN TRIANGLES

We will learn

- *The three reciprocal trigonometric ratios*
- *To relate the six trigonometric ratios to the unit circle*
- *To solve problems using trig ratios, properties of triangles, and the sine/cosine laws*
- *How to prove trigonometric identities*



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Chapter 5 – Trigonometric Equations

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.

Section 5.1

Pg. 280 – 282 # 3, 4, 5i,ii,iv, 6, 7, 8a, 12, 14, 15

Section 5.6

Pg. 318 – 320 #4, 5bd, 7 (If only you had a side of the right angle triangle…),
#9 (recall the meaning of angle of depression??), 10, 13

Section 5.7

Page 326 - 327 #4ad (do you "need" to use the cosine law?), 6, 8 – 10

Section 5.8

Pg. 332 – 334 #3ac, 4a, 6, 9, Bonus: 7 (this one is tricky!!!)

Unit Test Part 1 & HW Part 1 Due

Section 5.2

Pg 286 – 288 #3 – 9, 11, 13

Section 5.3/5.4

Pg. 299 – 301 #1 – 3 (*For #3, **READ** example 3, pg. 296*), 5, 6 (*see example 5.3.4 above*),
#8 – 10, 12

If you struggle with this stuff…ASK QUESTIONS in EDSBY!!! (and in class too!)

Section 5.5

Handout

Unit Test Part 2 & HW Part 2 Due

Chapter 5 – Trigonometric Ratios

5.1 – Trigonometric Ratios of Acute Angles

Learning Goal: We are learning to evaluate reciprocal trigonometric ratios.

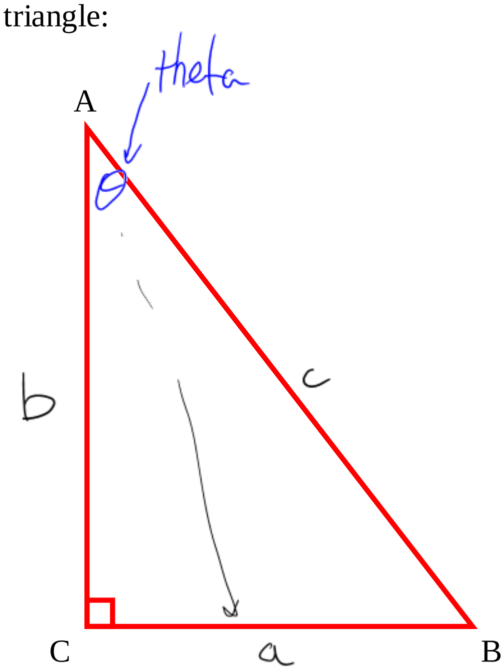
Recall from Grade 10 the mnemonic

SOH CAH TOA

ine pp yp os dj yp an pp dj

We use SOH CAH TOA to calculate the so-called "trig ratios" for a **right angle triangle**.

Consider the triangle:



The Trigonometric Ratios

bigger than one.

A

Primary Trig Ratios

$\sin\theta = \frac{opp}{hyp} = \frac{a}{c}$

$\cos\theta = \frac{adj}{hyp} = \frac{b}{c}$

less than 1

$\tan\theta = \frac{opp}{adj} = \frac{a}{b}$

Reciprocal Trig Ratios

$\frac{1}{\sin\theta} = \text{cosecant}\,\theta = \csc\theta = \frac{hyp}{opp}$

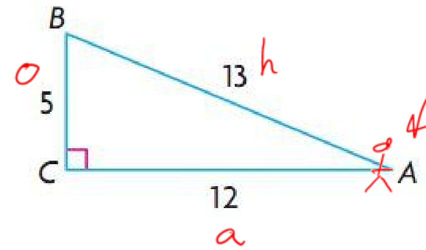
$\frac{1}{\cos\theta} = \text{secant}\,\theta = \sec\theta = \frac{hyp}{adj}$

$\frac{1}{\tan\theta} = \text{cotangent}\,\theta = \cot\theta = \frac{adj}{opp}$

Example 5.1.1

From your text, Pg. 280 #1

Given $\triangle ABC$, state the six trigonometric ratios for $\angle A$.



$\sin A = \frac{5}{13}$ $\csc A = \frac{13}{5}$

$\cos A = \frac{12}{13}$ $\sec A = \frac{13}{12}$

$\tan A = \frac{5}{12}$ $\cot A = \frac{12}{5}$

$\sin\theta = 0.7$

$\theta = \sin^{-1}(0.7)$

$\theta = 44.4^\circ$

$\frac{1}{2} = \frac{3}{6}$

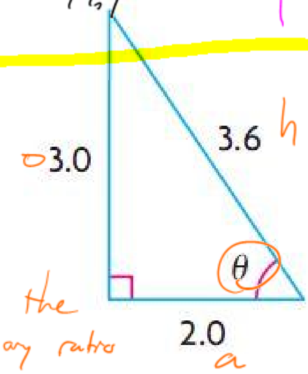
$\frac{2}{7} = \frac{6}{3}$

Example 5.1.2

For the given right triangle determine:

a) $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$.

b) the angle θ to the nearest degree.



a) $\csc\theta = \frac{3.6}{3} = 1.2$

$\sec\theta = \frac{3.6}{2} = 1.8$

$\cot\theta = \frac{2}{3}$

b) we can use any!!

$\csc\theta = 1.2$

$\frac{1}{\sin\theta} = \frac{1.2}{1}$ turn into the primary ratio

$\sin\theta = \frac{1}{1.2}$

$\theta = \sin^{-1}\left(\frac{1}{1.2}\right) = 56.4^\circ = 56^\circ$

Example 5.1.3

a) Determine the corresponding reciprocal ratio:

i) $\sin(\theta) = \frac{2}{5}$ ii) $\tan(\theta) = -3$

$\csc\theta = \frac{5}{2}$ $\cot\theta = \frac{-1}{3}$

$\cos(30^\circ) = \text{ratio}$

$= 0.866$

b) Calculate to the nearest hundredth: $\sec(34^\circ) = \frac{1}{\cos(34)} = 1.21$

c) Determine the value of θ to the nearest degree: $\csc(\theta) = \frac{2.46}{1}$

$\frac{1}{\sin\theta} = \frac{2.46}{1}$

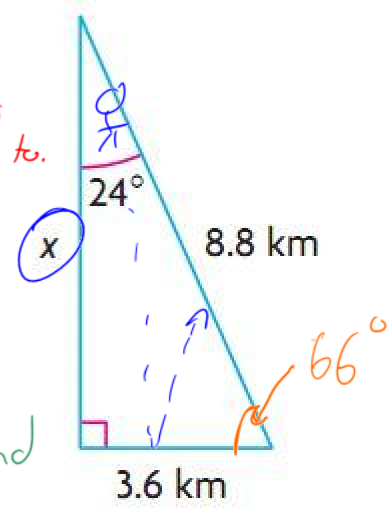
$\sin\theta = \frac{1}{2.46}$

$\theta = \sin^{-1}\left(\frac{1}{2.46}\right)$

$= 24^\circ$

Example 5.1.4

Given the right triangle, determine the unknown side using two different trig ratios: Don't use reciprocal ratios unless you really want to.



$$(8.8)\cos 24 = \frac{x}{8.8}(8.8)$$

$$8.04 = x$$

$$8 = x$$

$$\tan 24 = \frac{3.6}{x}$$

$$\cancel{x \tan 24 = 3.6}$$

Swap tan24 and x.

$$x = \frac{3.6}{\tan 24}$$

$$x = 8.08$$

$$x = 8.1$$

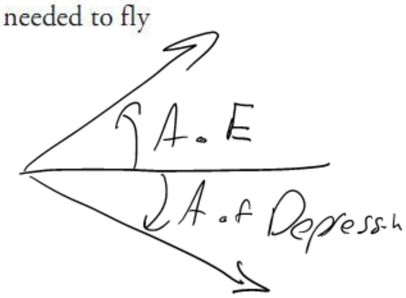
Example 5.1.5

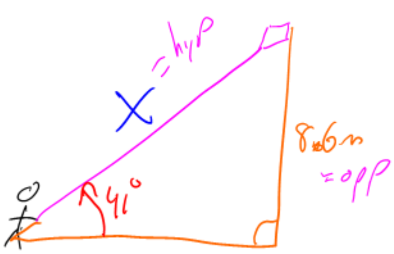
From your text, Pg. 282 #11

A kite is flying 8.6 m above the ground at an angle of elevation of 41°. Calculate the length of string, to the nearest tenth of a metre, needed to fly the kite using

a) a primary trigonometric ratio

~~**b)** a reciprocal trigonometric ratio~~





$$\sin 41 = \frac{8.6}{x}$$

$$x = \frac{8.6}{\sin 41}$$

$$x = 13.1\text{m.}$$

Success Criteria:

- I can use SohCahToa to determine the primary and reciprocal trigonometric ratios
- I can evaluate problems using the reciprocal trigonometric ratios
- I cannot use my calculator to directly solve a reciprocal trigonometric ratio

Chapter 5 – Trigonometric Ratios

5.6: The Sine Law

Learning Goal: We are learning to use the Sine law to solve non-right angle triangles.

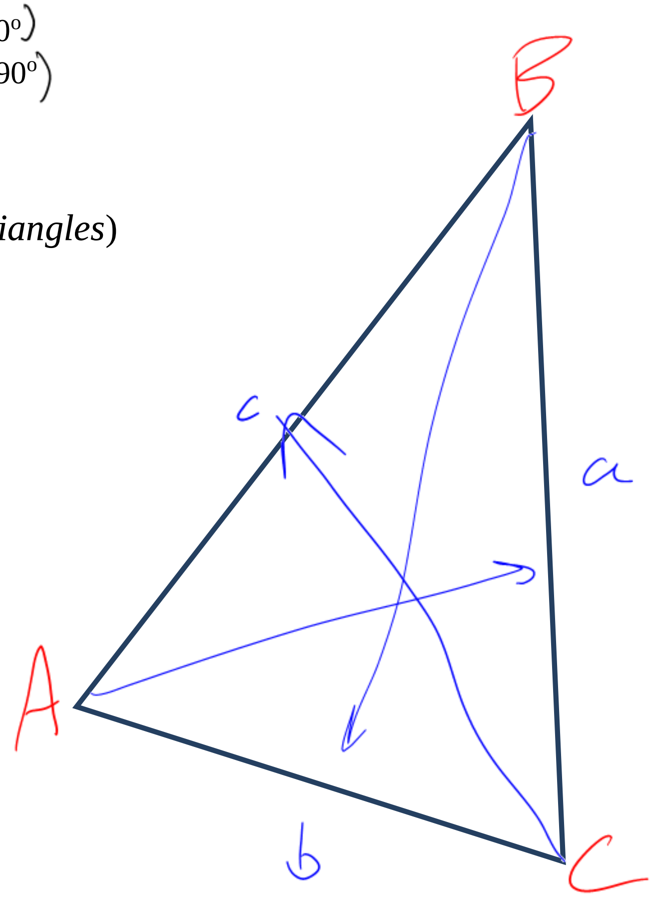
Last year you learned the Sine Law. It is a "formula" we can use to **solve triangles which are not right angle triangles**. There is one requirement to be able to use the Sine Law.

You Must Have an Angle With Its Corresponding Side!

So far we have been using Right Angle Triangles along with SOH CAH TOA to "solve" triangles. BUT right angle triangles aren't always the best triangle to use;

Sometime using a right angle triangle just can't be done. We then need to use so-called "**OBLIQUE TRIANGLES**". Oblique triangles come in two forms:

1) Acute (all angles are less than 90°)
2) Obtuse (one angle is more than 90°)



The Sine Law (*for oblique triangles*)

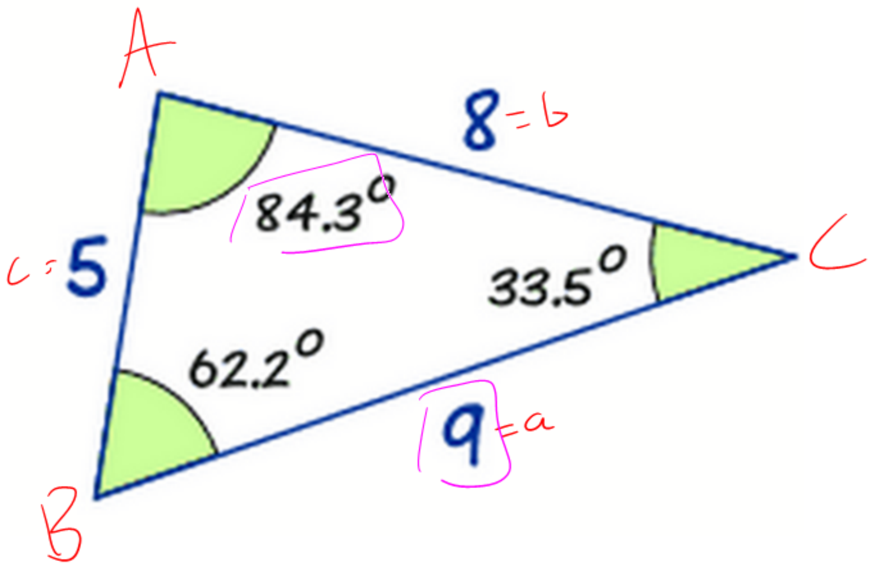
(There are **TWO FORMS** you should know!!)

Given the non-right triangle, ΔABC, then:

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

or

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$



$$\frac{\sin A}{a} = \frac{\sin(84.3)}{9}$$

$$= 0.11056$$

$$\frac{\sin B}{b} = \frac{\sin 62.2}{8}$$

$$= 0.11057$$

$$\frac{\sin C}{c} = \frac{\sin(33.5)}{5}$$

$$= 0.11039$$

Three Bears Theorem:

- The big angle is paired with the largest side
- the smallest angle is paired with the smallest side

Notes:

put the unknown on top

1) Memorize the SINE LAW!
2) If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
3) If we are trying to **find the length** of a side, use the second for of the law (**with sides on top**)
4) In order to use the Sine Law, you must have the correct information in the triangle. You must have:
   a) 3 pieces of information
   b) One "**CORRESPONDING PAIR**" – an angle with its opposite side (for example you might have side *a* and angle *A*)

Note: **There is a problem with the Sine Law**

~~Recall that for trig ratios, "sine" is positive in quadrants 1 *and* 2.~~

e.g. $\sin\left(51^\circ\right) = 0.777$ $\qquad$ $\sin\left(129^\circ\right) = 0.777$

$\sin\theta = 0.777$

$\theta = \sin^{-1}(0.777)$

$\theta = 51^\circ$

But it can also be 129°

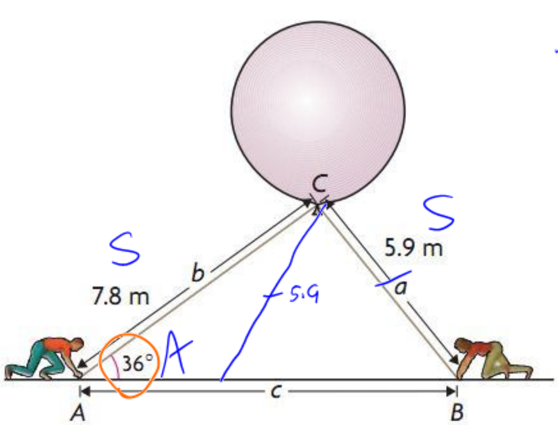
Consider Example 1 in your text: Pg. 312 – 314 .

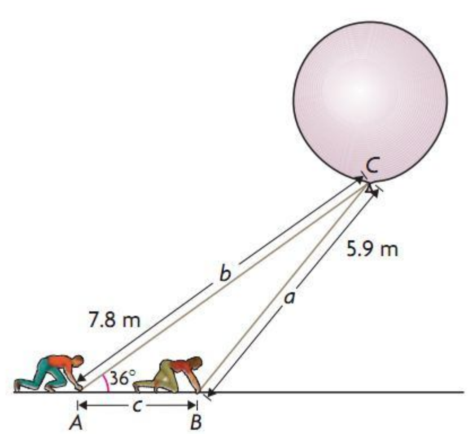
Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long.

The question we are asked is: **How far is Albert from Belle, to the nearest meter?**

Possible Pictures:

ASS





Both pictures describe the problem completely. So which is correct? Well…BOTH ARE **POSSIBLE** solutions. This is known as the "**AMBIGUOUS CASE**". Because both are possible solutions, you must find both.

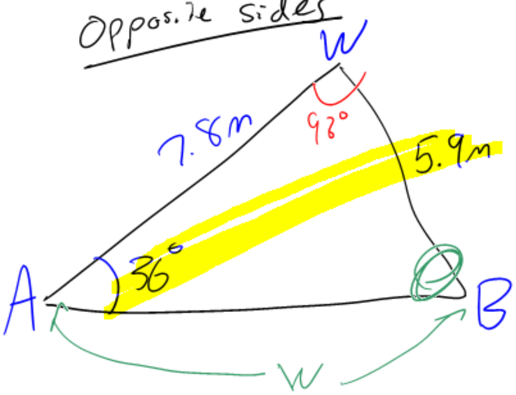
Note: If the **GIVEN ANGLE** is **ACUTE**, then this so called Ambiguous Case MAY APPLY. But, if the Given Angle is Obtuse, then the Ambiguous Case CANNOT APPLY. (*And* ***Sometimes***, *there is no triangle which solves the problem.*)

Why? If you are given an obtuse angle, the other angles MUST be acute.

Example 5.6.1 find all 6 pieces of information

Solve the triangles above.

Opposite sides



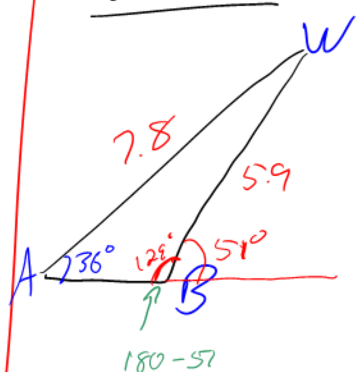
$A = 36°$ $a = 5.9m$

$B = \boxed{51°}$ $b = 7.8m$

$W = 93°$ $w = 10m$

$180°$

Same Side



$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{7.8} = (7.8)\frac{\sin 36}{5.9}$$

$$\sin B = 0.777$$

$$B = \sin^{-1}(0.777)$$

$$B = 51°$$

$$W = 180 - A - B$$
$$= 93°$$

$$\frac{w}{\sin W} = \frac{a}{\sin A}$$

$$\frac{w}{\cancel{\sin 93}} = \frac{5.9(\sin 93)}{\sin 36}$$

$$w = 10\ m.$$

$A = 36°$ $a = 5.9m$

$B = 129°$ $b = 7.8m$

$W = 15°$ $w = 2.6m$

$180°$

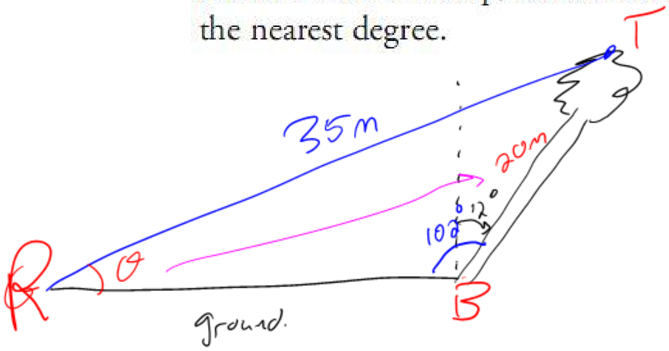
$$\frac{w}{\cancel{\sin 15}} = \frac{5.9(\sin 15)}{\sin 36}$$

$$w = 2.6m$$

Example 5.6.2

From your text: Pg. 319 #6

The trunk of a leaning tree makes an angle of 12° with the vertical. To prevent the tree from falling over, a 35.0 m rope is attached to the top of the tree and is pegged into level ground some distance away. If the tree is 20.0 m from its base to its top, calculate the angle the rope makes with the ground to the nearest degree.



unknown

$$\frac{\sin R}{r} = \frac{\sin B}{b}$$

$$(20)\frac{\sin R}{20} = (20)\frac{\sin 102}{35}$$

$$\sin R = 0.5589$$

$$R = \sin^{-1}(0.5589)$$

$$R = 34^\circ$$

Success Criteria

- I can recognize when the sine law applies and use it to solve for an unknown value
- I can identify, given S-S-A, that there will be two solutions (the ambiguous case)

Chapter 5 – Trigonometric Ratios

5.7: The Cosine Law

Learning Goal: We are learning to use the cosine law to solve non-right angle triangles.

The Cosine Law is another "formula" for solving Oblique Triangles. Remember, to "solve" a triangle you MUST be given **3 PIECES OF INFORMATION** about the triangle (and I should note that one of those given pieces **MUST BE A SIDE LENGTH**).

The main question you will have to be able to answer is this:

When do you use

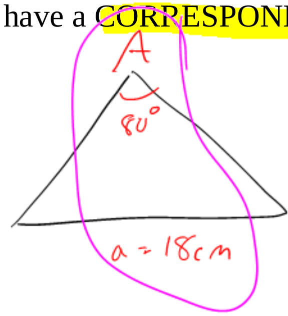
1) SOH CAH TOA

When you have a right triangle

2) The SINE LAW

When you have an oblique triangle

and you have a CORRESPONDING PAIR in the triangle



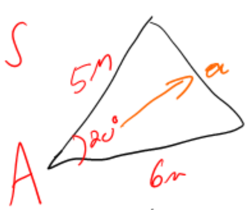
Then anything else.

3) The COSINE LAW

-you need an oblique Δ

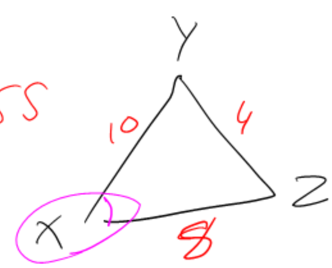
- you can't use Sine law

① SAS



2 sides and the angle inbetween

② SSS

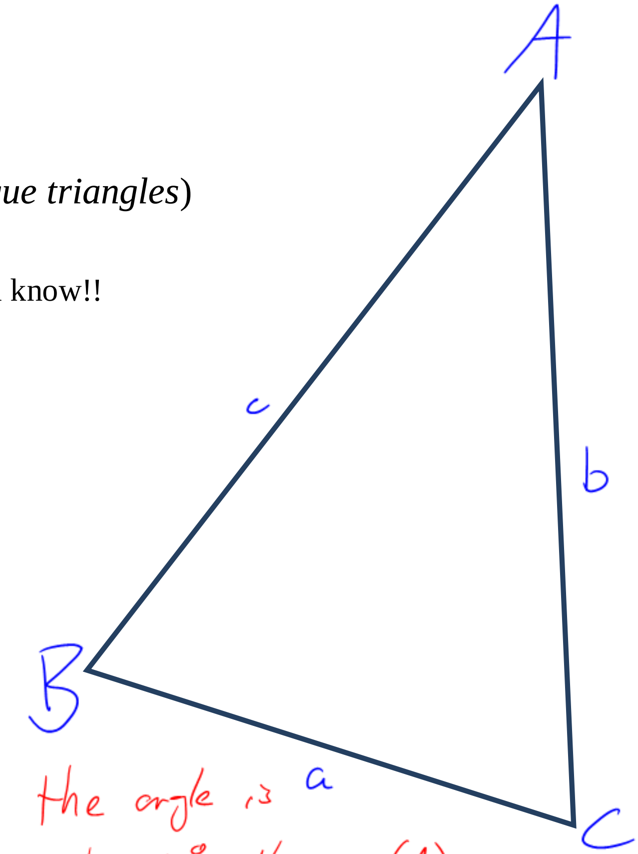


You can find any angle

The Cosine Law (*for oblique triangles*)

There are **THREE SIDE FORMS** you should know!!

Given the non-right triangle, ΔABC, then:



$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

or

one number

$$b^2 = a^2 + c^2 - 2ac\cos(B)$$

do this first.

or

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

If the angle is larger than 90°, then cos(A) = −#

ex: cos(100) = −0.17

Also, there are **THREE ANGLE FORMS** you should know!!

The formula you use depends on which side or angle you are looking for!!!

$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$

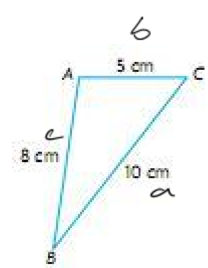
or

$$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$$

or

$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$$

e.g. Determine angle B



$$\cos B = \frac{5^2 - 10^2 - 8^2}{-2(10)(8)}$$

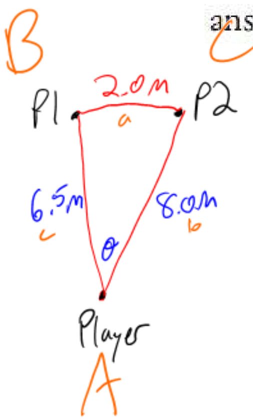
$$\cos B = \frac{+139}{+160}$$

$$B = \cos^{-1}\left(\frac{139}{160}\right) = 30^\circ$$

Example 5.7.1

From your text: Pg. 326 #5

The posts of a hockey goal are 2.0 m apart. A player attempts to score by shooting the puck along the ice from a point 6.5 m from one post and 8.0 m from the other. Within what angle θ must the shot be made? Round your answer to the nearest degree.



$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos(A) = \frac{2^2 - 8^2 - 6.5^2}{-2(8)(6.5)}$$

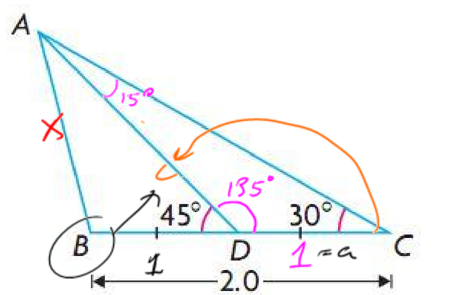
$$\cos A = \frac{+102.25}{+104} \Rightarrow A = \cos^{-1}\left(\frac{102.25}{104}\right) = 10.5^\circ$$

$$= 11^\circ$$

Example 5.7.2

From your text: Pg. 327 #7

Given $\triangle ABC$ at the right, $BC = 2.0$ and D is the midpoint of BC. Determine AB, to the nearest tenth, if $\angle ADB = 45^\circ$ and $\angle ACB = 30^\circ$.



In ΔACD

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 30} = \frac{1(\sin 30)}{\sin 15}$$

$$c = \frac{\sin(30)}{\sin(15)}$$

$$c = 1.93$$

In ΔADB SAS!!

$$d^2 = a^2 + b^2 - 2ab\cos D$$

$$d^2 = 1^2 + 1.93^2 - \boxed{2(1)(1.93)\cos 45}$$

$$d^2 = 1 + 3.72 - 2.73$$

$$\sqrt{d^2} = \sqrt{1.99}$$

$$d = 1.41$$

Success Criteria:

- I can use the cosine law, given S-A-S or S-S-S
- I can rearrange the cosine law to solve for an unknown angle

Chapter 5 – Trigonometric Ratios

5.8: 3D Problems

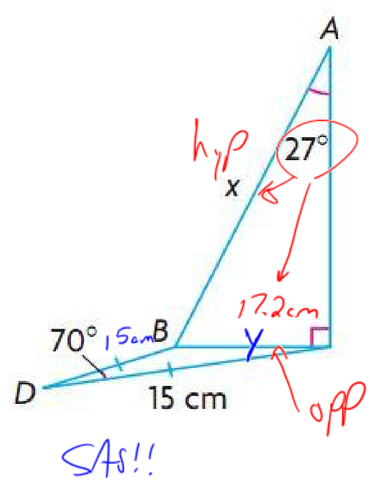
Learning Goal: We are learning to use trigonometry to solve 3-dimensional problems.

We will be using SOH CAH TOA, the Sine Law, and the Cosine Law for these problems. We'll jump right in by solving some problems since we already know how to use the various techniques! **One thing to keep in mind, though, is that these sorts of problems can be difficult to draw, or even simply visualize because we are working in 3D! Art specialists – rejoice!**

Example 5.8.1

From your text: Pg. 332 #4b

Solve for *x*



$y^2 = 15^2 + 15^2 - 2(15)(15)\cos 70$

$y^2 = 450 - 450\cos 70$

$y^2 = 450 - 153.9$

$y^2 = 296.1$

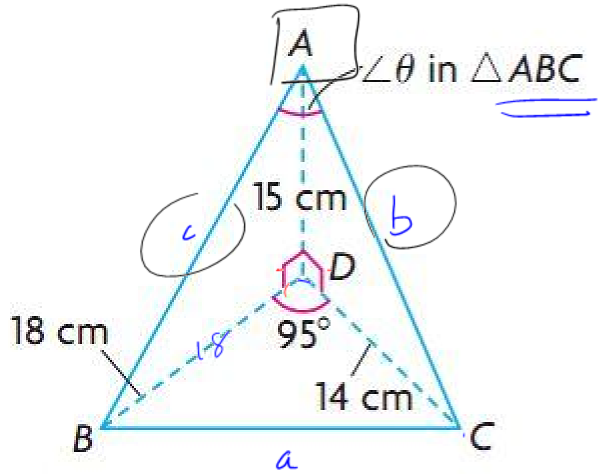
$y = 17.2\text{cm}$

$\boxed{\sin 27} = \dfrac{17.2}{\boxed{x}}$

$x = \dfrac{17.2}{\sin 27}$

$x = 37.9\text{cm}$ ✓

d) Solve for θ



In ΔBDC, SAS

$a^2 = 18^2 + 14^2 - 2(18)(14)\cos 95$ >90 ∴ −#

$\sqrt{a^2} = \sqrt{563.926}$

$a = 23.7\text{cm}$

In ΔACD, ~~SOHCAHTOA~~ Pythag!

$14^2 + 15^2 = b^2$

$\sqrt{421} = \sqrt{b^2}$

$20.5\text{cm} = b$

In ΔABD

$18^2 + 15^2 = c^2$

$\sqrt{549} = \sqrt{c^2}$

$23.4\text{cm} = c$

Now Find A from ΔABC

$\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc}$

$\cos A = \dfrac{23.7^2 - 20.5^2 - 23.4^2}{-2(20.5)(23.4)}$

$\cos A = \dfrac{-406.12}{-959.4}$

$\cos A = 0.4233$

$A = \cos^{-1}(0.4233)$

$A = 65°$

Example 5.8.2

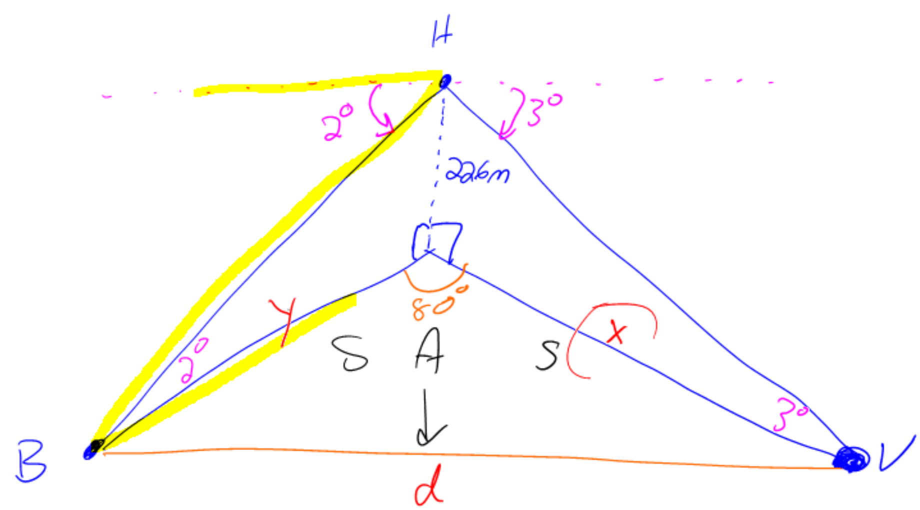
From your text: Pg. 333 #5

H

While Travis and Bob were flying a hot-air balloon from Beamsville to Vineland in southwestern Ontario, they decided to calculate the straight-line distance, to the nearest metre, between the two towns.

- From an altitude of 226 m, they simultaneously measured the angle of depression to Beamsville as 2° and to Vineland as 3°.
- They measured the angle between the lines of sight to the two towns as 80°. between the towns

Is there enough information to calculate the distance between the two towns? Justify your reasoning with calculations.



$\tan 3^\circ = \frac{226}{x}$

$x = \frac{226}{\tan 3}$

$x = 4312\,m$

$\tan 2^\circ = \frac{226}{y}$

$y = \frac{226}{\tan 2}$

$y = 6472\,m$

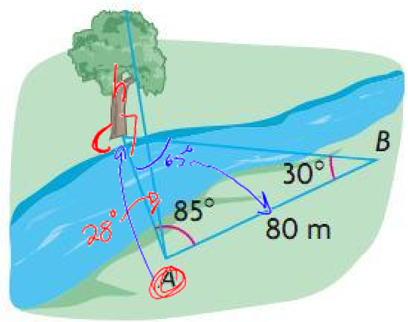
$d^2 = 4312^2 + 6472^2 - 2(4312)(6472)\cos 80$

$\sqrt{d^2} = \sqrt{50{,}788{,}036.93}$

$d = 7{,}126.6\,m$

Example 5.8.3

From your text: Pg. 334 #11



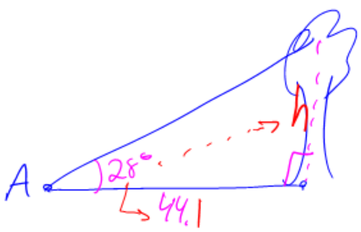
Bert wants to calculate the height of a tree on the opposite bank of a river. To do this, he lays out a baseline 80 m long and measures the angles as shown at the left. The angle of elevation from *A* to the top of the tree is 28°. Explain if this information helps Bert to calculate the height of the tree to the nearest metre. Justify your reasoning with calculations.

$C = 180 - 85 - 30$

$C = 65°$

$$\frac{b}{\cancel{\sin 30}} = \frac{80(\sin 30)}{\sin 65}$$

$b = 44.1\,m$



$$\tan 28° = \frac{h}{44.1}$$

$(44.1)\tan 28 = h$

$23.4\,m = h$

Success Criteria:

- I can sketch, to the best of my ability, a representation of the question
- I can identify the correct method to solve the unknown(s) in a given problem

Chapter 5 – Trigonometric Ratios

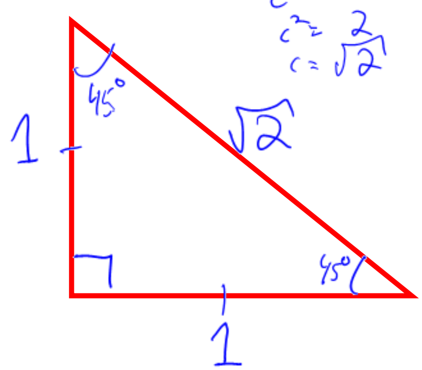
5.2 – Trigonometric Ratios and Special Triangles

Learning Goal: We are learning to find the EXACT values of sin, cos, and tan for specific angles.

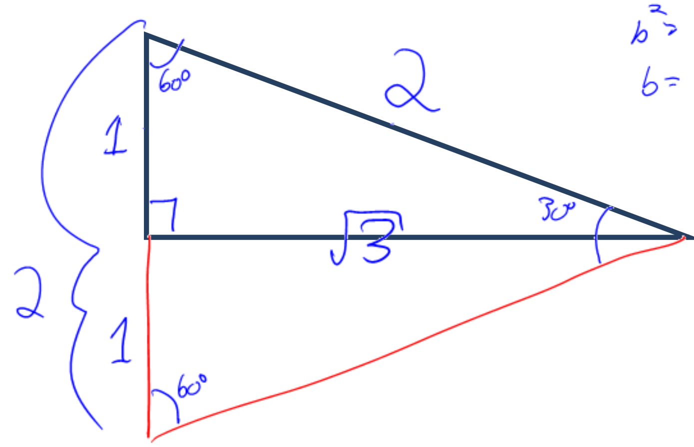
↳ no calculators! No decimals.

There are two "**Special Triangles**"

$c^2 = 1^2 + 1^2$
$c^2 = 2$
$c = \sqrt{2}$



$1^2 + b^2 = 2^2$
$b^2 = 3$
$b = \sqrt{3}$



MEMORIZE THESE!

rationalizing the denominator

The Primary Trigonometric Ratios of the Special Angles

$\sin(30^\circ) = \frac{1}{2}$ $\sin(60^\circ) = \frac{\sqrt{3}}{2}$ $\sin(45^\circ) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos(30^\circ) = \frac{\sqrt{3}}{2}$ $\cos(60^\circ) = \frac{1}{2}$ $\cos(45^\circ) = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan(30^\circ) = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $\tan(60^\circ) = \frac{\sqrt{3}}{1} = \sqrt{3}$ $\tan(45^\circ) = \frac{1}{1} = 1$

$= \frac{\sqrt{3}}{3}$

Example 5.2.1

Evaluate exactly

a) $\sin(45)\cdot\cos(60)$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{1}{2\sqrt{2}}$ you may stop here $\times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2(2)} = \frac{\sqrt{2}}{4}$

b) $\cos^2(30)+\sin^2(30)$

$(\cos 30)^2$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4} = 1$!!

c) $\tan(60)\cdot\cos(60)-\sin(60)$

$= \left(\frac{\sqrt{3}}{1}\right)\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} = 0$

d) $\tan(30)\cdot\frac{\sin(60)}{\cos(45)}$

$= \frac{1}{\sqrt{3}} \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$ ÷

$= \frac{1}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{2} \times \frac{\sqrt{2}}{1} = \frac{\sqrt{2}}{2}$

Example 5.2.2

Determine the angle θ (where $0 \le \theta \le 90^\circ$) given:

a) $\sec(\theta) = \frac{2}{\sqrt{3}}$

reciprocal of cosine

Flip

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$

b) $\tan(\theta) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

got rationalized...

$\theta = 30^\circ$

Success Criteria:

- I can draw the two special triangles
- I can identify the EXACT values for 30º, 45º, 60º, using the special triangles
- I can evaluate EXACTLY (no calculators…OR capes!!!) problems involving the special triangles
- I can understand how to rationalize the denominator

Chapter 5 – Trigonometric Ratios

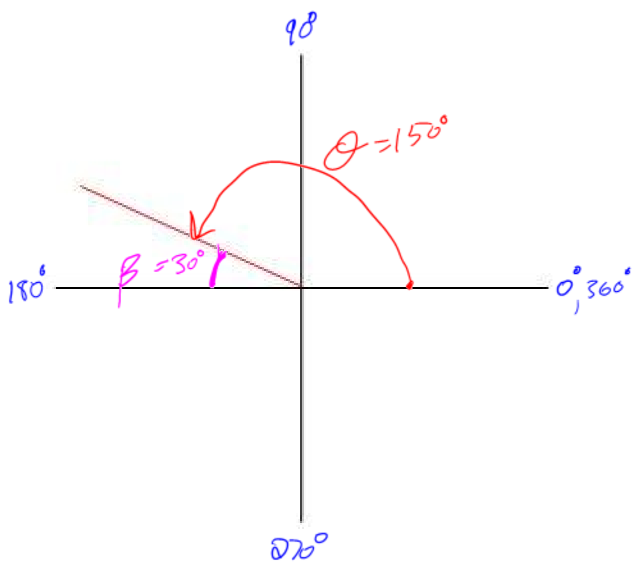
5.3 – 5.4 – Trigonometric Ratios for Angles Larger than 90º

Learning Goal: We are learning to use a Cartesian plane to evaluate trig ratios for angles between 0º and 360º.

Angles Larger than 90º

Consider the following sketch of the angle $\theta = 150^\circ$

β = beta



e.g. Calculate

$\sin(150) = 0.5$

$\sin(30) = 0.5$

$\cos(150) = -0.866$

$\cos(30) = 0.866$

$\tan(150) = -0.577$

$\tan(30) = 0.577$

between 0° and 360°

In this example, we call $\theta = 150^\circ$ the **PRINCIPAL ANGLE**, or the angle in standard position

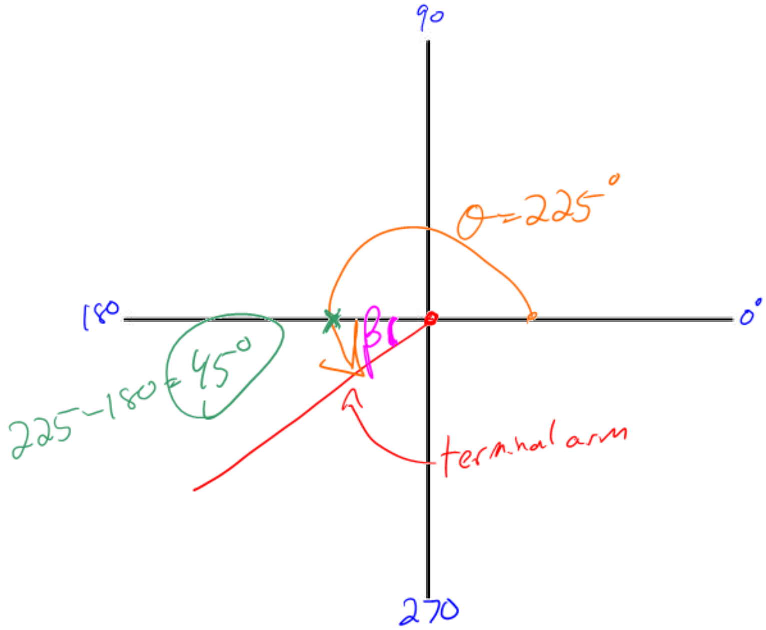
Note: The angle $\beta = 30^\circ$ is called the

RELATED ACUTE ANGLE

↳ less than 90°

Example 5.3.1

Sketch the angle of rotation $\theta = 225^\circ$ and determine the related acute angle. → always go from the x-axis



e.g. Calculate

$\sin(225) = -0.707$

$\sin(45) = 0.707$

$\cos(225) = -0.707$

$\cos(45) = 0.707$

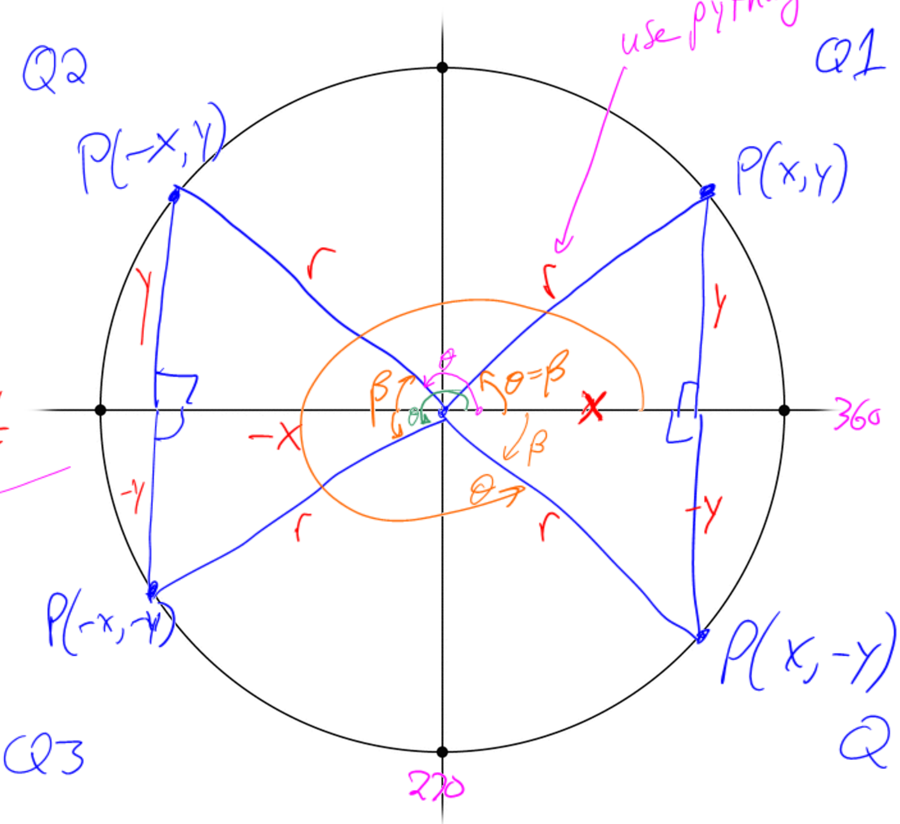
$\tan(225) = 1$

$\tan(45) = 1$

What is up with these signs??? (**BE CAREFUL WITH YOUR SIGNS!!!!!!!!!**)

Looking at the TRIG ratios on a Cartesian Plane

use pythagorean theorem, ∴ always positive



Q1:

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

Q2:

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{y}{-x} = -\frac{y}{x}$

Q3:

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{-y}{-x} = \frac{y}{x}$

Q4:

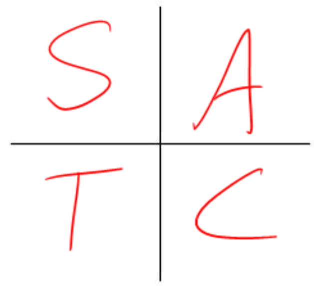
$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{-y}{x}$

osine ll ine an

The **CAST RULE** determines the sign (+ or -) of the trig ratio



We now have enough tools to calculate the trigonometric ratios of any angle!

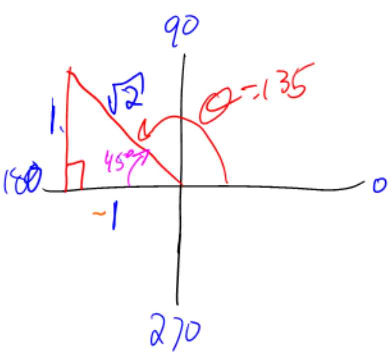
For any given angle θ we will:

1) Draw θ in **STANDARD POSITION** (i.e. draw the principal angle for θ)
2) Determine the **RELATED ACUTE ANGLE** (β) (between the terminal arm and the x-axis (also called the polar axis)
3) Use the related acute angle and the **CAST RULE** (and SOH CAH TOA) to determine the trig ratio (along with its sign…BE CAREFUL WITH YOUR SIGNS) in question

If the angle is 30, 45, or 60, use exact values

Example 5.3.2

Determine the trig ratio $\sin(135)$



$\beta = 180 - 135$

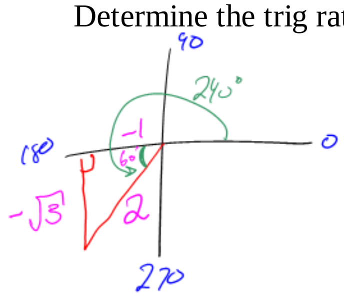
$\beta = 45°$

$\sin 45 = \frac{1}{\sqrt{2}}$

$\therefore \sin 135 = \frac{1}{\sqrt{2}}$

Because sine is positive in Q2

Determine the trig ratio tan (240)



$\beta = 240 - 180°$

$\beta = 60°$

$\tan 240 = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

Example 5.3.3 Q1

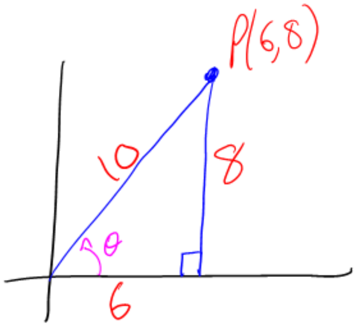
The point $P(x, y) = (6, 8)$ lies on the terminal arm (of length r) of an angle of rotation.

Sketch the angle of rotation.

Determine: a) the value of r

b) the primary trig ratios for the angle

c) the value of the angle of rotation in degrees, to two decimal places



a) $r^2 = 6^2 + 8^2$

$r^2 = 36 + 64$

$r^2 = 100$

$r = 10$

b) $\sin\theta = \frac{8}{10} = \frac{4}{5}$

$\cos\theta = \frac{6}{10} = \frac{3}{5}$

$\tan\theta = \frac{8}{6} = \frac{4}{3}$

c) $\sin\theta = \frac{4}{5}$

$\theta = \sin^{-1}\left(\frac{4}{5}\right)$ $\theta = 53.13^\circ$

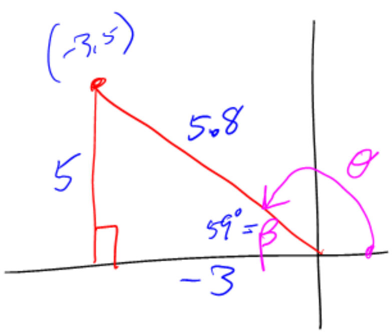
Example 5.3.4 Q2

The point $(-3, 5)$ lies on the terminal arm (of length r) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of r

b) the primary trig ratios for the angle

c) the value of the angle of rotation in degrees, to two decimal places



a) $r^2 = 3^2 + 5^2$

$r^2 = 9 + 25$

$r^2 = 34$

$r = \sqrt{34}$

$r = 5.8$

b) $\sin\theta = \frac{5}{5.8}$ $\Rightarrow \theta =$ acute

$\cos\theta = \frac{-3}{5.8}$ $\Rightarrow \theta =$ obtuse

$\tan\theta = \frac{5}{-3}$ $\Rightarrow \theta = -$ acute

c) Always calculate β, which means to use any ratio, but the positive ratio

$\tan\beta = \frac{5}{3}$

$\beta = \tan^{-1}\left(\frac{5}{3}\right)$

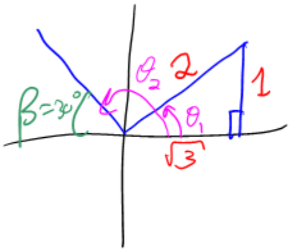
$\beta = 59^\circ$

$\theta = 180 - 59$

$\theta = 121^\circ$

Example 5.3.5 (going backwards!) Q1 + Q2

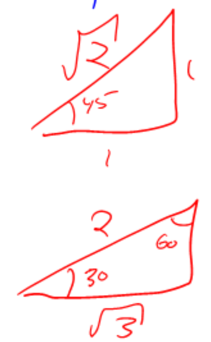
a) Given $\sin(\theta) = +\frac{1}{2}$ determine **BOTH** values of θ for $0^\circ \le \theta \le 360^\circ$



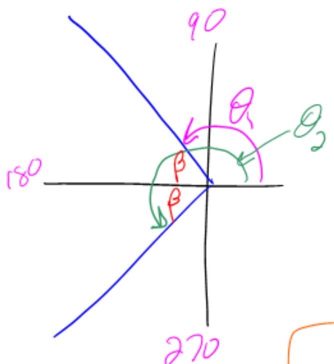
$\theta_1 = 30^\circ$ from the special Δ's

$\theta_2 = 180 - 30$

$= 150^\circ$



b) Given $\cos(\theta) = -0.5372$ determine **BOTH** values of θ for $0^\circ \le \theta \le 360^\circ$



$\cos\beta = 0.5372$

$\beta = \cos^{-1}(0.5372)$

$\beta = 58^\circ$

$\theta_1 = 180 - 58$

$\theta_1 = 122^\circ$

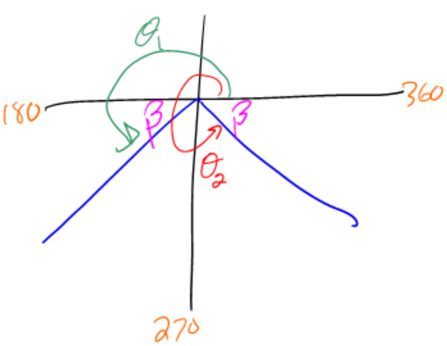
$\theta_2 = 180 + 58$

$\theta_2 = 238^\circ$

$\boxed{\cos 122 = -0.5299}$

c) Given ~~$\sin(\theta) = -0.4567$~~ determine BOTH values of θ for $0^\circ \le \theta \le 360^\circ$

$\csc\theta = -2.8531$



$\rightarrow \csc\beta = 2.8531$

$\frac{1}{\sin\beta} = \frac{2.8531}{1}$

$\sin\beta = \frac{1}{2.8531}$

$\beta = 21^\circ$

$\theta_1 = 180 + 21$

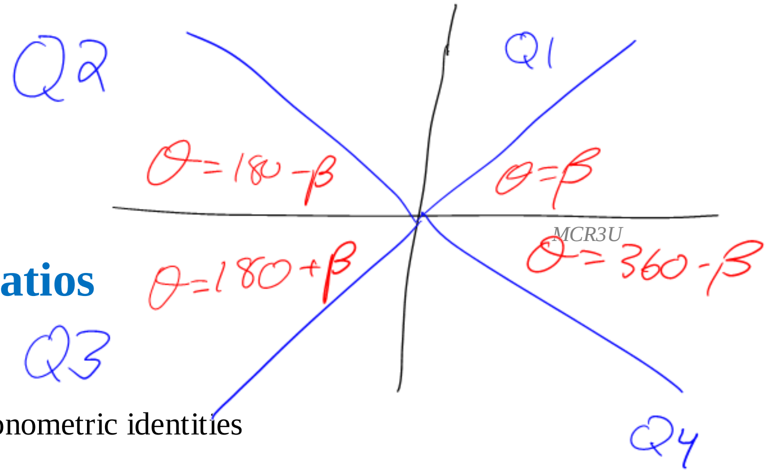
$= 201^\circ$

$\theta_2 = 360 - 21$

$= 339^\circ$

Success Criteria:

- I can identify a positive or negative angle based on the direction of rotation
- If the principal angle (Θ) lies in quadrants 2, 3, or 4 there is a related acute angle, β
- I can identify where a trigonometric ratio is + or – using the CAST Rule
- Every trigonometric ratio has two principal angles between 0º and 360º



Chapter 5 – Trigonometric Ratios

5.5 – Trigonometric Identities

Learning Goal: We are learning to prove trigonometric identities

Proving Trigonometric Identities is so much fun it's r i d i c u l o u s!

Let's start with a simple identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Recall:

Our second identity:

$$\sin^2\theta + \cos^2\theta = 1$$

When proving trig identities, it's helpful to keep a few things in your mind. Things such as:

- The Reciprocal Trig Identities
- Converting everything to sin and cos can be helpful
- Start with the side which has the most "stuff" to work with, and work toward the other side
- A few special formulas, which we need to find…

Example 5.5.1

Prove $\cos(x)\tan(x) = \sin(x)$

Example 5.5.2

Prove $1+\cot^2(x) = \csc^2(x)$

Example 5.5.3

From your text: Pg. 310 #8b

Prove $\dfrac{\tan^2\alpha}{1+\tan^2\alpha}=\sin^2\alpha$

Example 5.5.4

Prove $1-2\cos^2\phi=\sin^4\phi-\cos^4\phi$

Example 5.5.5

Prove $\sin\theta + \sin\theta\cot^2\theta = \csc\theta$

Example 5.5.6

Prove $\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$

Success Criteria:

- I can prove trig identities using a variety of strategies:
  - Using the reciprocal, quotient, and Pythagorean identities
  - Factoring
  - Converting to sin and cos
  - Common denominators
- I can recognize the proper form to proving trigonometric identities