Functions 11

Course Notes

Unit 3 – Quadratic Functions

FUNCTIONS TO THE MAX (OR MIN...AND SOMETIMES ZERO)

We will learn

- *the meaning of a zero, and how to find them algebraically*
- *to determine the max or min value of a quadratic algebraically and graphically*
- *to sketch parabolas (using transformations, zeroes, the vertex and y-intercept)*
- *to solve real-world problems, including linear-quadratic systems*

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MATH@TD

Chapter 3 – Quadratic Functions

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check.

Section 3.1

Pg. 145 – 147 #3, 4, 5bc, 6 (*expand*!), 7, 8, 9de, 12 (*tricky*!)

Section 3.2

Pg. 153 #1, 3, 4abc (*one method is fine*), 6 (*Desmos*), 7bc, 8, 9 (*try Partial Factoring*), 11 (*ask for help on c if you feel the need!*)

Section 3.4

Pg. 167 – 168 #3 – 5abc, 6 – 7acef, 8 - 13

Section 3.5

Pg 177 – 178 #1bc, 2bcd, 4abef, 6cd, 7 (*Hint: what is the height of the ball when it is on the ground?*), 9, 11 (#9 and 11 are tricky – ask for help!), 14

Section 3.6

Pg 185 – 186 #1 – 3abc, 4, 6 – 9 (*these are a bit tricky…ask for help!*), 15

Section 3.7

Page 192 #4 – 6, 8 – 10

Section 3.8

Pg198 – 199 #1ab, 2ab, 3, 4bcd, 6, 8, 11 (*tangent means touching at one point!*), 12

Chapter 3 – Quadratic Functions

3.1 – Properties of Quadratic Functions

→ highest exponent is 2.

Learning Goal: We are learning to represent and interpret quadratic functions in three different forms.

This lesson is a review of some of what we learned about quadratics in Grade 10. In Grade 10 we studied the **THREE FORMS** of quadratic functions and the **information** they give:

1) Standard Form - $f(x) = ax^2 + bx \boxed{+c}$

   Information

   $a > 0$, the parabola opens up ∴ min value

   $a < 0$, the parabola opens down ∴ max value

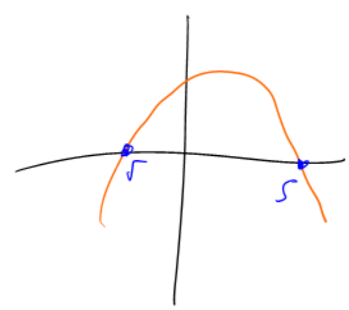
   $c \rightarrow$ is the y-intercept $f(0) = c$, $(0, c)$

2) Zeros (or Factored) Form - $f(x) = a(x - r)(x - s)$

   Information

   r and s are the zeros/x-intercepts

   $(r, 0)$ and $(s, 0)$

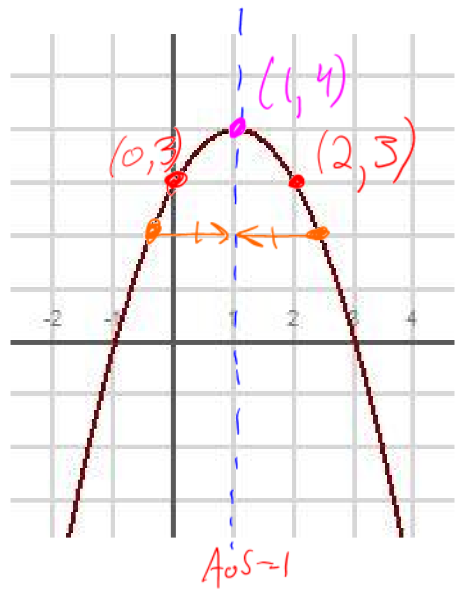


3) Vertex Form - $f(x) = a(x - h)^2 + k$

   Information

   Vertex is $(\overset{x}{h}, \overset{y}{k}) \Rightarrow f(h) = k$

Recall the concept of the axis of symmetry. AoS



A vertical line passing through the vertex. It is a "fold" line.

AoS = the average of 2 x-coordinate with the same y-coordinate

$$AoS = \frac{0+2}{2} = 1 = h$$

Example 3.1.1

Given the quadratic function $f(x) = \boxed{\frac{1}{2}}(x+3)^2 - 1$, state: > 0

a) The direction the parabola opens

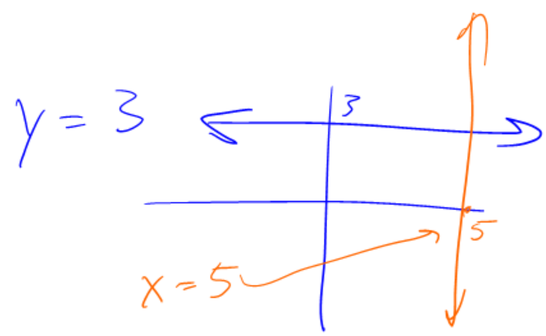
b) The coordinates of the vertex

c) The <u>equation</u> of the axis of symmetry

a) opens up.

b) $(\overset{h}{\underset{d}{-3}}, \overset{k}{\underset{c}{-1}})$

c) $x = -3$



Example 3.1.2

Given the quadratic function $g(x) = -2(x+3)(x-1)$, state

<0 , a , $=0$, $=0$

a) The direction the parabola opens

b) The zeros of the quadratic

c) The equation of the axis of symmetry

d) The coordinates of the vertex

e) The function in vertex form

Sketch the graph of the function.

a) opens down

b) $x = -3$, $x = 1$

c) $AoS = \frac{-3+1}{2} = \frac{-2}{2} = -1 = h$

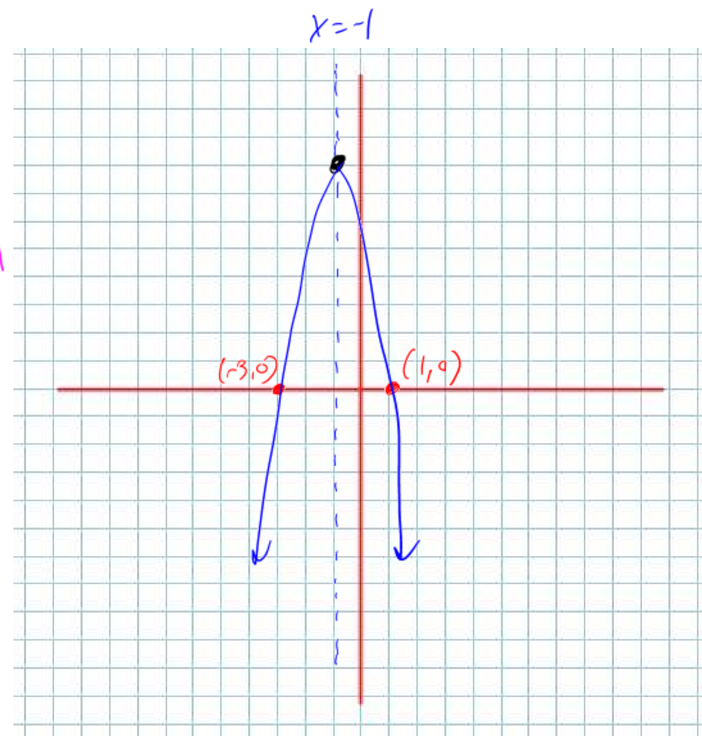
d) $f(h) = K$

$g(-1) = -2(-1+3)(-1-1)$

$= -2(2)(-2)$

$= 8$

$\therefore (-1, 8)$ (h, K)



e) $g(x) = a(x-h)^2 + K$

$\hookrightarrow g(x) = -2(x+1)^2 + 8$

Example 3.1.3

Given the two points $(4, 7)$, $(-5, 7)$ which are on a parabola, determine the equation of the axis of symmetry.

$AoS = \frac{4 + -5}{2} = \frac{-1}{2}$

$\therefore x = \frac{-1}{2}$ is the AoS

Example 3.1.4 (From Pg. 147 in your text)

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

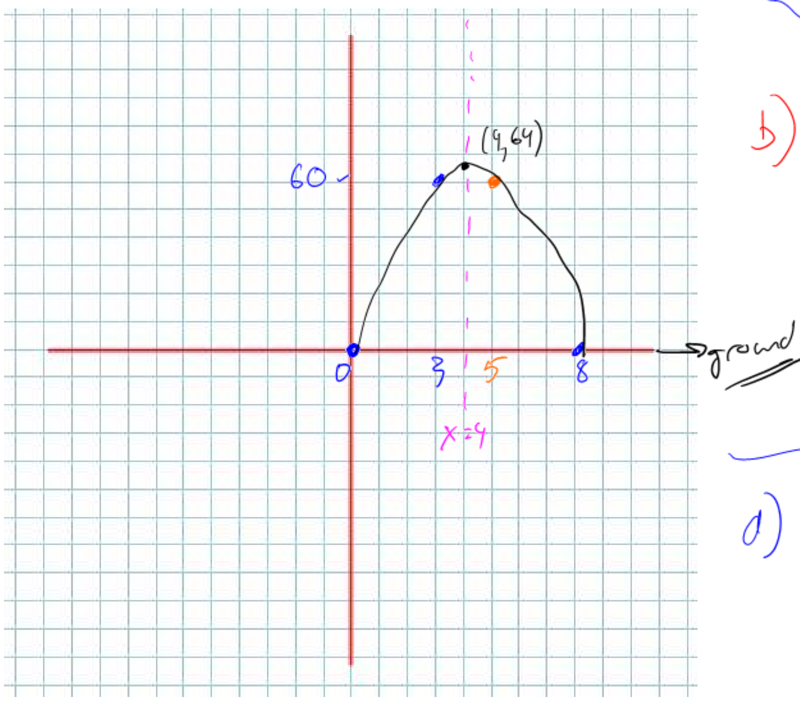
a) Graph the quadratic function.

b) How long will the rocket be in the air? How do you know?

c) How high will the rocket be after 3 s? t

d) What is the maximum height that the rocket will reach?

c) $h(3) = -4(3)^2 + 32(3)$
$= -36 + 96$
$= 60$ m.



b) $h(t) = -4t(t-8)$

$t=8$

$t=0$

The rocket will be in air for 8 seconds

d) AoS $= \frac{0+8}{2} = 4 = h$

$h(4) = -4(4)^2 + 32(4)$
$= -64 + 128$
$= 64$ m.

Success Criteria:

- I can recognize a quadratic function in standard, factored, and vertex form
- I can determine the zeros, direction of opening, axis of symmetry, vertex, domain and range from the graph of a parabola
- I can determine the equation of quadratic function from its parabola

Chapter 3 – Quadratic Functions

3.2 – The Maximum or Minimum of Quadratic Functions

Learning Goal: We are learning to determine the maximum/minimum value of a quadratic function.

One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). **Max/Min's** have so many **applications** in the real world that it's **ridiculous**.

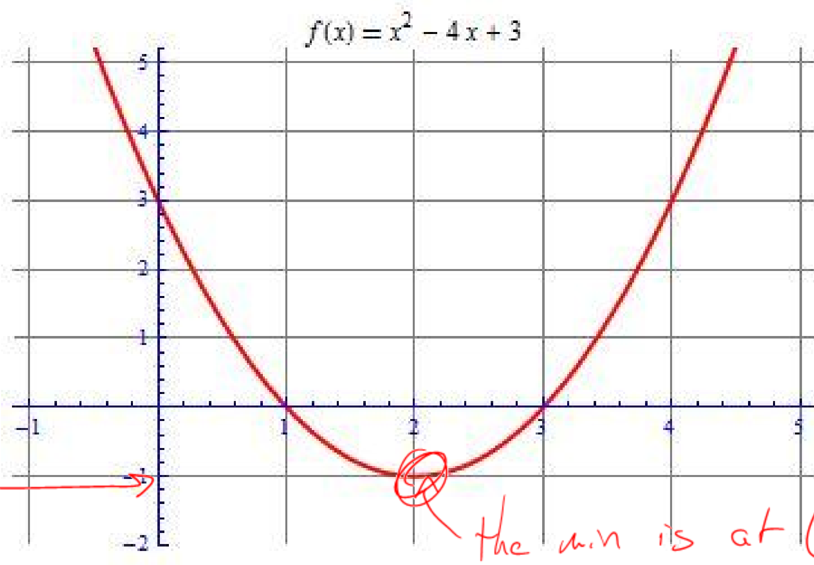
The **BIG QUESTION** we are faced with is this:

How do we find the Maximum or Minimum Value for some given Quadratic?

↳ the y or f(x) value.

Example 3.2.1

To find a minimum (or maximum) of a quadratic, **you are NOT allowed to**



there is the min

the min is at (2, -1)

Figure 3.2.1

or the min value is -1.

Figure 3.2.1b

$(h, \boxed{k}) \rightarrow$ the max/min value

$(x, f(x))$ ↳ how you get the max/min value.

So, we do need to find the vertex, but we also need to **KEEP IN MIND WHAT THE NUMBERS ASSOCIATED WITH THE VERTEX MEAN**.

In order to find the vertex using algebra, we will consider three techniques:

1) **USING THE ZEROS, TO FIND THE AXIS OF SYMMETRY**, and then the vertex (**this is the easiest technique**, ***assuming we can factor the quadratic***).
2) **COMPLETING THE SQUARE** to find the vertex (this is the toughest technique, but it's nice because you ***end up with the quadratic in vertex form***).
3) **USE PARTIAL FACTORING TO FIND THE AXIS OF SYMMETRY**, and then the vertex.

Note: **We can also use graphing calculators to find the max/min of a quadratic!**

Example 3.2.2

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.

$\cap$ ∴ max

↳ factor.

$f(x) = -3(x^2 + 4x - 5)$

$f(x) = -3(x+5)(x-1)$

$x = -5 \quad x = 1$

$AoS = \frac{-5+1}{2} = -2 = h$

Vertex $(-2, \_\_)$

$f(h) = k$

$f(-2) = -3(-2)^2 - 12(-2) + 15$

$= -12 + 24 + 15$

$= 27$

we have a max value of 27.

Example 3.2.3

COMPLETE THE SQUARE to find the vertex of the quadratic and state **where** the max (min) is and **what** the max (min) is.

$g(x) = 2x^2 + 8x - 5$

① Factor "a" from first two terms.

$g(x) = 2(x^2 + 4x + 0) - 5$

② $\left(\frac{b}{2}\right)^2 = \left(\frac{4}{2}\right)^2 = 4$

Add and subtract this #.

$g(x) = 2(\underbrace{x^2 + 4x + 4}_{\text{perfect square trinomial}} - 4) - 5$

③ Multiply "a" with -4.

$g(x) = 2(x^2 + 4x + 4) - 5 - 8$

③ Factor the ~~two~~ trinomial.

$g(x) = 2(x+2)^2 - 13$

∴ the min value is -13 at $x = -2$

Example 3.2.4

Using **PARTIAL FACTORING** determine the axis of symmetry. Then find the vertex and state the min or max value.

$$h(x) = \boxed{5x^2 + 15x} - 3$$

$= 0$

Force

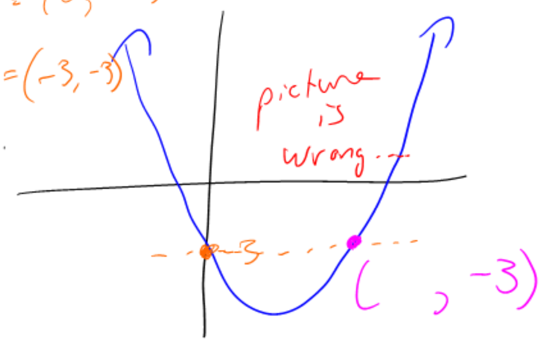
Factor "ax" from first two terms

$$h(x) = \boxed{5x(x+3)} - 3 \qquad = 0$$

$x = 0 \qquad x = -3$

y-int = (0, -3)

twin = (-3, -3)



$$AoS = \frac{0 + -3}{2} = \frac{-3}{2} \text{ or } -1.5 = h$$

$$k = h(-1.5) = 5(-1.5)^2 + 15(-1.5) - 3$$

$$= 11.25 - 22.5 - 3$$

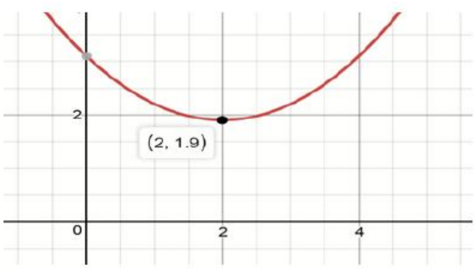
$$= -14.25 \text{ is the min value.}$$

This ALWAYS works.

Example 3.2.5

Using graphing technology, determine the max/min value of the quadratic

$$f(x) = 0.3x^2 - 1.2x + 3.1$$



Success Criteria:

- I can recognize when a function has a maximum or minimum value (based on "*a*")
- I can find the max/min (vertex) value using various methods (*partial factoring* ☺)

Chapter 3 – Quadratic Functions

3.4 – Operations with Radical Numbers

?

Learning Goal: We are learning to simplify and perform operations on radicals.

First we need to understand that **RADICALS** (*square roots, cube roots, etc*) **ARE NUMBERS**, and working with them should not induce any kind of fear in your spirit. So, **FEAR NOT!**

A COUPLE OF THINGS TO REMEMBER:

1) The square root of a square number is a nice integer.

e.g. $\sqrt{25} =$ 5

$\sqrt{49} =$ 7

2) The cube root of a cubed number is a nice integer

e.g. $\sqrt[3]{27} =$ 3

$\sqrt[3]{125} =$ 5

Now, if we don't have a radical with a perfect square (or cube as the case may be) we could use a calculator to find the root.

e.g. $\sqrt{24} = 4.898979485566356196394568114941 18...$

Non-repeating decimal an irrational number.

BUT the "**DECIMAL EXPANSION**" is **unending** and **doesn't repeat** and so we can only **APPROXIMATE THE VALUE** of $\sqrt{24}$ because of the need to **ROUND-OFF**. "**EXACT NUMBERS**" like $\sqrt{24}$ are sometimes preferred in mathematical solutions and so **we do need to know how to work with these radical NUMBERS.** Working with radical numbers means we'll be:

- adding/subtracting
- multiplying/dividing them.

Before beginning, there is one thing to keep in mind:

coefficient

$2x^5$ variable...

COEFFICIENTS WITH COEFFICIENTS, RADICALS WITH RADICALS

e.g. The number $2\sqrt{5}$ has a coefficient part of 2 and a radical part of $\sqrt{5}$

Such a number (with both a coefficient and a radical part) is called a mixed radical

Example 3.4.1

Multiply the following:

a) $\sqrt{5}\times\sqrt{3} = \sqrt{5\times 3} = \sqrt{15}$

b) $-2\sqrt{7}\times 3\sqrt{6} = -6\sqrt{42}$

c) $5\sqrt{10}\times 1\sqrt{5} = 5\sqrt{50}$

d) $\sqrt{2}\times\sqrt{2} = \sqrt{4} = 2$

Example 3.4.2

Simplify the following: turn into a mixed radical.

a) $\sqrt{50}$

$= \sqrt{25\times 2} = \sqrt{25}\times\sqrt{2}$

$= 5\sqrt{2}$

– Do any perfect square divide into the radical?

b) $-3\sqrt{27}$

$= -3\sqrt{9}\times\sqrt{3}$

$= -3(3)\times\sqrt{3} = -9\sqrt{3}$

c) $2\sqrt{50}\times\left(-3\sqrt{24}\right)$

$= 2\sqrt{25}\times\sqrt{2}\times\left(-3\sqrt{4}\times\sqrt{6}\right)$

$= 10\sqrt{2}\times\left(-6\sqrt{6}\right)$

$= -60\sqrt{12}$

$= -60\sqrt{4}\sqrt{3}$

$= -120\sqrt{3}$

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
$10^2 = 100$
$11^2 = 121$
$12^2 = 144$

$\longrightarrow 3x + 7x = 10x$

$3x + 7x^2$

Example 3.4.3

Add the following:

a) $3\sqrt{2} + 7\sqrt{2}$

$= 10\sqrt{2}$

b) $5\sqrt{7} - 3\sqrt{5} - 7\sqrt{7}$

$= -2\sqrt{7} - 3\sqrt{5}$

c) $2\sqrt{5} - 3\sqrt{20}$

$= 2\sqrt{5} - 3\sqrt{\cancel{4}^{\,2}}\sqrt{5}$

$= 2\sqrt{5} - 6\sqrt{5}$

$= -4\sqrt{5}$

d) $-3\sqrt{300} + \sqrt{243}$

$= -3\sqrt{\cancel{100}^{\,10}}\sqrt{3} + \sqrt{\cancel{81}^{\,9}}\sqrt{3}$

$= -30\sqrt{3} + 9\sqrt{3}$

$= -21\sqrt{3}$

Note: **We can only ADD OR SUBTRACT "LIKE" RADICALS.**

e.g. $2\sqrt{3}$ and $-5\sqrt{3}$ **ARE LIKE, but** $2\sqrt{5}$ and $3\sqrt{20}$ **ARE NOT** ***(or aren't they?.....)***

Rationalizing the Denominator

ex: $\dfrac{3}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}}{2}$

ex: $\dfrac{-6\sqrt{5}}{\sqrt{10}} \times \dfrac{\sqrt{10}}{\sqrt{10}}$

$= \dfrac{-6\sqrt{50}}{10}$

$= \dfrac{-6\sqrt{\cancel{25}^{\,5}}\sqrt{2}}{10}$

$= \dfrac{-\cancel{30}\sqrt{2}}{\cancel{10}}$

$= -3\sqrt{2}$

Example 3.4.4

Simplify:

a) $2\sqrt{3}\,(3\sqrt{2} - 5\sqrt{6})$

$= 6\sqrt{6} - 10\sqrt{18}$

$= 6\sqrt{6} - 10\sqrt{9}\sqrt{2}$

$= 6\sqrt{6} - 30\sqrt{2}$

FOIL

b) $(3\sqrt{12} - 5\sqrt{2})(2\sqrt{3} + 3\sqrt{2})$

$= 6\sqrt{36} + 9\sqrt{24} - 10\sqrt{6} - 15\sqrt{4}$

$\sqrt{4}\sqrt{6}$

$= 36 + 18\sqrt{6} - 10\sqrt{6} - 30$

$= 6 + 8\sqrt{6}$

c) $(5 - 2\sqrt{2})^2 \; (5 - 2\sqrt{2})$

$= 25 - 10\sqrt{2} - 10\sqrt{2} + 4\sqrt{4}$

$= 33 - 20\sqrt{2}$

Success Criteria:

- I can recognize "like" radicals. Totally awesome dude!
- I can write a radical in simplest form
- I can simplify radicals by adding, subtracting, multiplying, and dividing
- I can appreciate that a radical is an EXACT answer and therefore SUPERIOR to decimals

Chapter 3 – Quadratic Functions

3.5 – Solving Quadratic Equations

Learning Goal: We are learning to solve quadratic functions in different ways.

Note that last day we looked at section 3.6. We now go back to 3.5 as this is a better order for the concepts.

Before beginning we should look at the difference between a Quadratic **FUNCTION** and a Quadratic **EQUATION**. A function such as $f(x) = 3x^2 - 5x + 1$ has a graph with **infinitely** many points. On the other hand, a quadratic equation (in standard form) looks like:

$$3x^2 - 5x + 1 = 0$$

(*What is the difference between the function and the equation?*)

In section 3.6 we saw how to find the **zeroes** of quadratic functions, using the techniques of factoring, the quadratic formula or using graphing technology. As it turns out, solving a quadratic equation is **Exactly** The same as finding the zeros!

Quadratic equations, therefore can have 2, 1, or 0 **SOLUTIONS**.

Example 3.5.1

"stuff" = 0

Solve the equations:

a) $1x^2 - 5x - 14 = 0$

M: −14
A: −5
−7, +2

$(x-7)(x+2) = 0$

$x = 7$ and $x = -2$ are the solutions

b) $2x^2 + 5x = \boxed{2x + 4}$

$2x^2 + 3x - 4 = 0$ (a, b, c)

Not Factorable

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{41}}{4}$$

① $x = \dfrac{-3 + \sqrt{41}}{4}$

$x = 0.85$

② $x = \dfrac{-3 - \sqrt{41}}{4}$

$x = -2.35$

Example 3.5.2

Solve $-2.3x^2 - 1.32x = -1.45$

$\underbrace{-2.3}_{a}x^2 \underbrace{-1.32}_{b}x \underbrace{+1.45}_{c} = 0$

$$x = \frac{+1.32 \pm \sqrt{1.32^2 - 4(-2.3)(1.45)}}{2(-2.3)}$$

$$x = \frac{1.32 \pm \sqrt{15.0824}}{-4.6}$$

1 $x = \dfrac{1.32 + \sqrt{15.0824}}{-4.6}$

$x = -1.13$

2 $x = \dfrac{1.32 - \sqrt{15.0824}}{-4.6}$

$x = 0.56$

Example 3.5.3 (From your text: Pg. 178 #6a)

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

   a) $P(x) = -x^2 + 12x + 28$

$P(x) = 0$

Max/min = Partial Factor

Solving/hit the ground → = Q. Formula

Give an "x", evaluate

$$\frac{0}{-1} = \frac{-x^2 + 12x + 28}{-1}$$

$$0 = x^2 - 12x - 28$$

M: -28
A: -12
$-14, +2$

$$0 = (x - 14)(x + 2)$$

$x = 14$ and $\cancel{x = -2}$ inadmissible

14,000 $\quad$ $\cancel{-2,000}$

∴ They need to sell 14,000 items to break-even

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995. → 1995 ⇒ $t = 0$

a) What was the population in 1995?

b) What will be the population in 2010? ⇒ $P(t) = 450$

c) In what year will the population be at least 450 000? Explain your answer.

$t = ?$

a) In 1995, $t = 0$

$P(0) = \cancel{0.4(0)^2} + \cancel{10(0)} + 50$

$P(0) = 50$

∴ In 1995, the population was 50,000.

b) In 2010, $t = 15$

$P(15) = 0.4(15)^2 + 10(15) + 50$

$P(15) = 290$

∴ In 2010, the population is 290,000.

c) $P(t) = 450$, $t = ?$

$450 = 0.4t^2 + 10t + 50$

$0 = \underbrace{0.4}_{a}t^2 \underbrace{+10}_{b}t \underbrace{-400}_{c}$

$t = \dfrac{-10 \pm \sqrt{10^2 - 4(0.4)(-400)}}{2(0.4)}$

$t = \dfrac{-10 \pm \sqrt{740}}{0.8}$

① $t = \dfrac{-10 + \sqrt{740}}{0.8}$

$\boxed{t = 21.5}$

② $\cancel{t = \dfrac{-10 - \sqrt{740}}{0.8}}$

$\cancel{t = -46.5}$ inadmissible

Success Criteria:

- I can solve quadratic functions by factoring, then setting each factor equal to zero
- I can solve quadratic functions by using the quadratic formula

(1995 + 21.5)

∴ In the year 2016 the population is 450,000.

Chapter 3 – Quadratic Functions

3.6 – Zeroes of Quadratic Functions

Learning Goal: We are learning to determine the number of zeros of a quadratic function.

Before we begin, let's think about a couple of things…

Remember – **FUNCTIONS CAN BE DESCRIBED AS A SET OF ORDERED PAIRS**, where the "ordered pair" is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value.

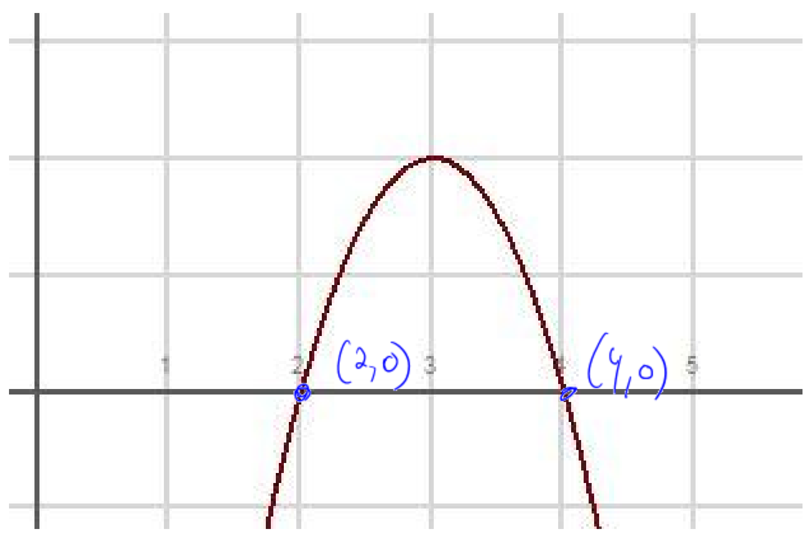
The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the "y" value) is the maximum.

When we talk about the ZEROS of a quadratic we need to understand what we mean by that. Consider the sketch of the graph of the quadratic function $f(x) = -2(x-3)^2 + 2$

Zero as a point

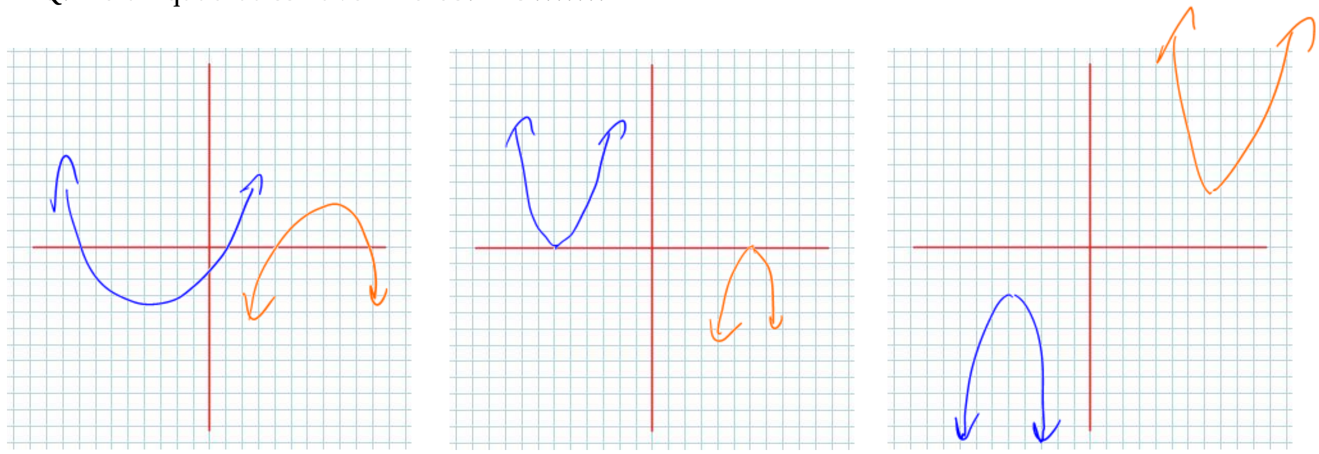
$(x, 0)$

$\boxed{x}$ is the zero



The zeros are $x=2$ and $x=4$.

Q. Do all quadratics have 2 zeros? NO!!!!!!!



2 zero   1 zero   0 zeros

Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

1) Writing the quadratic in zeros form (by factoring) $f(x) = a(x-r)(x-s)$
2) Writing the quadratic in vertex form, and doing some algebra (a bit nasty)
3) Using the quadratic formula (but the quadratic ***MUST BE IN STANDARD FORM*** - $f(x) = ax^2 + bx + c$)
4) Using graphing technology (lame, but legit)

Example 3.6.1

Determine the zeros:

a) $f(x) = x^2 - 3x - 4$

M: -4
A: -3
$-4, +1$

$f(x) = (x - \boxed{4})(x + 1)$

$x - (-1)$

$x = 4, \; x = -1$

b) $g(x) = 2x^2 + x - 1$

M: -2
A: $+1$
$2, -1$

$g(x) = \underbrace{2x^2 + 2x}_{2x} \underbrace{- x - 1}_{-1}$

$g(x) = (2x \boxed{-1})(x + 1)$

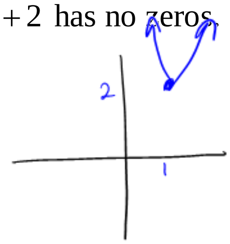
$x = \frac{1}{2}, \; x = -1$

Example 3.6.2

>0 h k

Determine why the quadratic $f(x) = 2(x-1)^2 + 2$ has no zeros.

The "a" and "k" need to have opposite signs to have 2 zeros.



$y=0$ $g(x)=0$

Example 3.6.3

Determine the zeros of $g(x) = -(x+1)^2 + 8$

$$\underset{-8}{0} = -(x+1)^2 + 8 \quad -8$$

$$+\sqrt{8} = +\sqrt{(x+1)^2}$$

$$\pm\sqrt{8} = x+1$$

$$\pm\sqrt{8} - 1 = x$$

Approximate form:

① $\sqrt{8} - 1 = x$

$1.82 = x$

② $-\sqrt{8} - 1 = x$

$-3.82 = x$

Example 3.6.4

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) $f(x) = \overset{a}{2}x^2 + \overset{b}{3}x - \overset{c}{7}$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{9 + 56}}{4}$$

$$x = \frac{-3 \pm \sqrt{65}}{4}$$

① $x = \dfrac{-3 + \sqrt{65}}{4}$

$x = 1.27$

② $x = \dfrac{-3 - \sqrt{65}}{4}$

$x = -2.77$

b) $g(x) = \overset{a}{3}x^2 \overset{b}{-2}x + \overset{c}{4}$

$$x = \frac{2 \pm \sqrt{2^2 - 4(3)(4)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{-44}}{6}$$

Cannot, do not try to square root negatives

$\therefore$ g(x) has no zeros.

The Discriminant

The Discriminant of the quadratic formula is called the **DISCRIMINANT** because it's very discriminating. ~ The b^2-4ac tells us how many zero a fn has.

The Discriminant is b^2-4ac

1) If $\underbrace{b^2-4ac}_{\text{positive}} > 0$, you have 2 zeros
2) If $b^2-4ac = 0$, you have 1 zero.
3) If $\underbrace{b^2-4ac}_{\text{negative}} < 0$, you have 0 zeros

Example 3.6.5

Determine the number of zeros using the discriminant:

a) $f(x) = \underset{a}{2}x^2 + \underset{b}{3}x \underset{c}{-2}$

$b^2-4ac = 3^2-4(2)(-2)$

$= 9+16$

$= 25 > 0$ ∴ 2 zeros.

b) $g(x) = \overset{a}{-}x^2 + \overset{b}{4}x - \overset{c}{4}$

$b^2-4ac = 4^2-4(-1)(-4)$

$= 16-16$

$= 0 = 0$ ∴ 1 zero!

c) $h(x) = \underset{a}{3}x^2 + \underset{b}{5}x + \underset{c}{6}$

$b^2-4ac = 5^2-4(3)(6)$

$= 25-72$

$= -47 < 0$, ∴ 0 zeros.

Success Criteria:

- I can recognize that a quadratic function may have 0, 1, or 2 zeros
- I can use the discriminant of the quadratic formula to determine the number of zeros

Chapter 3 – Quadratic Functions

3.7 – Families of Quadratic Functions

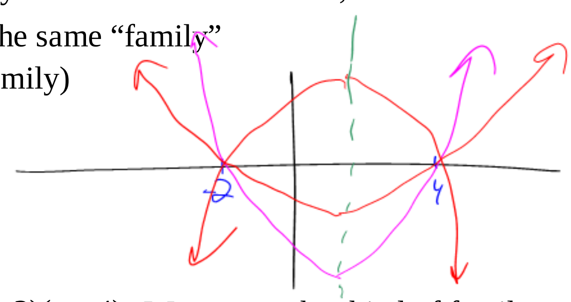
Learning Goal: We are learning the properties of families of quadratic functions.

Consider the two quadratic functions:

$f(x) = 2(x-3)^2 + 1$, and $g(x) = -3(x-3)^2 + 1$ **What's Different?** a

Clearly $f(x)$ and $g(x)$ are different functions, but they do share the same vertex, and the same axis of symmetry. These quadratics are said to be in the same "family"

(some might say they are in the same vertex family)



Next, consider $h(x) = 3(x+2)(x-4)$, and $f(x) = \frac{2}{3}(x+2)(x-4)$. We see another kind of family here because $h(x)$ and $f(x)$ share the same zeros, and the same axis of symmetry.

(some might say these quadratics are in the same zeroes family) **What's Different?** a

Finally consider the third form of a quadratic. Consider

$f(x) = -3x^2 - 2x + 7$

$g(x) = 2x^2 + 8x + 7$

the y-intercept is the same.

Example 3.7.1

Determine the equation of the quadratic with zeros $x = 3$, and $x = -1$ and that passes through the point $(5, 6)$.

r s

$x, f(x)$

The "a" is missing.

$$f(x) = a(x-r)(x-s)$$
$$f(x) = a(x-3)(x+1)$$
$$6 = a(5-3)(5+1)$$
$$6 = a(2)(6)$$
$$6 = 12a$$
$$\frac{6}{12} = a$$
$$\frac{1}{2} = a$$

$\therefore f(x) = \frac{1}{2}(x-3)(x+1)$

Example 3.7.2

Determine the equation of the quadratic function $f(x)$ with a $\boxed{\text{max value of } 3}$ and axis of symmetry with equation $x = -5$ if $\boxed{f(2) = -18}$.

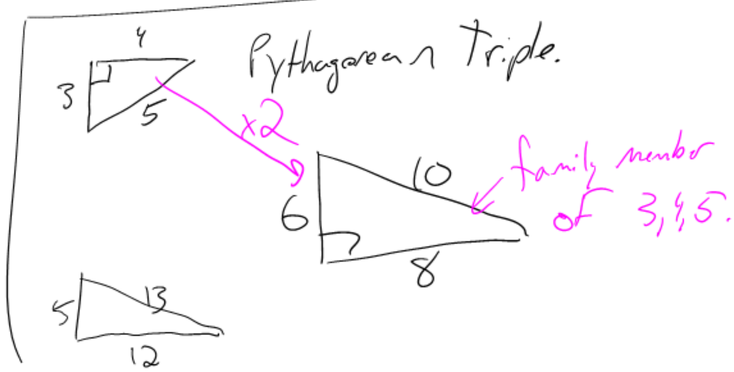
h x $f(x)$

$a < 0$ $f(x)$ or $y = k = 3$

Vertex is $(h, k) = (-5, 3)$

$$f(x) = a(x+5)^2 + 3$$
$$-18 - 3 = a(2+5)^2 + 3 - 3$$
$$-21 = a(7)^2$$
$$-21 = 49a$$
$$\frac{-21}{49} = a$$
$$\frac{-3}{7} = a$$

$\therefore f(x) = \frac{-3}{7}(x+5)^2 + 3$



Success Criteria:

- I can solve for "*a*" if given either the vertex or zeros

Chapter 3 – Quadratic Functions

3.8 – Linear-Quadratic Systems

Learning Goal: We are learning to solve problems involving the intersection of a linear and quadratic function.

Recall from Grade 10 that solving a **SYSTEM OF LINEAR EQUATIONS** could be interpreted to mean finding the point of intersection of the two lines. The solution to a **SoLE** is a point, (x, y). From an algebraic point of view, we have two techniques for solving a SoLE:

1) Substitution
2) Elimination

Example 3.8.1

Solve the SoLE

$$2x + 3y = 7 \quad (1)$$
$$x - 2y = -7 \quad (2)$$

Algebraically

By substitution:

$x = \boxed{2y - 7}$

$2(2y-7) + 3y = 7$

$4y - 14 + 3y = 7$ (+14, +14)

$\frac{7y}{7} = \frac{21}{7}$

$\boxed{y = 3}$

$x = 2(3) - 7$

$x = -1$

∴ The PoI is $(-1, 3)$

Graphically

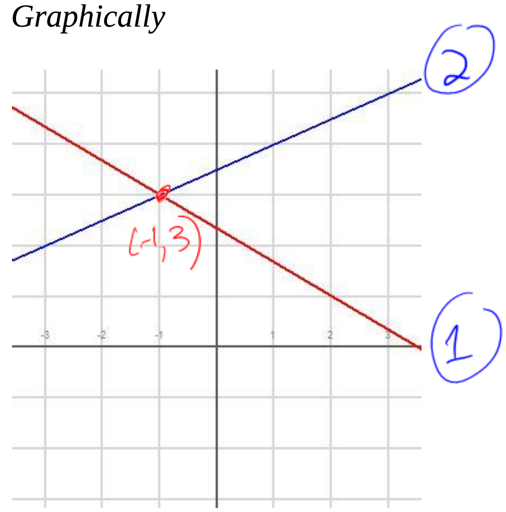


Figure 3.8.1

Solving a Linear-Quadratic System is more difficult, but we have the tools to succeed!
We will need to make use of (at least) one Property (or Rule) of Algebra:

THE TRANSITIVE PROPERTY OF EQUALITY

Rule: Given three numbers (or more generally, three mathematical objects) a, b, and c, and **if** $c = a$ and $c = b$, **then** $a = b$.

Example: If $\overset{c}{f(x)} = \overset{a}{-2x-4}$, and $\overset{c}{g(x)} = \overset{b}{x^2 - 3x - 10}$, and if $f(x) = g(x)$, then

$$\boxed{x^2 - 3x - 10 = -2x - 4}$$

+2x+4

Example 3.8.2

Solve the Linear-Quadratic System given directly above.

Goal: "stuff" = 0

$$x^2 - x - 6 = 0$$

M: −6
A: −1
−3, 2

$$(x-3)(x+2) = 0$$

$x = 3$ and $x = -2$

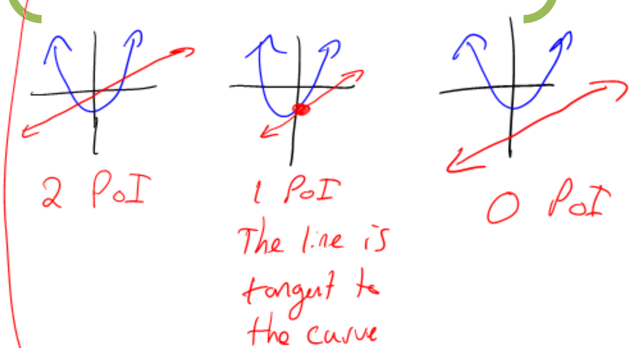
We are finding the points of intersection, ∴ we need x and y.

$f(3) = -2(3) - 4$	$f(-2) = -2(-2) - 4$
$f(3) = -10$	$f(-2) = 0$

∴ The PoIs are $(3, -10)$ and $(-2, 0)$

Note: **Solving a Linear-Quadratic System** is equivalent to **finding the solution(s) to a quadratic equation**.
For L-QS`s we can therefore have 0, 1, or 2 solutions.

We will apply the techniques for solving quadratic equations!



Example 3.8.3 (#2c, on Page 198 from your text)

Determine the point(s) of intersection of the two functions algebraically:

$f(x) = 3x^2 - 2x - 1, \;\; g(x) = -x - 6$

$3x^2 - 2x - 1 = -x - 6$

$+x +6$

$3x^2 - x + 5 = 0$

a b c

$$x = \frac{1 \pm \sqrt{1^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{-59}}{6} \rightarrow$$ there are no solutions and

$\therefore$ no PoI's.

Example 3.8.4

Determine the number of points of intersection without solving the System:

$f(x) = x^2 + 2x + 14, \;\; g(x) = 8x + 5$ (Hint: To solve this problem you must be very discriminating)

use the discriminant.

$x^2 + 2x + 14 = 8x + 5$

$-8x - 5$

$x^2 - 6x + 9 = 0$

$b^2 - 4ac = (-6)^2 - 4(1)(9)$

$= 36 - 36$

$= 0$

$\therefore$ one solution and one PoI.

Example 3.8.5 (#9 on Page 199 in your text)

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

constant $b^2 - 4ac < 0$

$$-3x^2 - x + 4 = 4x + \boxed{k} \quad \text{constant}$$

$$0 = \underbrace{3}_{a}x^2 \underbrace{+5}_{b}x \underbrace{+k-4}_{c}$$

$$b^2 - 4ac < 0$$

$$(5)^2 - 4(3)(k-4) < 0$$

$$25 - 12(k-4) < 0 \quad +12k$$

$$25 \boxed{-12k} + 48 < 0$$

$$73 < 12k$$

$$\frac{73}{12} < k$$

$$6.1 < k$$

$$\frac{-12k}{-12} < \frac{-73}{-12}$$

$$k > \frac{73}{12}$$

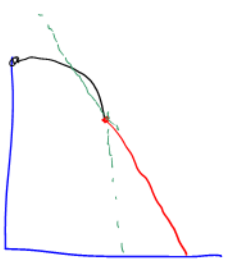
dividing by negatives causes the inequality to flip.

Example 3.8.6 (#10 in your text)

10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?



$$-4.9t^2 + t + 360 = -4t + 142$$

$$0 = 4.9t^2 - 5t - 218$$

QF: $t = \dfrac{5 \pm \sqrt{5^2 - 4(4.9)(-218)}}{2(4.9)}$

$$t = \frac{5 \pm 65.56}{9.8}$$

$$t_1 = \frac{5 + 65.56}{9.8} \qquad \cancel{t_2 = \frac{5 - 65.56}{9.8}}$$

$$\boxed{t_1 = 7.2} \qquad \cancel{t_2 = -6.2}$$

Inadmissible

∴ the daredevil opened his parachute 7.2 seconds after jumping.

Success Criteria:

- I can solve for the points of intersection by
  1. Making the functions equal to each other
  2. Solving for the zeros (x-coordinates) of the resulting quadratic function
  3. Substituting the zeros into the linear equation to determine the corresponding y-values
- I can identify when solutions are inadmissible