

Chapter 7.5 Arithmetic Series Classwork

Name: Solution by Hagen

1. Calculate the sum of the first 10 terms of each arithmetic series.

Use $S_n = \frac{n[2a + (n-1)d]}{2}$

a) $59 + 64 + 69 + \dots$

$$\begin{aligned} a &= 59 \\ d &= 5 \\ n &= 10 \\ S_{10} &= \frac{10[2(59) + (10-1)(5)]}{2} \\ S_{10} &= 5(118 + 45) \\ S_{10} &= 815 \end{aligned}$$

b) $-103 - 110 - 117 - \dots$

$$\begin{aligned} a &= -103 \\ d &= -7 \\ n &= 10 \\ S_{10} &= \frac{10[2(-103) + (10-1)(-7)]}{2} \\ S_{10} &= 5(-206 - 63) \\ S_{10} &= -1345 \end{aligned}$$

2. Calculate the sum of the first 20 terms of an arithmetic sequence with first term 18 and common difference 11.

$$\begin{aligned} n &= 20 \\ a &= 18 \\ d &= 11 \\ S_{20} &= \frac{20[2(18) + (20-1)(11)]}{2} \\ S_{20} &= 10[36 + 209] \\ S_{20} &= 2450 \end{aligned}$$

3. For each series, calculate t_{12} then S_{12} .

a) $-18 - 12 - 6 + 0 + \dots$

$$\begin{aligned} a &= -18 \\ d &= 6 \\ n &= 12 \\ t_{12} &= -18 + (12-1)(6) \\ t_{12} &= -18 + 66 \\ t_{12} &= 48 \\ S_{12} &= \frac{12(-18 + 48)}{2} \\ S_{12} &= 6(30) \\ S_{12} &= 180 \end{aligned}$$

b) $3.19 + 4.31 + 5.43 + 6.55 + \dots$

$$\begin{aligned} a &= 3.19 \\ d &= 1.12 \\ n &= 12 \\ t_{12} &= 3.19 + (12-1)(1.12) \\ t_{12} &= 3.19 + 12.32 \\ t_{12} &= 15.51 \\ S_{12} &= \frac{12(3.19 + 15.51)}{2} \\ S_{12} &= 6(18.7) \\ S_{12} &= 112.2 \end{aligned}$$

$t_n = a + (n-1)d$ Then use $S_n = \frac{n(a + t_n)}{2}$

4. Determine the sum of the first 20 terms of an arithmetic series in which the 15th term is 107 and terms decrease by 3.

$$\begin{aligned} n &= 20 \\ t_{15} &= 107 \\ d &= -3 \\ a &= ? \end{aligned}$$

$$\begin{aligned} 107 &= a + (15-1)(-3) \\ 107 &= a - 42 \\ 149 &= a \end{aligned}$$

$$\begin{aligned} S_{20} &= \frac{20[2(149) + (20-1)(-3)]}{2} \\ S_{20} &= 10(298 - 57) \\ S_{20} &= 2410 \end{aligned}$$

We need to know "n", so first use $t_n = a + (n-1)d$

5. Calculate the sums of these finite arithmetic series (example 2 in notes)

a) $1 + 6 + 11 + \dots + 96$

$a = 1$
 $d = 5$
 $t_n = 96$
 $n = ?$

Now:
 $96 = 1 + (n-1)(5)$
 $95 = (n-1)(5)$
 $19 = n-1$
 $20 = n$

$S_{20} = \frac{20(1+96)}{2}$
 $S_{20} = 10(97)$
 $S_{20} = 970$

b) $-31 - 38 - 45 - \dots - 136$

$a = -31$
 $d = -7$
 $t_n = -136$
 $n = ?$

$-136 = -31 + (n-1)(-7)$
 $-105 = (n-1)(-7)$
 $15 = n-1$
 $16 = n$

$S_{16} = \frac{16(-31-136)}{2}$
 $S_{16} = 8(-167)$
 $S_{16} = -1336$

6. Bricks are stacked in 20 rows such that each row has a fixed number of bricks more than the row above it. The top row has 5 bricks and the bottom row has 62 bricks. How many bricks are in the stack?

$S_{20} = \frac{20(5+62)}{2}$

$S_{20} = 10(67)$

$S_{20} = 670$

7. Jamal got a job working on an assembly line in a toy factory. On the 20th day of work, he assembled 137 toys. He noticed that since he started, every day he assembled 3 more toys than the day before. How many toys did Jamal assemble altogether during his first 20 days?

$a = ?$
 $137 = a + (20-1)(3)$
 $137 = a + 57$
 $80 = a$

$d = 3$
 $S_{20} = \frac{20[2(80) + (20-1)(3)]}{2}$

$S_{20} = 10(160 + 57)$

$S_{20} = 2170 \text{ toys}$

8. Given the arithmetic sequence below, determine the sum of the 20th term to the 39th term.

$5 + 12 + 19 + \dots$

Let's find t_{20} and t_{39} using $t_n = a + (n-1)d$
 $a = 5, d = 7$

$t_{20} = 5 + (20-1)(7)$ | $t_{39} = 5 + (39-1)(7)$

$t_{20} = 5 + 133$ | $t_{39} = 5 + 266$

$t_{20} = 138$ | $t_{39} = 271$

20 terms.

Now, shift the "a" to t_{20}

Do either $S_{20} = \frac{20[2(138) + (20-1)(7)]}{2}$

OR:

$S_{20} = \frac{20(t_{20} + t_{39})}{2}$

$S_{20} = 10(138 + 271)$

$S_{20} = 4090$