

Chapter 7.6 Geometric Series Classwork

Name: Solutions by Hagen

1. Calculate the sum of the first 7 terms of each geometric series. Use $S_n = \frac{a(r^n - 1)}{r - 1}$

a) $6 + 18 + 54 + \dots$

b) $8 - 24 + 72 + \dots$

$$\begin{aligned} a &= 6 \\ r &= 3 \\ n &= 7 \\ S_7 &= \frac{6(3^7 - 1)}{3 - 1} \\ S_7 &= \frac{6(2186)}{2} \\ S_7 &= 6558 \end{aligned}$$

$$\begin{aligned} a &= 8 \\ r &= -3 \\ n &= 7 \\ S_7 &= \frac{8((-3)^7 - 1)}{-3 - 1} \\ S_7 &= \frac{8(-2188)}{-4} \\ S_7 &= 4376 \end{aligned}$$

2. For each series, calculate t_6 then S_6 .

a) $21,000,000 + 4,200,000 + 840,000 + \dots$

b) $3.4 - 7.14 + 14.994 - \dots$

$$\begin{aligned} a &= 21,000,000 \\ r &= \frac{1}{5} = 0.2 \\ n &= 6 \\ t_6 &= 21,000,000(0.2)^{6-1} \\ t_6 &= 6720 \\ S_6 &= \frac{21,000,000(0.2^6 - 1)}{0.2 - 1} \\ S_6 &= \frac{-20,998,656}{-0.8} \\ S_6 &= 26,248,320 \end{aligned}$$

$$\begin{aligned} a &= 3.4 \\ r &= 2.1 \\ n &= 6 \\ t_6 &= 3.4(2.1)^{6-1} \\ t_6 &= 138.859434 \\ S_6 &= \frac{3.4(2.1^6 - 1)}{2.1 - 1} \\ S_6 &= 262.004374 \end{aligned}$$

3. Determine the sum of the first 7 terms of each geometric series in which:

a) the first term is 11 and the seventh term is 704

b) 5th term is 5 and 8th term is -40

$$\begin{aligned} t_7 &= \frac{704}{11} = \frac{11(r)^{7-1}}{11} \\ \sqrt[6]{64} &= \sqrt[6]{r^6} \\ 2 &= r \end{aligned}$$

$$S_7 = \frac{11(2^7 - 1)}{2 - 1}$$

$$S_7 = 11(127)$$

$$S_7 = 1397$$

$$\begin{aligned} &\text{Diagram showing terms 5, 6, 7, 8 with arrows indicating a common ratio } r. \\ &\text{Term 5 is 5, term 8 is -40. } r^3 = -8 \end{aligned}$$

$$r^3 = -8$$

$$r = -2$$

now need a or t_1

$$t_5 = 5 = a(-2)^{5-1}$$

$$5 = a(16)$$

$$\frac{5}{16} = a = 0.3125$$

$$\begin{aligned} \text{Now: } S_7 &= \frac{t_{7+1} - t_1}{r - 1} = \frac{-40 - 0.3125}{-3} = 13.4375 \\ &= S_7 \end{aligned}$$

4. Calculate the sums of these finite arithmetic series (example 2 in notes)

a) $960 + 480 + 240 + \dots + 15$

$$a = 960$$

$$r = \frac{1}{2}$$

$$t_n = 15$$

$$t_{n+1} = \frac{15}{2} = 7.5$$

$$S_n = \frac{7.5 - 960}{\frac{1}{2} - 1}$$

$$S_n = \frac{-952.5}{-0.5}$$

$$S_n = 1905$$

b) $-6 + 24 - 96 + \dots + 98304$

$$a = -6$$

$$r = -4$$

$$t_n = 98304$$

$$t_{n+1} = -393216$$

$$S_n = \frac{-393216 - (-6)}{-4 - 1}$$

$$S_n = \frac{-393210}{-5}$$

$$S_n = 78642$$

5. A ball is dropped from a height of 3 m and bounces on the ground. At the top of each bounce, the ball reaches 60% of its previous height. Calculate the total distance travelled by the ball when it hits the ground for the fifth time.

$$S_5 = \frac{3(0.6^5 - 1)}{0.6 - 1}$$

$$S_5 = 6.9168 \text{ m.}$$

6. A large company has a phone tree to contact its employees in case of an emergency factory shutdown. Each of the five senior managers calls three employees, who each call three other employees, and so on. If the tree consists of seven levels, how many employees does the company have?

$$S_7 = \frac{5(3^7 - 1)}{3 - 1}$$

$$S_7 = 5465 \text{ employees.}$$

7. In a geometric series, $t_1 = 23$, $t_3 = 92$, and the sum of all the terms is ~~6225~~ 94185. How many terms are in the series?

$$\begin{aligned} \text{First } t_3 &= 92 = \frac{23}{23} r^{3-1} \\ 4 &= r^2 \\ 2 &= r \end{aligned}$$

$$94185$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$94185 = \frac{23(2^n - 1)}{2 - 1}$$

$$\frac{94185}{23} = \frac{23(2^n - 1)}{23}$$

$$4095 = 2^n - 1$$

$$4096 = 2^n \Rightarrow n = 12$$