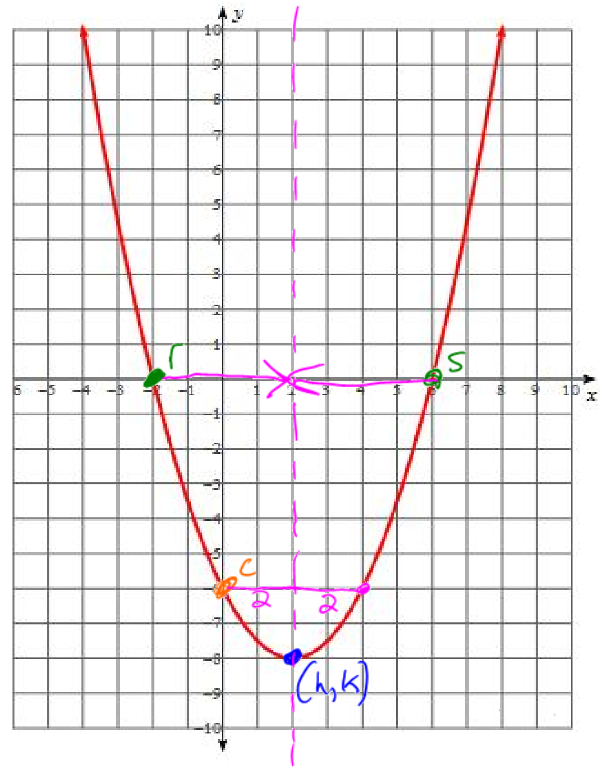
Our dear friend from Grade 10 is back. What are quadratics?



Vertex: (h,k)
↳ max or min

x-Intercepts/zeros/solutions/roots:
x=r and x=s
(r,0) (s,0)

y-intercept
y=c or (0,c)

Axis of Symmetry
x=h

$h = \frac{r+s}{2}$
h is the average of the zeros Or any two x's on the same horizontal line

Three forms (equations) of Quadratics:

$f(x) = a(x-h)^2 + k$
Vertex Form, (h,k)
useful for graphing

$f(x) = ax^2 + bx + c$
Standard Form
– has the y-intercepts
– great for solving quadratics

$f(x) = a(x-r)(x-s)$
Factored or Zeros form.
– gives the zeros x=r and x=s

If a>0, opens up

If a<0, opens down

Find the equation of the parabola:

Let's use Vertex Form.

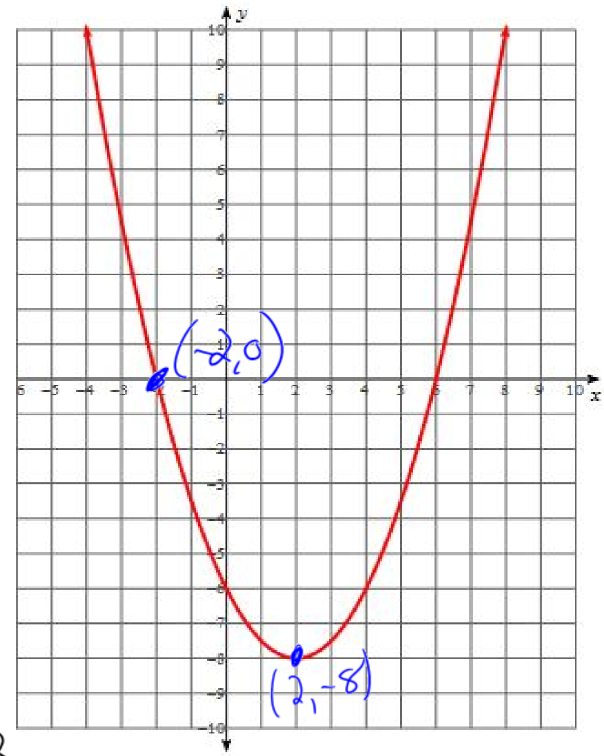
$h = 2$

$k = -8$

Now we need another point

$x = -2$

$y = 0$



$f(\cancel{x}) = a(x-h)^2 + k$

$0 = a\boxed{(-2-2)^2} - 8$

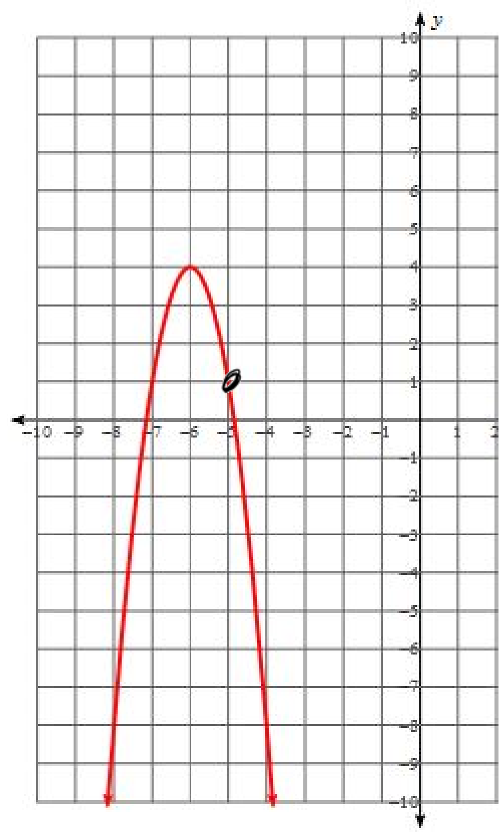
$+8 \qquad\qquad +8$

$\frac{8}{16} = \frac{a(16)}{16}$

$\frac{1}{2} = a$

$\therefore f(x) = \frac{1}{2}(x-2)^2 - 8$

Write the standard form of the parabola:



Convert to Standard Form:

$(x+4)(x+4)$

$f(x) = -2\boxed{(x-4)(x+7)}$

$f(x) = -2(x^2+7x-4x-28)$

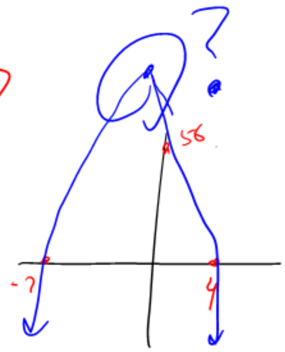
$f(x) = -2(x^2+3x-28)$

$f(x) = -2x^2-6x+56$

Zeros: $x=4$, $x=-7$

y-int: $(0, 56)$

Opens down



$g(x) = \frac{1}{2}\boxed{(x+4)^2}-6$

$g(x) = \frac{1}{2}(x^2+8x+16)-6$

$g(x) = \frac{1}{2}x^2+4x+8-6$

$g(x) = \frac{1}{2}x^2+4x+2$

State the direction of opening, the equation of axis and the vertex: (h,k)

$f(x) = \underbrace{3}_{a>0}(x+6)(x-2)$

Opens up.

Zeros: $x=-6$ r

$x=2$ s

$AoS = h = \frac{r+s}{2} = \frac{-6+2}{2} = \frac{-4}{2} = -2$ h

Vertex: $(\underset{x}{-2}, -48)$

$k = f(-2) = 3(-2+6)(-2-2)$

$f(-2) = 3(4)(-4)$

$f(-2) = -48$

Determine the equation of axis:

$(4, 3), (12, 3)$

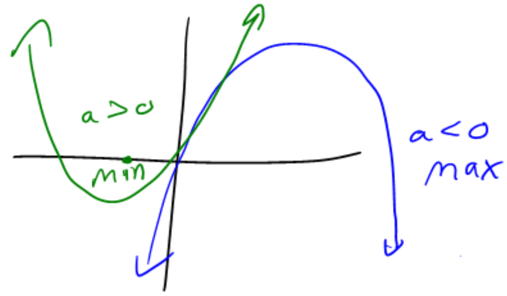
$AoS = \frac{4+12}{2} = \frac{16}{2} = 8$

$\boxed{x=8}$

The maximum/minimum is the y-coordinate of the vertex, or the *k*. A maximum occurs when *a*<0 and a minimum occurs when *a*>0.

Given vertex form:

$$f(x) = a(x-h)^2 + k$$



$$f(x) = -2(x+5)^2 - 8$$

a < 0   K

f(x) has a max of −8.

Given Zeros Form:

$$f(x) = a(x-r)(x-s)$$

zeros

AoS = h = $\frac{r+s}{2}$

Vertex is $(\overset{x}{h}, \overset{y}{k})$

f(h) = ________ = K

$$f(x) = 3(x+2)(x-8)$$

a > 0

r = −2, s = 8

$h = \frac{-2+8}{2} = \frac{6}{2} = 3$

Vertex of (3, −75)

$\boxed{\text{f(x) has a min of −75.}}$

f(3) = 3(3+2)(3−8) = 3(5)(−5) = −75 = K

Given Standard Form: $f(x) = ax^2 + bx + \underbrace{c}_{\text{y-int}}$

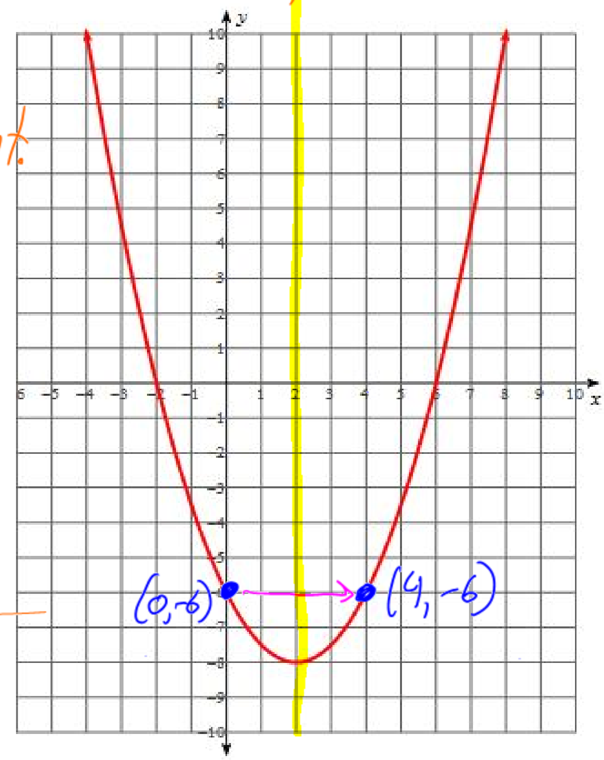
Partial Factoring.

All quadratics have a y-intercept.

$\rightarrow (0, c)$

Then they also have a symmetric partner with the same y-value

$\rightarrow (?, c)$



$f(x) = \boxed{ax^2+bx} + c$

↳ make this equal to zero

$x=0$ is one solution.

now we must find the other...

$f(x) = \underbrace{\overset{\neq 0}{2}x^2 + 10x}_{\text{factor } 2x} - 5$

$f(x) = \boxed{2x(x+5)} - 5$

$\underset{①}{x=0}$ $\quad$ $\underset{②}{x=-5}$ $\quad$ $=0$

① Factor ax from the first two terms

② Determine the second x point ~~that~~

we now have two points!

$(0, \overset{c}{-5})$ and $(-5, \overset{c}{-5})$

③ Calculate the AoS (h)

$AoS = h = \frac{0 + -5}{2} = -2.5$

④ Evaluate $f(h) =$

⑤ State the result

$f(-2.5) = 2(-2.5)^2 + 10(-2.5) - 5$

$= 12.5 - 25 - 5$

$K = -17.5$

$\therefore f(x)$ has a min of -17.5

$$f(x) = \underbrace{3x^2 + 48x}_{\text{factor } 3x} - 19$$

$$f(x) = 3x(x+16) \quad -19$$

$$\begin{array}{cc} \uparrow & \uparrow \\ 0 & -16 \end{array}$$

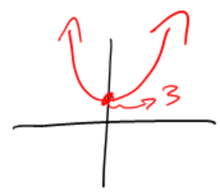
$$h = \frac{0 + -16}{2} = -8$$

$$f(-8) = 3(-8)^2 + 48(-8) - 19$$

$$= 192 - 384 \quad -19$$

$$K = -211$$

$\therefore$ f(x) has a min of -211.



$$f(x) = x^2 + \underline{3}$$

If you are missing the "bx" term, → both y-int and max/min value

$$f(x) = \underbrace{\tfrac{1}{2}x^2 - 5x}_{\frac{1}{2}x} + 8$$

$$f(x) = \tfrac{1}{2}x(x - 10) \quad + 8$$

$$\begin{array}{cc} \uparrow & \uparrow \\ 0 & 10 \end{array}$$

$$h = \frac{0 + 10}{2} = 5$$

$$f(5) = \tfrac{1}{2}(5)^2 - 5(5) + 8$$

$$= 12.5 - 25 \quad + 8$$

$$K = -4.5$$

$\therefore$ f(x) has a min of -4.5

$$f(x) = \underbrace{-2.2x^2 + 8.4x}_{-2.2x} + 5.76$$

$$f(x) = -2.2x(x - 3.82) + 5.76$$

0 3.82

$$h = \frac{0 + 3.82}{2} = 1.91$$

$$f(1.91) = -2.2(1.91)^2 + 8.4(1.91) + 5.76$$

$$= -8.03 + 16.04 + 5.76$$

$$= 13.77$$

$\therefore$ $f(x)$ has a max of 13.77

3.4 – Operations with Radicals → $\sqrt{\ }$

$1^2 =$ 1	$11^2 =$ 121
$2^2 =$ 4	$12^2 =$ 144
$3^2 =$ 9	$13^2 =$ 169
$4^2 =$ 16	$14^2 =$ 196
$5^2 =$ 25	$15^2 =$ 225
$6^2 =$ 36	$16^2 =$ 256
$7^2 =$ 49	$17^2 =$ 289
$8^2 =$ 64	$18^2 =$ 324
$9^2 =$ 81	$19^2 =$ 361
$10^2 =$ 100	$20^2 =$ 400

Simplifying Radicals:

1. $2\sqrt{48}$

$48 = \boxed{16} \times 3$

$= 2\sqrt{16 \times 3}$

$= 2\sqrt{16}\sqrt{3}$

$= 2(4)\sqrt{3}$

$= 8\sqrt{3}$

2. $-\sqrt{20}$

$= -\sqrt{4}\sqrt{5}$

$= -2\sqrt{5}$

3. $\frac{1}{8}\sqrt{320}$

64×5

$= \frac{1}{8}\cancel{\sqrt{64}}^{8}\sqrt{5}$

$= \frac{1}{\cancel{8}}(\cancel{8})\sqrt{5}$

$= \sqrt{5}$

4. $-3\sqrt{513}$

9×57

$= -3\cancel{\sqrt{9}}^{3}\sqrt{57}$

$= -9\sqrt{57}$

$\sqrt{x^2} = x^1$

$\sqrt{x^4} = x^2$

$\sqrt{x^{100}} = x^{50}$

$\sqrt{x^{33}}$

$= \sqrt{x^{32}x^1}$

Simplifying Radicals with variables:

1. $7\sqrt{288b^4}$

144×2

cut ~~variable~~ exponent in half

$= 7\sqrt{144b^4}\sqrt{2}$

$= 7(12b^2)\sqrt{2}$

$= 84b^2\sqrt{2}$

2. $-5\sqrt{45n^3}$

$= -5\sqrt{9n^2}\sqrt{5n}$

$= -15n\sqrt{5n}$

Adding and Subtracting Radicals:

1. $-2\sqrt{12} - 3\sqrt{8} + 3\sqrt{32} + 2\sqrt{27}$

$4\times3 \quad 4\times2 \quad 16\times2 \quad 9\times3$

$= -2\sqrt{\overset{2}{\cancel{4}}}\sqrt{3} - 3\sqrt{\overset{2}{\cancel{4}}}\sqrt{2} + 3\sqrt{\overset{4}{\cancel{16}}}\sqrt{2} + 2\sqrt{\overset{3}{\cancel{9}}}\sqrt{3}$

$= -4\sqrt{3} - 6\sqrt{2} + 12\sqrt{2} + 6\sqrt{3}$

$= 2\sqrt{3} + 6\sqrt{2}$

$2x^2 + x + 3x^2 - 4x$

$5x^2 - 3x$

We need like radicals

2. $2\sqrt{45} - \sqrt{8} - 2\sqrt{32} - 2\sqrt{18}$

$9\times5 \quad 4\times2 \quad 16\times2 \quad 9\times2$

$= 2\sqrt{\overset{3}{\cancel{9}}}\sqrt{5} - \sqrt{\overset{2}{\cancel{4}}}\sqrt{2} - 2\sqrt{\overset{4}{\cancel{16}}}\sqrt{2} - 2\sqrt{\overset{3}{\cancel{9}}}\sqrt{2}$

$= 6\sqrt{5} - 2\sqrt{2} - 8\sqrt{2} - 6\sqrt{2}$

$= 6\sqrt{5} - 16\sqrt{2}$

$(3x^2)(4x^3)$

$= 12x^5$

Multiplying Radicals:

1. $2\sqrt{6} \times 4\sqrt{5}$

multiply

$= 8\sqrt{30}$

2. $-3\sqrt{10} \times 6\sqrt{2}$

$= -18\sqrt{20}$

4×5

$= -18\sqrt{\overset{2}{\cancel{4}}}\sqrt{5} \quad = -36\sqrt{5}$

3. $(\sqrt{2} + \sqrt{3})(2\sqrt{2} - 5\sqrt{3})$

$= 2\sqrt{\overset{2}{\cancel{4}}} - 5\sqrt{6} + 2\sqrt{6} - 5\sqrt{\overset{3}{\cancel{9}}}$

$= 4 - 3\sqrt{6} - 15$

$= -11 - 3\sqrt{6}$

$$\left|\boxed{\frac{3}{5}} \times \frac{2}{2} = \boxed{\frac{6}{10}}\right|$$

Rationalizing the Denominator:

1. $\frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{4} = 2$

$= \frac{10\sqrt{2}}{2}$

$= 5\sqrt{2}$

2

1.41

$\times \quad 5$

7.05

2. $\frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{24}}{3} = \frac{\sqrt{4}\sqrt{6}}{3} = \frac{2\sqrt{6}}{3}$

3. $\frac{9\sqrt{63}}{7\sqrt{54}}$

4. $\frac{7+\sqrt{2}}{5\sqrt{40}}$

3.5 – Solving Quadratics

What is solving? It is simple to understand when the question itself is simple:

$$5x - 1 = 4$$
$$5x = 5$$
$$x = 1$$

$$3x^2 - 12 = 0$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

However, when we need to solve:
$x^2 - 6x + 8 = 0$ or $2x^2 + 5x - 5 = 7$,
it gets more complicated.

My definition
Solving for *x* means to find the value(s) of *x* to satisfy (make true) the equation/function. With a linear function, there is one main method to solve. A quadratic has 4 methods!!! The use of each method depends on the problem and how comfortable you are with the methods.

1. Graphing Calculator/Technology.
This is my least favourite method. While it works every time, you aren't "learning" or practicing your math. This method is great to check your work or to use on those pesky large questions. Standard Form is not required.

2. Solving from the Vertex Form (Completing the Square)
I very rarely use this method straight from the Standard Form. However, if your question is already in Vertex Form, this is a powerful method.

3. Factoring
This is not partial factoring. You need to be able to FULLY factor the quadratic. This is a very quick method if you can see that it is factorable. The solutions are then just in the factored form.

4. Quadratic Formula
This method works every time! It can even tell you if there are no solutions or just one solution. If you can master this formula, you will never sweat while solving quadratics! Standard Form is required.

Note
Standard Form is required in most cases. This may require you to move all the components to one side, leaving the other side equal to zero. Hence why this is called solving for the zeros.…

Examples of each method (expect technology):

2. Solving from the Vertex Form (Completing the Square)

$$f(x) = -3(x-4)^2 + 27$$

$$0 = -3(x-4)^2 + \cancel{27}$$

$-27 \qquad \cancel{-27}$

$$\frac{-27}{-3} = \frac{\cancel{-3}(x-4)^2}{\cancel{-3}}$$

$$\sqrt{9} = \sqrt{(x-4)^2}$$

$$\pm 3 \; {}^{+4} = x - 4 \quad {}^{+4}$$

$$\pm 3 + 4 = x$$

1 $x = 3 + 4$

$x = 7$

2 $x = -3 + 4$

$x = 1$

x-intercept of $(7,0)$ and $(1,0)$

2. Solving from the Vertex Form (Completing the Square)

$$g(x) = 4(x+5)^2 - 32$$

$$0 = 4(x+5)^2 - \cancel{32}$$

$+32 \qquad \cancel{+32}$

$$\frac{32}{4} = \frac{\cancel{4}(x+5)^2}{\cancel{4}}$$

$$\sqrt{8} = \sqrt{(x+5)^2}$$

$$\pm 2.83 = x + 5$$

$$\pm 2.83 - 5 = x$$

$$\sqrt{4}\sqrt{2} = \pm 2\sqrt{2} - 5$$

1 $x = 2.83 - 5$

$x = -2.17$

2 $x = -2.83 - 5$

$x = -7.83$

$x = 2\sqrt{2} - 5$

$x = -2\sqrt{2} - 5$

3. Factoring

$$f(x) = x^2 + 18x + 80$$

$0 = x^2 + 18x + 80$

$0 = (x+10)(x+8)$

$x = -10$

$x = -8$

$$g(x) = \boxed{2x^2 + 5x - 5 = 7} \quad -7 \quad -7$$

$$g(x) = 7$$

$2x^2 + 5x - 12 = 0$

$m: -24 \quad A: 5$

$-3, 8$

$\underbrace{2x^2 - 3x}_{x} \underbrace{+ 8x - 12}_{+4} = 0$

$(x+4)(2x-3) = 0$

$x = -4 \qquad x = \frac{3}{2}$

$2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

$2\left(\frac{3}{2}\right)^2 + 5\left(\frac{3}{2}\right) - 5$

$\cancel{2}\left(\frac{9}{\cancel{4}}\right) + \frac{15}{2} - 5$

$\frac{9}{2} + \frac{15}{2} - 5$

$\frac{24}{2} - 5$

$12 - 5 = 7$

4. Quadratic Formula

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$f(x) = \overset{a}{-3}x^2 \overset{b}{-8}x \overset{c}{+14}$$

$x = \dfrac{8 \pm \sqrt{8^2 - 4(-3)(14)}}{2(-3)}$

$x = \dfrac{8 \pm \sqrt{64 + 168}}{-6}$

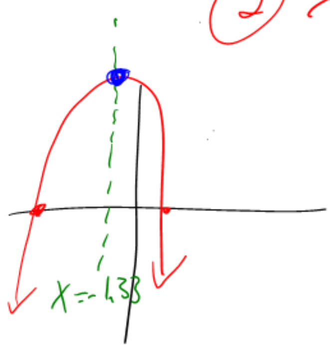
$x = \dfrac{8 \pm \sqrt{232}}{-6}$

$x = \dfrac{8 \pm 15.23}{-6}$

① $x = \dfrac{8 + 15.23}{-6} = -3.87$

② $x = \dfrac{8 - 15.23}{-6} = 1.21$

$AoS = h = \dfrac{1.21 + (-3.87)}{2}$



$(\underline{-1.33}_{h}, \boxed{19.33}_{k})$

4. Quadratic Formula

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$5x(x+3.4)=2x^2+9x-5$$

$$5x^2+17x=\boxed{2x^2+9x-5}$$

$-2x^2$ $\quad -9x \quad +5$

$$3x^2+8x+5=0$$

$a \quad b \quad c$

$$x=\frac{-8\pm\sqrt{8^2-4(3)(5)}}{2(3)}$$

$$x=\frac{-8\pm\sqrt{64-60}}{6}$$

$$x=\frac{-8\pm 2}{6}$$

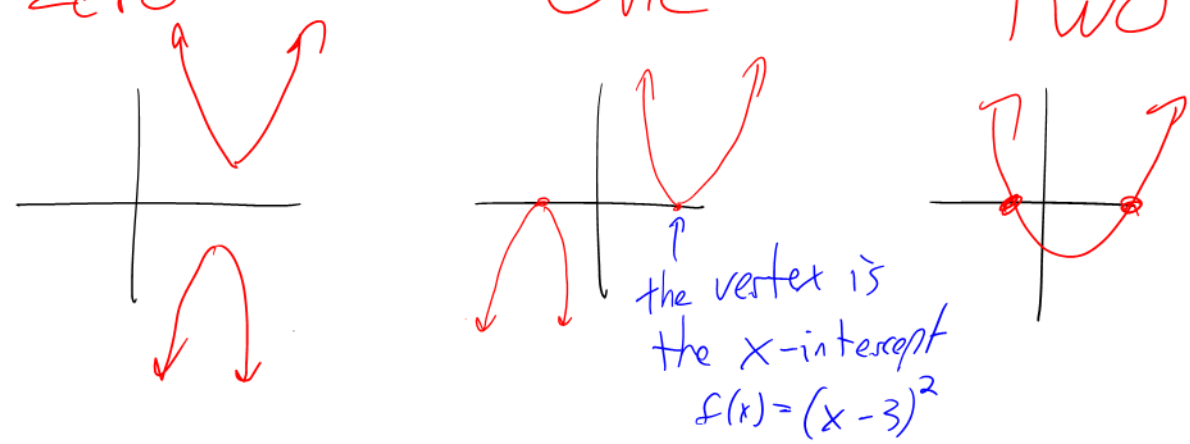
1. $x=\frac{-8+2}{6}=\frac{-6}{6}=-1$

2. $x=\frac{-8-2}{6}=\frac{-10}{6}=-1.67$

3.6 – Zeros of a Quadratic Function

Zero, One, or Two?

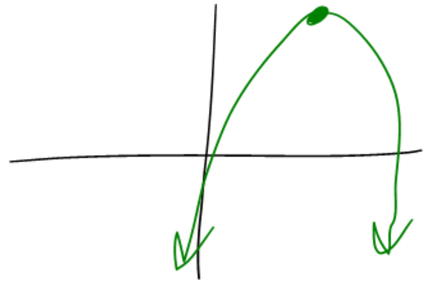
Zero $\qquad$ One $\qquad$ Two



Three methods to determine the number of zeros.

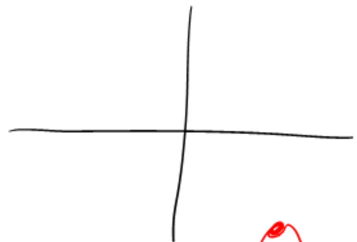
1. Graph it.
2. Vertex Form

$$f(x) = -3(x-4)^2 + 5$$



Twice

$$g(x) = -5(x-6)^2 - 10$$



No ~~Solution~~ zeros.

$$h(x) = 2(x+1)^2 - 8$$

below x-axis, then opens up ∴ 2 zeros

3. Algebra!

Recall: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To find the number of zeros, just look at the radical, or more commonly known as the discriminant.

If, $b^2 - 4ac > 0$, then there are 2 zeros. $\sqrt{9} = \pm 3$

If, $b^2 - 4ac = 0$, then there is 1 zero. $\sqrt{0} = \cancel{\pm} 0$

If, $b^2 - 4ac < 0$, then there are 0 zeros. $\sqrt{-9}$ = undefined - doesn't work.

Example:

$$f(x) = \overset{a}{-2}x^2 + \overset{b}{6}x - \overset{c}{9}$$

How many zeros.

$b^2 - 4ac$

$= 6^2 - 4(-2)(-9)$

$= 36 - 72$

$= -36 < 0$ $\therefore$ no zeros.

The curveball…

$$f(x) = \boxed{\overset{a}{-2}}x^2 \boxed{\overset{b}{+3}}x \boxed{\overset{c}{+k}}$$

Solve for *k* so that *f(x)* has only one solution.

$b^2 - 4ac = 0$

$3^2 - 4(-2)(k) = 0$

$9 + 8k = 0$

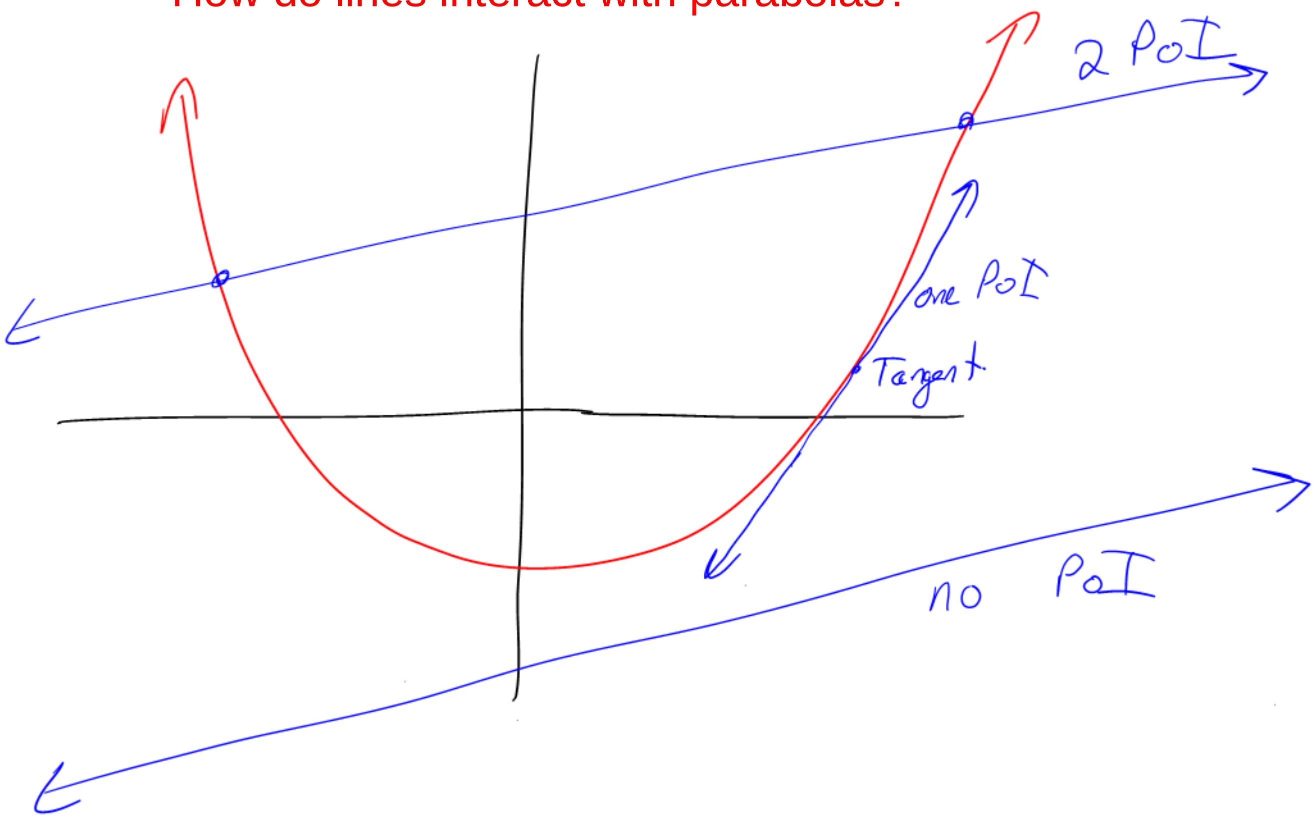
$8k = -9$

$k = \frac{-9}{8}$

$f(x) = -2x^2 + 3x - \frac{9}{8}$

this has one x-int.

How do lines interact with parabolas?



Example 1 $f(x) = -2x + 8 \quad g(x) = 4x^2 + 12x - 7$

~~How many points of intersection?~~

$f(x) = g(x)$

$-2x + 8 = 4x^2 + 12x - 7$

$+2x \quad -8 \qquad +2x \quad -8$

$0 = \underbrace{4x^2}_{a} \underbrace{+14x}_{b} \underbrace{-15}_{c}$

$x = \dfrac{-14 \pm \sqrt{14^2 - 4(4)(-15)}}{2(4)}$

$x = \dfrac{-14 \pm \sqrt{436}}{8} = \dfrac{-14 \pm 20.9}{8}$

① $x = \dfrac{-14 + 20.9}{8} = 0.86$

② $x = \dfrac{-14 - 20.9}{8} = -4.36$

PoI are:

$(0.86, \underline{6.28})$

$(-4.36, \underline{16.72})$

Plug into $f(x)$, the line.

Example 2 $f(x) = 3x + 5$ $g(x) = 2x^2 - 6x - 4$

Find the point(s) of intersection.

Example 3 $f(x) = 2x + k \quad g(x) = 3x^2 + 5x - 2$

Find the value of *k* so that *f(x)* and *g(x)* intersect only once.