

# Review of Technical Skills

## Part 1 Using the TI-83 Plus and TI-84 Graphing Calculators

<b>B-1</b>	Preparing the Calculator	577
<b>B-2</b>	Entering and Graphing Functions	577
<b>B-3</b>	Evaluating a Function	578
<b>B-4</b>	Changing Window Settings	578
<b>B-5</b>	Using the Split Screen	579
<b>B-6</b>	Using the TABLE Feature	579
<b>B-7</b>	Making a Table of Differences	580
<b>B-8</b>	Finding the Zeros of a Function	580
<b>B-9</b>	Finding the Maximum or Minimum Values of a Function	581
<b>B-10</b>	Graphing the Inverse of a Function	582
<b>B-11</b>	Creating Scatter Plots and Determining Lines and Curves of Best Fit Using Regression	583
<b>B-12</b>	Finding the Points of Intersection of Two Functions	584
<b>B-13</b>	Evaluating Trigonometric Ratios and Finding Angles	584
<b>B-14</b>	Graphing Trigonometric Functions	585
<b>B-15</b>	Evaluating Powers and Roots	585
<b>B-16</b>	Generating the Terms of a Sequence	586
<b>B-17</b>	Graphing Sequences	586
<b>B-18</b>	Finding the Sum and Cumulative Sum of a Series	590
<b>B-19</b>	Analyzing Financial Situations Using the TVM Solver	591
<b>B-20</b>	Creating Repayment Schedules Using the TVM Solver	595

## Part 2 Using a Spreadsheet

<b>B-21</b>	Introduction to Spreadsheets	599
<b>B-22</b>	Creating an Amortization Table	601

## Part 3 Using *The Geometer's Sketchpad*

<b>B-23</b>	Graphing Functions	602
<b>B-24</b>	Graphing Trigonometric Functions	604
<b>B-25</b>	Creating Geometric Figures	605
<b>B-26</b>	Measuring Sides and Angles, and Using the Calculator	606

## Part 4 Using Fathom

<b>B-27</b>	Creating a Scatter Plot and Determining the Equation of a Line or Curve of Good Fit	608
-------------	---	-----

# PART 1 USING THE TI-83 PLUS AND TI-84 GRAPHING CALCULATORS

## B-1 Preparing the Calculator

Before you graph any function, be sure to clear any information left on the calculator from the last time it was used. You should always do the following:

**1. Clear all data in the lists.**

Press **2nd** **+** **4** **ENTER**.

**2. Turn off all stat plots.**

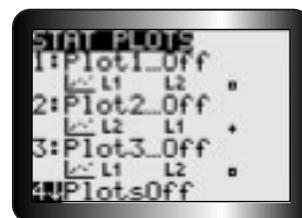
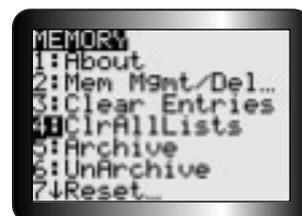
Press **2nd** **Y=** **4** **ENTER**.

**3. Clear all equations in the equation editor.**

Press **Y=**, then press **CLEAR** for each equation.

**4. Set the window so that the axes range from  $-10$  to  $10$ .**

Press **ZOOM** **6**. Press **WINDOW** to verify.

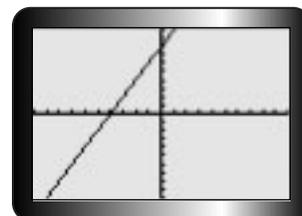


## B-2 Entering and Graphing Functions

Enter the equation of the function into the equation editor. The calculator will display the graph.

**1. Graph.**

To graph  $y = 2x + 8$ , press **Y=** **2** **X,T,θ,n** **+** **8** **GRAPH**. The graph will be displayed as shown.



**2. Enter all linear equations in the form  $y = mx + b$ .**

If  $m$  or  $b$  are fractions, enter them between brackets. For example, enter  $2x + 3y = 7$  in the form  $y = -\frac{2}{3}x + \frac{7}{3}$ , as shown.



**3. Press **GRAPH** to view the graph.**

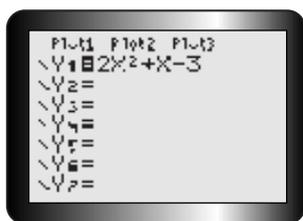
**4. Press **TRACE** to find the coordinates of any point on a graph.**

Use the left and right arrow keys to cursor along the graph.

Press **ZOOM** **8** **ENTER** **TRACE** to trace using integer intervals. If you are working with several graphs at the same time, use the up and down arrow to scroll between graphs.

## B-3 Evaluating a Function

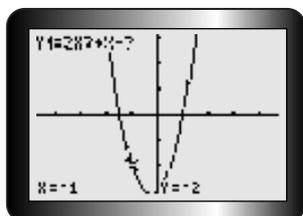
1. Enter the function into the equation editor.



To enter  $y = 2x^2 + x - 3$ , press  $\boxed{Y=}$   $\boxed{2}$   $\boxed{X, T, \theta, n}$   $\boxed{x^2}$   $\boxed{+}$   $\boxed{X, T, \theta, n}$   $\boxed{-}$   $\boxed{3}$ .

2. Use the value operation to evaluate the function.

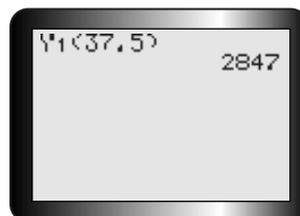
To find the value of the function at  $x = -1$ , press  $\boxed{2nd}$   $\boxed{TRACE}$   $\boxed{ENTER}$ , enter  $\boxed{(-)}$   $\boxed{1}$  at the cursor, then press  $\boxed{ENTER}$ .



3. Use function notation and the Y-VARS operation to evaluate the function.

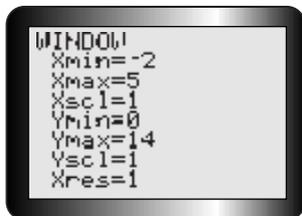
There is another way to evaluate the function, say at  $x = 37.5$ .

Press  $\boxed{CLEAR}$ , then  $\boxed{VAR}$ , then cursor right to **Y-VARS** and press  $\boxed{ENTER}$ . Press  $\boxed{1}$  to select **Y1**. Finally, press  $\boxed{(}$   $\boxed{3}$   $\boxed{7}$   $\boxed{.}$   $\boxed{5}$   $\boxed{)}$ , then  $\boxed{ENTER}$ .



## B-4 Changing Window Settings

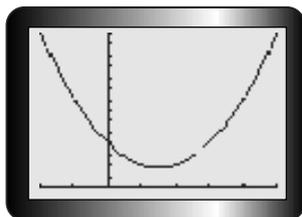
The window settings can be changed to show a graph for a given domain and range.



1. Enter the function  $y = x^2 - 3x + 4$  in the equation editor.
2. Use the WINDOW function to set the domain and range.

To display the function over the domain  $\{x \mid -2 \leq x \leq 5\}$  and range

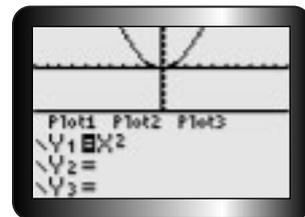
$\{y \mid 0 \leq y \leq 14\}$ , press  $\boxed{WINDOW}$   $\boxed{(-)}$   $\boxed{2}$   $\boxed{ENTER}$ , then  $\boxed{5}$   $\boxed{ENTER}$ , then  $\boxed{1}$   $\boxed{ENTER}$ , then  $\boxed{0}$   $\boxed{ENTER}$ , then  $\boxed{14}$   $\boxed{ENTER}$ , then  $\boxed{1}$   $\boxed{ENTER}$ , and  $\boxed{1}$   $\boxed{ENTER}$ .



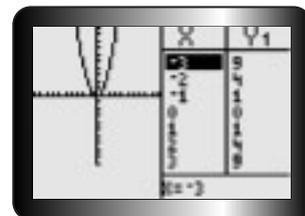
3. Press  $\boxed{GRAPH}$  to show the function with this domain and range.

## B-5 Using the Split Screen

To see a graph and the equation editor at the same time, press **MODE** and cursor to **Horiz**. Press **ENTER** to select this, then press **2nd** **MODE** to return to the home screen. Enter  $y = x^2$  in **Y1** of the equation editor, then press **GRAPH**.

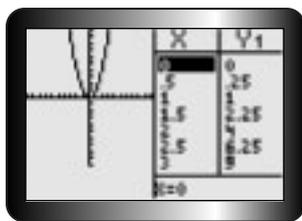


To see a graph and a table at the same time: press **MODE** and cursor to **G-T** (Graph-Table). Press **ENTER** to select this, then press **GRAPH**.



It is possible to view the table with different increments. For example, to see the table start at  $x = 0$  and increase in increments of 0.5, press **2nd** **WINDOW** and adjust the settings as shown.

Press **GRAPH**.



## B-6 Using the TABLE Feature

A function such as  $y = -0.1x^3 + 2x + 3$  can be displayed in a table of values.

### 1. Enter the function into the equation editor.

To enter  $y = -0.1x^3 + 2x + 3$ , press **Y=** **(-)** **.** **1**  
**X, T, θ, n** **^** **3** **+** **2** **X, T, θ, n** **+** **3**.

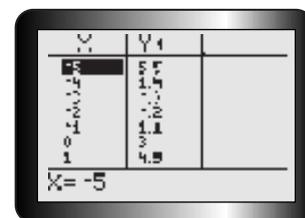
### 2. Set the start point and step size for the table.

Press **2nd** **WINDOW**. The cursor is alongside “TblStart=.” To start at  $x = -5$ , press **(-)** **5** **ENTER**. The cursor is now alongside “ΔTbl=” (Δ, the Greek capital letter delta, stands for “change in.”) To increase the  $x$ -value in steps of 1, press **1** **ENTER**.



### 3. To view the table, press **2nd** **GRAPH**.

Use **▲** and **▼** to move up and down the table. Notice that you can look at higher or lower  $x$ -values than the original range.

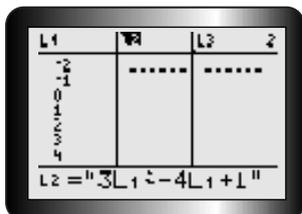


## B-7 Making a Table of Differences

To create a table with the first and second differences for a function, use the STAT lists.

1. Press **STAT** **1** and enter the  $x$ -values into L1.

For the function  $f(x) = 3x^2 - 4x + 1$ , use  $x$ -values from  $-2$  to  $4$ .

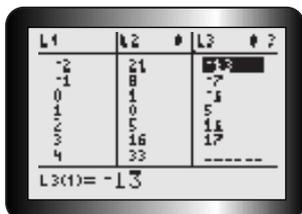


2. Enter the function.

Scroll right and up to select L2. Enter the function  $f(x)$ , using L1 as the

variable  $x$ . Press **ALPHA** **+** **3** **2nd** **1** **x<sup>2</sup>** **-** **4** **2nd** **1** **+** **1** **ALPHA** **+**.

3. Press **ENTER** to display the values of the function in L2.



4. Find the first differences.

Scroll right and up to select L3. Then press **2nd** **STAT**.

Scroll right to **OPS** and press **7** to choose **ΔList(**.

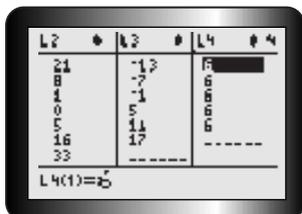
Enter L2 by pressing **2nd** **2** **)**.

Press **ENTER** to see the first differences displayed in L3.

5. Find the second differences.

Scroll right and up to select L4. Repeat step 4, using L3 in place of L2. Press

**ENTER** to see the second differences displayed in L4.



## B-8 Finding the Zeros of a Function

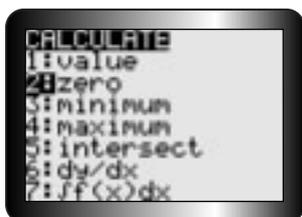
To find the zeros of a function, use the **zero** operation.

1. Start by entering  $y = -(x + 3)(x - 5)$  in the equation editor, then press

**GRAPH** **ZOOM** **6**.

2. Access the zero operation.

Press **2nd** **TRACE** **2**.



3. Use the left and right arrow keys to cursor along the curve to any point to the left of the zero.

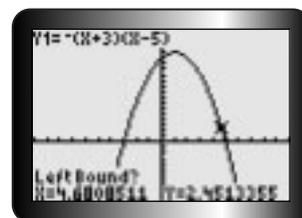
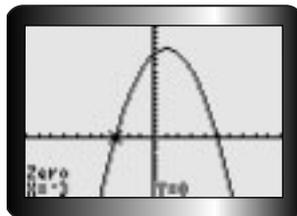
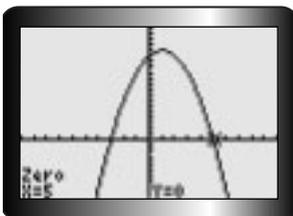
Press **ENTER** to set the left bound.

4. Cursor along the curve to any point to the right of the zero.

Press **ENTER** to set the right bound.

5. Press **ENTER** again to display the coordinates of the zero (the  $x$ -intercept).

6. Repeat to find the second zero.



## B-9 Finding the Maximum or Minimum Values of a Function

The least or greatest value can be found using the **minimum** operation or the **maximum** operation.

1. Enter  $y = -2x^2 - 12x + 30$ .

Graph it and adjust the window as shown. This graph opens downward, so it has a maximum.

2. Use the maximum operation.

Press **2nd** **TRACE** **4**. For parabolas that open upward, press

**2nd** **TRACE** **3** to use the **minimum** operation.

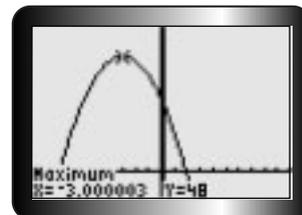
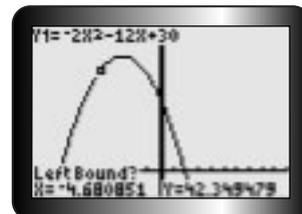
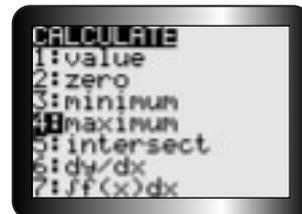
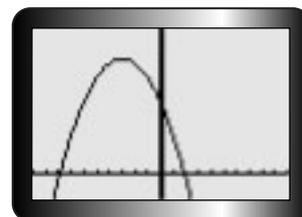
3. Use the left and right arrow keys to cursor along the curve to any point to the left of the maximum value.

Press **ENTER** to set the left bound.

4. Cursor along the curve to any point right of the maximum value.

Press **ENTER** to set the right bound.

5. Press **ENTER** again to display the coordinates of the optimal value.

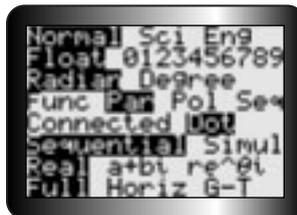


## B-10 Graphing the Inverse of a Function

Parametric equations allow you to graph any function and its inverse. For example, the function  $y = 2 - x^2$  with domain  $x \geq 0$  can be graphed using parametric mode. For a parametric equation, both  $x$  and  $y$  must be expressed in terms of a parameter,  $t$ . Replace  $x$  with  $t$ . Then  $x = t$  and  $y = 2 - t^2$ . The inverse of this function can now be graphed.

1. Clear the calculator and press **MODE**.

Change the setting to the parametric mode by scrolling down to the fourth line and to the right to **Par**, as shown on the screen. Press **ENTER**.



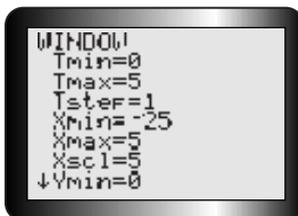
2. Enter the inverse function by swapping the parametric equations  $x = t$ ,  $y = 2 - t^2$  to  $x = 2 - t^2$ ,  $y = t$ .

Press **Y=**. At X1T=, enter **2** **-** **X, T, θ, n** **x<sup>2</sup>**

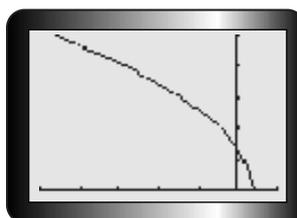
**ENTER**. At Y1T=, enter **X, T, θ, n**.

3. Press **WINDOW**.

The original domain,  $x \geq 0$ , is also the domain of  $t$ . Use window settings such as the ones shown to display the graph.



4. Press **GRAPH** to display the inverse function.



## B-11 Creating Scatter Plots and Determining Lines and Curves of Best Fit Using Regression

This table gives the height of a baseball above ground, from the time it was hit to the time it touched the ground.

Time (s)	0	1	2	3	4	5	6
Height (m)	2	27	42	48	43	29	5

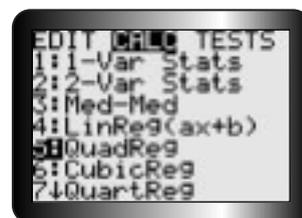
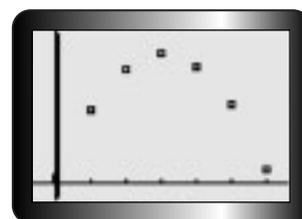
Create a scatter plot of the data.

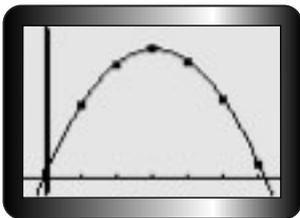
- Enter the data into lists.** To start press **STAT** **ENTER**. Move the cursor over to the first position in **L1** and enter the values for time. Press **ENTER** after each value. Repeat this for height in **L2**.
- Create a scatter plot.** Press **2nd** **Y=** and **1** **ENTER**. Turn on Plot 1 by making sure the cursor is over **On**, the **Type** is set to the graph type you prefer, and **L1** and **L2** appear after **Xlist** and **Ylist**.
- Display the graph.** Press **ZOOM** **9** to activate **ZoomStat**.
- Apply the appropriate regression analysis.** To determine the equation of the line or curve of best fit press **STAT** and scroll over to **CALC**. Press:
  - 4** to enable **LinReg(ax+b)**
  - 5** to enable **QuadReg**.
  - 0** to enable **ExpReg**.
  - ALPHA** **C** to enable **SinReg**.

Press **2nd** **1** **,** **2nd** **2** **,** **VARΣ**. Scroll over to **Y-VARS**. Press **1** twice. This action stores the equation of the line or curve of best fit into **Y1** of the equation editor.

- Display and analyze the results.**

Press **ENTER**. In this case, the letters  $a$ ,  $b$ , and  $c$  are the coefficients of the general quadratic equation  $y = ax^2 + bx + c$  for the curve of best fit.  $R^2$  is the percent of data variation represented by the model. In this case, the equation is about  $y = -4.90x^2 + 29.93x + 1.98$ .



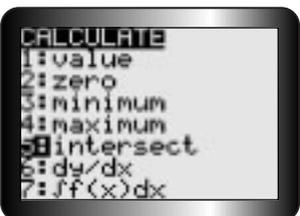


**Note:** In the case of linear regression, if  $r$  is not displayed, turn on the diagnostics function. Press **2nd** **0** and scroll down to **DiagnosticOn**. Press **ENTER** twice. Repeat steps 4 to 6.

**6. Plot the curve.**

Press **GRAPH**

## B–12 Finding the Points of Intersection of Two Functions



**1. Enter both functions into the equation editor.** In this case we will use  $y = 5x + 4$  and  $y = -2x + 18$ .

**2. Graph both functions.** Press **GRAPH**. Adjust the window settings until the point(s) of intersection are displayed.

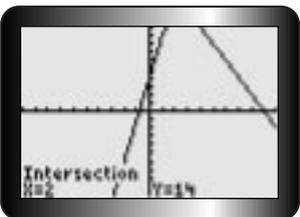
**3. Use the intersect operation.**

Press **2nd** **TRACE** **5**.

**4. Determine a point of intersection.** You will be asked to verify the two curves and enter a guess (optional) for the point of intersection. Press **ENTER** after each screen appears.

The point of intersection is exactly (2, 14).

**5. Determine any additional points of intersection.** Press **TRACE** and move the cursor close to the other point you wish to identify. Repeat step 4.



## B–13 Evaluating Trigonometric Ratios and Finding Angles



**1. Put the calculator in degree mode.**

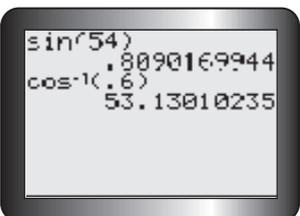
Press **MODE**. Scroll down and across to Degree. Press **ENTER**.

**2. Use the **SIN**, **COS**, or **TAN** key to calculate trigonometric ratios.**

To find the value of  $\sin 54^\circ$ , press **SIN** **5** **4** **)** **ENTER**.

**3. Use  $\text{SIN}^{-1}$ ,  $\text{COS}^{-1}$ , or  $\text{TAN}^{-1}$  to calculate angles.**

To find the angle whose cosine is 0.6, press **2nd** **COS** **.** **6** **)** **ENTER**.



## B-14 Graphing Trigonometric Functions

You can graph trigonometric functions in degree measure using the TI-83 Plus or TI-84 calculators. Graph the function  $y = \sin x$  for  $0^\circ \leq x \leq 360^\circ$ .

1. Put the calculator in degree mode.

Press **MODE**. Scroll down and across to **Degree**. Press **ENTER**.

2. Enter  $y = \sin x$  into the equation editor.

Press **Y=** **SIN** **(X, T,  $\theta$ , n)** **)**.

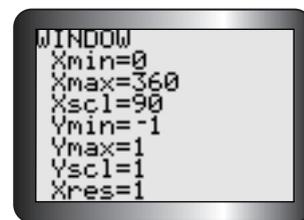
3. Adjust the window to correspond to the given domain.

Press **WINDOW**. Set **Xmin = 0**, **Xmax = 360**, and **Xscl = 90**. These settings display the graph from  $0^\circ$  to  $360^\circ$ , using an interval of  $90^\circ$  on the  $x$ -axis. In this case, set **Ymin = -1** and **Ymax = 1**, since the sine function lies between these values. However, if this fact is not known, this step can be omitted.

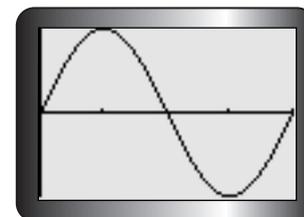
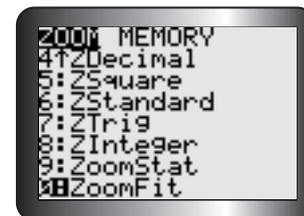
4. Graph the function using ZoomFit.

Press **ZOOM** **0**. The graph is displayed over the domain and the calculator determines the best values to use for **Ymax** and **Ymin** in the display window.

**Note:** You can use **ZoomTrig** (press **ZOOM** **7**) to graph the function in step 4. **ZoomTrig** will always display the graph in a window where **Xmin = -360°**, **Xmax = 360°**, **Ymin = -4**, and **Ymax = 4**.



step 3



step 4

## B-15 Evaluating Powers and Roots

1. Evaluate the power  $(5.3)^2$ .

Press **5** **.** **3** **x<sup>2</sup>** **ENTER**.

2. Evaluate the power  $7^5$ .

Press **7** **^** **5** **ENTER**.

3. Evaluate the power  $8^{-\frac{2}{3}}$ .

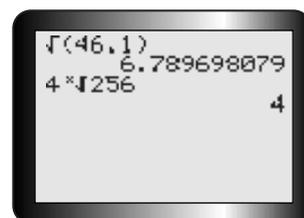
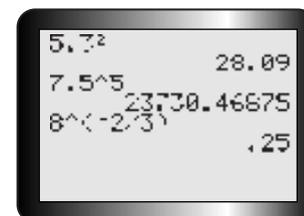
Press **8** **^** **(** **-** **2** **÷** **3** **)** **ENTER**.

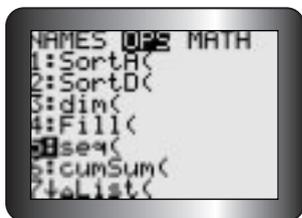
4. Evaluate the square root of 46.1.

Press **2nd** **x<sup>2</sup>** **4** **6** **.** **1** **)** **ENTER**.

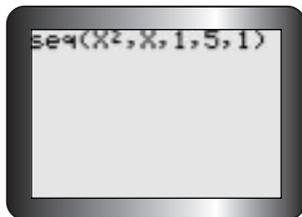
5. Evaluate  $\sqrt[4]{256}$ .

Press **4** **MATH** **5** **2** **5** **6** **ENTER**.

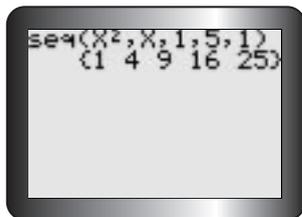




step 1



step 2



step 3

## B-16 Generating the Terms of a Sequence

Generate or list the first five terms of the sequence defined by  $t_n = n^2$ .

1. Select sequence from the List OPS menu.

Press **2nd** **STAT** **▶**. Scroll down to sequence and press **ENTER**.

2. Enter the information for sequence.

You will need to enter the following:

- the expression of the general term
- the variable  $n$  — let **X, T,  $\theta$ , n** represent  $n$
- the first position number
- the last position number
- the increment — the increment is 1, because the difference between each pair of consecutive natural numbers is always 1

Press **X, T,  $\theta$ , n** **x<sup>2</sup>** **,** **X, T,  $\theta$ , n** **,** **1** **,** **5** **,** **1** **)**.

3. Generate the first five terms of the sequence.

Press **ENTER**.

## B-17 Graphing Sequences

### Part 1: Creating a Table and Graphing — The General Term

Using the TI-83 or TI-84 Plus calculator, you can first create a table for a sequence and then a graph for the sequence. You will need the general term.

Create and graph the sequence defined by  $t_n = 3n - 1$ .

1. Change the graphing mode from function to sequence.

The graphing modes are listed on the fourth line of the MODE menu. Press

**MODE** and scroll down and across to **Seq**. Press **ENTER**.

2. Enter the general term into the sequence editor.

Press **Y=**. In this editor,  $u(n)$ ,  $v(n)$ , and  $w(n)$  represent the general terms of sequences. You can change the minimum value of  $n$  (**nMin**). In most cases, you will not need to change the value 1, because, when  $n = 1$ , the first term is generated. Scroll down to  $u(n)$  and position the cursor to the right of the equal sign. Press **3** **X, T,  $\theta$ , n** **-** **1**.



step 1



step 2

### 3. Adjust the window to display the required number of terms.

Press **WINDOW**. The setting **nMin** indicates the smallest  $n$ -value for the calculator to evaluate, while **nMax** indicates the largest  $n$ -value for the calculator to evaluate. **PlotStart=1** means that the graph starts at the first term. **PlotStep=1** means that each consecutive term will be plotted. You can change these settings, but use these window settings for this example.



step 3

### 4. Graph the sequence.

Press **GRAPH**.

### 5. Trace along the graph to identify specific terms of the sequence.

Press **TRACE**. Use **◀** and **▶** to move from point to point. The  $n$ -value, or position, and the  $x$ - and  $y$ -coordinates of each term are displayed at each point. The  $y$ -coordinate represents the value of the term.

### 6. View the terms of the sequence in a table.

- Press **2nd** **WINDOW**. Set **TblStart** to 1 and **ΔTbl** to 1.
- Press **2nd** **GRAPH** to display the table. Use the cursor keys to scroll through the table.

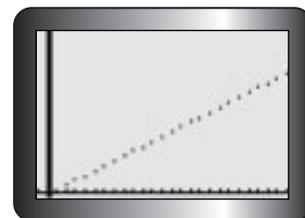


step 6 a)

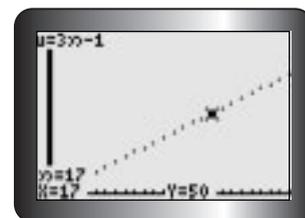
$n$	$u(n)$	
1	2	
2	5	
3	8	
4	11	
5	14	
6	17	
7	20	

$u(n)=2$

step 6 b)



step 4



step 5

**Note:** To see the graph and the table at the same time, use split-screen mode.

Press **MODE**, then scroll down and across to **G-T** (on the last line of the **MODE** menu). Press **ENTER** **GRAPH**.

## Part 2: Creating a Table and Graphing — The Recursive Formula

Using the TI-83 Plus or TI-84 calculator, you can graph recursive sequences in the same way, with one exception: you must specify an initial value or values for  $u(n\text{Min})$  in the sequence editor.

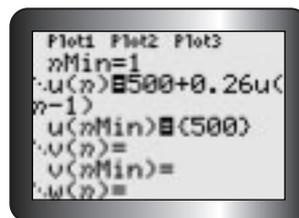
Graph the sequence  $t_1 = 500$ ,  $t_n = 500 + 0.26t_{n-1}$ .

1. Enter the recursive formula in the sequence editor and set the initial value.

Press  $\boxed{\text{Y=}}$ . Then, for the sequence  $u(n)$ , press  $\boxed{5} \boxed{0} \boxed{0}$   
 $\boxed{+} \boxed{0} \boxed{.} \boxed{2} \boxed{6} \boxed{2\text{nd}} \boxed{7} \boxed{(} \boxed{\text{X, T, } \theta, n}$   
 $\boxed{-} \boxed{1} \boxed{)}$ .

Set the initial value to 500. Position the cursor to the right of the equal

sign for  $u(n\text{Min})$  and press  $\boxed{2\text{nd}} \boxed{(} \boxed{5} \boxed{0} \boxed{0} \boxed{2\text{nd}}$   
 $\boxed{)} \boxed{\text{ENTER}}$ .



**Note:** You do not have to enter the braces ( $\boxed{2\text{nd}} \boxed{(}$  and  $\boxed{2\text{nd}} \boxed{)}$ ) around the initial value, or the first term. However, if you were to enter, for example,  $t^1 = 0$  and  $t^2 = 1$ , then you would press  $\boxed{2\text{nd}} \boxed{(}$   
 $\boxed{1} \boxed{,} \boxed{0} \boxed{2\text{nd}} \boxed{)} \boxed{\text{ENTER}}$ .

2. Set the window.

Press  $\boxed{\text{WINDOW}}$  and enter the values shown.



### 3. Draw the graph.

Press **GRAPH**.



### 4. View the terms of the sequence in a table.

Press **2nd** **WINDOW**. Set **TblStart** to 1 and **ΔTbl** to 1.

Press **2nd** **GRAPH** to display the table. Use the cursor keys to scroll through the table.

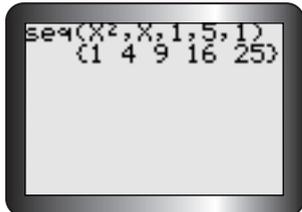
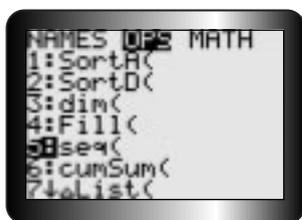
TABLE SETUP	
TblStart=	1
ΔTbl=	1
Indent:	Auto Ask
Depend:	Auto Ask

$n$	$u(n)$
1	500
2	630
3	663.8
4	672.59
5	674.87
6	675.47
7	675.62

$u(n)=500$

## B-18 Finding the Sum and Cumulative Sum of a Series

Find the sum and the cumulative sum of the first five terms of the sequence defined by  $t_n = n^2$ .



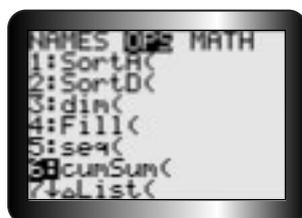
step 1



step 2



step 3



step 2

### Part 1: Finding the Sum of the Terms of a Series

#### 1. Generate the first five terms of the sequence.

- a) Select sequence from the List OPS menu. Press **2nd** **STAT** **▶**.  
Scroll down to **5:seq** and press **ENTER**.

- b) Enter the expression of the general term, the variable, the starting value of the variable, the ending value of the variable, and the increment, 1.  
Press **ENTER** to generate the first five terms, as shown.

#### 2. Select sum from the List MATH menu.

- Press **2nd** **STAT** **▶▶** and scroll down to **5:sum**.  
Press **ENTER**.

#### 3. Find the sum of the series.

- Use **Ans**, last answer, to insert the terms in **sum**.  
Press **2nd** **-** **)** **ENTER**. The sum is displayed.

### Part 2: Finding the Cumulative Sum of the Terms of a Series

The cumulative sum displays the progression of sums of the terms of a series.

#### 1. Generate the terms of the sequence again.

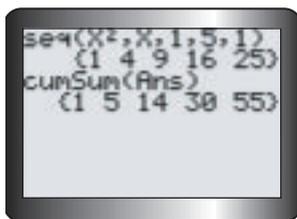
Follow step 1 of Part 1.

#### 2. Select cumulative sum from the LIST OPS menu.

- Press **2nd** **STAT** **▶** and scroll down to **6:cumSum**. Press **ENTER**.

#### 3. Find the cumulative sums of the terms of the sequence.

- Press **2nd** **-** **)** **ENTER**. The cumulative sums are displayed.



step 3

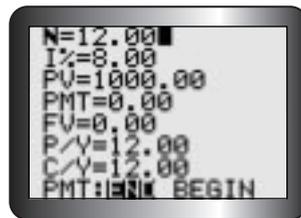
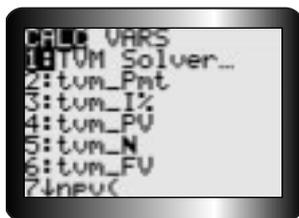
$$\begin{aligned} \leftarrow 1 &= 1 \\ 5 &= 1 + 4 \\ 14 &= 1 + 4 + 9 \\ 30 &= 1 + 4 + 9 + 16 \\ 55 &= 1 + 4 + 9 + 16 + 25 \end{aligned}$$

## B-19 Analyzing Financial Situations Using the TVM Solver

### Part 1: Introducing the TVM Solver

Press **MODE** and change the fixed decimal mode to 2, because most of the values that you are working with here represent dollars and cents. Scroll down to **Float**, across to **2**, and press **ENTER**.

Press **APPS** and then select **1:Finance**. From the Finance CALC menu, select **1:TVM Solver**. The screen that appears should be similar to the second one shown, but the values may be different.



You will notice eight variables on the screen.

- N** total number of payment periods, or the number of interest conversion periods for simple annuities
- I%** annual interest rate as a percent, not as a decimal
- PV** present or discounted value
- PMT** regular payment amount
- FV** future or accumulated value
- P/Y** number of payment periods per year
- C/Y** number of interest conversion periods per year
- PMT** Choose **BEGIN** if the payments are made at the beginning of the payment intervals. Choose **END** if the payments are made at the end of the payment intervals.

You may enter different values for the variables. Enter the value for money that is *paid* as a negative number, since the investment is a cash outflow; enter the value of money that is *received* as a positive number, since the money is a cash inflow. When you enter a whole number, you will see that the calculator adds the decimal and two zeros.

To solve for a variable, move the cursor to that variable and press **ALPHA** **ENTER**, and the calculator will calculate this value. A small shaded box to the left of the line containing the calculated value will appear.

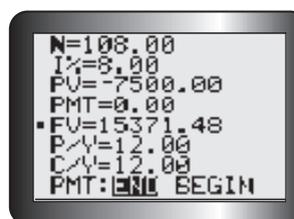
## Part 2: Determining Future Value and Present Value

### EXAMPLE 1

Find the future value or amount of \$7500 invested for nine years at 8%/a, compounded monthly.

#### Solution

The number of interest conversion periods,  $N$ , is  $9 \times 12 = 108$ ,  $I\% = 8$ , and  $PV = -7500$ . The value for present value,  $PV$ , is negative, because the investment represents a cash outflow.  $PMT = 0$  and  $FV = 0$ . The payments per year,  $P/Y$ , and the compounding periods per year,  $C/Y$ , are both 12. Open the **TVM Solver** and enter these values. Scroll to the line containing  $FV$ , the future value, and press **ALPHA** **ENTER**.



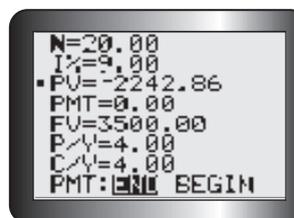
The investment will be worth \$15 371.48 after nine years.

### EXAMPLE 2

Maeve would like to have \$3500 at the end of five years, so she can visit Europe. How much money should she deposit now in a savings account that pays 9%/a, compounded quarterly, to finance her trip?

#### Solution

Open the **TVM Solver** and enter the values shown in the screen, except the value for  $PV$ . The value for  $FV$  is positive, because the future value of the investment will be “paid” to Maeve, representing a cash inflow. Scroll to the line containing  $PV$  and press **ALPHA** **ENTER** to get  $-\$2242.86$ . The solution for  $PV$  is negative, because Maeve must pay this money and the payment is a cash outflow.



## Part 3: Determining the Future or Accumulated Value of an Ordinary Simple Annuity

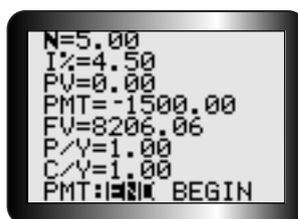
### EXAMPLE 3

Celia deposits \$1500 at the end of each year in a savings account that pays 4.5%/a, compounded annually. What will be the balance in the account after five years?

#### Solution

$N = 5$  and  $I\% = 4.5$ . Because there is no money in the account at the beginning of the term,  $PV = 0$ .  $PMT = -1500$ . The payment,  $PMT$ , is negative, because Celia makes a payment, which is a cash outflow.  $P/Y = 1$  and  $C/Y = 1$ . Open the **TVM Solver** and enter these values.

Scroll to the line containing **FV** and press **ALPHA** **ENTER**.



The balance in Celia's account at the end of the year will be \$8206.06.

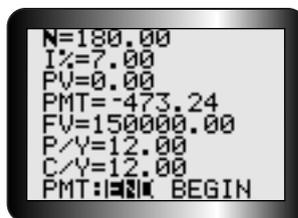
### EXAMPLE 4

Mr. Bartolluci would like to have \$150 000 in his account when he retires in 15 years. How much should he deposit at the end of each month in an account that pays 7%/a, compounded monthly?

#### Solution

Open the **TVM Solver** and enter the values shown, except for **PMT**. Note that  $N = 12 \times 15 = 180$ , and the future value, **FV**, is positive, since he will receive the money at some future time. Scroll to the line containing **PMT** and press

**ALPHA** **ENTER**.



Mr. Bartolluci must deposit \$473.24 at the end of each month for 15 years. The payment appears as  $-473.24$ , because it is a cash outflow.







- b) Use  $\Sigma\text{Int}(A, B, \text{roundvalue})$  to calculate the total interest that Eleanor will pay.

Press **2nd** **MODE** to return to the home screen. Press **APPS** and select **1:Finance** from the Finance CALC menu. Select  $\Sigma\text{Int}$  by scrolling down or by pressing **ALPHA** **MATH**.

Press **ENTER**.

Press **1** **,** **6** **0** **,** **6** **)** **ENTER**.

The sum of the interest paid from the first period to the 60th period is calculated.

By the end of the loan, Eleanor will have paid \$7129.71 in total interest. Eleanor will have paid  $\$7129.71 + \$18\,000 = \$25\,129.71$  in interest and principal for the truck. Note that the product of the payment, \$418.83, and the total number of payments, 60, is \$25 129.80. The difference of \$0.09 is due to rounding, because 418.828 515 3 was rounded to 418.83.



- c) Find the balance on the loan after the 30th payment. *roundvalue* must be consistent, that is, 6.

From the Finance CALC menu, select **bal** by scrolling or by pressing **9**.

Press **3** **0** **,** **6** **)** **ENTER**.

Eleanor still owes \$10 550.27 after the 30th payment. (Why is this amount not \$9000?)

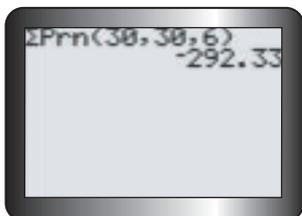


- d) Find the portion of the 30th payment that reduces the principal by calculating the sum of the principal paid from the 30th payment to the 30th payment. In the words, you are calculating the sum of only one item, the 30th payment. *roundvalue* is again 6. From the Finance CALC menu,

select  $\Sigma\text{Prn}$  by scrolling down or by pressing **0**. Press **3** **0**

**,** **3** **0** **,** **6** **)** **ENTER**.

The portion of the 30th payment that reduces the principal is \$292.33. The other portion of this payment, \$126.50, is interest.



## Part 3: Using the Finance Functions to Create Repayment Schedules

Use the functions described in parts 1 and 2 to create repayment schedules or amortization tables.

### EXAMPLE 2

Recall that Eleanor borrows \$18 000 to purchase a pickup truck. She will repay the loan with monthly payments. The term of the loan is five years. The interest rate is 14%/a compounded monthly.

- What will be the monthly outstanding balance on the loan after each of the first seven months?
- Create a repayment schedule for the first seven months of the loan.
- Use the repayment schedule to verify that the loan is completely paid after five years or 60 payments.

### Solution

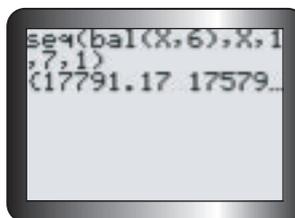
- Find the outstanding balance after each payment for the first seven months. You will combine **sequence** (List OPS menu) and **bal**.

From the home screen, press **2nd** **STAT** **▶** **5** to select sequence.

Then press **APPS** **ENTER** **9** to select **bal**.

Press **X, T,  $\theta$ , n** **,** **6** **)** **,** **X, T,  $\theta$ , n** **,** **1** **,** **7** **,** **1** **)**.

Press **ENTER** to calculate the sequence of balances, beginning with the first month, 1, and ending with the last month, 7. The increment for this sequence is 1, which is the last value entered. Recall that the increment is the change from payment number to payment number. Scroll right (**▶**) to see the other balances.

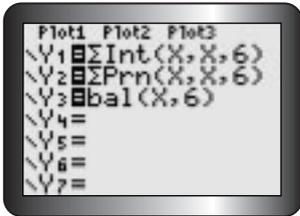




- b) Create a repayment, or an amortization, schedule for the first seven months by comparing the interest, principal, and balance in a table.

Begin by opening the equation editor. Press  $\boxed{Y=}$ . Clear **Y1** to **Y3**, if necessary. Store the interest portion of each payment in **Y1**. Move

the cursor to the right of **Y1** =. Press  $\boxed{\text{APPS}}$   $\boxed{\text{ENTER}}$   $\boxed{\text{ALPHA}}$   $\boxed{\text{MATH}}$  to select  $\Sigma\text{Int}$ . Press  $\boxed{X, T, \theta, n}$   $\boxed{,}$   $\boxed{X, T, \theta, n}$   $\boxed{,}$   $\boxed{6}$   $\boxed{)}$   $\boxed{\text{ENTER}}$ .



To store the principal portion of each payment in **Y2**, press  $\boxed{\text{APPS}}$

$\boxed{\text{ENTER}}$   $\boxed{0}$  to select  $\Sigma\text{Prn}$ . Press  $\boxed{X, T, \theta, n}$   $\boxed{,}$   $\boxed{X, T, \theta, n}$   $\boxed{,}$   $\boxed{6}$   $\boxed{)}$   $\boxed{\text{ENTER}}$ .

To store the outstanding balance after each payment in **Y3**, press  $\boxed{\text{APPS}}$

$\boxed{\text{ENTER}}$   $\boxed{9}$  to select **bal**. Press  $\boxed{X, T, \theta, n}$   $\boxed{,}$   $\boxed{6}$   $\boxed{)}$   $\boxed{\text{ENTER}}$ .

Before viewing the table, press  $\boxed{2\text{nd}}$   $\boxed{\text{WINDOW}}$ . Set **TblStart** to 1 and **ΔTbl** to 1. The table will start with payment 0 and the payment number will increase by 1 at each step.



X	Y1	Y2
1.00	-210.0	-208.8
2.00	-207.6	-211.3
3.00	-205.1	-213.7
4.00	-202.6	-216.2
5.00	-200.1	-218.7
6.00	-197.6	-221.1
7.00	-194.9	-223.6

Y2=-223.87956

Press  $\boxed{2\text{nd}}$   $\boxed{\text{GRAPH}}$  to see the amortization table. Notice that the interest portion and the principal portion of each payment appear as negative values. Each payment, which is a combination of interest and principal, is a cash outflow for Eleanor.

X	Y2	Y3
1.00	-208.8	17781
2.00	-211.3	17580
3.00	-213.7	17386
4.00	-216.2	17190
5.00	-218.7	16991
6.00	-221.1	16790
7.00	-223.6	16588

Y3=16486.030885

Scroll right to see the values for **Y3**, the outstanding balance.

The outstanding balance after seven payments is \$16 486.03.

X	Y2	Y3
54.00	-386.2	2413.5
55.00	-390.7	2022.8
56.00	-395.2	1627.6
57.00	-399.8	1227.7
58.00	-404.5	823.2
59.00	-409.2	414.0
60.00	-414.0	0.00025

Y3=2.5E-5

- c) Scroll or reset the table's start value to see other entries in the amortization table. Scroll up to the beginning of the table. Notice that a substantial portion of the \$418.83 payment is interest. Scroll down the table. At the end of the amortization, none of the payment is applied to the principal. The final outstanding balance is  $2.5E^{-5}$ . As a decimal, this value is \$0.000 25. Therefore, the amortization period of 60 payments is correct. The loan will be paid completely after five years or 60 payments.

## PART 2 USING A SPREADSHEET

### B–21 Introduction to Spreadsheets

A spreadsheet is a computer program that can be used to create a table of values and then graph the values. It is made up of cells that are identified by column letter and row number, such as A2 or B5. A cell can hold a label, a number, or a formula.

#### Creating a Table

Use spreadsheets to solve problems like this:

How long will it take to double your money if you invest \$1000 at 5%/a compounded quarterly?

To create a spreadsheet, label cell A1 as Number of Quarters, Cell B1 as Time (years), and cell C1 as Amount (\$). Enter the initial values of 0 in A2, 0 in B2, and 1000 in C2. Enter the formulas  $=A2+1$  in A3,  $=A3/4$  in B3, and  $=1000*(1.0125)^{A3}$  in C3 to generate the next values in the table.

	A	B	C
1	Number of Quarters	Time (years)	Amount (\$)
2	0	0	1000
3	$=A2+1$	$=A3/4$	$=1000*(1.0125^{A3})$
4			

Notice that an equal sign is in front of each formula, an asterisk (\*) is used for multiplication, and a caret (^) is used for exponents. Next, use the cursor to select cell A3 to C3 and several rows of cells below them. Use the **Fill Down** command. This command inserts the appropriate formula into each selected cell.

	A	B	C
1	Number of Quarters	Time (years)	Amount (\$)
2	0	0	1000
3	1	0.25	
4			

When the **Fill Down** command is used, the computer automatically calculates and enters the values in each cell, as shown below in the screen on the left.

Continue to select the cells in the last row of the table and use the **Fill Down** command to generate more values until the solution appears, as shown below in the screen on the right.

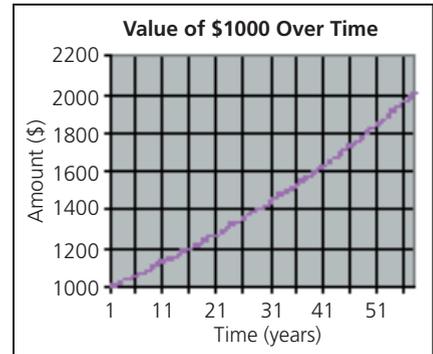
	A	B	C
1	Number of Quarters	Time (years)	Amount (\$)
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.16
5	3	0.75	1037.97
6	4	1	1050.94

	A	B	C
1	Number of Quarters	Time (years)	Amount (\$)
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.1563
⋮	⋮	⋮	⋮
56	54	13.5	1955.8328
57	55	13.75	1980.2807
58	56	14	2005.0342

## Creating a Graph

Use the spreadsheet's graphing command to graph the results. Use the cursor to highlight the portion of the table you would like to graph. In this case, Time versus Amount.

	A	B	C
1	<b>Number of Quarters</b>	<b>Time (years)</b>	<b>Amount (\$)</b>
2	0	0	1000
3	1	0.25	1012.50
4	2	0.5	1025.1563
⋮	⋮	⋮	⋮
56	54	13.5	1955.8328
57	55	13.75	1980.2807
58	56	14	2005.0342



Different spreadsheets have different graphing commands. Check your spreadsheet's instructions to find the proper command. This graph appears above on the right.

## Determining the Equation of the Curve of Best Fit

Different spreadsheets have different commands for finding the equation of the curve of best fit using regression. Check your spreadsheet's instructions to find the proper command for the type of regression that suits the data.

## B–22 Creating an Amortization Table

You have decided to purchase \$4000 in furniture for your apartment. Your monthly payment is \$520 and the interest rate 12%/a, compounded monthly. A spreadsheet can create a table of the outstanding balance and the graph.

### Create a Table of the Outstanding Balance

*You may find that the spreadsheet software that you are using may have different commands or techniques for achieving the same results.*

1. Create a spreadsheet with five columns, labelled Payment Number, Payment, Interest Paid, Principal Paid and Outstanding Balance in cells A1, B1, C1, D1 and E1, respectively.
2. In cells A2 and E2, enter 0 and 4000, respectively. Leave cell B2, C2, and D2 empty.
3. In cell A3, enter the expression =A2+1 and press **ENTER**. In cell B3, enter 520. In cell C3, enter the expression for interest, =E2\*0.01 and press **ENTER**. In cell D3, enter the expression =B3–C3 and press **ENTER**. In cell E3 enter = E2–D3 and press **ENTER**.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0				4000
3	=A2 + 1	520	=E2*0.01	=B3 – C3	=E2 – D3

4. Select cells A3 across to E3 and down to E11. Use the Fill Down command to complete the table. You can display financial data in different ways. Select cells B3 to E11 and click the \$ button or icon. Each number will appear with a dollar sign, a comma, and will be rounded to two decimal places.

	A	B	C	D	E
1	Payment Number	Payment	Interest Paid	Principal Paid	Outstanding Balance
2	0				\$4,000.00
3	1	\$520.00	\$40.00	\$480.00	\$3,520.00
4	2	\$520.00	\$35.20	\$484.80	\$3,035.20
5	3	\$520.00	\$30.35	\$489.65	\$2,545.55
6	4	\$520.00	\$25.46	\$494.54	\$2,051.01
7	5	\$520.00	\$20.51	\$499.49	\$1,551.52
8	6	\$520.00	\$15.52	\$504.48	\$1047.03
9	7	\$520.00	\$10.47	\$509.53	\$537.50
10	8	\$520.00	\$5.38	\$514.62	\$22.88
11	9	\$520.00	\$0.23	\$519.77	\$(496.89)

## PART 3 USING THE GEOMETER'S SKETCHPAD

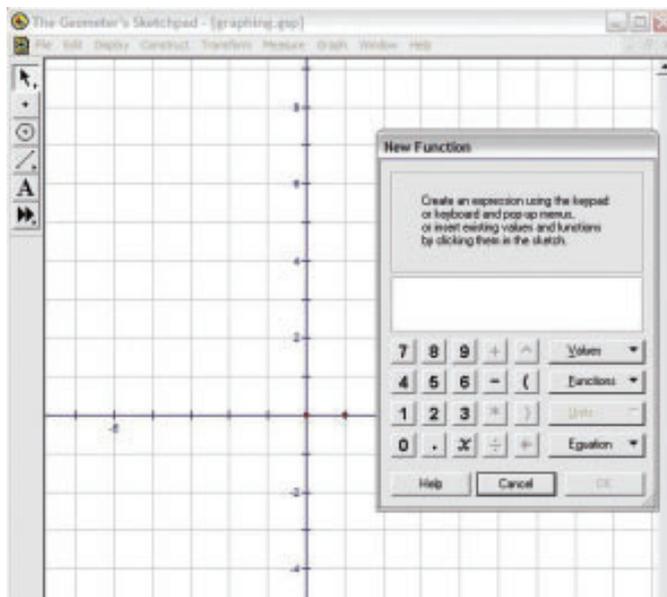
### B-23 Graphing Functions

**1. Turn on the grid.**

From the **Graph** menu, choose **Show Grid**.

**2. Enter the function.** From the **Graph** menu, choose **Plot New Function**.

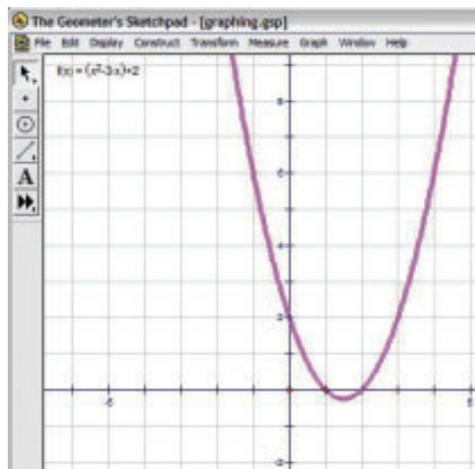
The function calculator should appear.



**3. Graph the function  $y = x^2 - 3x + 2$ .**

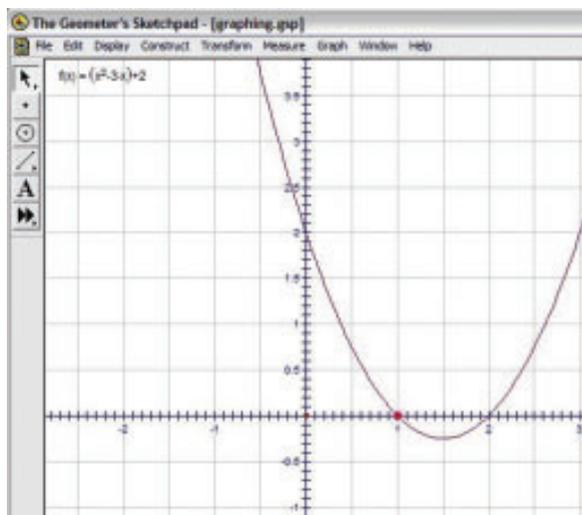
Use either the calculator keypad or the keyboard to enter " $x^2 - 3x + 2$ ".

Then press the "OK" button on the calculator keypad. The graph of  $y = x^2 - 3x + 2$  should appear on the grid.



#### 4. Adjust the origin and/or scale.

To adjust the origin, left-click on the point at the origin to select it. Then left-click and drag the origin as desired. To adjust the scale, left-click in blank space to deselect, then left-click on the point at (1, 0) to select it. Left-click and drag this point to change the scale.



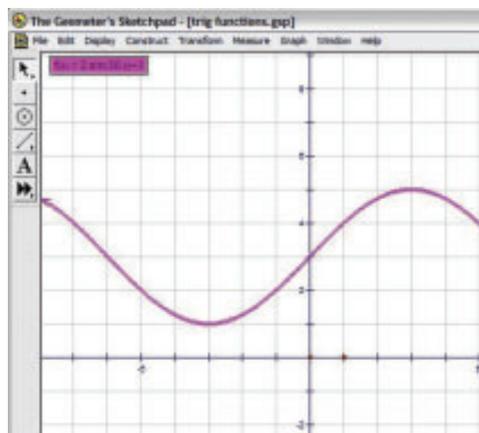
## B–24 Graphing Trigonometric Functions

### 1. Turn on the grid.

From the **Graph** menu, choose **Show Grid**.

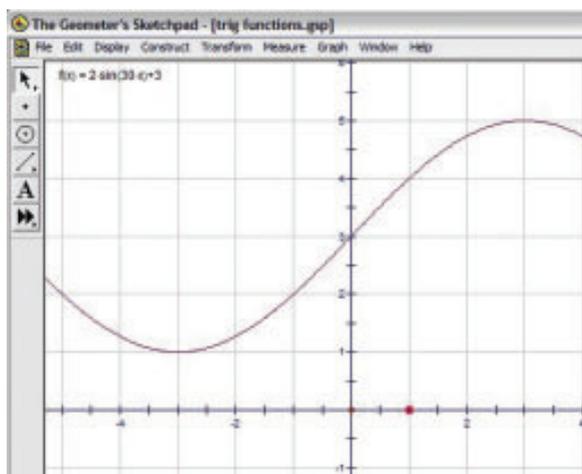
### 2. Graph the function $y = 2 \sin(30x) + 3$ .

- From the **Graph** menu, choose **Plot New Function**. The function calculator should appear.
- Use either the calculator keypad or the keyboard to enter “ $2 * \sin(30 * x) + 3$ ”. To enter “sin,” use the pull-down “Functions” menu on the calculator keypad.
- Click on the “OK” button on the calculator keypad.
- Click on “No” in the pop-up panel to keep degrees as the angle unit. The graph of  $y = 2 \sin(30x) + 3$  should appear on the grid.



### 3. Adjust the origin and/or scale.

Left-click on and drag either the origin or the point (1, 0).



## B–25 Creating Geometric Figures

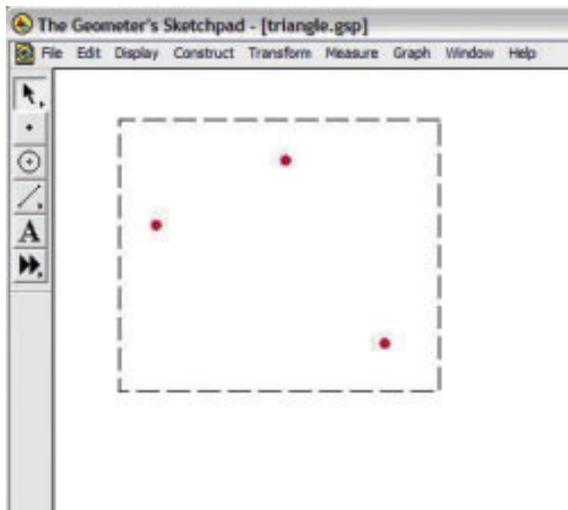
To draw a triangle, follow these steps.

**1. Plot the vertices of the figure.**

Choose the **Point** tool .

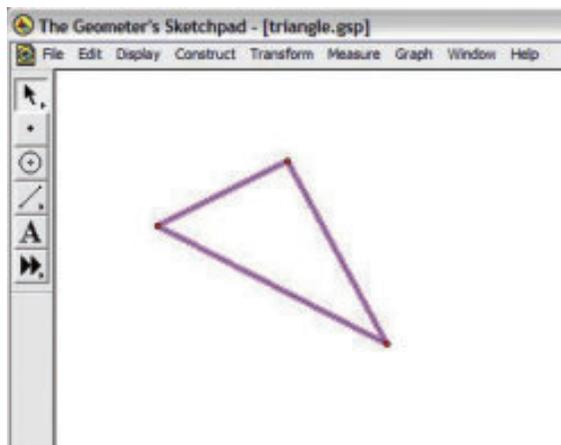
**2. Select the vertices.**

Choose the **Arrow Selection** tool . Left-click and drag around the three vertices to select them.



**3. Draw the sides of the figure.**

From the **Construct** menu, choose **Segments**. The sides of the triangle should appear.



## B-26 Measuring Sides and Angles, and Using the Calculator

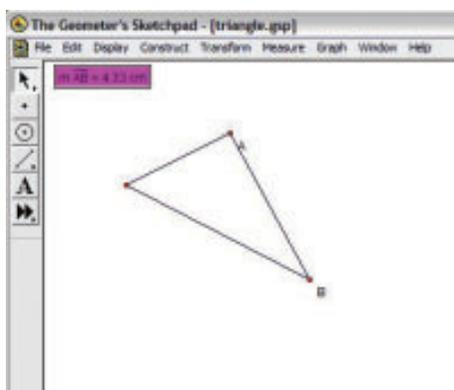
As an example, draw a triangle and verify that the sine law holds for it.

### 1. Draw a triangle.

See “Creating Geometric Figures” (B-25).

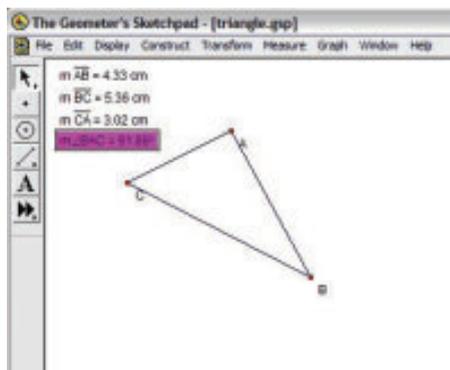
### 2. Find the measure of each side.

- Choose the **Arrow Selection**  tool.
- Left-click on any blank area to deselect.
- Left-click on a side to select it.
- From the **Measure** menu, choose **Length**.
- Repeat for the other two sides.

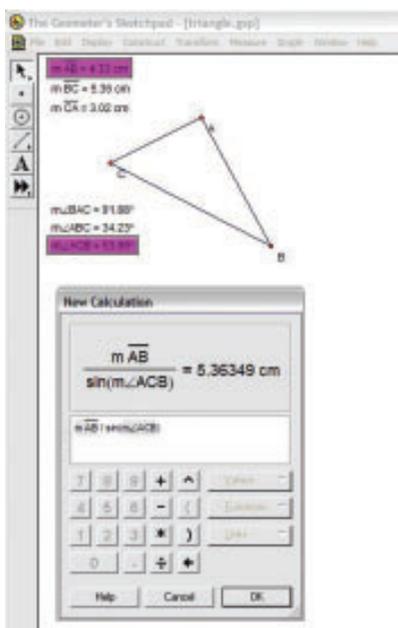


### 3. Find the measure of each angle.

- Left-click on any blank area to deselect.
- To measure  $BAC$ , select the vertices *in that order*: first  $B$ , then  $A$ , then  $C$ .
- From the **Measure** menu, choose **Angle**.
- Repeat for the other two angles.



4. For side  $AB$  and the opposite  $ACB$ , calculate the ratio  $\frac{AB}{\sin ACB}$ .
- From the Measure menu, choose Calculate.... The calculator keypad should appear.
  - Left-click on the measure “ $m\overline{AB}$ ” to select it.
  - Use the calculator keypad to enter “ $\div$ ”.
  - Use the “Functions” menu on the keypad to select “sin”.
  - Select the measure “ $m\angle ACB$ ”.
  - Press the “OK” button on the calculator keypad. The ratio should appear as a new measure.
5. Repeat step 4 for the other two pairs of corresponding sides and angles.

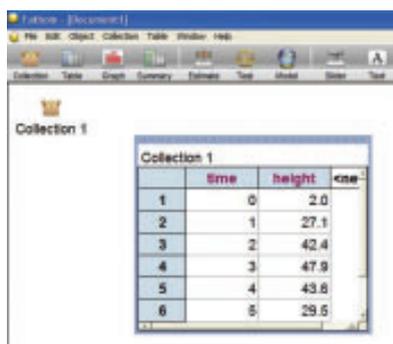


## PART 4 USING FATHOM

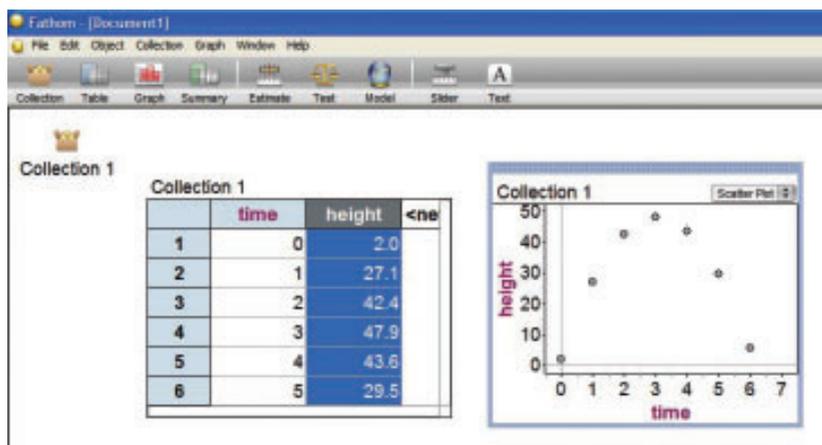
### B-27 Creating a Scatter Plot and Determining the Equation of a Line or Curve of Good Fit



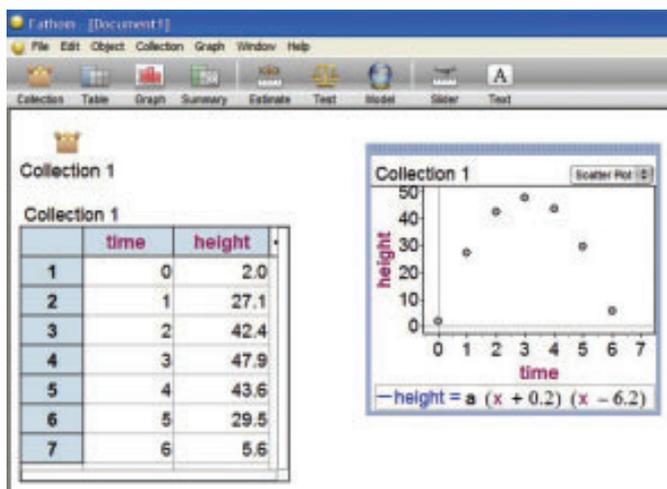
1. **Create a case table.** Drag a case table from the object shelf, and drop it in the document.
2. **Enter the Variables and Data.** Click <new>, type a name for the new variable or attribute and press Enter. (If necessary, repeat this step to add more attributes; pressing Tab instead of Enter moves you to the next column.) When you name your first attribute, Fathom creates an empty collection to hold your data (a little, empty box). The collection is where your data are actually stored. Deleting the collection deletes your data. When you add cases by typing values, the collection icon fills with gold balls. To enter the data click in the blank cell under the attribute name and begin typing values. (Press Tab to move from cell to cell.)



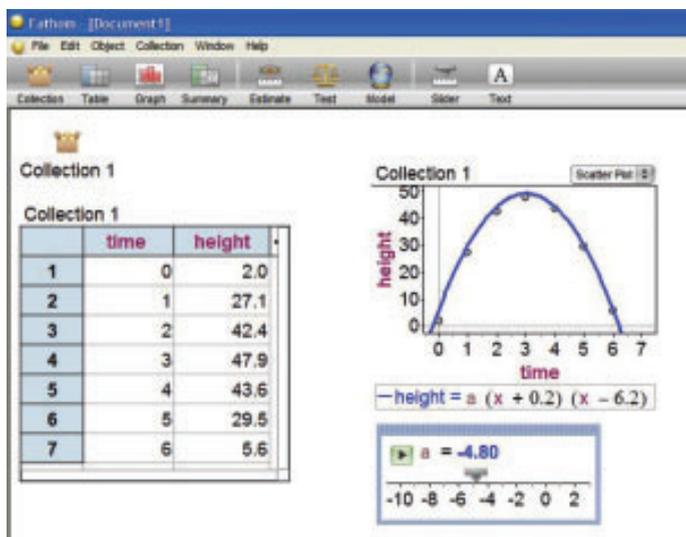
3. **Graph the data.** Drag a new graph from the object shelf at the top of the Fathom window, and drop it in a blank space in your document. Drag an attribute from the case table, and drop it on the prompt below and/or to the left of the appropriate axis in the graph.



4. **Create a function.** Right click the graph and select **Plot Function**. Enter your function using a parameter that can be adjusted to fit the curve to the scatter plot. (a was used in this case).



5. **Create a slider for the parameter(s) in your equation.** Drag a blank slider from the object shelf at the top of the Fathom window, and drop it in a blank space below your graph. Type in the letter of the parameter used in your function in step 4 over V1. Click on the number then adjust the value of the slider until you are satisfied with the fit.



The equation of a curve of good fit is  $y = -4.8(x + 0.2)(x - 6.2)$ .