Chapter 1: Introduction to Functions

Getting Started, p. 2

1. a) $3(x + y) - 5(x - y)$
$= 3x + 3y - 5x + 5y$
$= -2x + 8y$

b) $(4x - y)(4x + y)$
$= 16x^2 - y^2$

c) $\frac{1}{2}(x^2 + 1) - \frac{3}{2}(x^2 - 1)$
$= \frac{1}{2}x^2 + \frac{1}{2} - \frac{3}{2}x^2 + \frac{3}{2}$
$= -x^2 + 2$

d) $4x(x + 2) - 2x(x - 4)$
$= 4x^2 + 8x - 2x^2 + 8x$
$= 2x^2 + 16x$

2. a) $3(3 + (-5)) - 5(3 - (-5))$
$= 3(-2) - 5(8)$
$= -46$

b) $16(3)^2 - (-5)^2 = 119$

c) $-(3)^2 + 2 = -7$

d) $2(3)^2 + 16(3) = 66$

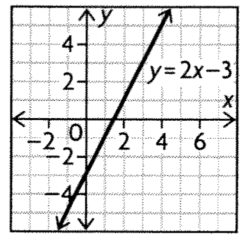
3. a) $5x - 8 = 7$
$5x = 15$
$x = 3$

b) $-2(x - 3) = 2(1 - 2x)$
$-2x + 6 = 2 - 4x$
$2x = -4$
$x = -2$

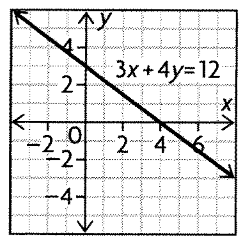
c) $\frac{5}{6}y - \frac{3}{4}y = -3$
$\frac{1}{12}y = -3$
$y = -36$

d) $\frac{x - 2}{4} = \frac{2x + 1}{3}$
$3x - 6 = 8x + 4$
$-5x = 10$
$x = -2$

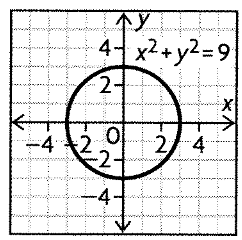
4. a)



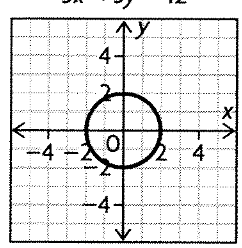
b)



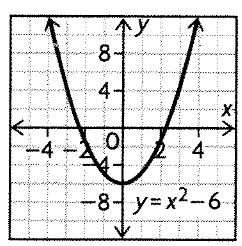
5. a)



b) $3x^2 + 3y^2 = 12$

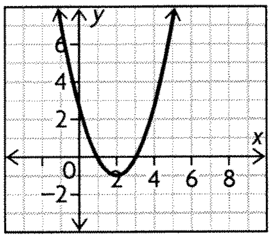


6. a)



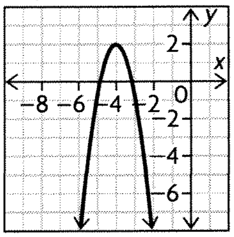
Axis of symmetry: $x = 0$; Vertex: $(0, -6)$

b) $y = (x-2)^2 - 1$



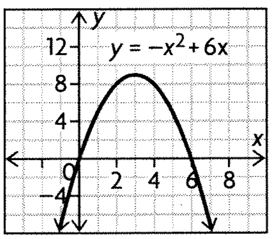
Axis of symmetry: $x = 2$; Vertex: $(2, -1)$

c) $y = -3(x+4)^2 + 2$



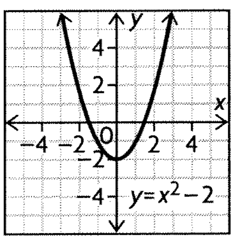
Axis of symmetry: $x = -4$; Vertex: $(-4, 2)$

d)



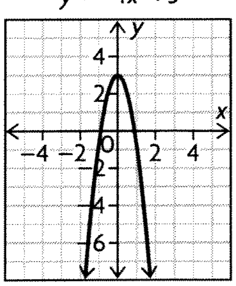
Axis of symmetry: $x = 3$; Vertex: $(3, 9)$

7. a) Translate down 2 units



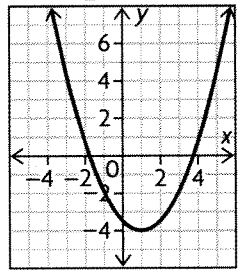
b) Reflect about x-axis, then vertical stretch, scale factor 4, then translate up 3 units.

$y = -4x^2 + 3$



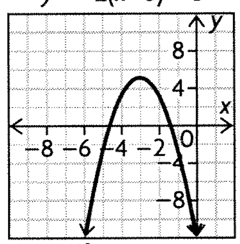
c) Vertical compression, scale factor 2, then translate right 1 unit and down 4 units.

$y = \frac{1}{2}(x-1)^2 - 4$



d) Reflect in x-axis, then vertical stretch, scale factor 2, then translate left 3 units and up 5 units.

$y = -2(x+3)^2 + 5$



8. a) $x^2 - 5x + 6 = 0$

$(x-2)(x-3) = 0$

$x = 2$ or $x = 3$

b) $3x^2 - 5 = 70$

$3x^2 = 75$

$x^2 = 25$

$x = \pm 5$

9. Some similarities among these relations are that none of the relations involve powers higher than 2. Some differences among these relations are that linear and quadratic relations are functions, while circles are not.

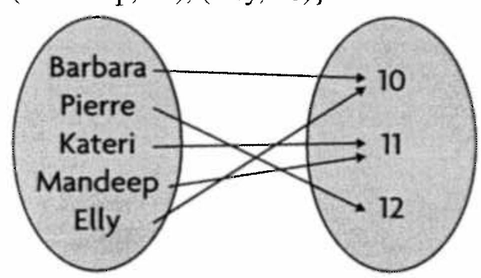
Property	Linear Relations	Circles	Quadratic Relations
Equation(s)	$y = mx + c$ or $Ax + By = C$	$(x - h)^2 + (y - k)^2 = r^2$	$y = ax^2 + bx + c$ or $y = a(x - b)^2 + k$
Shape of graph	Straight line	Circle	Parabola
Number of quadrants graph enters	2 or 3	1, 2, 3, or 4	1, 2, 3, or 4
Descriptive features of graph	Slope is constant; crosses each axis at most once	Graph has upper and lower parts	Graph has a single lowest or highest point (vartex); crosses y-axis once, x-axis 0, 1, or 2 times
Types of problems modelled by the relation	Direct and partial variation	Constant distance from a point	Some economic functions; motion of a Projected area

1.1 Relations and Functions, pp. 10–12

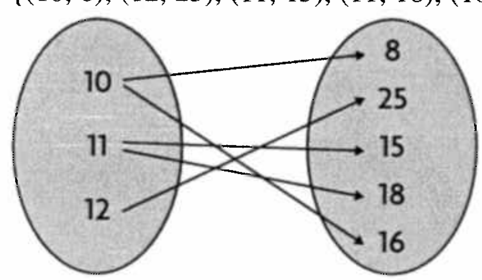
1. a) Function; each x-value has only one y-value
b) Not a function; for $x = 1$, $y = -3$ and 0
c) Not a Function; for $x = 0$, $y = 4$ and 1
d) Function; each x-value has only one y-value
2. a) Not a function; the vertical line at $x = 1$ intersects the graph at three points
b) Not a function; any vertical line with x greater than -4 intersects the graph at two points
c) Function; any vertical line intersects the graph at only one point
d) Not a function; every vertical line with x between -5 and 1 intersects the graph at two points
e) Function; every vertical line intersects the graph at only one point
f) Not a function; any vertical line with x less than 3 intersects the graph at two points
3. In the first equation, the substitution gives
$y = 6^2 - 5(6) = 6$
In the second, the substitution gives

$$\begin{aligned} 6 &= y^2 - 5y \\ 0 &= y^2 - 5y - 6 \\ y &= \frac{5 \pm \sqrt{5^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{5 \pm 7}{2} \\ &= \frac{12}{2} \text{ or } \frac{-2}{2} \\ &= 6 \text{ or } -1 \end{aligned}$$

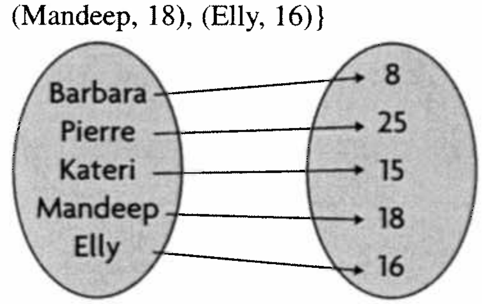
The first equation is a function because a single x-value gives a single y-value. The second equation is not because a single x-value gives a quadratic equation in y, which may have two solutions, as the example of $x = 6$ demonstrated.
4. a) {(Barbara, 10), (Pierre, 12), (Kateri, 11), (Mandeep, 11), (Elly, 10)}



{(10, 8), (12, 25), (11, 15), (11, 18), (10, 16)}

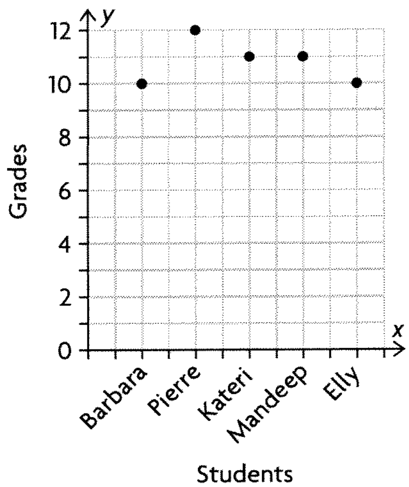


{(Barbara, 8), (Pierre, 25), (Kateri, 15), (Mandeep, 18), (Elly, 16)}

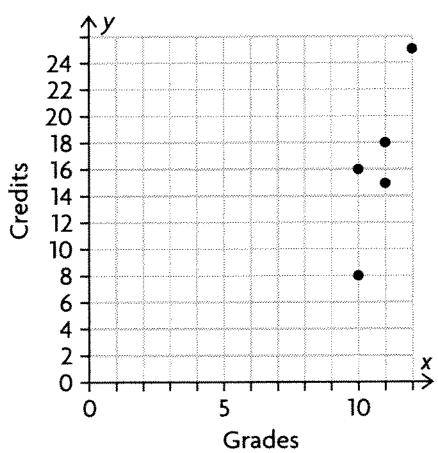


b) Students, Grades: domain = {Barbara, Pierre, Kateri, Mandeep, Elly}, range = {10, 11, 12}
Grades, Credits: domain = {10, 11, 12}, range = {8, 15, 16, 18, 25}
Students, Credits: domain = {Barbara, Pierre, Kateri, Mandeep, Elly}, range = {8, 15, 16, 18, 25}
c) Only grades–credits relation is not a function; it has repeated range values for single domain values.

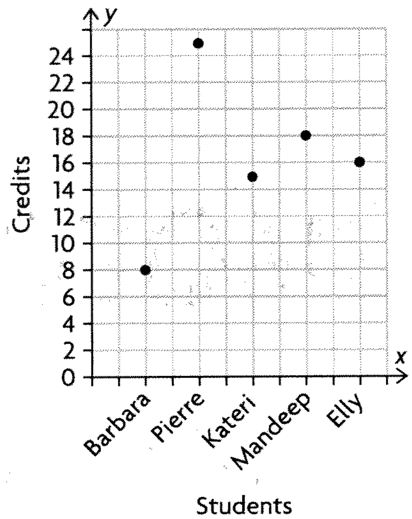
5. Students, Grades:



Grades, Credits:



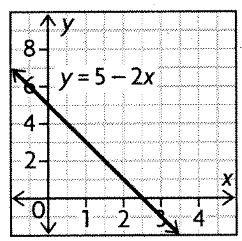
Students, Credits:



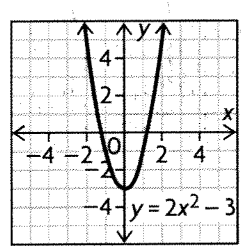
The graphs confirm that only the grades, credits relation fails to be a function because it is the only graph that fails the vertical line test.

6. $y = 3$: This is the graph of a horizontal line. It is a function because it passes the vertical line test (for any x-value, there is only one y-value; that is, $y = 3$). $x = 3$: This is the graph of a vertical line. It is not a function because it fails the vertical line test (at $x = 3$, one x value has an infinite number of y values.).

7. a) This relation is linear, so it is a function.

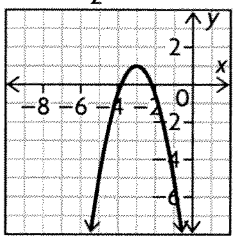


b) This relation is quadratic in x, so it is a function.



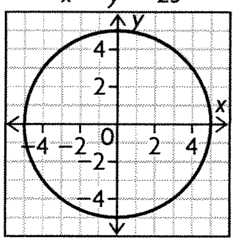
c) This relation is quadratic in x, so it is a function.

$y = -\frac{3}{2}(x + 3)^2 + 1$



d) This relation is a circle, so it is not a function.

$x^2 + y^2 = 25$



8. a) i) Substituting 0 for x gives

$$3(0) + 4y = 5$$
$$4y = 5$$
$$y = 1.25$$

Substituting -2 for x gives

$$3(-2) + 4y = 5$$
$$4y = 11$$
$$y = 2.75$$

ii) Substituting 0 for x gives

$$0^2 + y^2 = 4$$
$$y^2 = 4$$
$$y = \pm 2$$

Substituting -2 for x gives

$$(-2)^2 + y^2 = 4$$
$$y^2 = 0$$
$$y = 0$$

iii) Substituting 0 for x gives

$$0^2 + y = 2$$
$$y = 2$$

Substituting -2 for x gives

$$(-2)^2 + y = 2$$
$$y = -2$$

iv) Substituting 0 for x gives

$$0 + y^2 = 0$$
$$y^2 = 0$$
$$y = 0$$

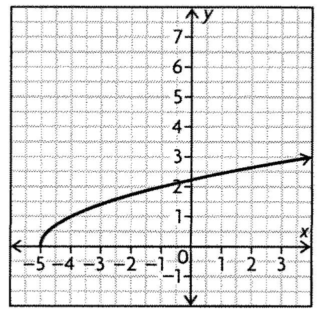
Substituting -2 for x gives

$$-2 + y^2 = 0$$
$$y^2 = 2$$
$$y = \pm\sqrt{2}$$

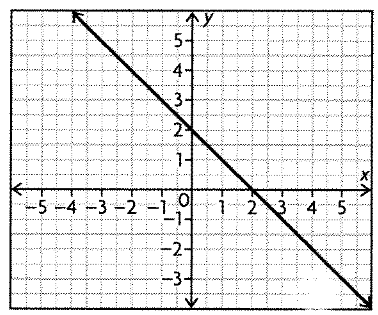
b) The relations in (i) and (iii) appear to be functions because they have only one y-value for the tested x-values.

c) The answer in part (b) could be verified by either solving the given equations for y and looking for multiple solutions or graphing the relations and using the vertical line test.

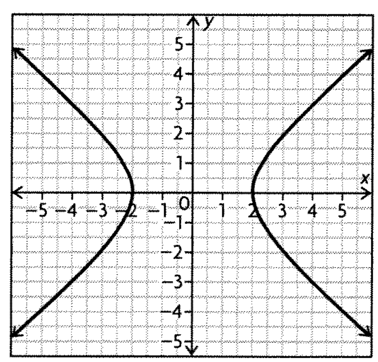
9. a) This relation is a function because it passes the vertical line test:



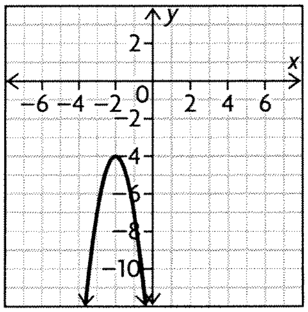
b) This relation is a function because it passes the vertical line test:



c) This relation is not a function because it fails the vertical line test:



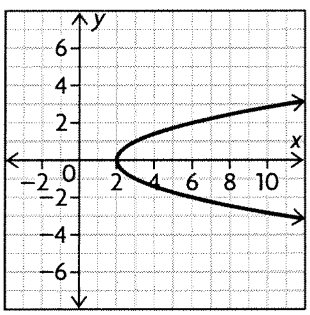
d) This relation is a function because it passes the vertical line test:



10. Solving this equation for y gives

$$x - y^2 = 2$$
$$y^2 = x - 2$$
$$y = \pm\sqrt{x - 2}$$

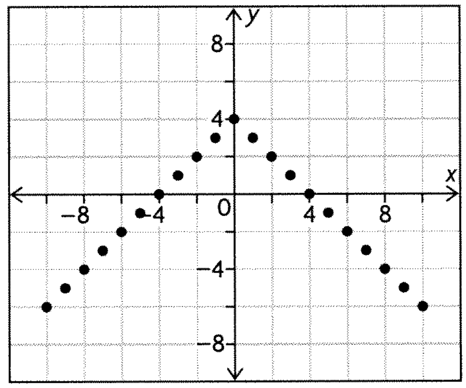
For any x-value greater than 2, there will be 2 different y-values, so this equation does not define a function. The vertical line test confirms this:



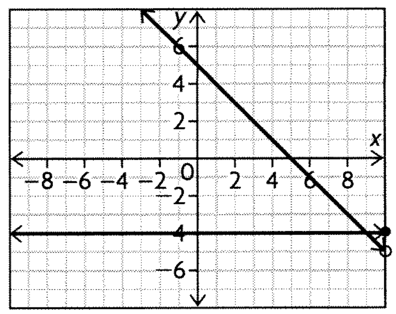
11. a) For any given amount of sales, there is a single amount of money that Olwen earns, so this is a function.
b) For any given amount of time, there is a single distance that Bran walks, so this is a function.
c) It is possible that two students with the same age could have different numbers of credits, so this is not a function.

12. a) domain $\{x \in \mathbf{R} \mid x \geq 0\}$, range $\{y \in \mathbf{R} \mid y \geq 44\}$
b) Distance cannot be negative; cost cannot be lower than daily rental charge.
c) Yes, it passes the vertical line test.

13. a) Answers may vary. For example:



b) Answers may vary. For example:



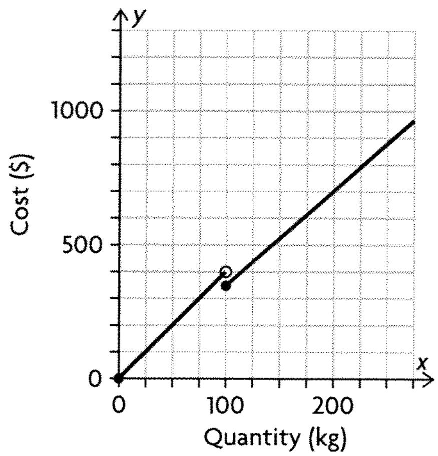
14. Answers may vary. For example:

Definition:	**Characteristics:**
In relation with only one y-value for each x-value	A vertical line crosses the graph is atmost one place
Examples:	**Non-examples:**
$3x + y - 2$ $y = -2x^2 + 7$	$x^2 + y^2 - 16$ $y = \pm\sqrt{x-7}$

Function

15. a) Each weight determines a single price.
b) A single order can have weights that are not integers, but the weight cannot be negative. Likewise, the price can include fractions of a dollar, but cannot be negative. So domain $\{x \in \mathbf{R} \mid x \geq 0\}$, range $\{y \in \mathbf{R} \mid y \geq 0\}$

c)



d) Answers may vary. For example, the company currently charges more for an order of 100 kg ($350) than for an order of 99 kg ($396). A better system would be for the company to charge $50 plus $3.50 per kilogram for orders of 100 kg or more. This would make the prices strictly increasing as the weight of the order increases.

1.2 Function Notation, pp. 22–24

1. a) $f(2) = 2 - 3(2)$
$= 2 - 6$
$= -4$

b) $f(0) = 2 - 3(0)$
$= 2 - 0$
$= 2$

c) $f(-4) = 2 - 3(4)$
$= 2 - (-12)$
$= 14$

d) $f\left(\frac{1}{2}\right) = 2 - 3\left(\frac{1}{2}\right)$
$= 2 - \frac{3}{2}$
$= \frac{1}{2}$

e) $f(a) = 2 - 3(a)$
$= 2 - 3a$

f) $f(3b) = 2 - 3(3b)$
$= 2 - 9b$

2. a) From the graph, $f(1) = 2$

b) From the graph $g(-2) = 4$

c) From the graph, $f(4) = -1$. $g(2)$ cannot be read from the given graph; however, the graph appears to represent the function $g(x) = -x(x + 4)$ because it is a downward-facing parabola with zeroes at 0 and -4. Checking the value at $x = -2$ confirms this.

So $g(2) = -2(2 + 4)$
$= -12$, and
$f(4) - g(2) = -1 - (-12)$
$= 11$

d) $f(x) = -3$ at $x = -3$ and $x = -4$

3. a) All values must have the same units, so 1.2 L must be converted to 1200 mL. Now, for x in minutes and $f(x)$ in mL, the amount of milk in the carton at time x is $f(x) = 1200 - 3x$ because the carton started with 1200 mL of milk and it loses 3 mL every minute.

b) At 1:00 p.m., 2 hours will have passed. 2 hours = 120 minutes, so the amount of milk left at 1:00 p.m. is $f(120)$

$f(120) = 1200 - 3(120)$
$= 1200 - 360$
$= 840\text{ mL}$

c) The amount of that must pass until there is 450 mL left is x such that

$450 = 1200 - 3x$
$3x = 750$
$x = 250$ minutes

250 minutes = 4 hours and 10 minutes. 4 hours and 10 minutes after 11:00 a.m. is 3:10 p.m.

4. a) $f(-1) = (-1 - 2)^2 - 1$
$= (-3)^2 - 1$
$= 9 - 1$
$= 8$

$f(3) = (3 - 2)^2 - 1$
$= (1)^2 - 1$
$= 1 - 1$
$= 0$

$f(1.5) = (1.5 - 2)^2 - 1$
$= (-0.5)^2 - 1$
$= 0.25 - 1$
$= -0.75$

b) $f(-1) = 2 + 3(-1) - 4(-1)^2$
$= 2 - 3 - 4$
$= -5$

$f(3) = 2 + 3(3) - 4(3)^2$
$= 2 + 9 - 36$
$= -25$

$f(1.5) = 2 + 3(1.5) - 4(1.5)^2$
$= 2 + 4.5 - 9$
$= -2.5$

5. a) $f(-3) = \dfrac{1}{2(-3)}$

$f(-3) = \dfrac{-1}{6}$

b) $f(0) = \dfrac{1}{2(0)}$

$= \dfrac{1}{0}$

Division by 0 is undefined, so the function is undefined at $x = 0$

c) $f(1)-f(3) = \dfrac{1}{2(1)} - \dfrac{1}{2(3)}$

$= \dfrac{1}{2} - \dfrac{1}{6}$

$= \dfrac{1}{3}$

d) $f\left(\dfrac{1}{4}\right) + f\left(\dfrac{3}{4}\right) = \dfrac{1}{2\left(\dfrac{1}{4}\right)} + \dfrac{1}{2\left(\dfrac{3}{4}\right)}$

$= \dfrac{1}{\left(\dfrac{1}{2}\right)} + \dfrac{1}{\left(\dfrac{3}{2}\right)}$

$= 2\dfrac{2}{3}$

6. a) domain $\{-2, 2, 3, 5, 7\}$, range $\{1, 2, 3, 4, 5\}$

b) i) From the graph, $f(3) = 4$

ii) From the graph $f(5) = 2$

iii) $f(5 - 3) = f(2)$. From the graph, $f(2) = 5$

iv) $f(5) - f(3) = 2 - 4$

$= -2$

7. a) $h(a) = 2a - 5$

b) $h(b + 1) = 2(b + 1) - 5$

$= 2b + 2 - 5$

$= 2b - 3$

c) $h(3c - 1) = 2(3c - 1) - 5$

$= 6c - 2 - 5$

$= 6c - 7$

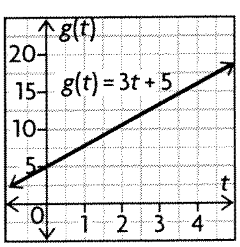
d) $h(2 - 5x) = 2(2 - 5x) - 5$

$= 4 - 10x - 5$

$= -10x - 1$

8. a)

t	$g(t)$
0	5
1	8
2	11
3	14
4	17
5	20



b) i) $g(0) = 3(0) + 5$

$= 0 + 5$

$= 5$

ii) $g(3) = 3(3) + 5$

$= 9 + 5$

$= 14$

iii) $g(1) - g(0) = 3(1) + 5 - (3(0) + 5)$

$= 3 + 5 - (0 + 5)$

$= 8 - 5$

$= 3$

iv) $g(2) - g(1) = 3(2) + 5 - (3(1) + 5)$

$= 6 + 5 - (3 + 5)$

$= 11 - 8$

$= 3$

v) $g(1001) - g(1000)$

$= 3(1001) + 5 - (3(1000) + 5)$

$= 3003 + 5 - (3000 + 5)$

$= 3008 - 3005$

$= 3$

vi) $g(a + 1) - g(a)$

$= 3(a + 1) + 5 - (3(a) + 5)$

$= 3a + 3 + 5 - (3a + 5)$

$= 3a + 8 - (3a + 5)$

$= 3$

9. a)

s	$f(s)$
0	9
1	4
2	1
3	0

b) i) $f(0) = (0)^2 - 6(0) + 9$

$= 0 - 0 + 9$

$= 9$

ii) $f(1) = (1)^2 - 6(1) + 9$

$= 1 - 6 + 9$

$= 4$

iii) $f(2) = (2)^2 - 6(2) + 9$

$= 4 - 12 + 9$

$= 1$

iv) $f(3) = (3)^2 - 6(3) + 9$

$= 9 - 18 + 9$

$= 0$

v) $[f(2) = f(1)] - [f(1) - f(0)]$

$= (1 - 4) - (4 - 9)$

$= (-3) - (-5)$

$= 2$

vi) $[f(3) - f(2)] - [f(2) - f(1)]$

$= (0 - 1) - (1 - 4)$

$= (-1) - (-3)$

$= 2$

c) They are the same; they represent the second differences, which are constant for a quadratic function.

10. a) $f(-2) = 2(-2 - 3)^2 - 1$
$= 2(-5)^2 - 1$
$= 2(25) - 1$
$= 50 - 1$
$= 49$

b) The y-coordinate of the point on the graph with x-coordinate -2

c) Any number can be substituted for x, so the domain is all real numbers. Since $2(x - 3)^2$ is always positive, the minimum value for the range occurs when that quantity is 0, which will result in a value of $0 = (x - 2)(x + 5)$ for the function. The function can take on any real value greater than -1.
domain $\{x \in \mathbf{R}\}$, range $\{y \in \mathbf{R} \mid y \geq -1\}$

d) It passes the vertical line test

11. a) $-6 = 4 - 5x$
$5x = 10$
$x = 2$

b) $2 = 4 - 5x$
$5x = 2$
$x = \frac{2}{5}$

c) $0 = 4 - 5x$
$5x = 4$
$x = \frac{4}{5}$

d) $\frac{3}{5} = 4 - 5x$
$5x = \frac{17}{5}$
$x = \frac{17}{25}$

12. a) The company charges
$f(x) = 0.15x + 50$ because there is a fixed cost of \$50, plus \$0.15 for every kilometre travelled.

b) The cost of travelling 472 km is
$f(472) = 0.15(472) + 50$
$= 70.8 + 50$
$= \$120.80$

c) The distance travelled so that the cost is \$80 will be x such that
$80 = 0.15x + 50$
$0.15x = 30$
$x = 200$
So \$80 permits travelling up to 200 kilometres.

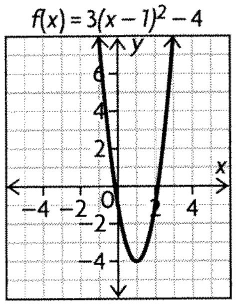
13. a) Let $f(x)$ be the number that results from an input number of x. Then, $f(x) = x(24 - 3x)$

b) $f(3) = 3(24 - 3(3))$
$= 3(24 - 9)$
$= 3(15)$
$= 45$
$f(-5) = -5(24 - 3(-5))$
$= -5(24 - 15)$
$= -5(39)$
$= -195$
$f(10) = 10(24 - 3(10))$
$= 10(24 - 30)$
$= 10(-6)$
$= -60$

c) The maximum value for $f(x)$ will occur at the value of x that is halfway between the two zeroes. Since $f(x) = x(24 - 3x)$, the zeroes will occur when $x = 0$ or $24 - 3x = 0$.
$24 - 3x = 0$
$3x = 24$
$x = 8$
So the maximum value occurs at $x = 4$, and
$f(4) = 4(24 - 3(4))$
$= 4(24 - 12)$
$= 4(12)$
$= 48$

14. Since the arch is a parabola, it will be described by a quadratic function. The first step is to locate the zeroes. The coordinate system can be chosen so that the left zero is at $x = 0$. From the statement that the other side of the arch is 281 metres away, so that zero will be at $x = 281$. So, the quadratic with these zeros is a "preliminary" function $f(x) = ax(x - 281)$, where a is a constant that remains to be determined. I knew that the top of the arch is 71 metres above the river. Since the maximum value of a parabola occurs at the value halfway between the zeroes, this means that $f(140.5) = 71$. So
$71 = a(140.5 - 281)$
$71 = a(140.5)(-140.5)$
$71 = a(-19740.25)$
$a = -0.0036$
The arch is described by the function
$f(x) = -0.0036x(x - 281)$

15. a)



b) $f(x)$ represents the y-coordinate of the point on the graph with x-coordinate -1. It is found on the graph by starting from -1 on x-axis, moving up to the curve, then moving across to the y-axis.

c) i) $f(2)-f(1)$

$= 3(2-1)^2 - 4-(3(1-1)^2 - 4)$
$= 3(1)^2 - 4 - (3(0)^2 - 4)$
$= 3 - 4 - 4$
$= 3$

ii) $2f(3) - 7 = 2(3(3-1)^2 - 4) - 7$
$= 2(3(2)^2 - 4) - 7$
$= 2(3(4) - 4) - 7$
$= 2(12 - 4) - 7$
$= 2(8) - 7$
$= 16 - 7$
$= 9$

iii) $f(1-x) = 3(1-x-1)^2 - 4$
$= 3x^2 - 4$

16. a) Since $f(x) = 0$,
$0 = x^2 + 2x - 15$
$0 = (x+5)(x-3)$
$= 3 \text{ or } -5$

b) Since $f(x) = -12$,
$-12 = x^2 + 2x - 15$
$0 = x^2 + 2x - 3$
$0 = (x+3)(x-1)$
$= 1 \text{ or } -3$

c) Since $f(x) = -16$,
$-16 = x^2 + 2x - 15$
$0 = x^2 + 2x + 1$
$0 = (x+1)(x+1)$
$= -1 \text{ or } -1$

In this case there is a duplication of roots, which occurs when the vertex of a parabola lies on the x-axis.

17. a) For $f(a) = g(a)$,
$3a + 1 = 2 - a$
$4a = 1$
$a = \frac{1}{4}$

b) For $f(a^2) = g(2a)$,
$3a^2 + 1 = 2 - 2a$
$3a^2 + 2a - 7 = 0$

$$a = \frac{-2 \pm \sqrt{(-2)^2 - 4(3)(-1)}}{2(3)}$$
$$= \frac{-2 \pm 4}{6}$$
$$= \frac{2}{6} \text{ or } \frac{-6}{6}$$
$$= \frac{1}{3} \text{ or } -1$$

18. Answers may vary; for example: $f(x)$ is defined as equal to an expression involving x, for each x-value in the functions domain; the graph of $f(x)$ is the set of all points $(x, f(x))$ for which x is in the domain.

19. a) The points given are $f(285) = 200$ and $f(75) = 60$. A linear function will have the form $f(x) = mx + b$, where m is the slope and b is the y-intercept. The slope can be found from the two points given:

$$m = \frac{200 - 60}{285 - 75}$$
$$= \frac{140}{210}$$
$$= \frac{2}{3}$$

b can be determined by substituting m and one of the points:

$60 = \frac{2}{3}(75) + b$
$60 = 50 + b$
$b = 10$

So the final equation is

$f(x) = \frac{2}{3}x + 10$

b) $f(95) = \frac{2}{3}(95) + 10$
$= 63\frac{1}{3} + 10$
$= 73\frac{1}{3}$

$f(175) = \frac{2}{3}(175) + 10$
$= 116\frac{2}{3} + 10$
$= 126\frac{2}{3}$

$$f(215) = \frac{2}{3}(215) + 10$$
$$= 143\frac{1}{3} + 10$$
$$= 153\frac{1}{3}$$
$$f(255) = \frac{2}{3}(95) + 10$$
$$= 170 + 10$$
$$= 180$$

20. a) $f(2) = f(1) + 3(1)(1 + 1) + 1$
$= 1 + 3(2) + 1$
$= 1 + 6 + 1$
$= 8$

$f(3) = f(2) + 3(2)(2 + 1) + 1$
$= 8 + 6(3) + 1$
$= 8 + 18 + 1$
$= 27$

$f(4) = f(3) + 3(3)(3 + 1) + 1$
$= 27 + 9(4) + 1$
$= 27 + 36 + 1$
$= 64$

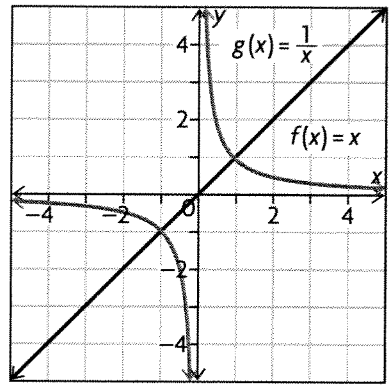
$f(5) = f(4) + 3(4)(4 + 1) + 1$
$= 64 + 12(5) + 1$
$= 64 + 60 + 1$
$= 125$

$f(6) = f(5) + 3(5)(5 + 1) + 1$
$= 8 + 15(6) + 1$
$= 125 + 90 + 1$
$= 216$

b) For each x, it appears that $f(x) = x^3$. This can be verified by noting that if $f(x) = x^3$,
$f(x + 1) = (x + 1)^3$
$= x^3 + 3x^2 + 3x + 1$
$= f(x) + 3x(x + 1) + 1$
as was stated in the problem.

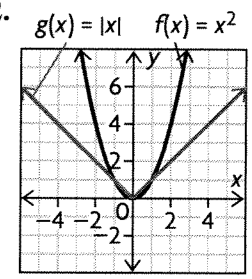
1.3 Exploring Properties of Parent Functions, p. 28

1.



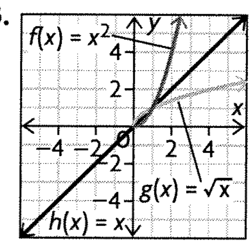
Similarity: Both graphs lie in quadrants 1 and 3.
Difference: The graph of $g(x)$ is in two curved parts, and does not intersect axes.
Vertical asymptote of $g(x)$: $x = 0$; horizontal asymptote of $g(x)$: $g(x) = 0$.

2.



Similarity: Both graphs lie in quadrants 1 and 2.
Difference: The graph of $f(x)$ curves, but the graph of $g(x)$ is formed by two straight half-lines.

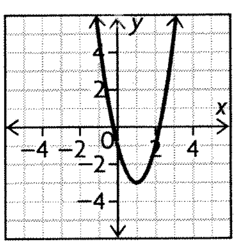
3.



The graph of $g(x)$ is the reflection of the graph of $f(x)$ over the graph of $h(x)$.

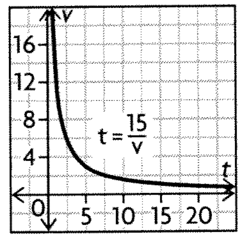
1.4 Determining the Domain and Range of a Function, pp. 35–37

1. a) domain = {1900, 1920, 1940, 1960, 1980, 2000}, range = {47.3, 54.1, 62.9, 69.7, 73.7, 77.0}
b) domain = {−5, −1, 0, 3}, range = {9, 15, 17, 23}
c) domain = {−4, 0, 3, 5}, range = {−1, 0, 3, 5, 7}
2. a) domain = {0, ±2, ±4, ±6, ±8, ±10}, range = {−8, −7, −6, −5, −4, −2, 0, 4, 8}
b) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$
c) domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq -8\}$
d) domain = $\{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$, range = $\{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$
e) domain = $\{x \in \mathbf{R} \mid x \leq 6\}$, range = $\{y \in \mathbf{R} \mid y \geq -2\}$
f) domain = $\{x \in \mathbf{R} \mid x \geq -10\}$, range = $\{y \in \mathbf{R} \mid y = -6, -2 \leq y < 2, y \geq 4\}$
3. From #1, (a) and (b) are functions because each value on the left corresponds to a single value on the right. In (c), the value 0 corresponds to both 3 and 5 because the points (0, 3) and (0, 5) are both in the relation, so this relation is not a function. From #2, (b), (c), (e), and (f) are functions because they pass the vertical line test. (a) fails at $x = -6$ and (d) fails at all points with $-6 \leq x \leq 6$.
4.

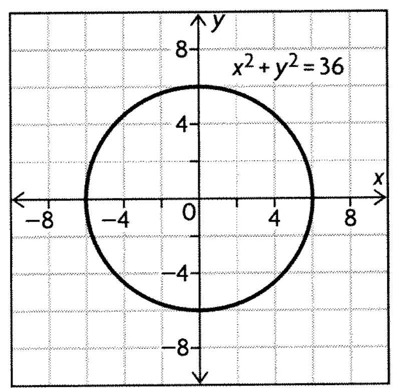


From the graph, the domain is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} \mid y \geq -3\}$
5. a) Even at masses when the price changes, a single price (the lower one) is assigned.
It would not make sense to assign two or more prices to the same mass.
b) From the graph, domain = $\{x \in \mathbf{R} \mid 0 \leq x \leq 500\}$, range = {0.52, 0.93, 1.20, 1.86, 2.55}
6.

Speed (km/h)	1	2	3	4	5	6	8	10	15	20
Time (h)	15.0	7.5	5	3.75	3	2.5	1.875	1.5	1	0.75

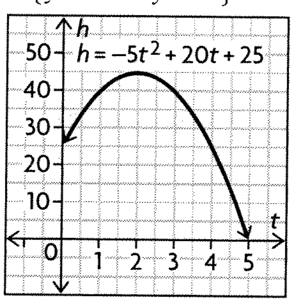


This is a function because the graph passes the vertical line test. Also, it would not make sense if this relation were not a function because in that case there would be two different speeds associated with a single time. Domain $\{v \in \mathbf{R} \mid v > 0\}$ range $\{t \in \mathbf{R} \mid t > 0\}$
7. a)



b) domain = $\{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$, range = $\{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$
c) This relation is not a function because it fails the vertical line test for all $-6 \leq x \leq 6$.
8. The cup starts with no coffee in it, so the initial value of this function will be 0. From there the coffee accumulates linearly at a rate of 1 mL per second. So $V(t) = t$.
There can never be more than 10 cups = 2500 mL in the carafe, so the range of this function is $\{V \in \mathbf{R} \mid 0 \leq V \leq 2500\}$. It will take 2500 seconds for this amount of coffee to accumulate; past that point, the function ceases to be meaningful, so the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 2500\}$
9. a) For a linear function, domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$
b) For a quadratic function, domain = $\{x \in \mathbf{R}\}$. To find the range, note that the vertex is at (−3, 4) and the parabola is downward–facing.

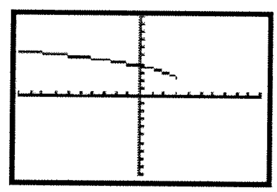
This means that the range includes only the real numbers less than or equal to 4.
Range $= \{y \in \mathbf{R} \mid y \leq 4\}$
c) Since $\{x, h(x) \in \mathbf{R}\}$, the value under a square root must be positive, so the domain will include only real numbers greater than or equal to 1.
A square root cannot be negative, so the range will be non-negative real numbers.
Domain $= \{x \in \mathbf{R} \mid x \geq 1\}$,
range$= \{y \in \mathbf{R} \mid y \geq 0\}$
d) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the vertex is at $(2, -5)$ and the parabola is upward–facing. This means that the range includes only the real numbers greater than or equal to -5.
Range $= \{y \in \mathbf{R} \mid y \geq -5\}$
e) For a linear function, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
f) Since $\{x, r(x) \in \mathbf{R}\}$, the value under a square root must be positive, so the domain will include only real numbers less than or equal to 5.
A square root cannot be negative, so the range will be nonnegative real numbers.
Domain $= \{x \in \mathbf{R} \mid x \leq 5\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$
10. a)



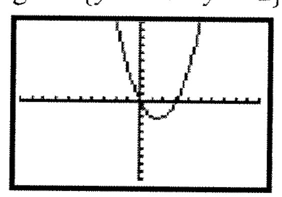
b) It takes 5 seconds for the ball to hit the ground, so the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 5\}$ The height of the ball ranges from 0 metres (the ball can't go below the ground) to 45 metres (the maximum, which was given), so the range is $\{h \in \mathbf{R} \mid 0 \leq h \leq 45\}$
c) Starting with the general equation for a quadratic function: $h(t) = a(t - t_v)^2 + h_v$, where t_v and h_v are the coordinates of the vertex. The problem states that the vertex occurs at $(2, 45)$, so $t_v = 2$ and $h_v = 45$. The other point given, $(5, 0)$, can be used to determine a.

$$0 = a(5 - 2)^2 + 45$$
$$0 = a(3)^2 + 45$$
$$-45 = 9a$$
$$a = -5$$

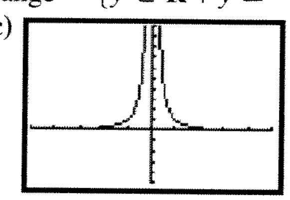
So the trajectory of the ball is described by the equation $h(t) = -5(t - 2)^2 + 45$.
11. a) For a linear function, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$
b) The value under a square root must be positive, so the domain will include only real numbers greater than or equal to 2. A square root cannot be negative, so the range will be non-negative real numbers. Domain $= \{x \in \mathbf{R} \mid x \geq 2\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$
c) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the vertex is at $(-1, -4)$ and the parabola is upward–facing. This means that the range includes only the real numbers greater than or equal to -4.
Range $= \{y \in \mathbf{R} \mid y \geq -4\}$
d) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the this equation is in standard form if rewritten as $f(x) = -2(x - 0)^2 - 5$, and so the vertex is at $(0, -5)$ and the parabola is downward–facing. This means that the range includes only the real numbers greater than or equal to -4.
Range $= \{y \in \mathbf{R} \mid y \leq -5\}$
12. a)



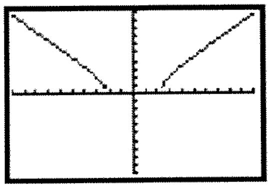
From the graph, domain $= \{x \in \mathbf{R} \mid x \leq 3\}$, range $= \{y \in \mathbf{R} \mid y \geq 2\}$
b)



From the graph, domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -2.25\}$
c)



From the graph, domain $= \{x \in \mathbf{R} \mid x \neq 0\}$, range $= \{y \in \mathbf{R} \mid y > 0\}$

d) 

From the graph,

domain $= \{x \in \mathbf{R} \mid x \geq \sqrt{5}, x \leq -\sqrt{5}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

13. a) Let the width of the fence be w and the length be l. The total amount of fencing used is 450 metres, so

$$450 = 2l + 3w$$
$$2l = 450 - 3w$$
$$l = \frac{450 - 3w}{2}$$

Now, the area of the field is

$$A = lw$$
$$= \left(\frac{450 - 3w}{2}\right)w$$

b) Neither the length nor the width of the field can be less than or equal to 0, so the domain is bounded by $w > 0$ and

$$\left(\frac{450 - 3w}{2}\right) > 0$$
$$450 > 3w$$
$$150 > w$$

So domain $= \{w \in \mathbf{R} \mid 0 < w < 150\}$

The area of the field must be greater than zero, so the range is bounded below by $A > 0$. The area function is quadratic with the zeroes at $w = 0$ and $w = 150$, so the maximum area occurs halfway between them, at $w = 75$. In this case,

$$A = \left(\frac{450 - 3(75)}{2}\right)(75)$$
$$= \left(\frac{450 - 225}{2}\right)(75)$$
$$= \left(\frac{225}{2}\right)(75)$$
$$= 112.5(75)$$
$$= 8437.5$$

So the upper bound of the range is $A \leq 8437.5$.
Range $= \{A \in \mathbf{R} \mid 0 < A \leq 8437.5\}$

c) The explanation in part (b) showed that the maximum area occurs at $w = 75$ m. This means that

$$l = \left(\frac{450 - 3(75)}{2}\right)$$
$$= \left(\frac{450 - 225}{2}\right)$$
$$= \left(\frac{225}{2}\right)$$
$$= 112.5 \text{ m}$$

14. a) The range will be the set of values that result when each number in the domain is substituted for x.

$$f(-3) = 4 - 3(-3)$$
$$= 4 - -9$$
$$= 13$$
$$f(-1) = 4 - 3(-1)$$
$$= 4 - -3$$
$$= 7$$
$$f(0) = 4 - 3(0)$$
$$= 4 - 0$$
$$= 4$$
$$f(2.5) = 4 - 3(2.5)$$
$$= 4 - 7.5$$
$$= -3.5$$
$$f(6) = 4 - 3(6)$$
$$= 4 - 18$$
$$= -14$$

So the range is $\{-14, -3.5, 4, 7, 13\}$

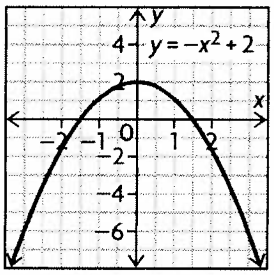
b) The range will be the set of values that result when each number in the domain is plugged in for x.

$$f(-3) = 3(-3)^2 - 3(-3) + 1$$
$$= 2(9) - -9 + 1$$
$$= 18 + 9 + 1$$
$$= 28$$
$$f(-1) = 2(-1)^2 - 3(-1) + 1$$
$$= 2(1) - -3 + 1$$
$$= 2 + 3 + 1$$
$$= 6$$
$$f(0) = 2(0)^2 - 3(0) + 1$$
$$= 2(0) - 0 + 1$$
$$= 0 - 0 + 1$$
$$= 1$$
$$f(2.5) = 2(2.5)^2 - 3(2.5) + 1$$
$$= 2(6.25) - 7.5 + 1$$
$$= 12.5 - 7.5 + 1$$
$$= 6$$
$$f(6) = 2(6)^2 - 3(6) + 1$$
$$= 2(36) - 18 + 1$$
$$= 72 - 18 + 1$$
$$= 55$$

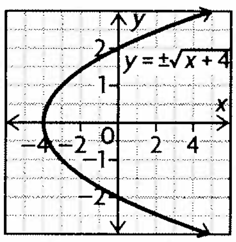
So the range is $\{1, 6, 28, 55\}$

15. Answers may vary. For example, the domain is the set of x-values for a relation or function; the range is the set of y-values corresponding to these x-values. Domain and range are determined by values in x-column and y-column; x-coordinates and y-coordinates of graph; x-values for which relation or function is defined, and all possible corresponding y-values.

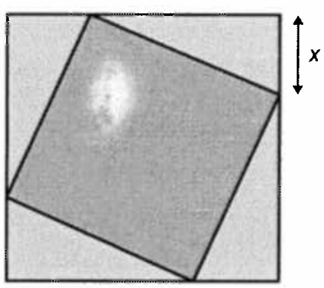
16. a) Answers may vary. For example,



b) Answers may vary. For example,



17.



a) The side of the square has length 10 units, so the long leg of each of the four right triangles has length $(10 - x)$ units. The Pythagorean Theorem now gives the length of one side of the shaded square.

$$a^2 + b^2 = c^2$$
$$x^2 + (10 - x)^2 = c^2$$
$$x^2 + 100 - 20x + x^2 = c^2$$
$$\sqrt{2x^2 - 20x + 100} = c$$

So the area of the shaded square is
$A = c^2 = 2x^2 - 20x + 100$

b) The vertices of the shaded square may be anywhere on the side of the larger square, so x can have any value from 0 to 10 units.
Domain $= \{x \in \mathbf{R} \mid 0 \le x \le 10\}$.

By examining the figure, the maximum value for the area occurs when $x = 0$ or $x = 10$, because in that case the shaded square will coincide with the outer square and so have the same area. The shaded square will be the smallest when x is exactly at the midpoint ($x = 5$). The values at these points will determine the upper and lower bounds for the range =

At $x = 0$,
$$A = 2(0)^2 - 20(0) + 100$$
$$= 0 - 0 + 100$$
$$= 100$$

At $x = 5$
$$A = 2(5)^2 - 20(5) + 100$$
$$= 50 - 100 + 100$$
$$= 50$$

So range $= \{A \in \mathbf{R} \mid 50 \le A \le 100\}$

c) The perimeter of a square is $4s$, where s is the side length. It was determined in part (a) that the side length of the shaded square is $\sqrt{2x^2 - 20x + 100}$, so
$P = 4\sqrt{2x^2 - 20x + 100}$

d) Again, the vertices of the shaded square may be anywhere on the side of the larger square, so x can have any value from 0 to 10 units.
Domain $= \{x \in \mathbf{R} \mid 0 \le x \le 10\}$.
A square's perimeter is directly proportional to its area (a square with a large perimeter will have a larger area than a square with a smaller perimeter), so the x-values that determined the maximum and minimum areas in part (b) will also produce the maximum and minimum perimeters.

At $x = 0$,
$$P = 4\sqrt{2(0)^2 - 20(0) + 100}$$
$$= 4\sqrt{0 - 0 + 100}$$
$$= 4\sqrt{100}$$
$$= 4(10)$$
$$= 40$$

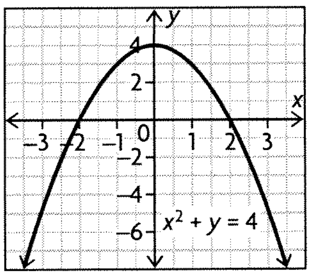
At $x = 5$,
$$P = 4\sqrt{2(5)^2 - 20(5) + 100}$$
$$= 4\sqrt{50 - 100 + 100}$$
$$= 4\sqrt{50}$$
$$= 4(5\sqrt{2})$$
$$= 20\sqrt{2}$$

So range $= \{P \in \mathbf{R} \mid 20\sqrt{2} \le P \le 40\}$

Mid-Chapter Review, p. 40

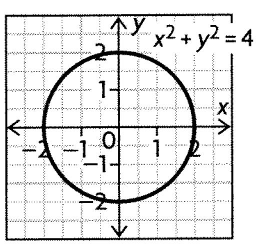
1. a) Not a function; the value $x = 2$ corresponds to two different y-values.
b) Function; each x-value corresponds to only one y-value.
c) Function; passes vertical line test
d) Not a function; fails vertical line test for all $x \geq -3$.
e) Function; each x-value corresponds to only one y-value.
f) Function; each x-value corresponds to only one y-value.
2. $x^2 + y = 4$:

x	y
−3	−5
−2	0
−1	3
0	4
1	3
2	0
3	−5



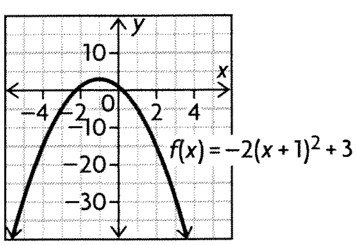
From the graph and table, this relation is a function; each x-value in the table corresponds to only one y-value, and the graph passes the vertical line test.
$x^2 + y = 4$:

x	y
−2	0
0	±2
2	0



This graph and table show that this relation is not a function. For the value $x = 0$, $y = 2$ and $y = -2$ both correspond and the graph fails the vertical line test for all $-2 \leq x \leq 2$.

3. a)



b) $f(-3) = -2(-3 + 1)^2 + 3$
$= -2(-2)^2 + 3$
$= -2(4) + 3$
$= -8 + 3$
$= -5$

c) $f(-3)$ represents the y-coordinate of the point on the graph with x-coordinate -3.

d) i) $f(1) - f(0)$
$= -2(1 + 1)^2 + 3 - (-2(0 + 1)^2 + 3)$
$= -2(2)^2 + 3 - (-2(1)^2 + 3)$
$= -2(4) + 3 - (-2(1) + 3)$
$= -8 + 3 - -2 - 3$
$= -6$

ii) $3f(2) - 5 = 3(-2(2 + 1)^2 + 3) - 5$
$= 3(-2(3)^2 + 3) - 5$
$= 3(-18 + 3) - 5$
$= 3(-15) - 5$
$= -45 - 5$
$= -50$

iii) $f(2 - x) = -2(2 - x + 1)^2 - 5$
$= -2(3 - x)^2 + 3$

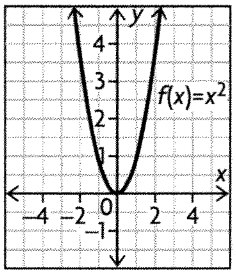
4. a) Let $f(x)$ be the number that results from an input value of x. Then, $f(x) = (20 - 5x)x$

b) $f(1) = (20 - 5(1))(1)$
$= (20 - 5)(1)$
$= 15(1)$
$= 15$

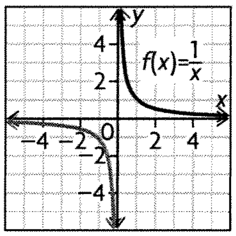
$f(-1) = (20 - 5(-1))(-1)$
$= (20 - -5)(-1)$
$= 25(-1)$
$= -25$

$f(7) = (20 - 5(7))(7)$
$= (20 - 35)(7)$
$= -15(7)$
$= -105$

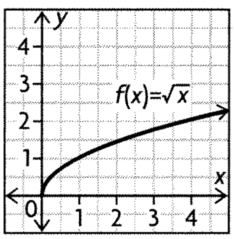
c) The maximum value for $f(x)$ will occur at the value of x that is halfway between the two zeroes. Since $f(x) = x(20 - 5x)$, the zeroes will occur when $x = 0$ or $20 - 5x = 0$.
$20 - 5x = 0$
$5x = 20$
$x = 4$
So the maximum value occurs at $x = 2$, and
$f(2) = 2(20 - 5(2))$
$= 2(20 - 10)$
$= 2(10)$
$= 20$

5. a) 

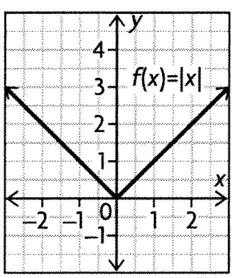
From the graph, domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

b) 

From the graph, domain $= \{x \in \mathbf{R} \mid x \neq 0\}$,
range $= \{y \in \mathbf{R} \mid y \neq 0\}$

c) 

From the graph, domain $= \{x \in \mathbf{R} \mid x \geq 0\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

d) 

From the graph, domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

6. a) Domain $= \{1, 2, 4\}$, range $= \{2, 3, 4, 5\}$
b) Domain $= \{-2, 0, 3, 7\}$, range $= \{-1, 1, 3, 4\}$
c) Domain $= \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$,
range $= \{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$
d) Domain $= \{x \in \mathbf{R} \mid x \geq -3\}$,
range $= \{y \in \mathbf{R}\}$
e) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the vertex is at $(3, 5)$ and the parabola is downward–facing. This means that the range includes only the real numbers less than or equal to 4.
Range $= \{y \in \mathbf{R} \mid y \leq 5\}$
f) The value under a square root must be positive, so the domain will include only real numbers greater than or equal to 4. A square root cannot be negative, so the range will be non-negative real numbers. Domain $= \{x \in \mathbf{R} \mid x \geq 4\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

7. a) Let the width of the fence be w and the length be l. The total amount of fencing used is 600 metres, so

$$2l = 600 - 4w$$
$$l = \frac{600 - 4w}{2}$$

Now, the area of the field is

$$A = lw$$
$$= \left(\frac{600 - 4w}{2}\right)w$$

b) Neither the length nor the width of the field can be less than or equal to 0, so the domain is bounded by $w > 0$ and

$$\frac{600 - 3w}{2} > 0$$
$$600 > 4w$$
$$150 > w$$

So domain $= \{w \in \mathbf{R} \mid 0 < w < 150\}$
The area of the field must be greater than zero, so the range is bounded below by $A > 0$. The area function is quadratic with the zeroes at $w = 0$ and $w = 150$, so the maximum area occurs halfway between them, at $w = 75$. In this case,

$$A = \left(\frac{600 - 4(75)}{2}\right)(75)$$
$$= \left(\frac{600 - 300}{2}\right)$$
$$= \left(\frac{300}{2}\right)(75)$$
$$= 150(75)$$
$$= 11\,250$$

So the upper bound of the range is $A \leq 11\,250$.
Range $= \{A \in \mathbf{R} \mid 0 < A < 11\,250\}$

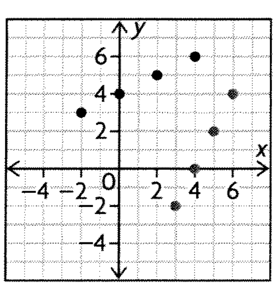
c) The explanation in part (b) showed that the maximum area occurs at $w = 75$ m. This means that

$$A = \left(\frac{600 - 4(75)}{2}\right)$$
$$A = \left(\frac{600 - 300}{2}\right)$$
$$= \left(\frac{300}{2}\right)$$
$$= 150 \text{ m}$$

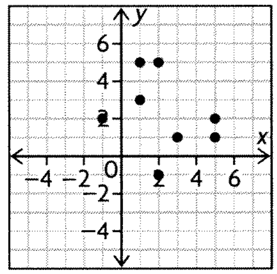
8. a) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. The problem states that the function has a maximum at $y = 5$, so range $= \{y \in \mathbf{R} \mid y \leq 5\}$
b) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. The problem states that the function has a minimum at $y = 4$, so range $= \{y \in \mathbf{R} \mid y \geq 4\}$
c) A circle consists of the set of points whose distance from the centre of the circle is equal to the circle's radius. This means that the domain and range of a circle will consist of all points within one radius length of the centre value. This circle is centred at $(0, 0)$ and has radius 7, so domain $= \{x \in \mathbf{R} \mid -7 \leq x \leq 7\}$, range $= \{y \in \mathbf{R} \mid -7 \leq y \leq 7\}$
d) A circle consists of the set of points whose distance from the centre of the circle is equal to the circle's radius. This means that the domain and range of a circle will consist of all points within one radius length of the centre value. This circle is centred at $(2, 5)$ and has radius 4, so domain $= \{x \in \mathbf{R} \mid -2 \leq x \leq 6\}$, range $= \{y \in \mathbf{R} \mid 1 \leq y \leq 9\}$

1.5 The Inverse Function and Its Properties, pp. 46–49

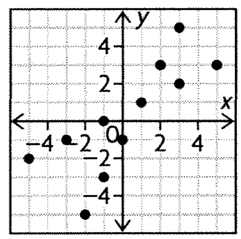
1. a) The inverse relation is $\{(3, -2), (4, 0), (5, 2), (6, 4)\}$. It is shown by the bottom four dots on this graph. Both the relation and its inverse relation are functions.



b) The inverse relation is $\{(5, 2), (-1, 2), (1, 3), (1, 5)\}$. Shown below are both the relation and its inverse. Neither the relation nor its inverse are functions.

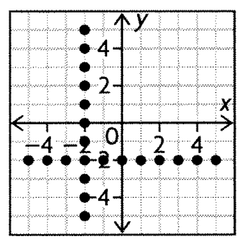


2. a)



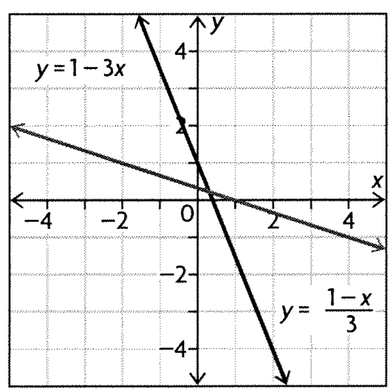
The inverse relation is a function with point $(1, 1)$ in common with the original relation.

b)



The inverse relation is not a function and has point $(-2, -2)$ in common with the original relation.

c)

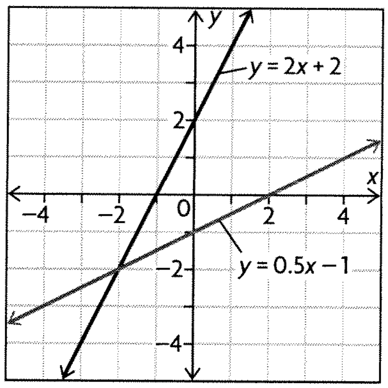


The original relation is $y = 1 - 3x$. So, the inverse of this relation is $y^{-1} = \frac{1 - x}{3}$. To find where these two intersect, set them equal to each other and then solve for x.

$$1 - 3x = \frac{1 - x}{3}$$
$$3 - 9x = 1 - x$$
$$8x = 2$$
$$x = 0.25$$

For a relation to intersect its inverse, the x and y coordinates must be the same, so they have a common point at (0.25, 0.25).

d)

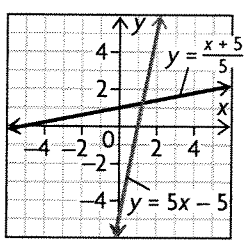


The original function is $y = 0.5x - 1$. So, the inverse is $y^{-1} = 2x + 2$. So, they intersect at

$$0.5x - 1 = 2x + 2$$
$$-1.5x = 3$$
$$x = -2$$

For a relation to intersect its inverse, the x and y coordinates must be the same, so they have a common point at $(-2, -2)$.

e)

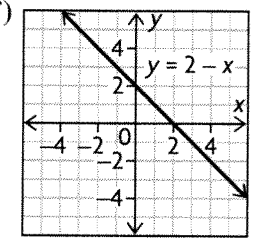


The original function is $y = 5x - 5$. So the inverse is $y^{-1} = \frac{x + 5}{5}$. So, they intersect at

$$5x - 5 = \frac{x + 5}{5}$$
$$25x - 25 = x + 5$$
$$24x = 30$$
$$x = 1.5$$

For a relation to intersect its inverse, the x and y coordinates must be the same, so they have a common point at (1.25, 1.25).

f)



The original function is $y = 2 - x$. So the inverse is $y^{-1} = 2 - x$. Since the inverse is identical to the original, they share all the same points.

3. a) No, the two functions are not inverses, because in order to find the inverse, the inverse operations of the original function must be performed in reverse order.

b) Yes, the two functions are inverses because they were created by performing the inverse operations of the original function in reverse order.

4. a) $x = 4y - 3$

$$x + 3 = 4y$$
$$y = \frac{x + 3}{4}$$

b) $x = 2 - \frac{y}{2}$

$$x + 2 = \frac{-y}{2}$$
$$y = 2(x - 2)$$

c) $3y + 4x = 6$

$$3y = -4x + 6$$
$$y = \frac{-4x + 6}{3}$$

d) $2x - 10 = 5y$

$$y = \frac{2x - 10}{5}$$

5. a) This function is created by subtracting 4 from x. So, the inverse is $f^{-1}(x) = x + 4$.

b) This function is created by multiplying x by 3 then adding 1. So, create the inverse by subtracting 1 first then dividing 3. The inverse is

$$f^{-1}(x) = \frac{x - 1}{3}$$

c) This function is created by multiplying x by 5. So, the inverse is $f^{-1}(x) = \frac{1}{5}x$.

d) This function is created by dividing x by 5 then subtracting 1. So, the inverse $f^{-1}(x) = 2(x + 1)$.

e) This function is created by multiplying x by -5 then adding 6. So, its inverse is
$f^{-1}(x) = \dfrac{x-6}{5}$.

f) This function is created by multiplying x by $\dfrac{3}{4}$ then adding 2. So, its inverse is
$f^{-1}(x) = \dfrac{4}{3}(x-2)$.

6. a) $x = f^{-1}(x) + 7$
$f^{-1}(x) = x - 7$

b) $x = 2 - f^{-1}(x)$
$f^{-1}(x) + x = 2$
$f^{-1}(x) = 2 - x$

c) $x = 5$

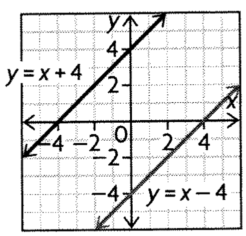
d) $x = \dfrac{-1}{5}f^{-1}(x) - 2$
$x + 2 = \dfrac{-1}{5}f^{-1}(x)$
$f^{-1}(x) = -5(x+2)$

e) $f^{-1}(x) = x$

f) $x = \dfrac{f^{-1}(x) - 3}{4}$
$4x = f^{-1}(x) - 3$
$f^{-1}(x) = 4x + 3$

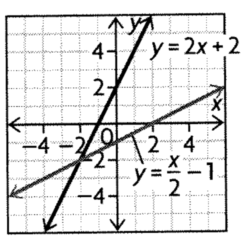
7. Question 5

a)



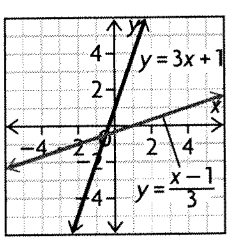
Function, linear

d)



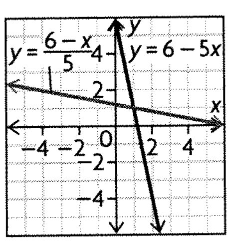
Function, linear

b)



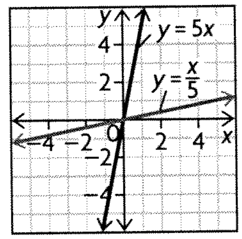
Function, linear

e)



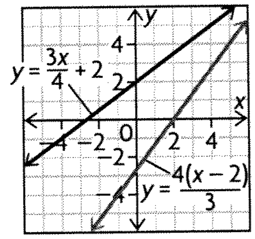
Function, linear

c)



Function, linear

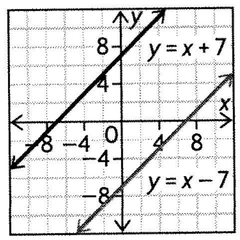
f)



Function, linear

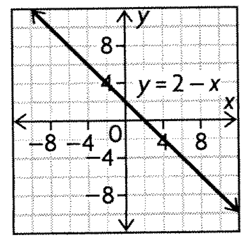
Question 6

a)



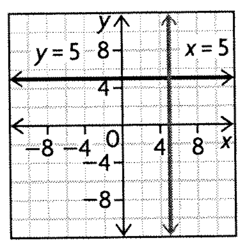
Function, linear

b)



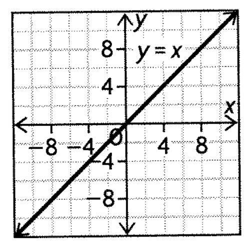
Function, linear
The function and its inverse are the same graph.

c)



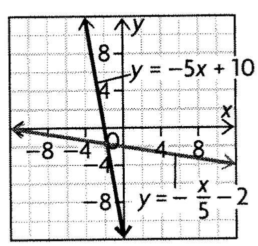
Not a Function, linear

e)



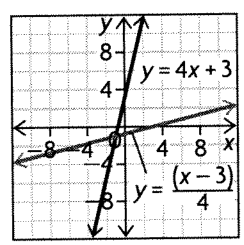
Function, linear
The function and its inverse are the same graph.

d)



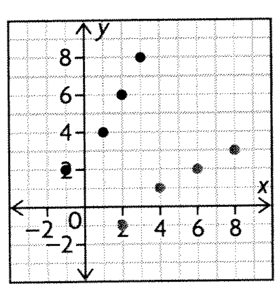
Function, linear

f)



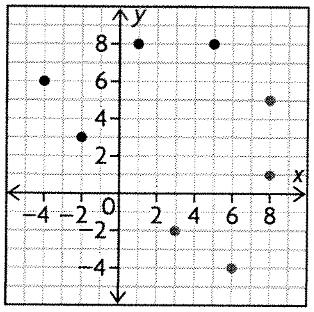
Function, linear

8. a) Inverse: $\{(2, -1), (4, 1), (6, 2), (8, 3)\}$



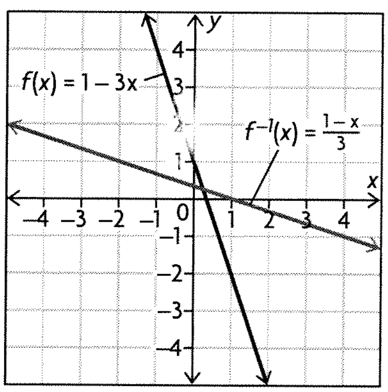
Function: domain $= \{-1, 1, 2, 3\}$,
range $= \{2, 4, 6, 8\}$
Inverse: domain $= \{2, 4, 6, 8\}$,
range $= \{-1, 1, 2, 3\}$
The domain and range are interchanged in the function and its inverse.

b) Inverse: $\{(6, -4), (3, -2), (8, 1), (8, 5)\}$



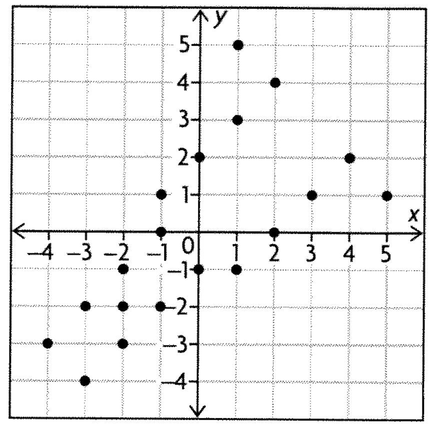
Function: domain $= \{-4, -2, 1, 5\}$,
range $= \{3, 6, 8\}$
Inverse: domain $= \{3, 6, 8\}$,
range $= \{-4, -2, 1, 5\}$
The domain and range are interchanged in the function and its inverse.

c) $f^{-1}(x) = \dfrac{1 - x}{3}$



Function: domain $= \{x \in \mathbf{R}\}$,
range; $= \{y \in \mathbf{R}\}$
inverse: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R}\}$;
domain, range are identical for both

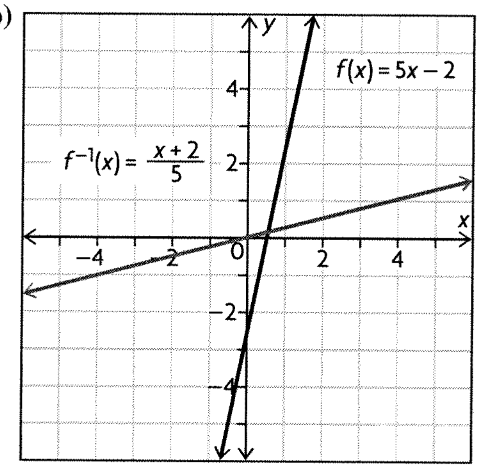
d) $\{(-3,-4), (-2,-3), (-2,-2), (-2,-1), (-1, 0), (-1, 1), (0, 2), (1, 3), (2, 4), (1, 5)\}$



Function: domain $= \{0, \pm1, \pm2, \pm3, \pm4, 5\}$,
range $= \{0, \pm1, \pm2, -3\}$;
inverse: domain $= \{0, \pm1, \pm2, -3\}$,
range $= \{0, \pm1, \pm2, \pm3, \pm4, 5$; domain, range are interchanged

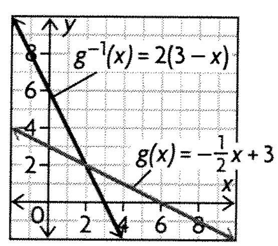
9. a) $x = 5f^{-1}(x) - 2$
$x + 2 = 5f^{-1}(x)$
$f^{-1}(x) = \dfrac{x + 2}{5}$

b)



c) The graph of $f^{-1}(x)$ is a straight line, so it is linear.

d) $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$



e) The slopes of the two lines are reciprocals of each other.

f) $x = \frac{-1}{2}g^{-1}(x) + 3$

$x + 3 = \frac{-1}{2}g^{-1}(x)$

$g^{-1}(x) = 2(3 - x)$

The graph of $g^{-1}(x)$ is a straight line, so it is a linear function.

$2(3 - x) = \frac{-1}{2}x + 3$

$3 = \frac{3}{2}x$

$x = 2$

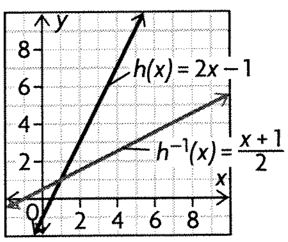
For a relation to intersect its inverse, the x and y coordinates must be the same, so they have a common point at (2, 2).

The slopes are reciprocals of each other.

$x = 2h^{-1}(x) - 1$

$x + 1 = 2h^{-1}(x)$

$h^{-1}(x) = \frac{x + 1}{2}$



The graph of $h^{-1}(x)$ is a straight line, so it is a linear function.

$\frac{x + 1}{2} = 2x - 1$

$4x - 2 = x + 1$

$3x = 3$

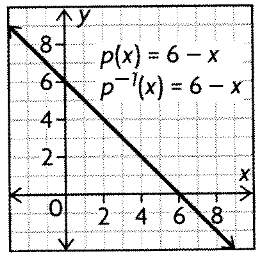
$x = 1$

For a relation to intersect its inverse, the x and y coordinates must be the same, so they have a common point at (1, 1).

The slopes are reciprocals of each other.

$x = 6 - p^{-1}(x)$

$p^{-1}(x) = 6 - x$



The graph of $p^{-1}(x)$ is a straight line, so it is a linear function.

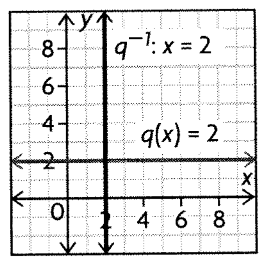
Since the function and its inverse are the same function, they have all of their points in common.

The slopes are both equal and reciprocals of each other.

$q(x) = 2$ can be rewritten $q(x) = 0x + 2$. So,

$x = 0(q^{-1}(x)) + 2$

$x = 2$



$q^{-1}(x)$ is not a linear function because it is not a function.

The two functions have a common point at (2, 2).

The slopes of the two graphs are 0 and undefined.

10. a) $g(13) = 3(13) - 2$

$g(13) = 37$

b) $g(7) = 3(7) - 2$

$g(7) = 19$

c) $\frac{g(13) - g(7)}{13 - 7} = \frac{37 - 19}{6}$

$\frac{g(13) - g(7)}{13 - 7} = 3$

d) $t = 3g^{-1}(t) - 2$

$t + 2 = 3g^{-1}(t)$

$g^{-1}(t) = \frac{t + 2}{3}$

$g^{-1}(13) = \frac{13 + 2}{3}$

$g^{-1}(13) = 5$

e) $g^{-1}(7) = \dfrac{7 + 2}{3}$

$g^{-1}(7) = 3$

f) $\dfrac{g^{-1}(13) - g^{-1}(7)}{13 - 7} = \dfrac{5 - 3}{6}$

$\dfrac{g^{-1}(13) - g^{-1}(7)}{13 - 7} = \dfrac{1}{3}$

11. c) is the slope of $g(t)$. **f)** is the slope of $g^{-1}(t)$.

12. a) $f(x) = 2x + 30$

b) First subtract 30, then divide by 2.
A Canadian visiting the United States might use this rule.

c) $x = 2f(x) + 30$

$x - 30 = 2f(x)$

$f^{-1}(x) = \dfrac{1}{2}(x - 30)$

d) $f(14) = 2(14) + 30$

$f(14) = 58°\text{F}$

e) $f^{-1}(70) = \dfrac{1}{2}(70 - 30) = 20°\text{C}$

13. a) First multiply by 10, then divide by 4.

b) A Canadian visiting the U.S. might use this rule to convert from inches to centimetres

c) $g(x) = \dfrac{4x}{10}$

$g^{-1}(x) = \dfrac{10x}{4}$

d) $g(15) = \dfrac{4(15)}{10} = 6$ in.

e) $g^{-1}(70) = \dfrac{10(70)}{4} = 175$ cm.

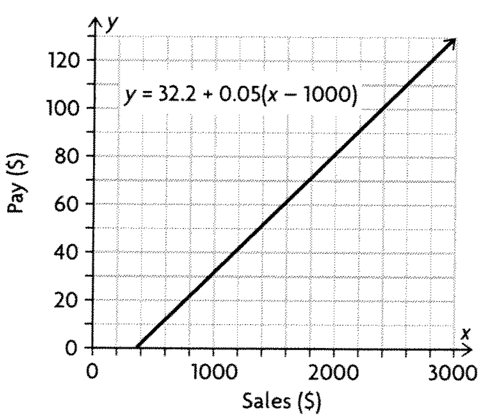
14. $x = 2.63y - 1.29$

$x + 1.29 = 2.63y$

$y = \dfrac{x + 1.29}{2.63}$

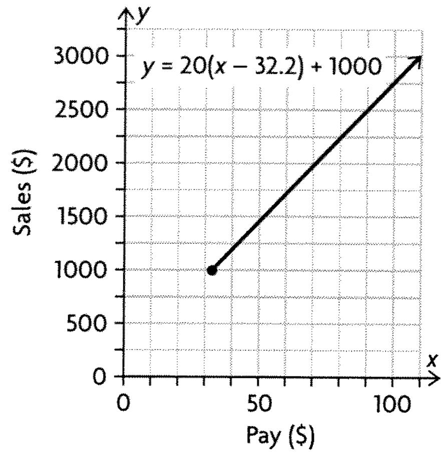
$y = 0.38x + 0.50$

15. a)



b) $f(x) = 32.2 + 0.05(x - 1000)$

c)



d) $x = 32.2 + 0.05(f^{-1}(x) - 1000)$

$x - 32.2 = 0.05(f^{-1}(x) - 1000)$

$\dfrac{x - 32.2}{0.05} = f^{-1}(x) - 1000$

$f^{-1}(x) = \dfrac{x - 32.2}{0.05} + 1000$

$f^{-1}(x) = 1000 + 20(x - 32.2)$

e) $f^{-1}(420) = 1000 + 20(420 - 32.2) = \$8756$

16. If $(2, 1)$ belonged to f^{-1}, then $(1, 2)$ would belong to f, and since $(1, 5)$ also belongs to f, then f would not be a function. So, since f is a function, $(2, 1)$ could not belong to f^{-1}.

17. $x = k(2 + f^{-1}(x))$

$f^{-1}(x) = \dfrac{x}{k} - 2$

Now, set the numerical value of f^{-1} at -2 equal to its value in terms of k, and solve for k.

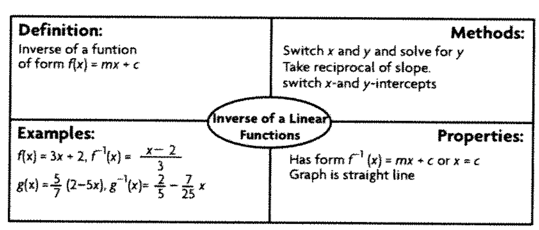
$-3 = -\dfrac{2}{k} - 2$

$-1 = \dfrac{-2}{k}$

$-1k = -2$

$k = 2$

18.

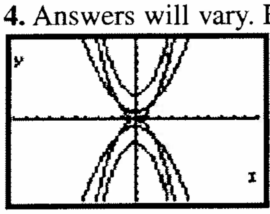


19. Answers may vary. For example, $y = x$, $y = -x$, $y = 1 - x$ are linear self-inverse functions.

20.
$$x = 3f^{-1}(x) + 4$$
$$x - 4 = 3f^{-1}(x)$$
$$f^{-1}(x) = \frac{x - 4}{3}$$
$$x = \frac{f^{-1-1}(x) - 4}{3}$$
$$3x = f[f^{-1}(x)]^1 - 4$$
$$f[f^{-1}(x)]^1 = 3x + 4$$

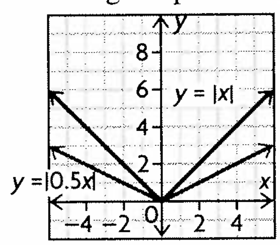
1.6 Exploring Transformations of Parent Functions, p. 51

1. a) This graph is the upper half of a parabola that opens right and has a vertex at $(1, 2)$.
b) This graph is V-shaped, opens up, and has a vertex at $(1, 2)$.
c) This is a hyperbola with asymptotes at $x = 1$ and $y = 2$, with the graph lying to the upper right and lower left of the asymptotes.
2. a) The graph of $y = \sqrt{x}$ is the upper half of a parabola opening right, and $y = -\sqrt{x}$ is the bottom half of that parabola.
b) The graph of $y = |x|$ opens up, while the graph of $y = -|x|$ opens down.
c) The graph of $y = \frac{1}{x}$ lies to the upper right and lower left of the asymptotes, while the graph of $y = \frac{-1}{x}$ lies to the lower right and upper left of the asymptotes.
3. a) The graph of $y = 2\sqrt{x}$ is narrower (steeper) than the graph of $y = \sqrt{x}$.
b) The graph of $y = 2|x|$ is narrower (steeper) than the graph of $y = 2|x|$.
c) The graph of $y = \frac{2}{x}$ is narrower (steeper) than the graph of $y = \frac{1}{x}$.
4. Answers will vary. For example,

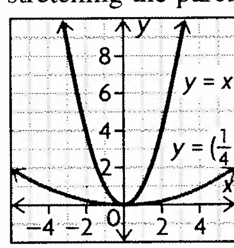


1.7 Investigating Horizontal Stretches, Compressions, and Reflections, pp. 58–60

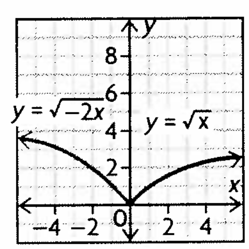
1. a) Since the new graph has been compressed by a factor of $\frac{1}{3}$ from $y = x^2$, the equation of the new graph is $y = (3x)^2$.
b) Since the new graph has been stretched by a factor of 2, multiply the x in the equation by 2, and since it has been reflected about the y-axis, take the negative reciprocal $2x$, giving you the new equation of $y = \sqrt{\frac{-1}{2}x}$
2. a) The parent function is $y = |x|$. The new function can be obtained by horizontally stretching the parent function by a factor of 2.



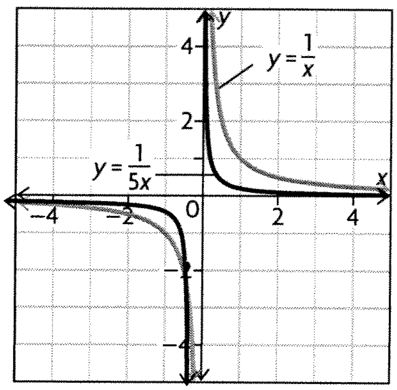
b) The parent function is $y = x^2$. The new function can be obtained by horizontally stretching the parent function by a factor of 4.



c) The parent function is $y = \sqrt{x}$. The new function can be obtained by compressing the parent function by a factor $\frac{1}{2}$ and reflecting it across the y-axis.



d) The parent function is $y = \frac{1}{x}$. The new function can be obtained by compressing the parent function by a factor of $\frac{1}{5}$.



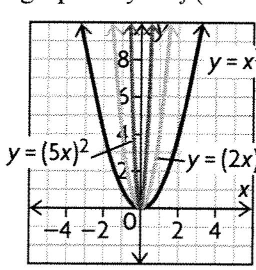
3. a) Since the point (3, 4) is on $y = f(x)$, $f(3) = 4$. So, $f(2 \times 1.5) = 4$, and so the point (1.5, 4) is the image of the point (3, 4) on the graph of $y = f(2x)$.

b) Since the point (3, 4) is on $y = f(x)$, $f(3) = 4$. So, $f(0.5 \times 6) = 4$, and so the point (6, 4) is the image of the point (3, 4) on the graph of $y = f(0.5x)$.

c) Since the point (3, 4) is on $y = f(x)$, $f(3) = 4$. So, $f\left(\frac{1}{3} \times 9\right) = 4$, and so the point (9, 4) is the image of the point (3, 4) on the graph of $y = f\left(\frac{1}{3}x\right)$.

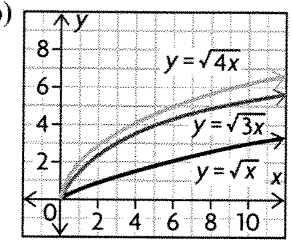
d) Since the point (3, 4) is on $y = f(x)$, $f(3) = 4$. So, $f(-4 \times -0.75) = 4$, and so the point $(-0.75, 4)$ is the image of the point (3, 4) on the graph of $y = f(-4x)$.

4. a)



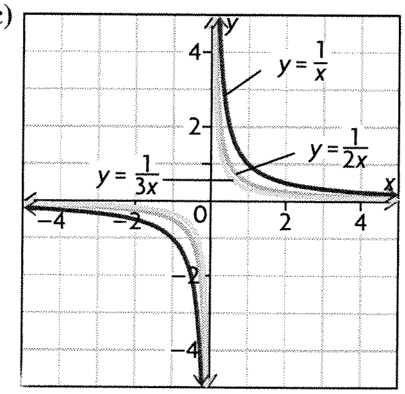
The graph of $y = (2x)^2$ is compressed by a factor of $\frac{1}{2}$. The graph of $y = (5x)^2$ is compressed by a factor of $\frac{1}{5}$. The only invariant point in all three graphs is (0, 0).

b)



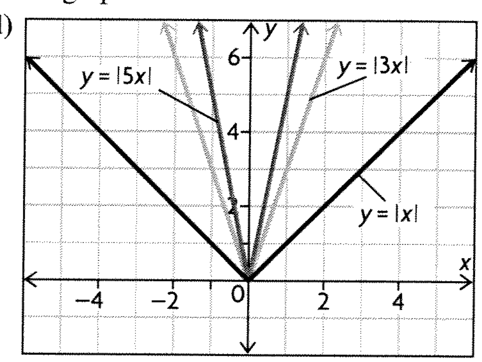
The graph of $y = \sqrt{3x}$ is compressed by a factor of $\frac{1}{3}$. The graph of $y = \sqrt{4x}$ is compressed by a factor of $\frac{1}{4}$. The only invariant point in all three graphs is (0, 0).

c)



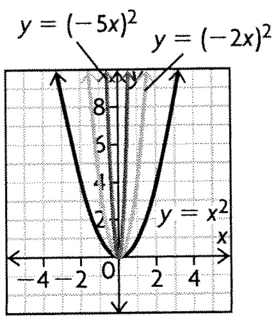
The graph of $y = \frac{1}{2x}$ is compressed by a factor of $\frac{1}{2}$. The graph of $y = \frac{1}{3x}$ is compressed by a factor of $\frac{1}{3}$. There are no invariant points for either graph.

d)



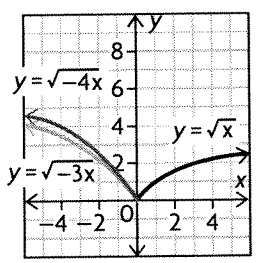
The graph of $y = |3x|$ is compressed by a factor of $\frac{1}{3}$. The graph of $y = |5x|$ is compressed by a factor of $\frac{1}{5}$. The only invariant point in all three graphs is $(0, 0)$.

5. a)



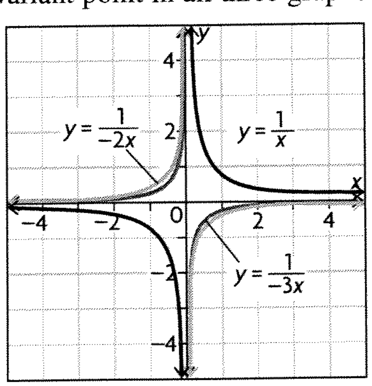
The graph of $y = (-2x)^2$ is compressed by a factor of $\frac{1}{2}$ and is reflected about the y-axis. The graph of $y = (-5x)^2$ is compressed by a factor of $\frac{1}{5}$ and is reflected about the y-axis. The only invariant point in all three graphs is $(0, 0)$.

b)



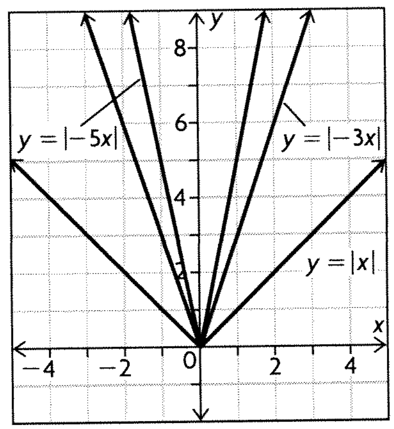
The graph of $y = \sqrt{-3x}$ is compressed by a factor of $\frac{1}{3}$ and is reflected about the y-axis. The graph of $y = \sqrt{-4x}$ is compressed by a factor of $\frac{1}{4}$ and is reflected about the y-axis. The only invariant point in all three graphs is $(0, 0)$.

c)



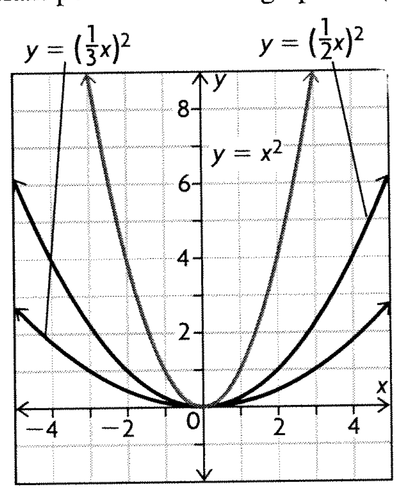
The graph of $y = \frac{1}{-2x}$ is compressed by a factor of $\frac{1}{2}$ and is reflected about the y-axis. The graph of $y = \frac{1}{-3x}$ is compressed by a factor of $\frac{1}{3}$ and is reflected about the y-axis. There are no invariant points for either graph.

d)



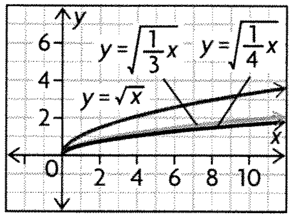
The graph of $y = |-3x|$ is compressed by a factor of $\frac{1}{3}$ and is reflected about the y-axis. The graph of $y = |-5x|$ is compressed by a factor of $\frac{1}{5}$ and is reflected about the y-axis. The only invariant point in all three graphs is $(0, 0)$.

6. a)



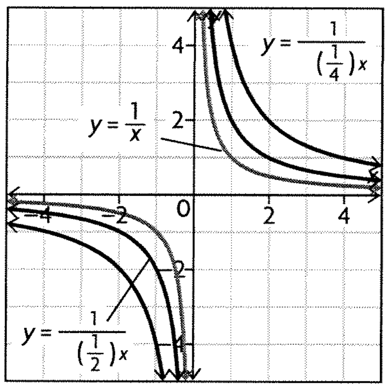
The graph of $y = \frac{1}{2}x^2$ is stretched horizontally by a factor of 2. The graph of $y = \frac{1}{3}x^2$ is stretched horizontally by a factor of 3. The only invariant point in all three graphs is $(0, 0)$.

b)



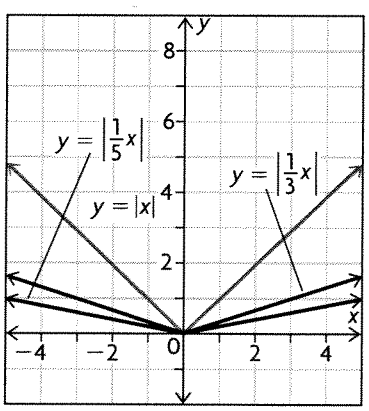
The graph of $y = \sqrt{\frac{1}{3}x}$ is stretched horizontally by a factor of 3. The graph of $y = \sqrt{\frac{1}{4}x}$ is stretched horizontally by a factor of 4. The only invariant point in all three graphs is $(0, 0)$.

c)



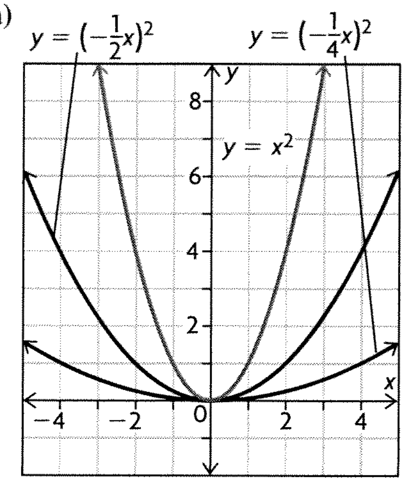
The graph of $y = \frac{1}{\frac{1}{2}}x$ is stretched horizontally by a factor of 2. The graph of $y = \frac{1}{\frac{1}{4}}x$ is stretched horizontally by a factor of 4. There are no invariant points.

d)



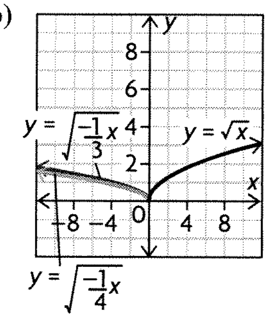
The graph of $y = \left|\frac{1}{3}x\right|$ is stretched horizontally by a factor of 3. The graph of $y = \left|\frac{1}{5}x\right|$ is stretched horizontally by a factor of 5. The only invariant point in all three graphs is $(0, 0)$.

7. a)



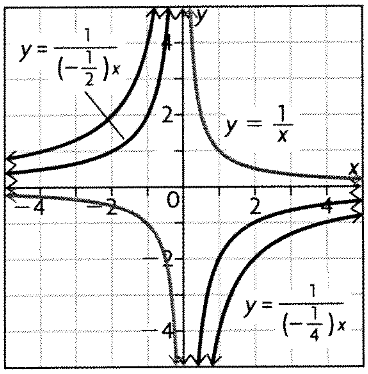
The graph of $y = \left(-\frac{1}{2}x\right)^2$ is stretched horizontally by a factor of 2 and is reflected about the y-axis. The graph of $y = \left(-\frac{1}{4}x\right)^2$ is stretched horizontally by a factor of 4 and is reflected about the y-axis. The only invariant point in all three graphs is $(0, 0)$.

b)



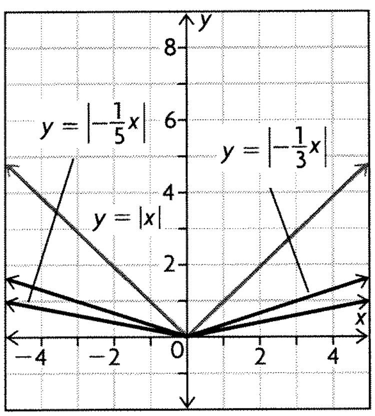
The graph of $y = \sqrt{\frac{-1}{3}x}$ is stretched horizontally by a factor of 3 and is reflected about the y-axis. The graph of $y = \sqrt{\frac{-1}{4}x}$ is stretched horizontally by a factor of 4 and is reflected about the y-axis. The only invariant point in all three graphs is $(0, 0)$.

c)



The graph of $y = \frac{1}{\frac{1}{-2}}x$ is stretched horizontally by a factor of 2 and is reflected about the y-axis.
The graph of $y = \frac{1}{\frac{1}{-4}}x$ is stretched horizontally by a factor of 4 and is reflected about the y-axis.
There are no invariant points.

d)



The graph of $y = \left|\frac{-1}{3}x\right|$ is stretched horizontally by a factor of 3 and is reflected about the y-axis.
The graph of $y = \left|\frac{-1}{5}x\right|$ is stretched horizontally by a factor of 5 and is reflected about the y-axis.
The only invariant point in all three graphs is $(0, 0)$.

8. a) The parent function is $f(x) = |x|$ and it has been compressed by a factor of 2, so the equation of the new function is $f(x) = |2x|$.

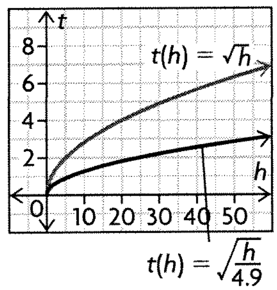
b) The parent function is $f(x) = \frac{1}{x}$ and it has been reflected about y-axis and stretched horizontally by a factor of 2, so the equation of the new function is $f(x) = \frac{1}{-2x}$.

c) The parent function is $f(x) = x^2$ and it has been stretched horizontally by a factor 4, so the equation of the new function is
$f(x) = \left(\frac{1}{4}x\right)^2$.

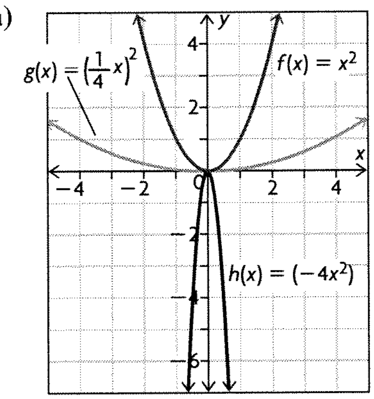
d) The parent function is $y = \sqrt{x}$ and it has been reflected about the y-axis and compressed by a factor of 3, so the equation of the new function is $y = \sqrt{-3x}$.

9. a) The domain of the function is the set of non-negative real numbers. The range of the function is the set of non-negative real numbers

b)

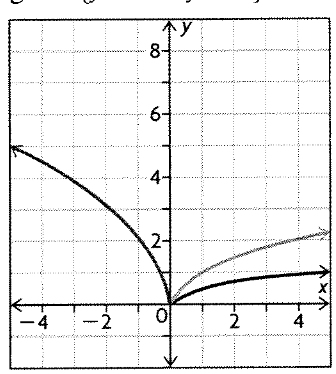


10. a)



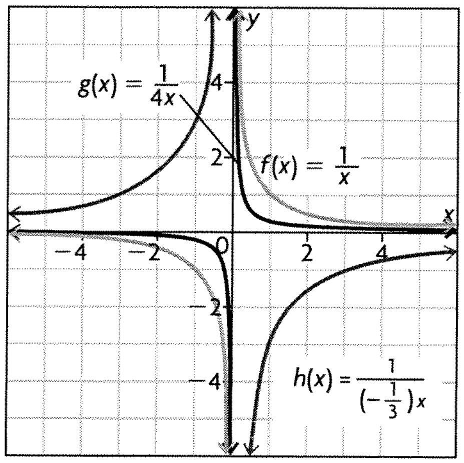
$g(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b)



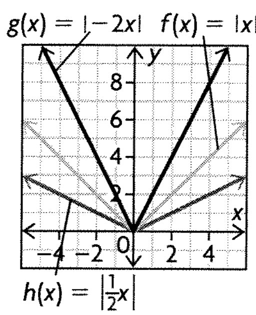
$g(x)$: domain $= \{x \in \mathbf{R} \mid x \geq 0\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R} \mid x \leq 0\}$,
range $= \{y \in \mathbf{R} \mid y \leq 0\}$

c)



$g(x)$: domain $= \{x \in \mathbf{R} \mid x \neq 0\}$,
range $= \{y \in \mathbf{R} \mid y \neq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R} \mid x \neq 0\}$,
range $= \{y \in \mathbf{R} \mid y \neq 0\}$

d)



$g(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

11. a) $k = \frac{1}{4}$.

b) $k = 2$

c) $k = -1$

d) $k = -5$

12. a) If $f(x) = x^2 - x - 6$, and $y = f(2x)$, then $y = 4x^2 - 2x - 6 = (2x - 3)(2x + 2)$. So, it has x-intercepts at

$2x - 3 = 0$ and $2x + 2 = 0$

$2x = 3 \qquad 2x = -2$

$x = 1.5 \qquad x = -1$

b) If $f(x) = x^2 - x - 6$, and $y = f\left(\frac{1}{3}x\right)$, then $y = \frac{1}{9}x^2 - \frac{1}{3}x - 6 = \left(\frac{1}{3}x - 3\right)\left(\frac{1}{3}x + 2\right)$. So, it has x-intercepts at

$\frac{1}{3}x - 3 = 0$ and $\frac{1}{3}x + 2 = 0$

$\frac{1}{3}x = 3 \qquad \frac{1}{3}x = -2$

$x = 9 \qquad x = -6$

c) If $f(x) = x^2 - x - 6$, and $y = f(-3x)$, then $y = 9x^2 + 3x - 6 = (3x + 3)(3x - 2)$. So, it has x-intercepts at

$3x + 3 = 0$ and $3x - 2 = 0$

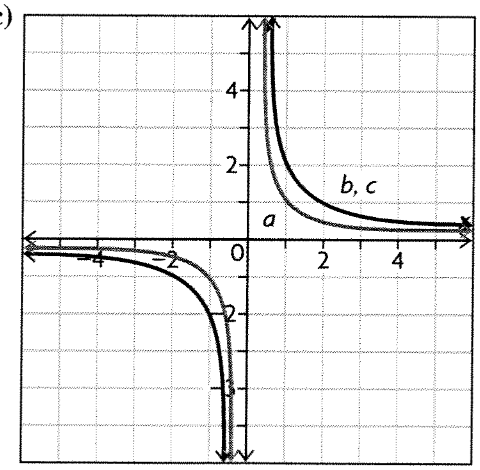
$3x = -3 \qquad 3x = 2$

$x = -1 \qquad x = \frac{2}{3}$

13. a) For $k > 1$, the effect is a horizontal compression with scale factor $\frac{1}{k}$; for $0 < k < 1$, the effect is a horizontal stretch with a scale factor of $\frac{1}{k}$; for $k < 0$, the effect is a reflection about the y-axis and a horizontal compression or stretch with a scale factor of $\frac{1}{|k|}$. Apply these transformations to the graph of $y = f(x)$ to sketch the graph of $y = f(kx)$.

b) Answers may vary. For example, a horizontal compression or stretch is equivalent to a vertical stretch or compression, respectively; scale factors are reciprocals of each other for some functions but not for others.

14. a)–c)



c) The horizontal and vertical stretches give the same graph.

d) $y = \frac{1}{\frac{1}{2}x}$, $y = 2\left(\frac{1}{x}\right)$ Both equations simplify to $y = \frac{2}{x}$.

15. The two transformations required are a translation of 4 units left, then a horizontal compression of factor $\frac{1}{2}$. The order the two are applied does matter.

1.8 Using Transformations to Graph Functions of the Form $y = af[k(x - d)] + c$, pp. 70–73

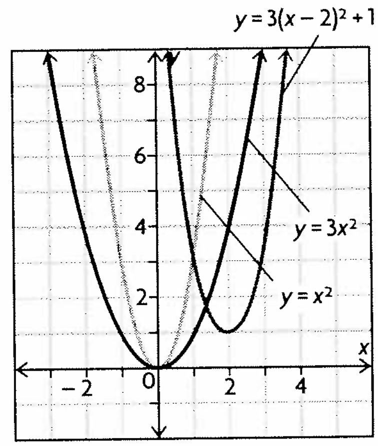
1. A) Multiplying the entire equation by 5 results in a vertical stretch of factor 5.
B) Multiplying the value inside the parent function by -1 results in a reflection over the y-axis.
C) Multiplying the value inside the parent function by 3 results in a horizontal compression of factor $\frac{1}{3}$.
D) Subtracting 2 from x results in a translation 2 units to the right.
E) Adding 4 to the entire equation results in a translation 4 units up.

2. Divide the x-coordinates by 3: C
Multiply the y-coordinates by 5: A
Multiply the x-coordinates by -1: B
Add 4 to the y-coordinate: E
Add 2 to the x-coordinate: D

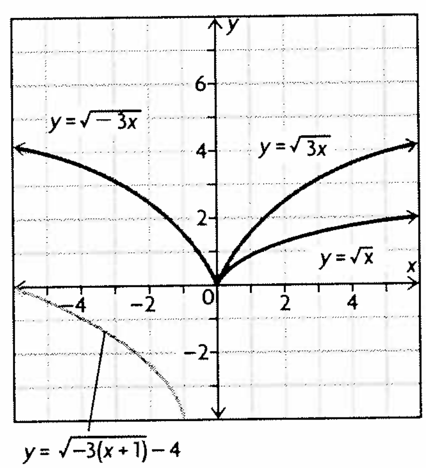
3. $f(3x)$ is a horizontal compression of factor 3, and so results in the point $\left(\frac{1}{3}, 1\right)$.
$f(-3x)$ is the reflection of $f(3x)$over the y-axis, and so results in the point $\left(-\frac{1}{3}, 1\right)$
$5f(-3x)$ is a vertical stretch of factor 5 of $f(-3x)$, and so results in the point $\left(-\frac{1}{3}, 5\right)$
$5f[-3(x - 2)] + 4$ is a translation of $5f(-3x)$ 2 units to the right and 4 units up, and so results in the point $\left(1\frac{2}{3}, 9\right)$

4. a) Vertical stretch, factor 3, then translation 1 unit down
b) Translation 2 units right and 3 units up
c) Horizontal compression of factor $\frac{1}{2}$, then translation 5 units down
d) Reflection in x-axis, horizontal stretch with factor 2, and then translation 2 units down
e) Vertical compression of factor $\frac{2}{3}$, then translation 3 units left and 1 unit up
f) Vertical stretch with factor 4, reflection in y-axis, and then translation 4 units down

5. a)

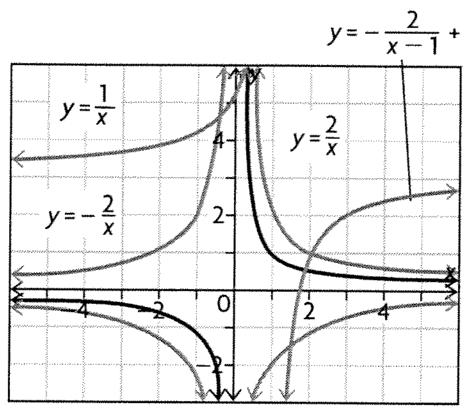


b)

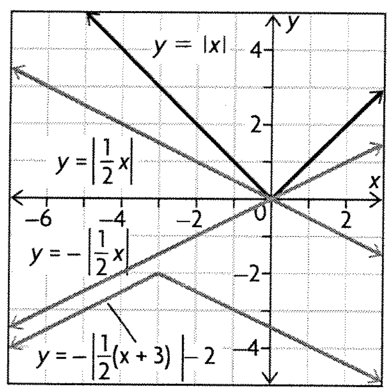


$f(x)$	$f(3x)$	$f(-3x)$	$5f(-3x)$	$5f[-3(x - 2)] + 4$
(1, 1)	$\left(\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, 1\right)$	$\left(-\frac{1}{3}, 5\right)$	$\left(1\frac{2}{3}, 9\right)$

c)



d)

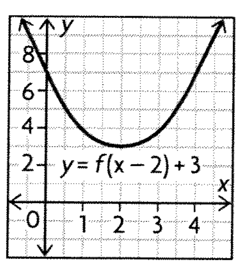


6. a) Horizontal stretch of factor 3, then a translation 4 units to the left.

b) Vertical stretch of factor 2, then a reflection over the y-axis, then a translation 3 units to the right and 1 unit up.

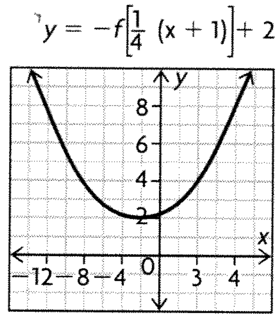
c) Vertical stretch of factor 3, then a reflection over the x-axis, then a horizontal compression of factor $\frac{1}{2}$, then a translation 1 unit to the right and 3 units down.

7. a) Transformation from parent graph: translation 2 units to the right and 3 units up.



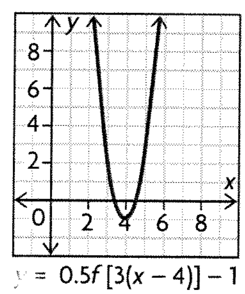
The vertex is now at (2, 3), and the parabola still opens upward, so domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq 3\}$.

b) Transformation from parent graph: reflection in the x-axis, horizontal stretch of factor 4, translation 1 unit to the left and 2 units up.



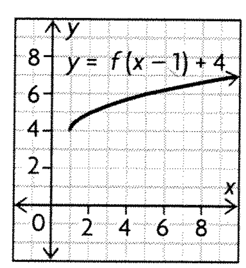
The vertex is now at $(-1, 2)$, and the parabola now opens downward, so domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \leq 2\}$.

c) Transformation from parent graph: vertical compression of factor $\frac{1}{2}$, horizontal compression of factor $\frac{1}{3}$, translation 4 units to the right and 1 unit down.



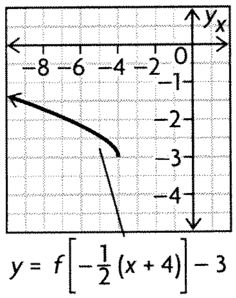
The vertex is now at $(4, -1)$, and the parabola still opens upward, so domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R} \mid y \geq -1\}$.

8. a) Transformation from parent graph: translation 1 unit to the right and 4 units up.



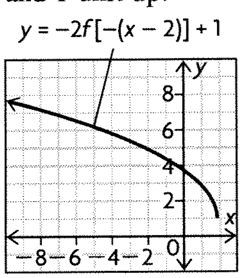
The usual domain and range for a square root graph is all non-negative real numbers, but this graph has been translated 1 unit to the right and 4 units up, so domain = $\{x \in \mathbf{R} \mid x \geq 1\}$, range = $\{y \in \mathbf{R} \mid y \geq 4\}$

b) Transformation from parent graph: reflection in the y-axis, horizontal stretch of factor 2, translation 4 units to the left and 3 units down.



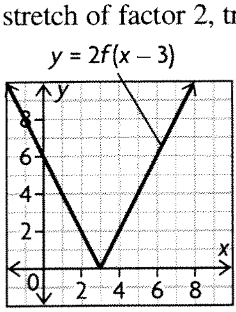
The usual domain and range for a square root graph is all non-negative real numbers, but this graph has been reflected in the y-axis and translated 4 units to the left and 3 units down, so domain $= \{x \in \mathbf{R} \mid x \leq -4\}$, range $= \{y \in \mathbf{R} \mid y \geq -3\}$

c) Transformation from parent graph: reflection in the x-axis, vertical stretch of factor 2, reflection in the y-axis, translation 2 units to the right and 1 unit up.



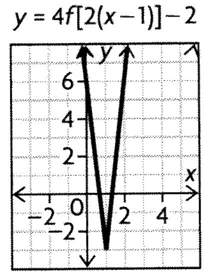
The usual domain and range for a square root graph is all non-negative real numbers, but this graph has been reflected in the x- and y-axes and translated 2 units to the right and 1 unit up, so domain $= \{x \in \mathbf{R} \mid x \leq 2\}$, range $= \{y \in \mathbf{R} \mid y \leq 1\}$

9. a) Transformation from parent graph: vertical stretch of factor 2, translation 3 units to the right.



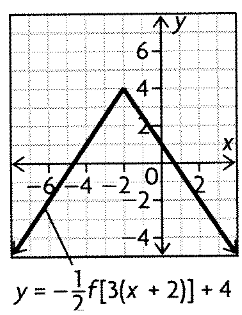
The vertex is now at (3, 0), and the graph still opens upward, so domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$.

b) Transformation from parent graph: vertical stretch of factor 4, horizontal compression of factor $\frac{1}{2}$, translation 1 unit to the right and 2 units down.



The vertex is now at $(1, -2)$ and the graph still opens upward, so domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -2\}$.

c) Transformation from parent graph: reflection in the x-axis, vertical compression of factor, horizontal compression of factor, translation 2 units to the left and 4 units up.



The vertex is now at $(-2, -4)$, and the graph now opens downward, so domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 4\}$.

10. a) Translation 2 units to the right.

b) Translation 2 units up.

c) Vertical compression of factor $\frac{1}{2}$

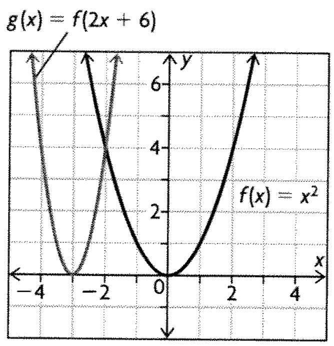
d) Vertical stretch of factor 2

e) Horizontal compression of factor $\frac{1}{2}$

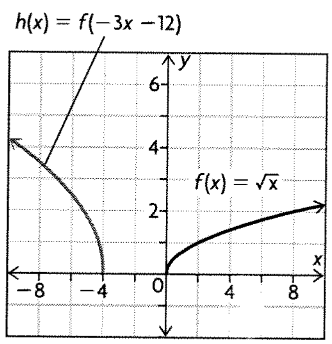
f) Reflection over the x-axis

11. To determine the transformations necessary to form this graph, the equation must be rewritten. Factoring out a 2 inside the function gives $f(2x + 6) = f[2(x + 3)]$

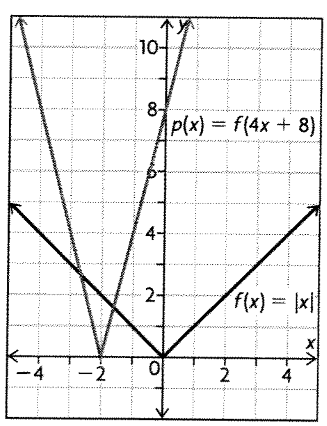
So the new function will be the old function under a horizontal compression of factor $\frac{1}{2}$ and a translation 3 units to the left:



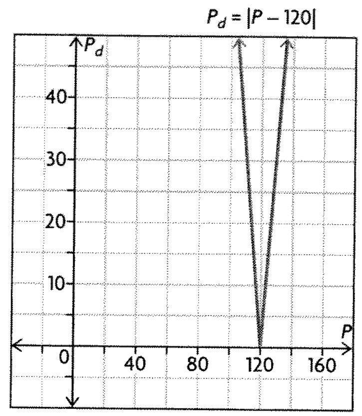
12. To determine the transformations necessary to form this graph, the equation must be rewritten. Factoring out a -3 inside the function gives
$f(-3x - 12) = f[-3(x + 4)]$
So the new function will be the old function under a reflection in the y-axis, a horizontal compression of factor $\frac{1}{3}$, and a translation 4 units to the left:



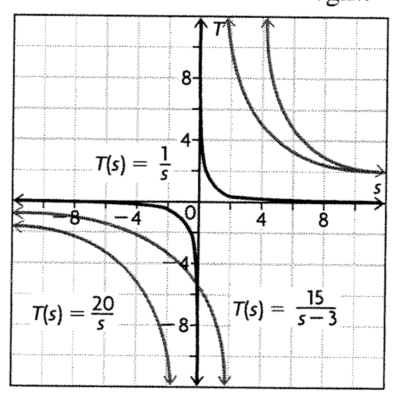
13. To determine the transformations necessary to form this graph, the equation must be rewritten. Factoring out a 4 inside the function gives
$f(4x + 8) = f[4(x + 2)]$
So the new function will be the old function under a horizontal compression of factor $\frac{1}{4}$ and a translation 2 units to the left:



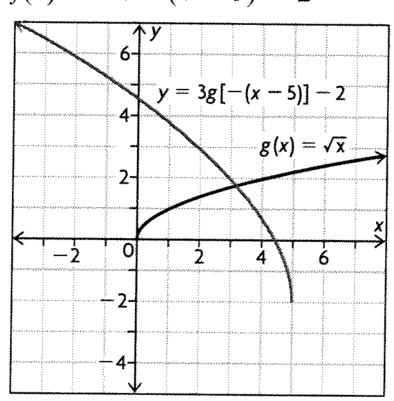
14. From the given information, $P_d = |P - 120|$, so the graph will be the parent graph $P_d = |P|$ translated 120 units to the right:



15. The graph of $T(s) = \frac{20}{s}$ is the graph of the parent function $T(s) = \frac{1}{s}$ under a vertical stretch of factor 20. The graph of $T(s) = \frac{15}{s - 3}$ is the graph of the parent function $T(s) = \frac{1}{s}$ under a vertical stretch of factor 15 and a translation 3 units to the right:



16. The equation for this function is
$f(x) = 3\sqrt{-(x - 5)} - 2$



17. $g(x) = 3f[-(x + 1)] + 2$

18. a) C; parent graph is $y = \frac{1}{x}$, asymptotes are translated 2 units right and 1 unit up, and graph has been reflected in one of the axes

b) E; parent graph is $y = |x|$, and vertex is translated 3 units right and 2 units down

c) A; parent graph is $y = \sqrt{x}$, graph has been reflected in y-axis, and vertex is translated 3 units left and 2 units down

d) G; parent graph is $y = x^2$, and vertex is translated 2 units right and 3 units down

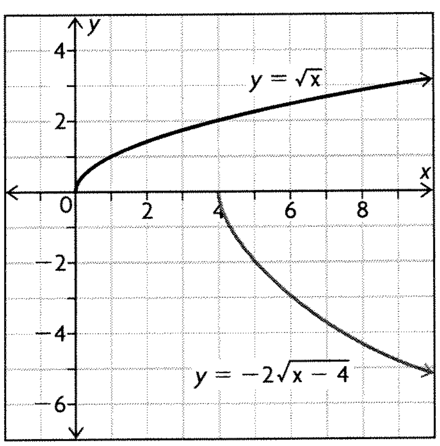
e) F; parent graph is $y = \frac{1}{x}$, asymptotes are translated 3 units down, and graph has been reflected in one of the axes

f) D; parent graph is $y = |x|$, graph has been reflected in y-axis, and vertex is translated 4 units left and 2 units up

g) H; parent graph is $y = \sqrt{x}$, graph has been reflected in x- and y-axes, and vertex is translated 1 unit right and 1 unit up

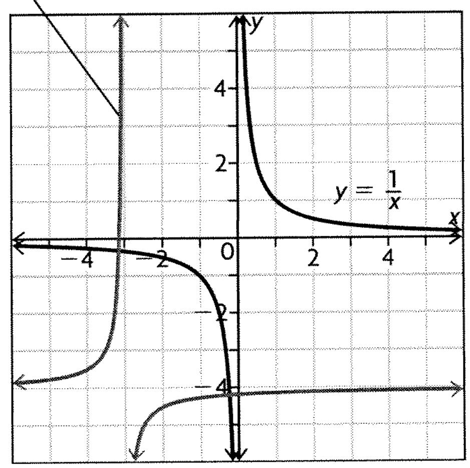
h) B; parent graph is $y = x^2$, graph has been reflected in y-axis, and vertex is translated 4 units left and 1 unit up

19. a) A vertical stretch of factor 2 and a reflection over the x-axis means that $a = -2$. A translation 4 units to the right means that $d = 4$. No further transformations are given, so $k = 1$ and $c = 0$. Graph:



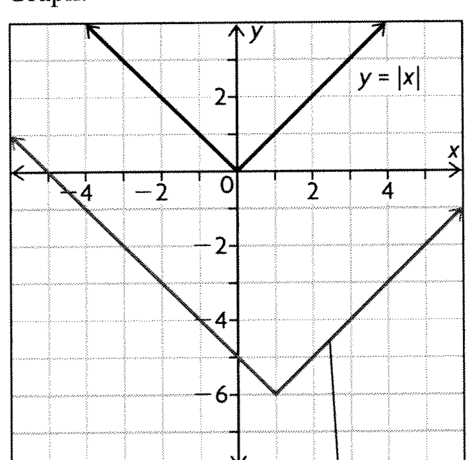
The usual domain and range for a square root graph is all non-negative real numbers, but this graph has been reflected in the x-axis and translated 4 units to the right, so domain $= \{x \in \mathbf{R} \mid x \geq 4\}$, range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b) A vertical compression of factor $\frac{1}{2}$ means that $a = \frac{1}{2}$. A reflection in the y-axis means that $k = -1$. A translation 3 units left and 4 units down means that $c = -3$ and $d = -4$. Graph:



The usual domain and range for $y = \frac{1}{x}$ is all real numbers excluding zero. Under the translation 3 units left and 4 units down, domain $= \{x \in \mathbf{R} \mid x \neq -3\}$, range $= \{y \in \mathbf{R} \mid y \neq -4\}$

c) A horizontal compression of factor $\frac{1}{3}$ means that $k = \frac{1}{3}$. A vertical stretch of factor 3 means that $a = 3$. A translation 1 unit right and 6 units down means that $c = 1$ and $d = -6$. Graph:



The vertex is now at $(1, -6)$, and the graph still opens upward, so domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -6\}$.

20. a) The x-intercepts are the points at which $f(x) = 0$. Given
$0 = (x - 2)(x + 5)$, the solutions are
$0 = x - 2$ or $0 = x + 5$
$x = 2,$ $x = -5$

b) This equation is the first equation under a vertical stretch of factor 4 and a reflection over the x-axis. Neither of these transformations affect the y-value of the function at any point along the x-axis, so the zeroes remain at $x = 2$ and $x = -5$.

c) This equation is the first equation under a horizontal stretch of factor 3 and a reflection in the y-axis. This will multiply the roots by 3 and change the sign of each, so the new zeroes are $x = -6$ and $x = 15$.

d) This equation is the first equation under a translation 2 units to the left. This will shift the zeroes to $x = -4$ and $x = 3$.

21. A. Sketch parent function; B. Apply reflections in x-axis if $a < 0$ and in y-axis if $k < 0$; apply vertical stretch or compression with factor $|a|$, and horizontal stretch or compression with factor $\frac{1}{|k|}$; D. Translate c units right (or $-c$ units left if $c < 0$) and d units up (or $-d$ units down if $d < 0$). Transformations in steps B and C can be done in any order, but must precede translation in step D.

22. a) Reflection in the x-axis, vertical compression of factor $\frac{1}{4}$ (or horizontal stretch of factor 2), then translation 3 units to the left and 1 unit up.

b) $y = -\frac{1}{4}(x + 3)^2 + 2$, or

$$y = -\left[\frac{1}{2}(x + 3)\right]^2 + 2$$

23. Answers may vary. For example, graphs are both based on a parabola, but open in different directions, and graph of $g(x)$ is only an upper half-parabola. $g(x)$ is the reflection of the right half of the graph of $f(x)$ in the line $y = x$.

Chapter Review, pp. 76–77

1. a) Domain $= \{-3, -1, 0, 4\}$, range $= \{0, 1, 5, 6\}$; not a function, because two y-values are assigned to $x = 0$.

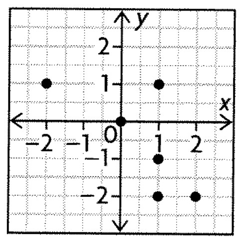
b) Domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$; function because each x-value has only one y-value assigned.

c) Domain $= \{x \in \mathbf{R} \mid x \geq -4\}$, range $= \{y \in \mathbf{R}\}$; not a function, because each $x > -4$ has two y-values assigned.

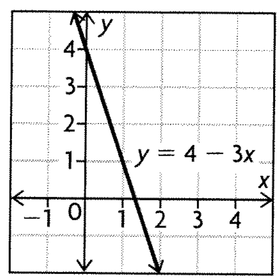
d) Domain $\{x \in \mathbf{R} \mid -4 \leq x \leq 4\}$, range $= \{y \in \mathbf{R} \mid -4 \leq y \leq 4\}$; not a function, because each x except $x = \pm 4$ has two y-values assigned.

2. The vertical line test, which states that if any vertical line intersects a graph in more than one place, that graph does not represent a function.

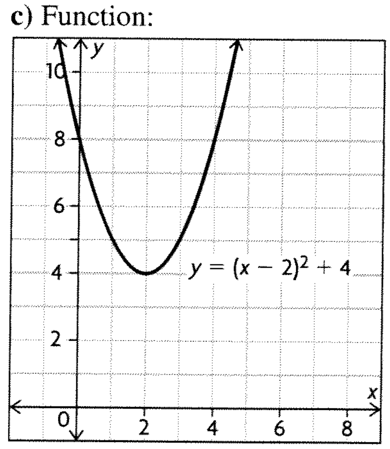
a) Not a function:



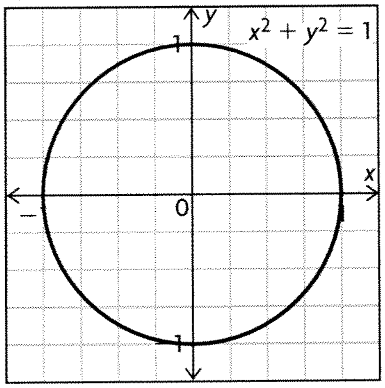
b) Function:



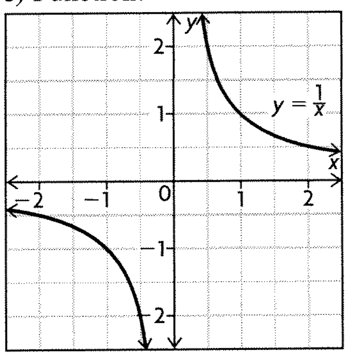
c) Function:



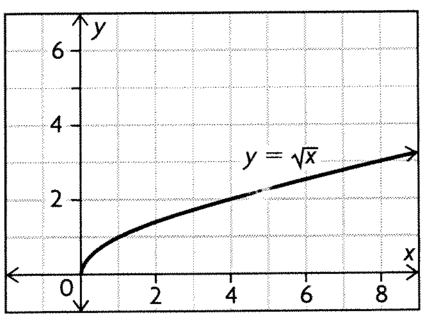
d) Not a function:



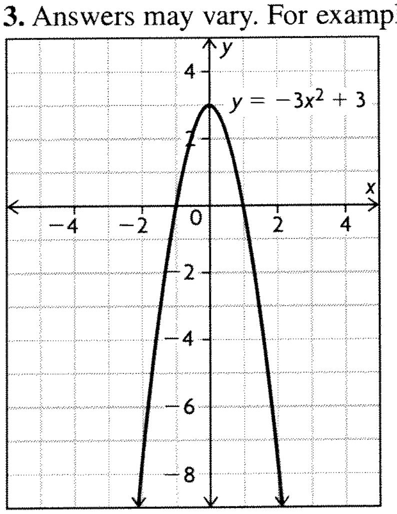
e) Function:



f) Function:



3. Answers may vary. For example:



4. a) $f(-1) = (-1)^2 + 3(-1) - 5$
$= 1 + -3 - 5$
$= -7$

b) $f(0) = (0)^2 + 3(0) - 5$
$= 0 + 0 - 5$
$= -5$

c) $g\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) - 3$
$= 1 - 3$
$= -2$

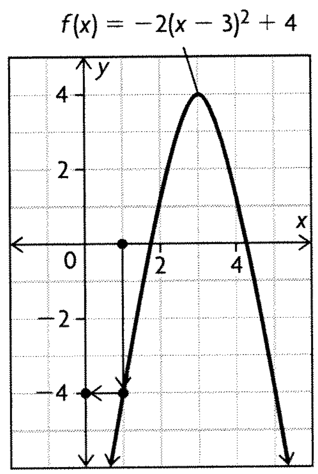
d) $f(2b) = (2b)^2 + 3(2b) - 5$
$= 4b^2 + 6b - 5$

e) $g(1 - 4a) = 2(1 - 4a) - 3$
$= 2 - 8a - 3$
$= -1 - 8a$

f) At the point where $f(x) = g(x)$,
$x^2 + 3x - 5 = 2x - 3$
$x^2 + x - 2 = 0$

$$x = \frac{-1 \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$$
$$= \frac{-1 \pm 3}{2}$$
$$= \frac{2}{2} \text{ or } \frac{-4}{2}$$
$$= 1 \text{ or } -2$$

5. a), b)



a) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the vertex is at (3, 4) and the parabola is downward-facing. This means that the range includes only the real numbers less than or equal to 4.
Range $= \{y \in \mathbf{R} \mid y \leq 4\}$

b) $f(1)$ represents the y-coordinate of the point with x-coordinate 1.

c) i) $f(3) - f(2)$

$= -2(3-3)^2 + 4 - (-2(2-3)^2 + 4$
$= -2(0)^2 + 4 - (-2(-1)^2 + 4)$
$= -2(0) + 4 - (-2(1) + 4)$
$= 0 + 4 - -2 - 4$
$= 2$

ii) $2f(5) + 7 = 2(-2(5-3)^2 + 4) + 7$
$= 2(-2(2)^2 + 4) + 7$
$= 2(-2(4) + 4) + 7$
$= 2(-8 + 4) + 7$
$= 2(-4) + 7$
$= -1$

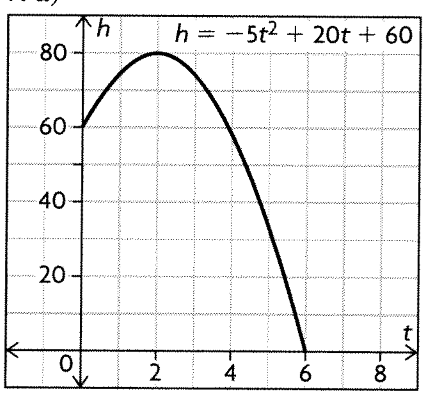
iii) $f(2 - x) = -2(1 - x - 3)^2 + 4$
$= -2(-2 - x)^2 + 4$

6. The x-values where $f(x) = 8$ are the values for which

$8 = x^2 - 4x + 3$
$0 = x^2 - 4x - 5$

$$x = \frac{4 \pm \sqrt{(4)^2 - 4(1)(-5)}}{2(1)}$$
$$= \frac{4 \pm 6}{2}$$
$$= \frac{10}{2} \text{ or } \frac{-2}{2}$$
$$= 5 \text{ or } -1$$

7. a)



b) It takes 6 seconds for the ball to hit the ground, so the domain is $\{t \in \mathbf{R} \mid 0 \le t \le 6\}$ The height of the ball ranges from 0 metres (the ball can't go below the ground) to 80 metres (the maximum, which was given), so the range is $\{h \in \mathbf{R} \mid 0 \le h \le 80\}$

c) Starting with the general equation for a quadratic function: $h(t) = a(t - t_v)^2 + h_v$, where t_v and h_v are the coordinates of the vertex. The problem states that the vertex occurs at $(2, 80)$, so $t_v = 2$ and $h_v = 80$. The other point given, $(6, 0)$, can be used to determine a.

$0 = a(6-2)^2 + 80$
$0 = a(4)^2 + 80$
$-80 = 16a$
$a = -5$

So the trajectory of the ball is described by the equation $h(t) = -5(t-2)^2 + 80$.

8. a) For a quadratic function, domain $= \{x \in \mathbf{R}\}$. To find the range, note that the vertex is at $(1, 3)$ and the parabola is downward-facing. This means that the range includes only the real numbers less than or equal to 4. Range $= \{y \in \mathbf{R} \mid y \ge 3\}$

b) The value under a square root must be positive, so the domain will include only real numbers greater than or equal to -2. A square root cannot be negative, so the range will be non-negative real numbers.

Domain $= \{x \in \mathbf{R} \mid x \ge -2\}$,
range $= \{y \in \mathbf{R} \mid y \ge 0\}$

9. a) Let the width of the fence be w and the length be l. The total amount of fencing used is 540 metres, so

$540 = 2l + 3w$
$2l = 540 - 3w$
$$l = \frac{540 - 3w}{2}$$

Now, the area of the field is

$A = lw$
$$= \left(\frac{540 - 3w}{2}\right)w$$

b) Neither the length nor the width of the field can be less than or equal to 0, so the domain is bounded by $w > 0$ and

$$\frac{540 - 3w}{2} > 0$$
$540 > 3w$
$180 > w$

So domain $= \{w \in \mathbf{R} \mid 0 < w < 180\}$ The area of the field must be greater than zero, so the range is bounded below by $A > 0$. The area function is quadratic with the zeroes at $w = 0$ and $w = 180$, so the maximum area occurs halfway between them, at $w = 90$. In this case,

$$A = \left(\frac{540 - 3(90)}{2}\right)(90)$$
$$= \left(\frac{540 - 270}{2}\right)(90)$$
$$= \left(\frac{270}{2}\right)(90)$$
$$= 135(90)$$
$$= 12\,150$$

So the upper bound of the range is $A \leq 12\,150$.

Range $= \{A \in \mathbf{R} \mid 0 < A \leq 12\,150\}$

c) The explanation in part (b) showed that the maximum area occurs at $w = 90$ m. This means that

$$l = \left(\frac{540 - 3(90)}{2}\right)$$
$$= \left(\frac{540 - 270}{2}\right)$$
$$= \left(\frac{270}{2}\right)$$
$$= 135 \text{ m}$$

10. a) Graph $y = 2x - 5$, and reflect it in the line $y = x$ to get graph of inverse. Use graph to determine the slope-intercept form of inverse; slope is 0.5 and y-intercept is 2.5, so $f^{-1}(x) = 0.5x + 2.5$.

b) Switch x and y, then solve for y:

$$x = \frac{y + 3}{7}$$
$$7x = y + 3$$
$$y = 7x - 3$$

c) Reverse operations: for f, divide by 2 and subtract from 4, so for f^{-1}, subtract from 4 (operation is self-inverse) and multiply by 2. Therefore, $f^{-1}(x) = 2(4 - x)$.

11. a) $f(x) = 30x + 15\,000$

b) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 15\,000\}$; number of people cannot be negative, and income cannot be less than corporate sponsorship.

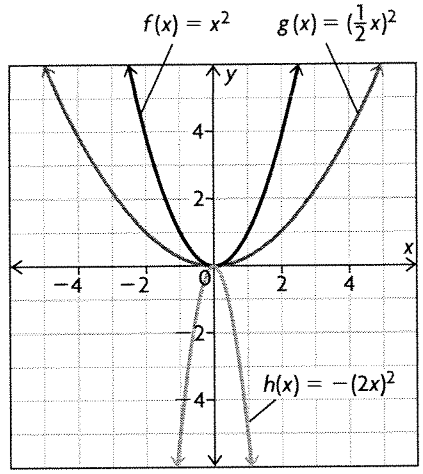
c)
$$x = 30f^{-1}(x) + 15\,000$$
$$x - 15\,000 = 30f^{-1}(x)$$
$$f^{-1}(x) = \frac{x - 15\,000}{30}$$

domain $= \{x \in \mathbf{R} \mid x \geq 15\,000\}$

12. a) The parent function is $y = \sqrt{x}$. The new function is compressed by a factor of $\frac{1}{4}$, so the equation of the new function is $y = \sqrt{4x}$.

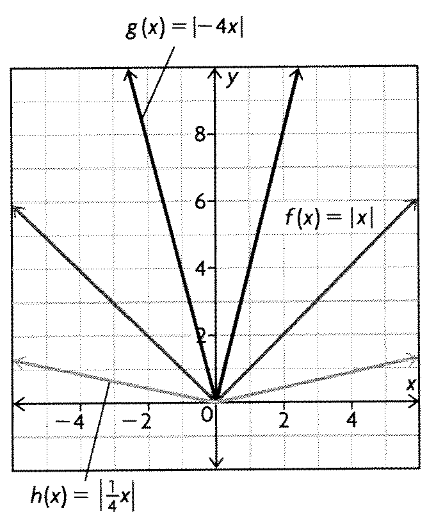
b) The parent function is $y = \frac{1}{x}$. The new function is reflected over the y-axis and stretched horizontally by a factor of 5. So, the equation of the new function is $y = \frac{1}{\frac{-1}{5}x}$.

13. a)



$f(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$g(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \leq 0\}$

b)



$f(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;

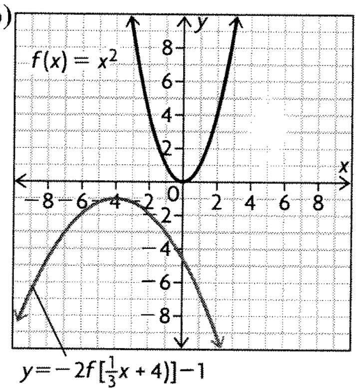
$g(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$;
$h(x)$: domain $= \{x \in \mathbf{R}\}$,
range $= \{y \in \mathbf{R} \mid y \geq 0\}$

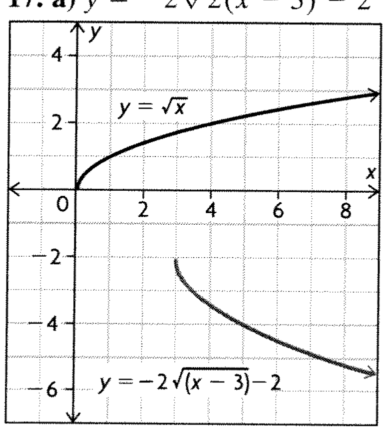
14. a) Yes, the translations must be done last.
b) Yes, if the order of the two translations were switched, it would produce the same result.

15. To find the image point, apply the transformations found in the new equation to the original point.
Step 1: Apply the vertical stretch with factor of 3 to the point to get $(1, 12)$.
Step 2: Reflect in the y-axis to get $(-1, 12)$.

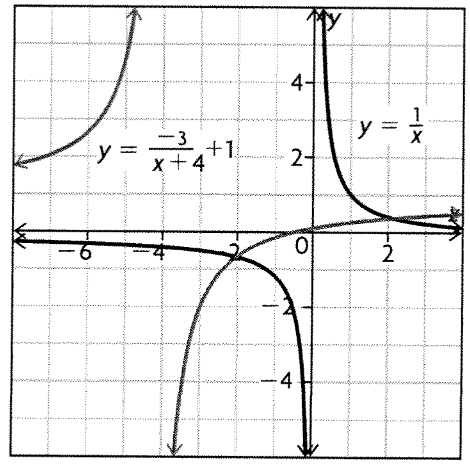
Step 3: Compress horizontally by a factor of 4 to get $(-4, 12)$.
Step 4: Translate 1 unit left and 2 units down to get $(-5, 10)$.

16. a) First the equation must be rewritten as $y = -2f\left[\frac{1}{3}(x + 12)\right] - 1$. To get this equation from the old one, first reflect in the x-axis, then stretch vertically by a factor of 2, stretch horizontally by a factor of 3, then translate left 4 units and down 1 unit.

b)


17. a) $y = -2\sqrt{2(x - 3)} - 2$


domain $= \{x \in \mathbf{R} \mid x \geq 3\}$,
range $= \{y \in \mathbf{R} \mid y \leq -2\}$

b) $y = \dfrac{-3}{x + 4} + 1$


domain $= \{x \in \mathbf{R} \mid x \neq -4\}$,
range $= \{y \in \mathbf{R} \mid y \neq 1\}$

18. a) The x-intercepts are the points at which $f(x) = 0$. Given
$0 = (x - 4)(x + 3)$, the solutions are
$0 = x - 4$ or $0 = x + 3$
$x = 4$, $x = -3$

b) This equation is the first equation under a vertical stretch of factor 2 and a reflection in the x-axis. Neither of these transformations affect the y-value of the function at any point along the x-axis, so the zeroes remain at $x = 4$ and $x = -3$.

c) This equation is the first equation under a horizontal stretch of factor 2 and a reflection in the y-axis. This will multiply the roots by 2 and change the sign of each, so the new zeroes are $x = -8$ and $x = 6$.

d) This equation is the first equation under a reflection in the y-axis and a translation 1 unit to the left. This will shift the zeroes to $x = -5$ and $x = 2$.

19. a) This is a vertical stretch by a factor of 2, so it expands the upper bound of the range by a factor of 2.
Domain $= \{x \in \mathbf{R} \mid x \geq -4\}$,
range $= \{y \in \mathbf{R} \mid y < -2\}$

b) This is a reflection in the y-axis, so it will change the sign of the bound of the domain, and the direction of the inequality.
Domain $= \{x \in \mathbf{R} \mid x \leq 4\}$,
range $= \{y \in \mathbf{R} \mid y < -1\}$

c) This is a vertical stretch of 3, followed by translations of left 1 unit and up 4 units.
Domain = $\{x \in \mathbf{R} \mid x \geq -5\}$,
range = $\{y \in \mathbf{R} \mid y < 1\}$

d) First, rewrite the equation $y = -2f(-x - 5) + 1$. This is a reflection in both the x- and y-axes, so it will change the signs of the bounds of the domain and range, and the direction of their inequalities. There is also a vertical stretch by a factor of 2, followed by translations of 5 right and 1 up.
Domain = $\{x \in \mathbf{R} \mid x \leq -1\}$,
range = $\{y \in \mathbf{R} \mid y > 3\}$

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1. a) Domain = $\{-5, -2, 0, 3\}$, range = $\{-1, 1, 7\}$; function, because each x-value has only one y-value assigned.

b) The value under a square root must be positive, so the domain will include only real numbers greater than or equal to -2. A square root cannot be negative, so the range will be non-negative real numbers. Domain = $\{x \in \mathbf{R} \mid x \geq -2\}$range = $\{y \in \mathbf{R} \mid y \geq 0\}$. This is a function because each x-value has only one y-value.

2. a) For each bulb, the cost of using it is represented by a fixed cost plus an hourly rate. So if $f(x)$ is the cost of using the incandescent bulb and $g(x)$ is the cost of using the fluorescent bulb, then
$f(x) = 0.004x + 0.65$, and
$g(x) = 0.001x + 3.50$

b) The time that a bulb is used cannot be negative, so domain = $\{x \in \mathbf{R} \mid x \geq 0\}$ for both bulbs. The minimum cost for using each bulb is the cost of the bulb itself and there is no upper cost for either, so range = $\{f \in \mathbf{R} \mid y \geq 0.65\}$ for the incandescent bulb, domain = $\{x \in \mathbf{R} \mid x \geq 0\}$ and range = $\{g \in \mathbf{R} \mid y \geq 3.50\}$ for the fluorescent bulb.

c) The fluorescent bulb will be cheaper for

$$\begin{aligned} 0.004x + 0.65 &> 0.001x + 3.50 \\ 0.003x &> 2.85 \\ x &> 950 \end{aligned}$$

So the fluorescent bulb is cheaper when it is used for more than 950 hours.

d) Using the bulb for 6 hours per day for a year is a total of 2190 hours. The difference in price will be

$$\begin{aligned} f(2190) - g(2190) &= 0.002(2190) + 0.65 \\ &\quad - (0.001(2190) + 3.50) \\ &= 8.76 + 0.65 \\ &\quad - (2.19 + 3.50) \\ &= 3.72 \end{aligned}$$

So the fluorescent bulb is \$3.72 cheaper over the course of a year.

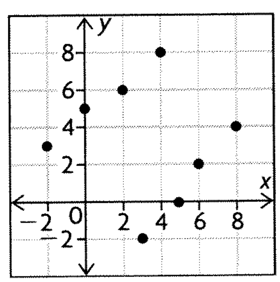
3. a) The usual domain and range for $y = \frac{1}{x}$ is all real numbers excluding zero. Under the translation 2 units to the right, domain = $\{x \in \mathbf{R} \mid x \neq 2\}$, range = $\{y \in \mathbf{R} \mid y \neq 0\}$

b) The value under a square root must be positive, so the domain will include only real numbers less-than or equal to 3. A square root cannot be negative, so the range will be the real number less than or equal to -4. Domain = $\{x \in \mathbf{R} \mid x \leq 3\}$, range = $\{y \in \mathbf{R} \mid y \geq -4\}$

c) The vertex is at $(-1, 3)$, and the graph opens downward, so domain = $\{x \in \mathbf{R}\}$ range = $\{y \in \mathbf{R} \mid y \leq 3\}$.

4. The inverse of a linear function is either the linear function obtained by reversing the operations of the original function, or if the original function is $f(x) = c$ constant, the relation $x = c$. Domain and range are exchanged for the inverse.

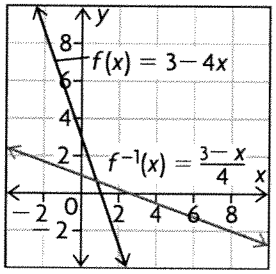
5. a) $\{(3, -2), (5, 0), (6, 2), (8, 4)\}$



Function: domain = $\{-2, 0, 2, 4\}$,
range = $\{3, 5, 6, 8\}$;
Inverse: domain $\{3, 5, 6, 8\}$, range $\{-2, 0, 2, 4\}$

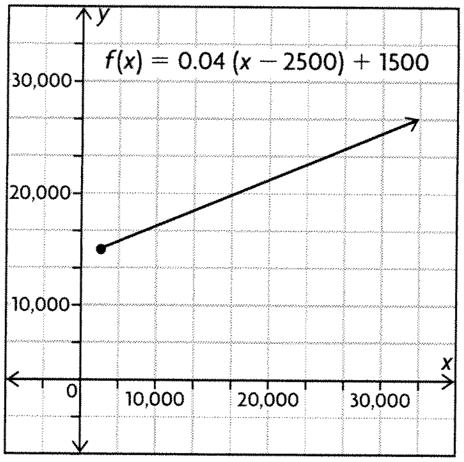
b)

$$\begin{aligned} x &= 3 - 4f^{-1}(x) \\ x - 3 &= -4f^{-1}(x) \\ f^{-1}(x) &= \frac{3 - x}{4} \end{aligned}$$



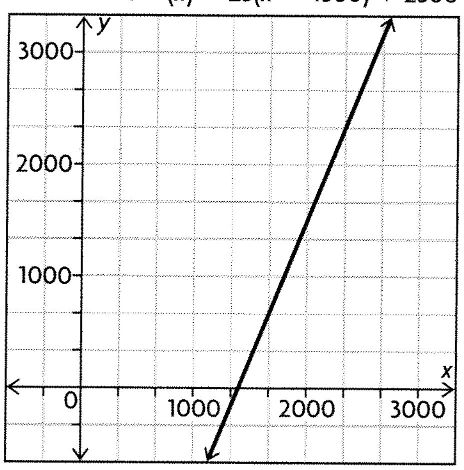
Function: domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$; inverse: domain = $\{x \in \mathbf{R}\}$, range = $\{y \in \mathbf{R}\}$

6. a)



b) $f(x) = 0.04(x - 2500) + 1500$ for $x \geq 2500$

c) $f^{-1}(x) = 25(x - 1500) + 2500$



d)
$$x = 0.04(f^{-1}(x) - 2500) + 1500$$
$$x - 1500 = 0.04(f^{-1}(x) - 2500)$$
$$25(x - 1500) = f^{-1}(x) - 2500$$
$$f^{-1}(x) = 25(x - 1500) + 2500$$
$$x \geq 1500$$

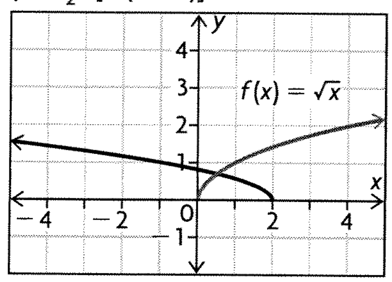
e) $f^{-1}(1740) = 25(1740 - 1500) + 2500$ for $= \$51\,000$

7. a) $k = \frac{1}{5}$

b) $k = -3$

8. a) A vertical stretch of factor $\frac{1}{2}$ means that $a = \frac{1}{2}$. A translation 2 units to the right means that $d = 2$. A reflection in the y-axis means $k = -1$. No further transformations are given, so $c = 0$.

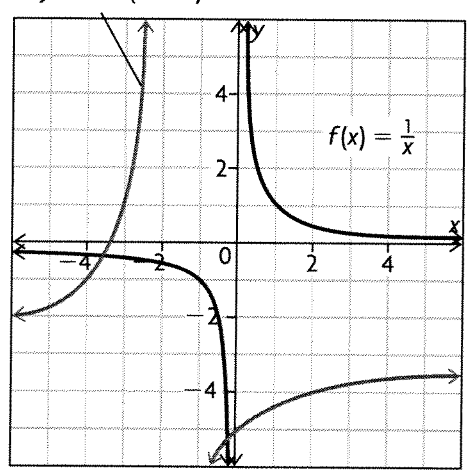
$y = \frac{1}{2}f[-(x - 2)]$



The usual domain and range for a square root graph is all non-negative real numbers, but this graph has been reflected in the y-axis and translated 2 units to the right, so domain = $\{x \in \mathbf{R} \mid x \leq 2\}$, range = $\{y \in \mathbf{R} \mid y \geq 0\}$

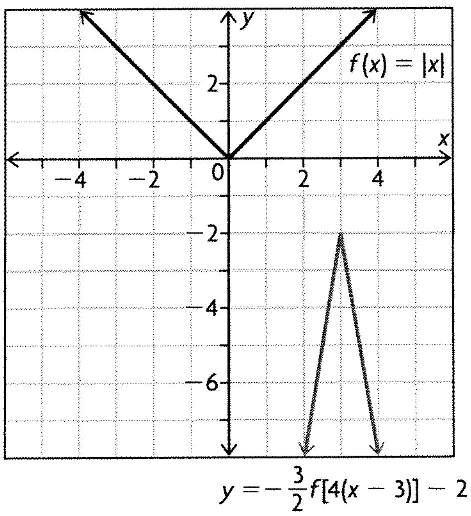
b) A vertical stretch of factor 4 and a reflection over the x-axis means that $a = -4$. A translation 2 units left and 3 units down means that $c = -3$ and $d = -2$. No further transformations are given, so $k = 1$

$y = -4f(x + 2) - 3$



The usual domain and range for $y = \frac{1}{x}$ is all real numbers excluding zero. Under the translation 2 units left and 3 units down, domain = $\{x \in \mathbf{R} \mid x \neq -2\}$, range = $\{y \in \mathbf{R} \mid y \neq -3\}$

c) A horizontal compression of factor $\frac{1}{4}$ means that $k = 4$. A vertical stretch of factor $\frac{3}{2}$ and a reflection in the x-axis means that $a = -\frac{3}{2}$. A translation 3 units right and 2 units down means that $c = -2$ and $d = 3$.



The vertex is now at $(3, -2)$ and the graph opens downward, so domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq -2\}$