Chapter 2: Equivalent Algebraic Expressions

Getting Started, p. 82

1. a) $4x - 7$ has 2 terms so it is a binomial; the greatest degree is 1, so $4x - 7$ has degree 1.
b) 3 has 1 term so it is a monomial; the degree of a constant (3) is 0.
c) $7 + 2x^2$ has 2 terms so it is a binomial; the greatest degree is 2 so $7 + 2x^2$ has a degree of 2.
d) $x^2 - 3xy + y^2$ has 3 terms so it is a trinomial; the greatest degree is 2, so $x^2 - 3xy + y^2$ has a degree of 2.
2. a) Group the like terms and add.
$$\begin{aligned}(2x + 3) + (7x - 5) &= 2x + 7x + 3 - 5\\ &= 9x - 2\end{aligned}$$
b) Find the additive inverse of $2x^2 - 3x + 10$. Then group the like terms and add.
$$\begin{aligned}&(4x^2 - 7x + 1) - (2x^2 - 3x + 10)\\ &= (4x^2 - 7x + 1) + (-2x^2 + 3x - 10)\\ &= 4x^2 - 2x^2 - 7x + 3x + 1 - 10\\ &= 2x^2 - 4x - 9\end{aligned}$$
c) To multiply two binomials, find the sum of the products of the first terms, the outer terms, the inner terms, and the last terms.
$$\begin{aligned}&(2x - 3)(4x + 5)\\ &= (2x)(4x) + (2x)(5)\\ &\quad + (-3)(4x) + (-3)(5)\\ &= 8x^2 + 10x - 12x - 15\\ &= 8x^2 - 2x - 15\end{aligned}$$
d) The square of a difference
$(a - b)^2 = a^2 - 2ab + b^2$.
$$\begin{aligned}(2x - 1)^2 &= (2x)^2 - 2(2x)(1) + (1)^2\\ &= 4x^2 - 4x + 1\end{aligned}$$
3. Factoring is the opposite of expanding. To expand a polynomial, you multiply using the distributive property. To factor, you try to determine the polynomials that multiply together to give you the given polynomial. For example, $(x + 2)(3x - 1)$ expands to $3x^2 + 5x - 2$; $3x^2 + 5x - 2$ factors to $(x + 2)(3x - 1)$
4. a) Find the GCF for $6xy^3 - 8x^2y^3$. Then factor out the GCF from $6xy^3 - 8x^2y^3$.
$$6xy^3 = 2 \cdot 3 \cdot x \cdot y \cdot y \cdot y$$
$$8x^2y^3 = 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot y \cdot y \cdot y$$
$$\text{GCF} = 2 \cdot x \cdot y \cdot y \cdot y \text{ or } 2xy^3$$
$$6xy^3 - 8x^2y^3 = 2xy^3(3 - 4x)$$
b) You need to find two integers whose product is 10 and whose sum is -7. The factors of 10 are 1 and 10, -1 and -10, 2 and 5, and -2 and -5. The sum of -2 and -5 is -7, so select the factors -2 and -5. Group terms that have a common monomial factor. Then factor the GCF from each group. Finally, use the distributive property.
$$\begin{aligned}a^2 - 7a + 10 &= a^2 + (-2 + -5)a + 10\\ &= a^2 - 2a - 5a + 10\\ &= (a^2 - 2a) + (-5a + 10)\\ &= a(a - 2) - 5(a - 2)\\ &= (a - 2)(a - 5)\end{aligned}$$
c) You need to find two integers whose product is $12(-10)$, and whose sum is 7.
$12(-10) = -1 \cdot 2 \cdot 2 \cdot 3 \cdot 2 \cdot 5$. Try factors of -8 and 15.
$$\begin{aligned}12n^2 + 7n - 10 &= 12n^2 + 15n - 8n - 10\\ &= (12n^2 + 15n) - (8n + 10)\\ &= 3n(4n + 5) - 2(4n + 5)\\ &= (4n + 5)(3n - 2)\end{aligned}$$
d) $9 - 25x^2$ is a difference of squares. The difference of squares rule states that
$a^2 - b^2 = (a - b)(a + b)$. So,
$9 - 25x^2 = (3 - 5x)(3 + 5x)$.
e) Find two integers whose product is 8 and whose sum is 5. Factors of 8 are 1 and 8, -1 and -8, 2 and 4, and -2 and -4. None of the factor pairs have a sum of 5. So, it is not possible to factor $x^2 + 5x + 8$.
f) Find two integers whose product is -36 and whose sum is -5. Factors of -36 are -1 and 36, 1 and -36, -2 and 18, 2 and -18, -3 and 12, 3 and -12, -4 and 9, 4 and -9, and -6 and 6. The sum of 4 and -9 is -5.
$$\begin{aligned}y^2 - 5y - 36 &= (y^2 - 9y) + (4y - 36)\\ &= y(y - 9) + 4(y - 9)\\ &= (y - 9)(y + 4)\end{aligned}$$

5. a) Rename the fractions using the LCD, 12. Then add.

$$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12}$$
$$= \frac{11}{12}$$

b) Rename the fractions using the LCD, 10. Then subtract.

$$-\frac{2}{5} - \frac{1}{10} = -\frac{4}{10} - \frac{1}{10}$$
$$= \frac{2}{5} - \frac{5}{10} \text{ or } -\frac{1}{2}$$

c) Multiply the numerators and multiply the denominators. Then simplify.

$$\left(-\frac{12}{25}\right)\left(-\frac{10}{9}\right) = \frac{-12 \cdot -10}{25 \cdot 9}$$
$$= \frac{120}{225} \div 15$$
$$= \frac{8}{15}$$

d) Find the inverse of the divisor and multiply.

$$\left(\frac{4}{3}\right) \div \left(-\frac{2}{15}\right) = \left(\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{3}}}\right) \times \left(-\frac{\overset{5}{\cancel{15}}}{\underset{-1}{\cancel{2}}}\right)$$
$$= -\frac{10}{1} \text{ or } -10$$

6. a) You can multiply powers with the same base by adding exponents.

$$\left(\frac{2x^2}{3}\right)\left(\frac{5x^3}{4}\right) = \frac{2x^2 \cdot 5x^3}{3 \cdot 4}$$
$$= \frac{\overset{5}{\cancel{10}}x^{2+3}}{\underset{6}{\cancel{12}}}$$
$$= \frac{5x^5}{6}$$

b) To divide by a fraction, multiply by its inverse. You can divide powers with the same base by subtracting exponents.

$$\left(\frac{3x}{2}\right) \div \left(\frac{x^3}{5}\right) = \left(\frac{3x}{2}\right) \times \left(\frac{5}{x^3}\right)$$
$$= \frac{15x}{2x^3}$$
$$= \frac{15x^{1-3}}{2}$$
$$= \frac{15x^{-2}}{2} \text{ or } \frac{15}{2x^2}$$

c) You can multiply powers with the same base by adding exponents.

$$(2x^2y^3)(4xy^2) = (2 \cdot 4)(x^2 \cdot x)(y^3 \cdot y^2)$$
$$= 8(x^{2+1})(y^{3+2})$$
$$= 8x^3y^5$$

d) To divide powers with the same base, subtract exponents.

$$(25x^5y^3) \div (5x^2y) = \frac{25x^5y^3}{5x^2y}$$
$$= 5x^{5-2}y^{3-1}$$
$$= 5x^3y^2$$

7. a) $f(x) = x$

There are no values where the function is undefined. The domain is the set of all real numbers: $\{x \in \mathrm{R}\}$

b) $g(x) = 2x^2$

There are no values where the function is undefined. The domain is the set of all real numbers: $\{x \in \mathrm{R}\}$

c) $m(x) = \sqrt{x}$

The value of x must be positive, so the function is undefined for $x < 0$.

Domain: $\{x \in \mathrm{R} \mid x \geq 0\}$

d) $h(x) = \frac{1}{x}$

Since division by 0 is undefined, the function is undefined for $x = 0$.

Domain: $\{x \in \mathrm{R} \mid x \neq 0\}$

e) $j(x) = \frac{3}{x - 4}$

$4 - 4 = 0$, so the function is undefined for $x = 4$.

Domain: $\{x \in \mathrm{R} \mid x \neq 4\}$

f) $m(x) = \sqrt{x + 10}$

$x + 10$ must be ≥ 0, so the function is undefined for $x < -10$.

Domain: $\{x \in \mathrm{R} \mid x \geq -10\}$

8.

Definition:	**Characteristics:**
A polynomial is any algebraic expression that contains one or more terms	• usually contains variables • can contain both like and unlike terms • exponents must be whole numbers
Polynomial	
Examples:	**Non-examples:**
$3x^2$ $4x - 3$ $5x^2 - 2xy + 6y$	$\frac{5-x}{3+x}$ $5x^{-2} + 4x + 3$ $\sqrt{x}$

2.1 Adding and Subtracting Polynomials, pp. 88–90

1. a) Collect like terms. Then simplify.
$(3x^2 - 7x + 5) + (x^2 - x + 3)$
$= 3x^2 + x^2 - 7x - x + 5 + 3$
$= 4x^2 - 8x + 8$
b) Collect like terms. Then simplify.
$(x^2 - 6x + 1) - (-x^2 - 6x + 5)$
$= x^2 - 6x + 1 + x^2 + 6x - 5$
$= x^2 + x^2 - 6x + 6x + 1 - 5$
$= 2x^2 - 4$
c) $(2x^2 - 4x + 3) - (x^2 - 3x + 2) + (x^2 - 1)$
$= 2x^2 - 4x + 3 - x^2 + 3x - 2 + x^2 - 1$
$= 2x^2 - x^2 + x^2 - 4x + 3x + 3 - 2 - 1$
$= 2x^2 - x$
2. Simplify by collecting like terms.
$f(x) = (2x - 1) - (3 - 4x) + (x + 2)$
$= 2x - 1 - 3 + 4x + x + 2$
$= 2x + 4x + x - 1 - 3 + 2$
$= 7x - 2$
$g(x) = (-x + 6) + (6x - 9) - (-2x - 1)$
$= -x + 6 + 6x - 9 + 2x + 1$
$= -x + 6x + 2x + 6 - 9 + 1$
$= 7x - 2$
Both simplify to the same expression, so they are equivalent functions.
3. Substitute the value 1 for x. Then compare the results.
$f(x) = 2(x - 3) + 3(x - 3)$
$f(1) = 2(1 - 3) + 3(1 - 3)$
$= 2(-2) + 3(-2)$
$= -4 - 6$
$= -10$
$g(x) = 5(2x - 6)$
$g(1) = 5(2 \cdot 1 - 6)$
$= 5(-4)$
$= -20$
$-10 \neq -20$, so the functions are not equivalent.
4. a) $(2a + 4c + 8) + (7a - 9c - 3)$
$= 2a + 7a + 4c - 9c + 8 - 3$
$= 9a - 5c + 5$
b) $(3x + 4y - 5z) + (2x^2 + 6z)$
$= 2x^2 + 3x + 4y - 5z + 6z$
$= 2x^2 + 3x + 4y + z$
c) $(6x + 2y + 9) + (-3x - 5y - 8)$
$= 6x - 3x + 2y - 5y + 9 - 8$
$= 3x - 3y + 1$
d) $(2x^2 - 7x + 6) + (x^2 - 2x - 9)$
$= 2x^2 + x^2 - 7x - 2x + 6 - 9$
$= 3x^2 - 9x - 3$
e) $(-4x^2 - 2xy) + (6x^2 - 3xy + 2y^2)$
$= -4x^2 + 6x^2 - 2xy - 3xy + 2y^2$
$= 2x^2 - 5xy + 2y^2$
f) $(x^2 + y^2 + 8) + (4x^2 - 2y^2 - 9)$
$= x^2 + 4x^2 + y^2 - 2y^2 + 8 - 9$
$= 5x^2 - y^2 - 1$
5. a) $(m - n + 2p) - (3n + p - 7)$
$= m - n + 2p - 3n - p + 7$
$= m - 4n + p + 7$
b) $(-6m - 2q + 8) - (2m + 2q + 7)$
$= -6m - 2q + 8 - 2m - 2q - 7$
$= -8m - 4q + 1$
c) $(4a^2 - 9) - (a^3 + 2a - 9)$
$= 4a^2 - 9 - a^3 - 2a + 9$
$= -a^3 + 4a^2 - 2a$
d) $(2m^2 - 6mn + 8n^2) - (4m^2 - mn - 7n^2)$
$= 2m^2 - 6mn + 8n^2 - 4m^2 + mn + 7n^2$
$= -2m^2 - 5mn + 15n^2$
e) $(3x^2 + 2y^2 + 7) - (4x^2 - 2y^2 - 8)$
$= 3x^2 + 2y^2 + 7 - 4x^2 + 2y^2 + 8$
$= -x^2 + 4y^2 + 15$
f) $5x^2 - (2x^2 - 30) - (-20)$
$= 5x^2 - 2x^2 + 30 + 20$
$= 3x^2 + 50$
6. a) $(2x - y) - (-3x + 4y) + (6x - 2y)$
$= 2x - y + 3x - 4y + 6x - 2y$
$= 11x - 7y$
b) $(3x^2 - 2x) + (x^2 - 7x) - (7x + 3)$
$= 3x^2 - 2x + x^2 - 7x - 7x - 3$
$= 4x^2 - 16x - 3$
c) $(2x^2 + xy - y^2) - (x^2 - 4xy - y^2) + (3x^2 - 5xy)$
$= 2x^2 + xy - y^2 - x^2 + 4xy + y^2 + 3x^2 - 5xy$
$= 4x^2$
d) $(xy - xz + 4yz) + (2x - 3yz) - (4y - xz)$
$= xy - xz + 4yz + 2x - 3yz - 4y + xz$
$= 2x + xy - 4y + yz$
e) $\left(\frac{1}{2}x + \frac{1}{3}y\right) - \left(\frac{1}{5}x - y\right)$
$= \frac{1}{2}x + \frac{1}{3}y - \frac{1}{5}x + y$

$= \frac{1}{2}x - \frac{1}{5}x + \frac{1}{3}y + y$

$= \frac{3}{10}x + \frac{4}{3}y$

f) $\left(\frac{3}{4}x + \frac{1}{2}y\right) - \left(\frac{2}{3}x + \frac{1}{4}y - 1\right)$

$= \frac{3}{4}x + \frac{1}{2}y - \frac{2}{3}x - \frac{1}{4}y + 1$

$= \frac{1}{12}x + \frac{1}{4}y + 1$

7. i) Simplify the expressions.

$(3x^2 - x) - (5x^2 - x) = -2x^2$
$\neq -2x^2 - 2x$

They do not simplify to the same expression.

ii) Substitute a value for x. For example, if $x = 1$,

$(3x^2 - x) - (5x^2 - x) = (3 - 1) - (5 - 1)$
$= -2$

but $-2x^2 - 2x = -2 - 2$
$= -4$

The expressions result in different values, so they are not equivalent.

8. a) $f(x) = (2x^2 + 7x - 2) - (3x + 7)$
$= 2x^2 + 7x - 2 - 3x - 7$
$= 2x^2 + 4x - 9$

$g(x) = (x^2 + 12) + (x^2 + 4x - 17)$
$= x^2 + 12 + x^2 + 4x - 17$
$= 2x^2 + 4x - 5$

$f(x) = 2x^2 + 4x - 9$ and
$g(x) = 2x^2 + 4x - 5$
$\therefore f(x) \neq g(x)$

b) $s_1(t) = (t + 2)^3 \quad s_2(t) = t^3 + 8$
$s_1(1) = (1 + 2)^3 \quad s_2(1) = 1^3 + 8$
$= 27 \quad = 9$

$s_1(1) = 27$ and $s_2(1) = 9 \ \therefore \ s_1(t) \neq s_2(t)$

c) Substituting -1 for x:

$y_1 = (x - 1)(x)(x + 2) \quad y_2 = 3x(x^2 - 1)$
$= (-2)(-1)(1) \quad = -3(0)$
$= 2 \quad = 0$
$\therefore y_1 \neq y_2$

d) $f(n) = 0.5n^2 + 2n - 3 + (1.5n^2 - 6)$
$= 2n^2 + 2n - 9$

$g(n) = n^2 - n + 1 - (-n^2 - 3n + 10)$
$= 2n^2 + 2n - 9$
$\therefore f(n) = g(n)$

e) If $p = 1$ and $q = 1$, then

$y_1 = 3p(q - 2) + 2p(q + 5)$
$= 3(-1) + 2(6)$
$= 9$

$y_2 = p(q + 4)$
$= 5$
$\therefore y_1 \neq y_2$

f) Substituting 2 for m:

$f(m) = m(5 - m) - 2(2m - m^2)$
$f(2) = 2(3) - 2(0)$
$= 6$

$g(m) = 4m^2(m - 1) - 3m^2 + 5m$
$g(2) = 16(1) - 12 + 10$
$= 14$
$\therefore f(m) \neq g(m)$

9. Answers will vary. For example, $f(x) = 2x$ and $g(x) = x^2$

10. a) $25 - x - y$

b) $6x + 2y + 25 - x - y = 5x + y + 25$

c) $x = 13$ and from part **a,**

$25 - x - y = 7$
$25 - 13 - y = 7$
$5 = y$

from part **b,**

$5x + y + 25 = 65 + 5 + 25$
$= 95$

Kosuke's score was 95.

11. $s = 7x + 9y - 2(2x + 3y - 1)$
$= 3x + 3y + 2$

12. a) $P(x) = R(x) - C(x)$
$= -50x^2 + 2500x - (150x + 9500)$
$= -50x^2 + 2350x - 9500$

b) $P(x) = -50(12)^2 + 2350(12) - 9500$
$= -7200 + 28\,200 - 9500$
$= 11\,500$

The profit will be \$11 500.

13. a) cannot be determined
b) cannot be determined
c) not equivalent
d) cannot be determined
e) equivalent

14. a) yes; when two functions are equivalent, their graphs are identical. One of Ramy's graphs represented two of the functions.
b) Replace variables with numbers and simplify.

15. a) If x represents the first number, then the equation representing the calendar sums is:

$x + (x + 7) + (x + 14) + (x + 15) + (x + 16) = 5(x + 14) - 18$

Simplify both expressions:

$5x + 52 = 5x + 70 - 18$
$= 5x + 52$

b) $5x + 52 = 112$

$5x = 60$

$x = 12$

The corner number is $(x + 14)$ or 26.

c) $5x - 18$

16. a) $19 + 20 + 21 + 22 + 23$

b) $n = (m - 2) + (m - 1) + m + (m + 1) + (m + 2)$

c) $91 = 7m; m = 13$

$91 = 10 + 11 + 12 + 13 + 14 + 15 + 16$

17. a) Both functions are linear; a pair of linear functions intersect at only one point, unless they are equivalent; since the functions are equal at two values, they must be equivalent.

b) Both functions are quadratic; a pair of quadratic functions intersect at most in two points, unless they are equivalent; since the functions are equal at three values, they must be equivalent.

2.2 Multiplying Polynomials, pp. 95–97

1. a) Use the distributive property.

$2x(3x - 5x^2 + 4y) = 6x^2 - 10x^3 + 8xy$

b) To multiply two binomials, multiply the first, outer, inner, and last terms.

$(3x - 4)(2x + 5)$

$= (3x)(2x) + (3x)(5) + (-4)(2x) + (-4)(5)$

$= 6x^2 + 15x - 8x - 20$

$= 6x^2 + 7x - 20$

c) The square of a binomial $a + b$ is $a^2 + 2ab + b^2$.

$(x + 4)^2 = x^2 + 8x + 16$

d) Multiply each of the three terms in the trinomial by each of the terms in the binomial.

$(x + 1)(x^2 + 2x - 3)$

$= x(x^2 + 2x - 3) + 1(x^2 + 2x - 3)$

$= (x^3 + 2x^2 - 3x) + (x^2 + 2x - 3)$

$= x^3 + 2x^2 + x^2 - 3x + 2x - 3$

$= x^3 + 3x^2 - x - 3$

2. a) No; for $x = 1$,

$(3x + 2)^2 = (3(1) + 2)^2$

$= 5^2$

$= 25$

$9x^2 + 4 = 9(1)^2 + 4$

$= 9 + 4$

$= 13$

b) $(3x + 2)^2 = (3x + 2)(3x + 2)$

$= (3x)(3x) + 2(2)(3x) + 2(2)$

$= 9x^2 + 12x + 4$

3. a) $(2x + 4)(3x^2 + 6x - 5)$

$= 2x(3x^2) + 2x(6x) + 2x(-5) + 4(3x^2) + 4(6x) + 4(-5)$

$= 6x^3 + 12x^2 - 10x + 12x^2 + 24x - 20$

$= 6x^3 + 24x^2 + 14x - 20$

b) $(2x + 4)(3x^2 + 6x - 5)$

$= 3x^2(2x) + 3x^2(4) + 6x(2x) + 6x(4) + (-5)(2x) + (-5)(4)$

$= 6x^3 + 12x^2 + 12x^2 + 24x - 10x - 20$

$= 6x^3 + 24x^2 + 14x - 20$

4. a) $5x(5x^2 + 3x - 4)$

$= 5x(5x^2) + 5x(3x) + 5x(-4)$

$= 25x^3 + 15x^2 - 20x$

b) $(x - 6)(2x + 5)$

$= x(2x) + x(5) + (-6)(2x) + (-6)(5)$

$= 2x^2 + 5x - 12x - 30$

$= 2x^2 - 7x - 30$

c) $(x + 3)(x - 3) + (5x - 6)(3x - 7)$

$= (x^2 - 9) + (15x^2 - 35x - 18x + 42)$

$= 16x^2 - 53x + 33$

d) $4(n - 4)(3 + n) - 3(n - 5)(n + 8)$

$= 4(n^2 - n - 12) - 3(n^2 + 3n - 40)$

$= 4n^2 - 4n - 48 - 3n^2 - 9n + 120$

$= n^2 - 13n + 72$

e) $3(2x - 1)^2 - 5(4x + 1)^2$

$= 3(4x^2 - 4x + 1) - 5(16x^2 + 8x + 1)$

$= 12x^2 - 12x + 3 - 80x^2 - 40x - 5$

$= -68x^2 - 52x - 2$

f) $2(3a + 4)(a - 6) - (3 - a)^2 + 4(5 - a)$

$= 2(3a^2 - 14a - 24) - (9 - 6a + a^2) + 20 - 4a$

$= 6a^2 - 28a - 48 - 9 + 6a - a^2 + 20 - 4a$

$= 5a^2 - 26a - 37$

5. a) $4x(x + 5)(x - 5) = 4x(x^2 - 25)$

$= 4x^3 - 100x$

b) $-2a(a + 4)^2 = -2a(a^2 + 8a + 16)$

$= -2a^3 - 16a^2 - 32a$

c) $(x + 2)(x - 5)(x - 2)$

$= (x + 2)(x^2 - 7x + 10)$

$= x(x^2 - 7x + 10) + 2(x^2 - 7x + 10)$

$= x^3 - 7x^2 + 10x + 2x^2 - 14x + 20$

$= x^3 - 5x^2 - 4x + 20$

d) $(2x + 1)(3x - 5)(4 - x)$
$= (6x - 7x - 5)(4 - x)$
$= 4(6x^2 - 7x - 5) - x(6x^2 - 7x - 5)$
$= 24x^2 - 28x - 20 - 6x^3 + 7x^2 + 5x$
$= -6x^3 + 31x^2 - 23x - 20$

e) $(9a - 5)^3$
$= (9a - 5)(9a - 5)(9a - 5)$
$= (9a - 5)(81a^2 - 90a + 25)$
$= 9a(81a^2 - 90a + 25)$
$\quad - 5(81a^2 - 90a + 25)$
$= 729a^3 - 810a^2 + 225a - 405a^2$
$\quad + 450a - 125$
$= 729a^3 - 1215a^2 + 675a - 125$

f) $(a - b + c - d)(a + b - c - d)$
$= a(a + b - c - d) - b(a + b - c - d)$
$\quad + c(a + b - c - d) - d(a + b - c - d)$
$= a^2 + ab - ac - ad - ab - b^2 + bc$
$\quad + bd + ac + bc - c^2 - cd - ad - bd$
$\quad + cd + d^2$
$= a^2 - 2ad - b^2 + 2bc - c^2 + d^2$

6. a) yes
Let $x = 1$:
$(3x - 2)(2x - 1) = (1)(1) = 1$
$3x(2x - 1) - 2(2x - 1) = 3(1) - 2(1)$
$= 1$

b) yes
Let $x = 1$:
$(x - 4)(2x^2 + 5x - 6) = (-3)(1)$
$= -3$
$2x^2(x - 4) + 5x(x - 4) - 6(x - 4)$
$= 2(-3) + 5(-3) - 6(-3)$
$= -3$

c) no
Let $x = 1$:
$(x + 2)(3x - 1) - (1 - 2x)^2$
$= (3)(2) - (-1)^2$
$= 5$
$x^2 + 9x - 3 = 1 + 9 - 3$
$= 7$

d) yes
$2(x - 3)(2x^2 - 4x + 5)$
$= (2x - 6)(2x^2 - 4x + 5)$
$= 4x^3 - 8x^2 + 10x - 12x^2 + 24x - 30$
$= 4x^3 - 20x^2 + 34x - 30$

e) no
$(4x + y - 3)^2$
$= 16x^2 + 4xy - 12x + 4xy + y^2 - 3y$
$\quad - 12x - 3y + 9$
$= 16x^2 + 8xy - 24x + y^2 - 6y + 9$

f) yes
Let $x = y = 1$
$3(y - 2x)^3 = 3(-1)^3$
$= -3$
$- 24x^3 + 36x^2y - 18xy^2 + 3y^3$
$= -24 + 36 - 18 + 3$
$= -3$

7. All real numbers. Expressions are equivalent.

8. a) Start with the first two factors.
$19(5x + 7)(3x - 2) = (95x + 133)(3x - 2)$
$= 285x^2 + 209x - 266$
Start with the last two factors.
$19(5x + 7)(3x - 2) = 19(15x^2 + 11x - 14)$
$= 285x^2 + 209x - 266$

9. a) $2\pi r^2 + 2\pi rh$
$= 2\pi(2x + 1)^2 + 2\pi(2x + 1)(2x - 1)$
$= 2\pi(4x^2 + 4x + 1) + 2\pi(4x^2 - 1)$
$= 8\pi x^2 + 8\pi x + 2\pi + 8\pi x^2 - 2\pi$
$= 16\pi x^2 + 8\pi x$

b) $\pi r^2 h = \pi(2x + 1)^2(2x - 1)$
$= \pi(2x + 1)(4x^2 - 1)$
$= \pi(8x^3 + 4x^2 - 2x - 1)$
$= 8\pi x^3 + 4\pi x^2 - 2\pi x - \pi$

10. a) yes
Let $x = 1$:
$(x - 3)^2 = 4$ and $(3 - x)^2 = 4$

b) no; $x - 3 = -(3 - x)$. A negative number squared is positive (the same); a negative number cubed is negative (different).

11. a) $(x^2 + 2x - 1)^2$
$= (x^2 + 2x - 1)(x^2 + 2x - 1)$
$= x^4 + 2x^3 - x^2 + 2x^3 + 4x^2$
$\quad - 2x - x^2 - 2x + 1$
$= x^4 + 4x^3 + 2x^2 - 4x + 1$

b) $(2 - a)^3 = (2 - a)(4 - 4a + a^2)$
$= 8 - 8a + 2a^2 - 4a + 4a^2 - a^3$
$= 8 - 12a + 6a^2 - a^3$

c) $(x^3 + x^2 + x + 1)(x^3 - x^2 - x - 1)$
$= x^6 - x^5 - x^4 - x^3 + x^5 - x^4 - x^3 - x^2$
$\quad + x^4 - x^3 - x^2 - x + x^3 - x^2 - x - 1$
$= x^6 - x^4 - 2x^3 - 3x^2 - 2x - 1$

d) $2(x + 1)^2 - 3(2x - 1)(3x - 5)$
$= 2(x^2 + 2x + 1) - 3(6x^2 - 13x + 5)$
$= 2x^2 + 4x + 2 - 18x^2 + 39x - 15$
$= -16x^2 + 43x - 13$

12. $A_1 = \frac{1}{2}xy \quad A_2 = \frac{1}{2}(2x)\left(\frac{1}{2}y\right)$

$A_2 - A_1 = \frac{1}{2}xy - \frac{1}{2}(2x)\left(\frac{y}{2}\right)$

$= \frac{1}{2}xy - \frac{1}{2}xy$
$= 0$

13. a) $E = \frac{1}{2}(m + x)v^2$
$= \frac{1}{2}mv^2 + \frac{1}{2}xv^2$

b) $E = \frac{1}{2}m(v + y)^2$
$= \frac{1}{2}m(v^2 + 2vy + y^2)$
$= \frac{1}{2}mv^2 + mvy + \frac{1}{2}my^2$

14. a) 6; $2 \times 3 = 6$; $(x^7 + x^6)(x^9 + x^4 + 1)$ has 6 terms.
b) Multiply the number of terms in each polynomial.

15. a) i) 8 **ii)** 12 **iii)** 6 **iv)** 1
b) i) 8 **ii)** 96 **iii)** 384 **iv)** 512
c) i) 8 **ii)** $12(n - 2)$
iii) $6(n - 2)^2$ **iv)** $(n - 2)^3$
d) same answers

16. a) Answers may vary. For example, 115:
$11^2 + 11 = 132$
$115^2 = 13\,225$
b) $(10x + 5)^2 = 100x^2 + 100x + 25$ and $(x^2 + x)100 + 25$ are both the same

2.3 Factoring Polynomials, pp. 102–104

1. a) $x^2 - 6x - 27$ is a factorable trinomial of the form $ax^2 + bx + c$, where $a = 1$. Find two numbers whose sum is -6 and whose product is -27. Use -9 and 3.
$x^2 - 6x - 27 = (x - 9)(x + 3)$
b) $25x^2 - 49$ can be factored as the difference of squares.
$25x^2 - 49 = (5x + 7)(5x - 7)$
c) $4x^2 + 20x + 25$ can be factored as perfect squares.
$4x^2 + 20x + 25 = (2x + 5)^2$
d) $6x^2 - x - 2$ is a trinomial of the form $ax^2 + bx + c$, where $a \neq 1$, and has no common factor. Find two numbers whose sum is -1 and whose product is $(6)(2) = 12$. Use 3 and -4 to "decompose" the middle term.
$6x^2 - x - 2 = 6x^2 + 3x - 4x - 2$
$= 3x(2x + 1) - 2(2x + 1)$
$= (2x + 1)(3x - 2)$

2. a) $ac + bc - ad - bd$
$= (ac + bc) - (ad + bd)$
$= c(a + b) - d(a + b)$
$= (c - d)(a + b)$
b) $x^2 + 2x + 1 - y^2 = (x^2 + 2x + 1) - y^2$
$= (x + 1)^2 - y^2$
$= (x + y + 1)(x - y + 1)$
c) $x^2 - y^2 - 10y - 25$
$= (x)^2 - (y^2 - 10y - 25)$
$= (x)^2 - (y - 5)^2$
$= (x - y - 5)(x + y + 5)$

3. a) Find two numbers whose sum is -3 and whose product is -28. Use -7 and 4.
$x^2 - 3x - 28 = (x - 7)(x + 4)$
b) $36x^2 - 25$ is a difference of squares.
$36x^2 - 25 = (6x - 5)(6x + 5)$
c) $9x^2 - 42x + 49$ is a perfect square.
$9x^2 - 42x + 49 = (3x - 7)^2$
d) Find two numbers whose sum is -7 and whose product is $2(-15) = -30$. Use 3 and -10.
$2x^2 - 7x - 15 = 2x^2 + 3x - 10x - 15$
$= x(2x + 3) - 5(2x + 3)$
$= (2x + 3)(x - 5)$

4. a) $4x^3 - 6x^2 + 2x$
$= 2x(2x^2 - 3x + 1)$
$= 2x(2x^2 - x - 2x + 1)$
$= 2x(x(2x - 1) - (2x - 1))$
$= 2x(2x - 1)(x - 1)$
b) $3x^3y^2 - 9x^2y^4 + 3xy^3$
$= 3xy^2(x^2 - 3xy^2 + y)$
c) $4a(a + 1) - 3(a + 1) = (a + 1)(4a - 3)$
d) $7x^2(x + 1) - x(x + 1) + 6(x + 1)$
$= (x + 1)(7x^2 - x + 6)$
e) $5x(2 - x) + 4x(2x - 5) - (3x - 4)$
$= 10x - 5x^2 + 8x^2 - 20x - 3x + 4$
$= 3x^2 - 13x + 4$
$= 3x^2 - 12x - x + 4$
$= 3x(x - 4) - 1(x - 4)$
$= (x - 4)(3x - 1)$
f) $4t(t^2 + 4t + 2) - 2t(3t^2 - 6t + 17)$
$= 4t^3 + 16t^2 + 8t - 6t^3 + 12t^2 - 34t$
$= -2t^3 + 28t^2 - 26t$
$= -2t(t^2 - 14t + 13)$

$= -2t(t^2 - 13t - t + 13)$
$= -2t(t(t - 13) - (t - 13))$
$= -2t(t - 13)(t - 1)$

5. a) $x^2 - 5x - 14 = (x - 7)(x + 2)$

b) $x^2 + 4xy - 5y^2 = (x + 5y)(x - y)$

c) $6m^2 - 90m + 324 = 6(m^2 - 15m + 54)$
$= 6(m - 6)(m - 9)$

d) $2y^2 + 5y - 7 = 2y^2 + 7y - 2y - 7$
$= y(2y + 7) - (2y + 7)$
$= (2y + 7)(y - 1)$

e) $8a^2 - 2ab - 21b^2$
$= 8a^2 - 14ab + 12ab - 21b^2$
$= 2a(4a - 7b) + 3b(4a - 7b)$
$= (4a - 7b)(2a + 3b)$

f) $16x^2 + 76x + 90$
$= 2(8x^2 + 38x + 45)$
$= 2(8x^2 + 20x + 18x + 45)$
$= 2(4x(2x + 5) + 9(2x + 5))$
$= 2(2x + 5)(4x + 9)$

6. a) $x^2 - 9 = (x - 3)(x + 3)$

b) $4n^2 - 49 = (2n - 7)(2n + 7)$

c) $x^8 - 1 = (x^4 + 1)(x^4 - 1)$
$= (x^4 + 1)(x^2 + 1)(x^2 - 1)$
$= (x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

d) $9(y - 1)^2 - 25$
$= (3(y - 1) - 5)(3(y - 1) + 5)$
$= (3y - 3 - 5)(3y - 3 + 5)$
$= (3y - 8)(3y + 2)$

e) $3x^2 - 27(2 - x)^2$
$= 3(x^2 - 9(2 - x)^2)$
$= 3(x - 3(2 - x))(x + 3(2 - x))$
$= 3(x - 6 + 3x)(x + 6 - 3x)$
$= 3(4x - 6)(-2x + 6)$
$= 3(2)(2x - 3)(-2)(x - 3)$
$= -12(2x - 3)(x - 3)$

f) $-p^2q^2 + 81 = -(p^2q^2 - 81)$
$= -(pq + 9)(pq - 9)$

7. a) $ax + ay + bx + by$
$= a(x + y) + b(x + y)$
$= (x + y)(a + b)$

b) $2ab + 2a - 3b - 3$
$= 2a(b + 1) - 3(b + 1)$
$= (b + 1)(2a - 3)$

c) $x^3 + x^2 - x - 1 = x^2(x + 1) - (x + 1)$
$= (x + 1)(x^2 - 1)$
$= (x + 1)(x + 1)(x - 1)$
$= (x + 1)^2(x - 1)$

d) $1 - x^2 + 6x - 9 = -(x^2 - 6x + 8)$
$= -(x - 4)(x - 2)$
$= (4 - x)(x - 2)$

e) $a^2 - b^2 + 25 + 10a$
$= a^2 + 10a + 25 - b^2$
$= (a + 5)^2 - b^2$
$= (a + 5 - b)(a + 5 + b)$

f) $2m^2 + 10m + 10n - 2n^2$
$= 2(m^2 + 5m + 5n - n^2)$
$= 2(m^2 - n^2 + 5m + 5n)$
$= 2((m - n)(m + n) + 5(m + n))$
$= 2(m + n)(m - n + 5)$

8. a) No
$(x - y)(x^2 + y^2) = x(x^2 + y^2) - y(x^2 + y^2)$
$= x^3 + xy^2 - x^2y - y^2$
$\neq x^3 - y^3$

9. a) $2x(x - 3) + 7(3 - x)$
$= 2x(x - 3) - 7(x - 3)$
$= (x - 3)(2x - 7)$

b) $xy + 6x + 5y + 30 = x(y + 6) + 5(y + 6)$
$= (y + 6)(x + 5)$

c) $x^3 - x^2 - 4x + 4 = x^2(x - 1) - 4(x - 1)$
$= (x - 1)(x^2 - 4)$
$= (x - 1)(x - 2)(x + 2)$

d) $y^2 - 49 + 14x - x^2$
$= y^2 - (x^2 - 14x + 49)$
$= y^2 - (x - 7)^2$
$= (y - (x - 7))(y + (x - 7))$
$= (y - x + 7)(y + x - 7)$

e) $6x^2 - 21x - 12x + 42$
$= 3(2x^2 - 7x - 4x + 14)$
$= 3(2x^2 - 4x - 7x + 14)$
$= 3(2x(x - 2) - 7(x - 2))$
$= 3(x - 2)(2x - 7)$

f) $12m^3 - 14m^2 - 30m + 35$
$= 2m^2(6m - 7) - 5(6m - 7)$
$= (6m - 7)(2m^2 - 5)$

10. $f(n) = 2n^3 + n^2 + 6n + 3$
$= n^2(2n + 1) + 3(2n + 1)$
$= (n^2 + 3)(2n + 1)$

Since n is a natural number, $2n + 1$ is always odd and greater than 1. Because $(2n + 1)$ is a factor of $f(n)$, the condition is always true.

11. a) $a^2 = c^2 - b^2$
$= (c - b)(c + b)$

b) $(b + 3) + b = 11; b = 4$ m
$c = 4 + 3 = 7$ m
$a^2 = c^2 - b^2$
$= 49 - 16$
$= 33$
$a = \sqrt{33}$ m

12. Saturn:
a) i) $\pi r_2^2 - \pi r_1^2 = \pi(r_2 - r_1)(r_2 + r_1)$
ii) $\pi r_3^2 - \pi r_1^2 = \pi(r_3 - r_1)(r_3 + r_1)$
iii) $(\pi r_3^2 - \pi r_1^2) - (\pi r_2^2 - \pi r_1^2)$
$= \pi r_3^2 - \pi r_1^2 - \pi r_2^2 + \pi r_1^2$
$= \pi r_3^2 - \pi r_2^2$
$= \pi(r_3 - r_2)(r_3 + r_2)$
b) The area of the region between the inner ring and outer ring
13. 1. Always do common factor first.
2. Do difference of squares for 2 square terms separated by a minus sign.
3. Do simple trinomials for 3 terms with $a = 1$ or a prime.
4. Do complex trinomials for 3 terms with $a \neq 1$ or a prime.
5. Do grouping for a difference of squares for 4 or 6 terms with 3 or 4 squares.
6. Do incomplete squares for 3 terms when you can add a square to allow factoring; for example,
1. $5x + 10 = 5(x + 2)$
2. $4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$
3. $x^2 - x - 20 = (x - 5)(x + 4)$
4. $12x^2 - x - 20 = (4x + 5)(3x - 4)$
5. $x^2 + 6x + 9 - y^2$
$= (x + 3 + y)(x + 3 - y)$
6. $x^4 + 5x^2 + 9 = (x^2 + 3 + x)(x^2 + 3 - x)$
14. a) $x^4 + 3x^2 + 36$
$= x^4 + 12x^2 + 36 - 12x^2 + 3x^2$
$= (x^2 + 6)^2 - 9x^2$
$= (x^2 + 6 - 3x)(x^2 + 6 + 3x)$
$= (x^2 - 3x + 6)(x^2 + 3x + 6)$
b) $x^4 - 23x^2 + 49$
$= x^4 - 14x^2 + 49 + 14x^2 - 23x^2$
$= (x^2 - 7)^2 - 9x^2$
$= (x^2 - 7 - 3x)(x^2 - 7 + 3x)$
$= (x^2 - 3x - 7)(x^2 + 3x - 7)$
15. a) $(2^2 - 1)(2^4 + 2^2 + 2^0)$
$= 2^6 + 2^4 + 2^2 - 2^4 - 2^2 - 2^0$
$= 2^6 - 1$
and
$(2^3 - 1)(2^3 + 2^0) = 2^6 + 2^3 - 2^3 - 2^0$
$= 2^6 - 1$
b) $35 = 5 \times 7$
$\therefore\ 2^{35} - 1 = (2^5 - 1)(2^{30} - 2^{25} + 2^{20} + 2^{15} + 2^{10} + 2^5 + 2^0)$ or
$2^{35} - 1 = (2^7 - 1)(2^{28} + 2^{21} + 2^{14} + 2^7 + 2^0)$
c) Yes. If m is composite, then let $m = a \times b$, where a and/or b cannot equal 1.

$$2^{m-1} = 2^{ab-1} = \frac{2^{ab}}{2^1} = \frac{(2^a)^b}{2^1}$$

This result will always have two factors:
$(2^{a-1})(2^a)^{b-1}$
Neither of these will ever equal 1, so 2^{m-1} is composite.
16. a) $x^4 - 1 = (x - 1)(x^3 + x^2 + x + 1)$
b) $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$
c) $x^n - 1 = (x - 1)(x^{(n-1)} + x^{(n-2)} + \ldots + x^0)$
d) $x^n - y^n = (x - y)(x^{(n-1)}y^0 + x^{(n-2)}y^1 + x^{(n-3)}y^2 + \ldots + x^0y^{(n-1)})$

Mid-Chapter Review, p. 107

1. a) $(4a^2 - 3a + 2) - (-2a^2 - 3a + 9)$
$= 4a^2 - 3a + 2 + 2a^2 + 3a - 9$
$= 6a^2 - 7$
b) $(2x^2 - 4xy + y^2) - (4x^2 + 7xy - 2y^2) + (3x^2 + 6y^2)$
$= 2x^2 - 4xy + y^2 - 4x^2 - 7xy + 2y^2 + 3x^2 + 6y^2$
$= x^2 - 11xy + 9y^2$
c) $-(3d^2 - 2cd + d) + d(2c - 5d) - 3c(2c + d)$
$= -3d^2 + 2cd - d + 2cd - 5d^2 - 6c^2 - 3cd$
$= -6c^2 + cd - 8d^2 - d$
d) $3x(2x + y) - 4x[5 - (3x + 2)]$
$= 6x^2 + 3xy - 20x + 12x^2 + 8x$
$= 18x^2 - 12x + 3xy$
e) $2a(3a - 5b + 4) - 6(3 - 2a - b)$
$= 6a^2 - 10ab + 8a - 18 + 12a + 6b$
$= 6a^2 + 20a - 10ab + 6b - 18$
f) $7x(2x^2 + 3y - 3) - 3x(9 - 2x + 4y)$
$= 14x^3 + 21xy - 21x - 27x + 6x^2 - 12xy$
$= 14x^3 + 6x^2 - 48x + 9xy$
2. a) No, for example, $g(0) = -32$ and $h(0) = 32$
b) Yes, $f(x) = 2x^2 - 7x + 5$ and $g(x) = 2x^2 - 7x + 5$
c) Yes,
$h(x) = (x + 4)(x - 4)(x + 7) = (x^2 - 16)(x + 7) = d(x)$
d) No, for example, $b(0) = 1$ and $c(0) = -1$
3. The resulting polynomial will be a cubic because you add the exponents of the highest terms (linear = 1 and quadratic = 2).

4. f(sum of children's ages)
$= (5x - 99) - (x + x - 5)$
$= 3x - 94$

5. a) $2(x - 5)(3x - 4) = (2x - 10)(3x - 4)$
$= 6x^2 - 38x + 40$

b) $(3x - 1)^3 = (3x - 1)(3x - 1)(3x - 1)$
$= (3x - 1)(9x^2 - 6x + 1)$
$= 27x^3 - 18x^2 + 3x - 9x^2 + 6x - 1$
$= 27x^3 - 27x^2 + 9x - 1$

c) $2(x^2 - 3x + 4)(-x^2 + 3x - 4)$
$= (2x^2 - 6x + 8)(-x^2 + 3x - 4)$
$= -2x^4 + 6x^3 - 8x^2 + 6x^3 - 18x^2 + 24x - 8x^2 + 24x - 32$
$= -2x^4 + 12x^3 - 34x^2 + 48x - 32$

d) $(5x - 4)(3x - 5) - (2x - 3)^2$
$= 15x^2 - 37x + 20 - 4x^2 + 12x - 9$
$= 11x^2 - 25x + 11$

e) $3(2x - 5) - 9(4x - 5)$
$= 6x - 15 - 36x + 45$
$= -30x + 30$

f) $-(x - y)^3$
$= -(x - y)(x - y)(x - y)$
$= -(x - y)(x^2 - 2xy + y^2)$
$= -(x^3 - 2x^2y + xy^2 - x^2y + 2xy^2 - y^3)$
$= -x^3 + 3x^2y - 3xy^2 + y^3$

6. a) $P_1 = 2l + 2w$
$P_2 = 2(l + 2) + 2(w - 1)$
$= 2l + 2w + 2$
$P_2 - P_1 = (2l + 2w + 2) - (2l + 2w)$
$= 2$

b) $A_1 = lw$
$A_2 = (l + 2)(w - 1)$
$= lw + 2w - l - 2$
$A_2 - A_1 = (lw + 2w - l - 2) - lw$
$= 2w - l - 2$

7. a) $x(x - 2) - 3(x - 2)$
$= (x - 2)(x - 3)$

b) $x^2 - 11x + 28 = (x - 7)(x - 4)$

c) $3a^2 - 10a - 8 = 3a^2 - 12a + 2a - 8$
$= 3a(a - 4) + 2(a - 4)$
$= (3a + 2)(a - 4)$

d) $30x^2 - 9x - 3$
$= 3(10x^2 - 3x - 1)$
$= 3(10x^2 - 5x + 2x - 1)$
$= 3(5x(2x - 1) + (2x - 1))$
$= 3(5x + 1)(2x - 1)$

e) $16 - 25x^2 = (4 - 5x)(4 + 5x)$

f) $4(2 - a)^2 - 81$
$= (2(2 - a) - 9)(2(2 - a) + 9)$
$= -(5 + 2a)(13 - 2a)$

8. a) $2n - 6m + 5n^2 - 15mn$
$= 2(n - 3m) + 5n(n - 3m)$
$= (n - 3m)(2 + 5n)$

b) $y^2 + 9 - 6y - x^2 = y^2 - 6y + 9 - x^2$
$= (y - 3)^2 - x^2$
$= (y - 3 - x)(y - 3 + x)$

c) $y - b - (y - b)^2 = (y - b) - (y - b)(y - b)$
$= (y - b)(1 - y + b)$

d) $2x^2 - 8y^2 + 8x + 8$
$= 2(x^2 + 4x + 4 - 4y^2)$
$= 2((x + 2)^2 - 4y^2)$
$= 2(x + 2 - 2y)(x + 2 + 2y)$

e) $w^2 + wb - aw - ab$
$= w^2 - aw + wb - ab$
$= w(w - a) + b(w - a)$
$= (w - a)(w + b)$

f) $ab + b^2 + 6a + 6b$
$= b(a + b) + 6(a + b)$
$= (a + b)(b + 6)$

9. $25x^2 + 20x + 4 = 25x^2 + 10x + 10x + 4$
$= 5x(5x + 2) + 2(5x + 4)$
$= (5x + 2)^2$
$P = 4(5x + 2)$
$= 20x + 8$

10. Many answers are possible; for example, $k = -60, -42, -34, -20, -14, -4, 0, 6, 8, 10$. All answers for k are of the form $k = 11b - 3b^2, b \in I$

2.4 Simplifying Rational Functions, pp. 112–114

1. a) $\frac{6 - 4t}{2}$
$= \frac{\cancel{2}(3 - 2t)}{\cancel{2}}$
$= 3 - 2t$

b) $\frac{9x^2}{6x^3}$
$= \frac{\cancel{3x^2}(3)}{\cancel{3x^2}(2x)}$
$= \frac{3}{2x}; x \neq 0$

c) $\dfrac{7a^2b^3}{21a^4b}$

$= \dfrac{\cancel{7a^2b}(b^2)}{\cancel{7a^2b}(3a^2)}$

$= \dfrac{b^2}{3a^2}; a \neq 0, b \neq 0$

2. a) $\dfrac{5(x + 3)}{(x + 3)(x - 3)}$

$= \dfrac{5\cancel{(x + 3)}}{\cancel{(x + 3)}(x - 3)}$

$= \dfrac{5}{(x - 3)}$

$x + 3 \neq 0$ and $x - 3 \neq 0$

$x \neq -3, 3$

b) $\dfrac{6x - 9}{2x - 3}$

$= \dfrac{3\cancel{(2x - 3)}}{\cancel{2x - 3}}$

$= 3$

$2x - 3 \neq 0$

$2x \neq 3$

$x \neq \dfrac{3}{2}$

c) $\dfrac{4a^2b - 2ab^2}{(2a - b)^2}$

$= \dfrac{2ab\cancel{(2a - b)}}{(2a - b)\cancel{(2a - b)}}$

$= \dfrac{2ab}{2a - b}$

$2a - b \neq 0$

$2a \neq b$

$a \neq \dfrac{1}{2}b$

3. a) $\dfrac{(x - 1)(x - 3)}{(x + 2)(x - 1)}$

$= \dfrac{\cancel{(x - 1)}(x - 3)}{(x + 2)\cancel{(x - 1)}}$

$= \dfrac{(x - 3)}{(x + 2)}$

$x + 2 \neq 0$ and $x - 1 \neq 0$

$x \neq -2, 1$

b) $\dfrac{5x^2 + x - 4}{25x^2 - 40x + 16}$

$= \dfrac{5x^2 + 5x - 4x - 4}{25x^2 - 20x - 20x + 16}$

$= \dfrac{5x(x + 1) - 4(x + 1)}{5x(5x - 4) - 4(5x - 4)}$

$= \dfrac{\cancel{(5x - 4)}(x + 1)}{\cancel{(5x - 4)}(5x - 4)}$

$= \dfrac{(x + 1)}{(5x - 4)}$

$5x - 4 \neq 0$

$5x \neq 4$

$x \neq \dfrac{4}{5}$

c) $\dfrac{x^2 - 7xy + 10y^2}{x^2 + xy - 6y^2}$

$= \dfrac{(x - 5y)\cancel{(x - 2y)}}{(x + 3y)\cancel{(x - 2y)}}$

$= \dfrac{(x - 5y)}{(x + 3y)}$

$x + 3y \neq 0$ and $x - 2y \neq 0$

$x \neq -3y, 2y$

4. a) $\dfrac{14x^3 - 7x^2 + 21x}{7x}$

$= \dfrac{\cancel{7x}(2x^2 - x + 3)}{\cancel{7x}}$

$= 2x^2 - x + 3$

$7x \neq 0$

$x \neq 0$

b) $\dfrac{-5x^3y^2}{10xy^3}$

$= \dfrac{\cancel{5xy^2}(-x^2)}{\cancel{5xy^2}(2y)}$

$= \dfrac{-x^2}{2y}$

$x \neq 0, y \neq 0$

c) $\dfrac{2t(5 - t)}{5t^2(t - 5)}$

$= \dfrac{-2t\cancel{(t - 5)}}{5t^2\cancel{(t - 5)}}$

$= \dfrac{-2t}{5t^2}$

$5t^2 \neq 0$ and $t - 5 = \neq 0$

$t \neq 0, 5$

d) $\dfrac{5ab}{15a^4b - 10a^2b^2}$

$= \dfrac{\cancel{5ab}}{\cancel{5ab}(3a^3 - 2ab)}$

$= \dfrac{1}{a(3a^2 - 2b)}$

$a \neq 0$ and $3a^2 - 2b \neq 0$ and $b \neq 0$

$3a^2 \neq 2b$

$a^2 \neq \frac{2b}{3}$

$a \neq \sqrt{\frac{2b}{3}}$

e) $\frac{2x^2 + 10x}{-3x - 15}$

$= \frac{2x\cancel{(x+5)}}{-3\cancel{(x+5)}}$

$= -\frac{2x}{3}$

$x \neq -5$

f) $\frac{2ab - 6a}{9a - 3ab}$

$= \frac{2\cancel{a}\cancel{(b-3)}}{-3\cancel{a}\cancel{(b-3)}}$

$= -\frac{2}{3}$

$a \neq 0$ and $b - 3 \neq 0$

$b \neq 3$

5. a) $\frac{a + 4}{a^2 + 3a - 4}$

$= \frac{\cancel{a+4}}{\cancel{(a+4)}(a - 1)}$

$= \frac{1}{a - 1}$

$a \neq -4, 1$

b) $\frac{x^2 - 9}{15 - 5x}$

$= \frac{(x + 3)\cancel{(x-3)}}{-5\cancel{(x-3)}}$

$= -\frac{x + 3}{5}$

$x \neq 3$

c) $\frac{x^2 - 5x + 6}{x^2 + 3x - 10}$

$= \frac{(x - 3)\cancel{(x-2)}}{(x + 5)\cancel{(x-2)}}$

$= \frac{(x - 3)}{(x + 5)}$

$x \neq -5, 2$

d) $\frac{10 + 3p - p^2}{25 - p^2}$

$= \frac{p^2 - 3p - 10}{p^2 - 25}$

$= \frac{(p + 2)\cancel{(p-5)}}{(p + 5)\cancel{(p-5)}}$

$= \frac{p + 2}{p + 5}$

$p \neq -5, 5$

e) $\frac{t^2 - 7t + 12}{t^3 - 6t^2 + 9t}$

$= \frac{\cancel{(t-3)}(t - 4)}{t\cancel{(t-3)}(t - 3)}$

$= \frac{(t - 4)}{t(t - 3)}$

$t \neq 0, 3$

f) $\frac{6t^2 - t - 2}{2t^2 - t - 1}$

$= \frac{6t^2 + 3t - 4t - 2}{2t^2 - 2t + 1t - 1}$

$= \frac{3t(2t + 1) - 2(2t + 1)}{2t(t - 1) + 1(t - 1)}$

$= \frac{(3t - 2)\cancel{(2t+1)}}{(2t + 1)\cancel{(t-1)}}$

$= \frac{3t - 2}{t - 1}$

$t \neq -\frac{1}{2}, 1$

6. a) All reals; $x \neq 0$

b) All reals; $x \neq 0, 2$

c) All reals; $x \neq -5, 5$

d) $\frac{1}{x^2 - 1}$

$= \frac{1}{(x + 1)(x - 1)}$

All reals; $x \neq -1, 1$

e) All reals

f) $\frac{x - 1}{x^2 - 1}$

$= \frac{\cancel{x-1}}{\cancel{(x-1)}(x + 1)}$

$= \frac{1}{x + 1}$

All reals; $x \neq -1, 1$

7. a) $g(x) = \frac{6x^2 + 3x - 21}{3}$

$= \frac{\cancel{3}(2x^2 + x - 7)}{\cancel{3}}$

Yes, they are equivalent.

b) $j(x) = \frac{3x^3 + 5x^2 + x}{x}$

$= \frac{\cancel{x}(3x^2 + 5x + 1)}{\cancel{x}}$

While the equations are the same, they are not equivalent because the domains are different.

8. a) First, find the length of the base in relation to the perimeter.

$(30x + 10) - 2(9x + 3)$
$= (30x + 10) - (18x + 6)$
$= 12x + 4$

Now, set this value in a ratio with the perimeter.

$$\frac{12x + 4}{30x + 10} = \frac{4\cancel{(3x + 1)}}{10\cancel{(3x + 1)}} = \frac{2}{5}$$

$x > -\frac{1}{3}$

b) $x \le -\frac{1}{3}$ would imply sides of length 0 or less, therefore this would not be a triangle.

9. First, substitute 5 and 4 in for the radii and 2 and 3 for the heights of the cones respectively. Then set the formulas in ratio form and simplify.

$$\frac{\frac{1}{3}\cancel{\pi}5^2(2)}{\frac{1}{3}\cancel{\pi}4^2(3)}$$

$$\frac{25(2)}{16(3)} = \frac{50}{48} = \frac{25}{24}$$

10. a) $\frac{20t^3 + 15t^2 - 5t}{5t}$

$= \frac{\cancel{5t}(4t^2 + 3t - 1)}{\cancel{5t}}$
$= 4t^2 + 3t - 1$
$= 4t^2 + 4t - 1t - 1$
$= 4t(t + 1) - 1(t + 1)$
$= (4t - 1)(t + 1)$
$t \ne 0$

b) $\frac{5(4x - 2)}{8(2x - 1)^2}$

$= \frac{5(2)\cancel{(2x - 1)}}{8(2x - 1)\cancel{(2x - 1)}}$
$= \frac{10}{8(2x - 1)}$
$= \frac{5}{4(2x - 1)}$

$2x - 1 \ne 0$
$2x \ne 1$
$x \ne \frac{1}{2}$

c) $\frac{x^2 - 9x + 20}{16 - x^2}$

$= \frac{(x - 5)(x - 4)}{x^2 - 16}$
$= \frac{(x - 5)\cancel{(x - 4)}}{(x + 4)\cancel{(x - 4)}}$
$= \frac{(x - 5)}{(x + 4)}$
$x \ne -4, 4$

d) $\frac{2x^2 - xy - y^2}{x^2 - 2xy + y^2}$

$= \frac{2x^2 - 2xy + xy - y^2}{(x - y)(x - y)}$
$= \frac{2x(x - y) + y(x - y)}{(x - y)(x - y)}$
$= \frac{(2x + y)\cancel{(x - y)}}{(x - y)\cancel{(x - y)}}$
$= \frac{2x + y}{x - y}$
$x \ne y$

11. $l = 6w$
Area $= 6w^2$
Perimeter $= 14w$
So, the ratio is:

$$= \frac{6w^2}{14w} = \frac{\cancel{2} \cdot 3 \cdot \cancel{w} \cdot w}{\cancel{2} \cdot 7 \cdot \cancel{w}} = \frac{3w}{7}$$

12. Example 1:

$\frac{(3x - 2)(x - 4)}{(x - 4)}$
$= \frac{(3x - 2)\cancel{(x - 4)}}{\cancel{(x - 4)}}$
$= 3x - 2$

Example 2:

$\frac{5(3x - 2)(x - 4)}{5(x - 4)}$
$= \frac{\cancel{5}(3x - 2)\cancel{(x - 4)}}{\cancel{5(x - 4)}}$
$= 3x - 2$

13. $\frac{5}{(x - 1)(x - 2)(x - 3)}$

$x \ne 1, 2, 3$

14. a) i) $x \ne -1, 4$

$= \frac{(2x + 1)(x + 1)}{(x - 4)(x + 1)}$

$= \frac{(2x+1)\cancel{(x+1)}}{(x-4)\cancel{(x+1)}}$

$= \frac{(2x+1)}{(x-4)}$

ii) $x \neq 0, 4$

$= \frac{x(2x+1)}{x(x-4)}$

$= \frac{\cancel{x}(2x+1)}{\cancel{x}(x-4)}$

$= \frac{(2x+1)}{(x-4)}$

iii) $x \neq \frac{2}{3}, 4$

$= \frac{(2x+1)(3x-2)}{(x-4)(3x-2)}$

$= \frac{(2x+1)\cancel{(3x-2)}}{(x-4)\cancel{(3x-2)}}$

$= \frac{(2x+1)}{(x-4)}$

iv) $x \neq -\frac{1}{2}, 4$

$= \frac{(2x+1)\cancel{(2x+1)}}{(x-4)\cancel{(2x+1)}}$

$= \frac{(2x+1)(2x+1)}{(x-4)(2x+1)}$

$= \frac{(2x+1)}{(x-4)}$

b) yes; $\frac{(2x+1)(x-4)}{(x-4)^2}$

15. yes; $\frac{(x+1)(x+2)}{(x+1)(x+3)}$ and $\frac{(x+4)(x+2)}{(x+4)(x+3)}$

16. a) $\lim_{x\to\infty} \frac{50x+73}{x^2-10x-400}$

$= \lim_{x\to\infty} \left(\frac{\frac{50x}{x^2} + \frac{73}{x^2}}{\frac{x^2}{x^2} - \frac{10x}{x^2} - \frac{400}{x^2}} \right)$

$= \lim_{x\to\infty} \left(\frac{\frac{50}{x} + \frac{73}{x^2}}{1 - \frac{10}{x} - \frac{400}{x^2}} \right)$

$= \frac{0+0}{1-0-0}$

$= \frac{0}{1}$

$= 0$

$\lim_{x\to\infty} f(x) = 0$

b) $\lim_{x\to\infty} \left(\frac{4x^3-100}{5x^3+87x+28} \right)$

$= \lim_{x\to\infty} \left(\frac{\frac{4x^3}{x^3} - \frac{100}{x^3}}{\frac{5x^3}{x^3} + \frac{87x}{x^3} + \frac{28}{x^3}} \right)$

$= \frac{4-0}{5-0-0}$

$= \frac{4}{5}$

$\lim_{x\to\infty} g(x) = \frac{4}{5}$

c) $\lim_{x\to\infty} \left(\frac{-7x^2+3x}{200x+9999} \right)$

$= \lim_{x\to\infty} \left(\frac{\frac{-7x^2}{x^2} + \frac{3x}{x^2}}{\frac{200x}{x^2} + \frac{9999}{x^2}} \right)$

$= \frac{-7+0}{0+0}$

$= \frac{-7}{0}$

$\lim_{x\to\infty} h(x) = -\infty$

17. a) $\frac{-2(1+t^2)^2 + 2t(2)(1+t^2)(2t)}{(1+t^2)^4}$

$= \frac{-2(1+2t^2+t^4) + 8t^2(1+t^2)}{(1+t^2)^4}$

$= \frac{-2-4t^2-2t^4+8t^2+8t^4}{(1+t^2)^4}$

$= \frac{6t^4+4t^2-2}{(1+t^2)^4}$

$= \frac{2(3t^4+2t^2-1)}{(1+t^2)^4}$

$= \frac{2(3t^4+3t^2-1t^2-1)}{(1+t^2)^4}$

$= \frac{2(3t^2(t^2+1) - 1(t^2+1))}{(1+t^2)^4}$

$= \frac{2(3t^2-1)\cancel{(t^2+1)}}{\cancel{(t^2+1)}(t^2+1)^3}$

$= \frac{2(3t^2-1)}{(1+t^2)^3}$; no restrictions

b) $\dfrac{2(2x+1)(2)(3x-2)^3-(2x+1)^2(3)(3x-2)^2}{(3x-2)^6}$

$= \dfrac{(8x+4)(3x-2)^3-(4x^2+4x+1)(3)(3x-2)^2}{(3x-2)^6}$

$= \dfrac{(8x+4)(3x-2)^3-(12x^2+12x+3)(3x-2)^2}{(3x-2)^6}$

$= \dfrac{(8x+4)(3x-2)^3-(12x^2+6x+6x+3)(3x-2)^2}{(3x-2)^6}$

$= \dfrac{(8x+4)(3x-2)^3-[6x(2x+1)+3(2x+1)](3x-2)^2}{(3x-2)^6}$

$= \dfrac{(8x+4)(3x-2)^3-(6x+3)+(2x+1)(3x-2)^2}{(3x-2)^6}$

$= \dfrac{\cancel{(3x-2)^2}[(8x+4)(3x-2)-(6x+3)(2x+1)]}{\cancel{(3x-2)^2}(3x-2)^4}$

$= \dfrac{(24x^2-4x-8)-(12x^2+12x+3)}{(3x-2)^4}$

$= \dfrac{12x^2-16x-11}{(3x-2)^4}$

$= \dfrac{12x^2-22x+6x-11}{(3x-2)^4}$

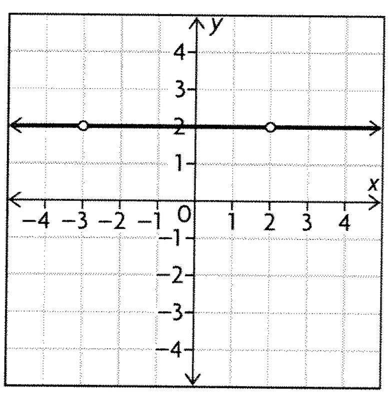
$= \dfrac{2x(6x-11)+1(6x-11)}{(3x-2)^4}$

$= \dfrac{(2x+1)(6x-11)}{(3x-2)^4}$

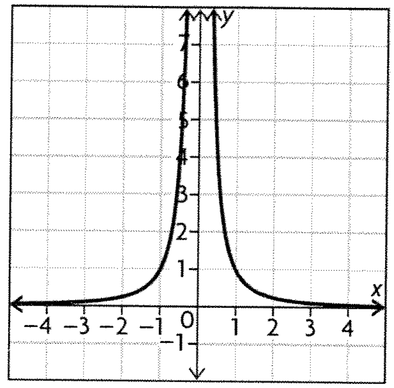
$x \neq \dfrac{2}{3}$

2.5 Exploring Graphs of Rational Functions, p. 116

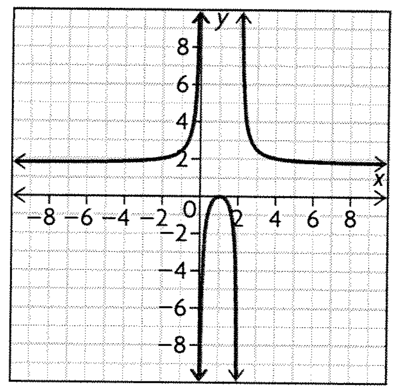
1. For example, $y = \dfrac{2(x-2)(x+3)}{(x-2)(x+3)}$



2. For example, $y = \dfrac{1}{x^2}$



3. For example, $y = \dfrac{2}{x(x-2x)} + 2$



2.6 Multiplying and Dividing Rational Expressions, pp. 121–123

1. a) $\dfrac{2}{3} \times \dfrac{5}{8}$

$= \dfrac{1}{3} \times \dfrac{5}{4} = \dfrac{5}{12}$

b) $\dfrac{6x^2y}{5y^3} \times \dfrac{xy}{8}$

$= \dfrac{6x^3y^2}{40y^3}$

$= \dfrac{\cancel{2y^2}(3x^3)}{\cancel{2y^2}(20y)}$

$= \dfrac{3x^3}{20y}$

$5y^3 \neq 0$

$y \neq 0$

c) $\dfrac{(x+1)(x-5)}{(x+4)} \times \dfrac{(x+4)}{2(x-5)}$

$= \dfrac{(x+1)\cancel{(x-5)}}{\cancel{(x+4)}} \times \dfrac{\cancel{(x+4)}}{2\cancel{(x-5)}}$

$= \dfrac{(x + 1)}{2}$

$x + 4 \neq 0$ and $x - 5 \neq 0$

$x \neq -4, 5$

d) $\dfrac{x^2}{2x + 1} \times \dfrac{6x + 3}{5x}$

$= \dfrac{\cancel{x}(x)}{\cancel{2x + 1}} \times \dfrac{3\cancel{(2x + 1)}}{\cancel{x}(5)}$

$= \dfrac{3x}{5}$

$2x + 1 \neq 0$ and $5x \neq 0$

$x \neq -\dfrac{1}{2}, 0$

2. a) $\dfrac{2x}{3} \div \dfrac{x^2}{5}$

$= \dfrac{2x}{3} \times \dfrac{5}{x^2}$

$= \dfrac{\cancel{x}(2)}{3} \times \dfrac{5}{\cancel{x}(x)}$

$= \dfrac{10}{(3x)}$

$x^2 \neq 0$

$x \neq 0$

b) $\dfrac{x - 7}{10} \div \dfrac{2x - 14}{25}$

$= \dfrac{x - 7}{10} \times \dfrac{25}{2x - 14}$

$= \dfrac{\cancel{x - 7}}{\cancel{5}(2)} \times \dfrac{\cancel{5}(5)}{2\cancel{(x - 7)}}$

$= \dfrac{5}{4}$

$2x - 14 \neq 0$

$x \neq 7$

c) $\dfrac{3x(x - 6)}{(x + 2)(x - 7)} \div \dfrac{(x - 6)}{(x + 2)}$

$= \dfrac{3x\cancel{(x - 6)}}{\cancel{(x + 2)}(x - 7)} \times \dfrac{\cancel{(x + 2)}}{\cancel{(x - 6)}}$

$= \dfrac{(3x)}{(x - 7)}$

$x + 2 \neq 0$ and $x - 7 \neq 0$ and $x - 6 \neq 0$

$x \neq -2, 6, 7$

d) $\dfrac{x^2 - 1}{x - 2} \div \dfrac{x + 1}{12 - 6x}$

$= \dfrac{x^2 - 1}{x - 2} \times \dfrac{12 - 6x}{x + 1}$

$= \dfrac{(x - 1)\cancel{(x + 1)}}{\cancel{x - 2}} \times \dfrac{-6\cancel{(x - 2)}}{\cancel{x + 1}}$

$= -6(x - 1)$

$x - 2 \neq 0$ and $x + 1 \neq 0$

$x \neq -1, 2$

3. a) $\dfrac{(x + 1)^2}{x^2 + 2x - 3} \times \dfrac{(x - 1)^2}{x^2 + 4x + 3}$

$= \dfrac{(x + 1)\cancel{(x + 1)}}{(x + 3)\cancel{(x - 1)}} \times \dfrac{\cancel{(x - 1)}(x - 1)}{(x + 3)\cancel{(x + 1)}}$

$= \dfrac{(x + 1)(x - 1)}{(x + 3)(x + 3)}$

$= \dfrac{(x - 1)^2}{(x + 3)^2}$

$x + 3 \neq 0$ and $x - 1 \neq 0$ and $x + 1 \neq 0$

$x \neq -3, -1, 1$

b) $\dfrac{2x + 10}{x^2 - 4x + 4} \div \dfrac{x^2 - 25}{x - 2}$

$= \dfrac{2x + 10}{x^2 - 4x + 4} \times \dfrac{x - 2}{x^2 - 25}$

$= \dfrac{2\cancel{(x + 5)}}{(x - 2)\cancel{(x - 2)}} \times \dfrac{\cancel{x - 2}}{(x - 5)\cancel{(x + 5)}}$

$= \dfrac{2}{(x - 2)(x - 5)}$

$x - 2 \neq 0$ and $x - 5 \neq 0$ and $x + 5 \neq 0$

$x \neq -5, 2, 5$

4. a) $\dfrac{2x^2}{7} \times \dfrac{21}{x}$

$= \dfrac{\cancel{x}(2x)}{\cancel{7}} \times \dfrac{\cancel{7}(3)}{\cancel{x}}$

$= 3(2x)$

$= 6x$

$x \neq 0$

b) $\dfrac{7a}{3} \div \dfrac{14a^2}{5}$

$= \dfrac{7a}{3} \times \dfrac{5}{14a^2}$

$= \dfrac{\cancel{7a}}{3} \times \dfrac{5}{\cancel{7a}(2a)}$

$= \dfrac{5}{3(2a)}$

$= \dfrac{5}{6a}$

$a \neq 0$

c) $\dfrac{2x^3y}{3xy^2} \times \dfrac{9x}{4x^2y}$

$= \dfrac{18x^4y}{12x^3y^3}$

$= \dfrac{\cancel{6x^3y}(3x)}{\cancel{6x^3y}(2y^2)}$

$= \frac{3x}{2y^2}$

$x \neq 0$ and $y \neq 0$

d) $\frac{3a^2b^3}{2ab^2} \div \frac{9a^2b}{14a^2}$

$= \frac{3a^2b^3}{2ab^2} \times \frac{14a^2}{9a^2b}$

$= \frac{42a^4b^3}{18a^3b^3}$

$= \frac{\cancel{6a^3b^3}(7a)}{\cancel{6a^3b^3}(3)}$

$= \frac{7a}{3}$

$a \neq 0$ and $b \neq 0$

5. a) $\frac{2(x+1)}{3} \times \frac{x-1}{6(x+1)}$

$= \frac{2\cancel{(x+1)}}{3} \times \frac{x-1}{6\cancel{(x+1)}}$

$= \frac{2(x-1)}{18}$

$= \frac{\cancel{2}(x-1)}{\cancel{2}(9)}$

$= \frac{(x-1)}{9}$

$x + 1 \neq 0$

$x \neq -1$

b) $\frac{3a-6}{a+2} \div \frac{a-2}{a+2}$

$= \frac{3a-6}{a+2} \times \frac{a+2}{a-2}$

$= \frac{3\cancel{(a-2)}}{\cancel{a+2}} \times \frac{\cancel{a+2}}{\cancel{a-2}}$

$= 3$

$a + 2 \neq 0$ and $a - 2 \neq 0$

$a \neq -2, 2$

c) $\frac{2(x-2)}{9x^3} \times \frac{12x^4}{2-x}$

$= \frac{2\cancel{(x-2)}}{\cancel{3x^3}(3)} \times \frac{\cancel{3x^3}(4x)}{-1\cancel{(x-2)}}$

$= -\frac{8x}{3}$

$x - 2 \neq 0$ and $x \neq 0$

$x \neq 0, 2$

d) $\frac{3(m+4)^2}{2m+1} \div \frac{5(m+4)}{7m+14}$

$= \frac{3(m+4)^2}{2m+1} \times \frac{7m+14}{5(m+4)}$

$= \frac{3(m+4)\cancel{(m+4)}}{2m+1} \times \frac{7(m+2)}{5\cancel{(m+4)}}$

$= \frac{21(m+4)(m+2)}{5(2m+1)}$

$2m + 1 \neq 0$ and $m + 4 \neq 0$ and $7m + 14 \neq 0$

$m \neq -\frac{1}{2}, -2, -4$

6. a) $\frac{(x+1)(x-3)}{(x+2)^2} \times \frac{2(x+2)}{(x-3)(x+3)}$

$= \frac{(x+1)\cancel{(x-3)}}{(x+2)\cancel{(x+2)}} \times \frac{2\cancel{(x+2)}}{\cancel{(x-3)}(x+3)}$

$= \frac{2(x+1)}{(x+2)(x+3)}$

$x + 2 \neq 0$ and $x - 3 \neq 0$ and $x + 3 \neq 0$

$x \neq -3, -2, 3$

b) $\frac{2(n^2 - 7n + 12)}{n^2 - n - 6} \div \frac{5(n-4)}{n^2 - 4}$

$= \frac{2(n^2 - 7n + 12)}{n^2 - n - 6} \times \frac{n^2 - 4}{5(n-4)}$

$= \frac{2\cancel{(n-4)}\cancel{(n-3)}}{\cancel{(n+2)}\cancel{(n-3)}} \times \frac{\cancel{(n+2)}(n-2)}{5\cancel{(n-4)}}$

$= \frac{2(n-2)}{5}$

$n + 2 \neq 0$ and $n - 2 \neq 0$ and $n - 3 \neq 0$ and $n - 4 \neq 0$

$n \neq -2, 2, 3, 4$

c) $\frac{2x^2 - x - 1}{x^2 - x - 6} \times \frac{6x^2 - 5x + 1}{8x^2 - 14x + 5}$

$= \frac{2x^2 - 2x + 1x - 1}{(x+2)(x-3)} \times \frac{6x^2 - 3x - 2x + 1}{8x^2 - 10x - 4x + 5}$

$= \frac{2x(x-1) + 1(1x-1)}{(x+2)(x-3)}$

$\times \frac{3x(2x-1) - 1(2x-1)}{-2x(4x+5) - 1(4x+5)}$

$= \frac{\cancel{(2x+1)}(x-1)}{(x+2)(x-3)} \times \frac{(3x-1)(2x-1)}{\cancel{(2x+1)}(4x+5)}$

$= \frac{(x-1)(3x-1)(2x-1)}{(x+2)(x-3)(4x+5)}$

$x + 2 \neq 0$ and $x - 3 \neq 0$ and $2x + 1 \neq 0$ and $4x + 5 \neq 0$

$x \neq -2, -\frac{1}{2}, -\frac{5}{4}, 3$

d) $\frac{9y^2 - 4}{4y - 12} \div \frac{9y^2 + 12y + 4}{18 - 6y}$

$= \frac{9y^2 - 4}{4y - 12} \times \frac{18 - 6y}{9y^2 + 12y + 4}$

$$= \frac{(3y-2)(3y+2)}{4(y-3)} \times \frac{-6(y-3)}{9y^2+6y+6y+4}$$

$$= \frac{(3y-2)(3y+2)}{4(y-3)}$$

$$\times \frac{-6(y-3)}{3y(3y+2)+2(3y+2)}$$

$$= \frac{(3y-2)\cancel{(3y+2)}}{4\cancel{(y-3)}} \times \frac{-6\cancel{(y-3)}}{\cancel{(3y+2)}(3y+2)}$$

$$= \frac{-3(3y-2)}{2(3y+2)}$$

$y - 3 \neq 0$ and $3y + 2 \neq 0$

$y \neq 3, -\frac{2}{3}$

7. a) $\dfrac{x^2-5xy+4y^2}{x^2+3xy-28y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

$$= \frac{\cancel{(x-4y)}\cancel{(x-y)}}{(x+7y)\cancel{(x-4y)}} \times \frac{\cancel{(x+y)}(x+y)}{\cancel{(x-y)}\cancel{(x+y)}}$$

$$= \frac{x+y}{x+7y}$$

$x + 7y \neq 0$ and $x + y \neq 0$ and $x - y \neq 0$ and $x - 4y \neq 0$

$x \neq -7y, -y, y, 4y$

b) $\dfrac{2a^2-12ab+18b^2}{a^2-7ab+10b^2} \div \dfrac{4a^2-12ab}{a^2-7ab+10b^2}$

$$= \frac{2a^2-12ab+18b^2}{a^2-7ab+10b^2} \times \frac{a^2-7ab+10b^2}{4a^2-12ab}$$

$$= \frac{2(a^2-6ab+9b^2)}{(a-5b)(a-2b)} \times \frac{(a-5b)(a-2b)}{4a(a-3b)}$$

$$= \frac{2(a-3b)\cancel{(a-3b)}}{\cancel{(a-5b)}\cancel{(a-2b)}} \times \frac{\cancel{(a-5b)}\cancel{(a-2b)}}{4a\cancel{(a-3b)}}$$

$$= \frac{(a-3b)}{2a}$$

$a - 5b \neq 0$ and $a - 2b \neq 0$ and $4a \neq 0$ and $a - 3b \neq 0$

$a \neq 0, 2b, 3b, 5b$

c) $\dfrac{10x^2+3xy-y^2}{9x^2-y^2} \div \dfrac{6x^2+3xy}{12x+4y}$

$$= \frac{10x^2+3xy-y^2}{9x^2-y^2} \times \frac{12x+4y}{6x^2+3xy}$$

$$= \frac{10x^2+5xy-2xy-y^2}{(3x+y)(3x-y)} \times \frac{4(3x+y)}{3x(2x+y)}$$

$$= \frac{5x(2x+y)-y(2x+y)}{(3x+y)(3x-y)} \times \frac{4(3x+y)}{3x(2x+y)}$$

$$= \frac{(5x-y)\cancel{(2x+y)}}{\cancel{(3x+y)}(3x-y)} \times \frac{4\cancel{(3x+y)}}{3x\cancel{(2x+y)}}$$

$$= \frac{4(5x-y)}{3x(3x-y)}$$

$3x + y \neq 0$ and $3x - y \neq 0$ and $3x \neq 0$ and $2x + y \neq 0$

$x \neq -\frac{1}{3}y, -\frac{1}{2}y, 0, \frac{1}{3}y$

d) $\dfrac{15m^2+mn-2n^2}{2n-14m} \times \dfrac{7m^2-8mn+n^2}{5m^2+7mn+2n^2}$

$$= \frac{15m^2+6mn-5mn-2n^2}{2(n-7m)}$$

$$\times \frac{7m^2-7mn-1mn+n^2}{5m^2+5mn+2mn+2n^2}$$

$$= \frac{3m(5m+2n)-1n(5m+2n)}{2(n-7m)}$$

$$\times \frac{7m(m-n)-1n(m-n)}{5m(m+1n)+2n(m+n)}$$

$$= \frac{(3m-1n)\cancel{(5m+2n)}}{-2\cancel{(7m-n)}}$$

$$\times \frac{\cancel{(7m-n)}(m-n)}{\cancel{(5m+2n)}(m+n)}$$

$$= \frac{-(3m-n)(m-n)}{2(m+n)}$$

$7m - n \neq 0$ and $5m + 2n \neq 0$ and $m + n \neq 0$

$m \neq \frac{1}{7}n, -\frac{2}{5}n, -n$

8. $\dfrac{x^2+x-6}{(2x-1)^2} \times \dfrac{x(2x-1)^2}{x^2+2x-3} \div \dfrac{x^2-4}{3x}$

$$= \frac{x^2+x-6}{(2x-1)^2} \times \frac{x(2x-1)^2}{x^2+2x-3} \times \frac{3x}{x^2-4}$$

$$= \frac{\cancel{(x+3)}\cancel{(x-2)}}{\cancel{(2x-1)}\cancel{(2x-1)}} \times \frac{x\cancel{(2x-1)}\cancel{(2x-1)}}{\cancel{(x+3)}(x-1)}$$

$$\times \frac{3x}{(x+2)\cancel{(x-2)}}$$

$$= \frac{3x^2}{(x-1)(x+2)}$$

$2x - 1 \neq 0$ and $x + 3 \neq 0$ and $x - 1 \neq 0$ and $x + 2 \neq 0$ and $x - 2 \neq 0$ and $3x \neq 0$

$x \neq \frac{1}{2}, -3, 1, -2, 2, 0$

9. Area $= \frac{1}{2} \times$ base $\times$ height

$$= \frac{1}{2} \times \frac{4x^2}{x^2-16x+63} \times \frac{5x-35}{x+3}$$

$$= \frac{1}{2} \times \frac{4x^2}{(x-9)\cancel{(x-7)}} \times \frac{5\cancel{(x-7)}}{x+3}$$

$$= \frac{10x^2}{(x-9)(x+3)}$$

$x - 9 \neq 0$ and $x + 3 \neq 0$ and $x - 7 \neq 0$

$x \neq 9, -3, 7$

10. Volume = mass ÷ density

$$= \frac{p+1}{3p+1} \div \frac{p^2-1}{9p^2+6p+1}$$
$$= \frac{p+1}{3p+1} \times \frac{9p^2+6p+1}{p^2-1}$$
$$= \frac{p+1}{3p+1} \times \frac{9p^2+3p+3p+1}{(p-1)(p+1)}$$
$$= \frac{p+1}{3p+1} \times \frac{3p(3p+1)+1(3p+1)}{(p-1)(p+1)}$$
$$= \frac{\cancel{p+1}}{\cancel{3p+1}} \times \frac{(3p+1)\cancel{(3p+1)}}{(p-1)\cancel{(p+1)}}$$
$$= \frac{(3p+1)}{(p-1)}$$

$p - 1 \neq 0$ and $p + 1 \neq 0$ and $3p + 1 \neq 0$

$p \neq 1, -1, -\frac{1}{3}$

11. If $x = y$, then Liz is dividing by zero.

12. a) Then you can simplify and cancel common factors.

b) Sometimes you cannot factor and you need to take into account the factors you cancel because they could make the denominator equal to 0.

c) Yes. Dividing is the same as multiplying by the reciprocal.

13. First divide the numerator:

$$\frac{m^2-mn}{6m^2+11mn+3n^2} \div \frac{m^2-n^2}{2m^2-mn-6n^2}$$
$$= \frac{m^2-mn}{6m^2+11mn+3n^2} \times \frac{2m^2-mn-6n^2}{m^2-n^2}$$
$$= \frac{m(m-n)}{6m^2+9mn+2mn+3n^2} \times \frac{2m^2-4mn+3mn-6n^2}{(m-n)(m+n)}$$
$$= \frac{m(m-n)}{3m(2m+3n)+n(2m+3n)} \times \frac{2m(m-2n)+3n(m-2n)}{(m-n)(m+n)}$$
$$= \frac{m\cancel{(m-n)}}{(3m+n)\cancel{(2m+3n)}} \times \frac{\cancel{(2m+3n)}(m-2n)}{\cancel{(m-n)}(m+n)}$$
$$= \frac{m(m-2n)}{(3m+n)(m+n)}$$

Now, divide this result by the denominator:

$$\frac{m(m-2n)}{(3m+n)(m+n)} \div \frac{4m^2-7mn-2n^2}{3m^2+7mn+2n^2}$$
$$= \frac{m(m-2n)}{(3m+n)(m+n)} \times \frac{3m^2+7mn+2n^2}{4m^2-7mn-2n^2}$$
$$= \frac{m(m-2n)}{(3m+n)(m+n)} \times \frac{3m^2+6mn+1mn+2n^2}{4m^2-8mn+1mn-2n^2}$$
$$= \frac{m(m-2n)}{(3m+n)(m+n)} \times \frac{3m(m+2n)+1n(m+2n)}{4m(m-2n)+1n(m-2n)}$$
$$= \frac{m\cancel{(m-2n)}}{\cancel{(3m+n)}(m+n)} \times \frac{\cancel{(3m+1n)}(m+2n)}{(4m+1n)\cancel{(m-2n)}}$$
$$= \frac{m(m+2n)}{(m+n)(4m+n)}$$

$m + n \neq 0$ and $4m + n \neq 0$ and $3m + n \neq 0$ and $m - 2n \neq 0$ and $m - n \neq 0$ and $m + 2n \neq 0$ and $2m + 3n \neq 0$

$m \neq -n, -\frac{1}{4}n, -\frac{1}{3}n, 2n, n, -2n, -\frac{3}{2}n$

14. Let F_1 = Mercury's force and F_2 = Pluto's force.

$$F_1 = \frac{m_1 m_2}{(102.1r)^2}$$
$$F_2 = \frac{m_1(2.2m_2)}{r^2}$$

Find the number that when multiplied by F1, the forces will be equal.

$$x \times \left(\frac{m_1 m_2}{(102.1r)^2}\right) = \frac{m_1(2.2m_2)}{r^2}$$
$$x = \frac{m_1(2.2m_2)}{r^2} \div \frac{m_1 m_2}{(102.1r)^2}$$
$$= \frac{m_1(2.2m_2)}{r^2} \times \frac{(102.1r)^2}{m_1 m_2}$$
$$= \frac{2.2m_1 m_2}{r^2} \times \frac{10\,424.41r^2}{m_1 m_2}$$
$$= 22\,933.7$$

The force is 22 933.7 times greater between the sun and Mercury than it is between the sun and Pluto.

2.7 Adding and Subtracting Rational Expressions, pp. 128–130

1. a) $\frac{1}{3} + \frac{5}{4}$

$= \frac{1(4)}{3(4)} + \frac{5(3)}{4(3)}$

$= \frac{4}{12} + \frac{15}{12}$

$= \frac{19}{12}$

b) $\frac{2x}{5} + \frac{6x}{2}$

$= \frac{2x(2)}{5(2)} + \frac{6x(5)}{2(5)}$

$= \frac{4x}{10} + \frac{30x}{10}$

$= \frac{34x}{10}$

$= \frac{17x}{5}$

c) $\frac{5}{4x^2} + \frac{1}{7x^3} = \frac{5(7x)}{4x^2(7x)} + \frac{1(4)}{7x^3(4)}$

$= \frac{35x}{28x^3} + \frac{4}{28x^3}$

$= \frac{35x + 4}{28x^3}$

$28x^3 \neq 0$

$x \neq 0$

d) $\frac{2}{x} + \frac{6}{x^2}$

$= \frac{2(x)}{x(x)} + \frac{6}{x^2}$

$= \frac{2x}{x^2} + \frac{6}{x^2}$

$= \frac{2x + 6}{x^2}$

$x^2 \neq 0$

$x \neq 0$

2. a) $\frac{5}{9} - \frac{2}{3}$

$= \frac{5}{9} - \frac{2(3)}{3(3)}$

$= \frac{5}{9} - \frac{6}{9}$

$= -\frac{1}{9}$

b) $\frac{5y}{3} - \frac{y}{2}$

$= \frac{5y(2)}{3(2)} - \frac{y(3)}{2(3)}$

$= \frac{10y}{6} - \frac{3y}{6}$

$= \frac{7y}{6}$

c) $\frac{5}{3x^2} - \frac{7}{5}$

$= \frac{5(5)}{3x^2(5)} - \frac{7(3x^2)}{5(3x^2)}$

$= \frac{25}{15x^2} - \frac{21x^2}{15x^2}$

$= \frac{25 - 21x^2}{15x^2}$

$15x^2 \neq 0$

$x \neq 0$

d) $\frac{6}{3xy} - \frac{5}{y^2}$

$= \frac{6(y)}{3xy(y)} - \frac{5(3x)}{y^2(3x)}$

$= \frac{6y}{3xy^2} - \frac{15x}{3xy^2}$

$= \frac{6y - 15x}{3xy^2}$

$3xy^2 \neq 0$

$x, y \neq 0$

3. a) $\frac{3}{x - 3} - \frac{7}{5x - 1}$

$= \frac{3(5x - 1)}{(x - 3)(5x - 1)} - \frac{7(x - 3)}{(5x - 1)(x - 3)}$

$= \frac{3(5x - 1) - 7(x - 3)}{(x - 3)(5x - 1)}$

$= \frac{15x - 3 - 7x + 21}{(x - 3)(5x - 1)}$

$= \frac{8x + 18}{(x - 3)(5x - 1)}$

$x - 3 \neq 0$ and $5x - 1 \neq 0$

$x \neq 3, \frac{1}{5}$

b) $\frac{2}{x + 3} + \frac{7}{x^2 - 9}$

$= \frac{2}{x + 3} + \frac{7}{(x - 3)(x + 3)}$

$$= \frac{2(x-3)}{(x+3)(x-3)} + \frac{7}{(x-3)(x+3)}$$
$$= \frac{2(x-3)+7}{(x+3)(x-3)}$$
$$= \frac{2x-6+7}{(x+3)(x-3)}$$
$$= \frac{2x+1}{(x+3)(x-3)}$$
$x + 3 \neq 0$ and $x - 3 \neq 0$

$x \neq -3, 3$

c) $\frac{5}{x^2-4x+3} - \frac{9}{x^2-2x+1}$
$$= \frac{5}{(x-3)(x-1)} - \frac{9}{(x-1)(x-1)}$$
$$= \frac{5(x-1)}{(x-3)(x-1)(x-1)} - \frac{9(x-3)}{(x-3)(x-1)(x-1)}$$
$$= \frac{5(x-1)-9(x-3)}{(x-3)(x-1)(x-1)}$$
$$= \frac{5x-5-9x+27}{(x-3)(x-1)(x-1)}$$
$$= \frac{-4x+22}{(x-3)(x-1)(x-1)}$$
$x - 1 \neq 0$ and $x - 3 \neq 0$

$x \neq 1, 3$

4. a) $\frac{2}{(x^2-9)} + \frac{3}{(x-3)}$

Substitute 5 for x.
$$= \frac{2}{(5^2-9)} + \frac{3}{(5-3)}$$
$$= \frac{2}{25-9} + \frac{3}{2}$$
$$= \frac{2}{16} + \frac{3}{2}$$
$$= \frac{1}{8} + \frac{3(4)}{2(4)}$$
$$= \frac{1}{8} + \frac{12}{8}$$
$$= \frac{13}{8}$$

b) Simplify the original expression:
$$\frac{2}{(x^2-9)} + \frac{3}{(x-3)}$$
$$= \frac{2}{(x-3)(x+3)} + \frac{3}{(x-3)}$$
$$= \frac{2}{(x-3)(x+3)} + \frac{3(x+3)}{(x-3)(x+3)}$$
$$= \frac{2+3(x+3)}{(x-3)(x+3)}$$
$$= \frac{2+3x+9}{(x-3)(x+3)}$$
$$= \frac{3x+11}{(x-3)(x+3)}$$
$x - 3 \neq 0$ and $x + 3 \neq 0$

$x \neq 3, -3$

c) Substitute $x = 5$ into $\frac{3x+11}{(x-3)(x+3)}$
$$= \frac{3(5)+11}{(5-3)(5+3)}$$
$$= \frac{15+11}{(2)(8)}$$
$$= \frac{26}{16}$$
$$= \frac{13}{8}$$

The answers to a) and c) are equal.

5. a) $\frac{2x}{3} + \frac{3x}{4} - \frac{x}{6}$
$$= \frac{2x(4)}{3(4)} + \frac{3x(3)}{4(3)} - \frac{x(2)}{6(2)}$$
$$= \frac{8x}{12} + \frac{9x}{12} - \frac{2x}{12}$$
$$= \frac{8x+9x-2x}{12}$$
$$= \frac{15x}{12}$$
$$= \frac{5x}{4}$$

b) $\frac{3}{t^4} + \frac{1}{2t^2} - \frac{3}{5t}$
$$= \frac{3(10)}{t^4(10)} + \frac{1(5t^2)}{2t^2(5t^2)} - \frac{3(2t^3)}{5t(2t^3)}$$
$$= \frac{30}{10t^4} + \frac{5t^2}{10t^4} - \frac{6t^3}{10t^4}$$
$$= \frac{30+5t^2-6t^3}{10t^4}$$
$10t^4 \neq 0$

$t \neq 0$

c) $\frac{2x}{3y} - \frac{x^2}{4y^3} + \frac{3}{5y^4}$

$= \frac{2x(20y^3)}{3y(20y^3)} - \frac{x^2(15y)}{4y^3(15y)} + \frac{3(12)}{5y^4(12)}$

$= \frac{40xy^3}{60y^4} - \frac{15x^2y}{60y^4} + \frac{36}{60y^4}$

$= \frac{40xy^3 - 15x^2y + 36}{60y^4}$

$60y^4 \neq 0$

$y \neq 0$

d) $\frac{n}{m} + \frac{m}{n} - m$

$= \frac{n(n)}{m(n)} + \frac{m(m)}{n(m)} - \frac{m(mn)}{1(mn)}$

$= \frac{n^2 + m^2 - m^2n}{mn}$

$m \neq 0, n \neq 0$

6. a) $\frac{7}{a-4} + \frac{2}{a}$

$= \frac{7(a)}{(a-4)(a)} + \frac{2(a-4)}{a(a-4)}$

$= \frac{7a + 2(a-4)}{a(a-4)}$

$= \frac{7a + 2a - 8}{a(a-4)}$

$= \frac{9a - 8}{a(a-4)}$

$a \neq 0$ and $a - 4 \neq 0$

$a \neq 0, 4$

b) $\frac{4}{3x-2} + 6$

$= \frac{4}{3x-2} + \frac{6(3x-2)}{1(3x-2)}$

$= \frac{4 + 6(3x-2)}{1(3x-2)}$

$= \frac{4 + 18x - 12}{3x - 2}$

$= \frac{18x - 8}{3x - 2}$

$3x - 2 \neq 0$

$x \neq \frac{2}{3}$

c) $\frac{5}{x+4} + \frac{7}{x+3}$

$= \frac{5(x+3)}{(x+4)(x+3)} + \frac{7(x+4)}{(x+3)(x+4)}$

$= \frac{5(x+3) + 7(x+4)}{(x+3)(x+4)}$

$= \frac{5x + 15 + 7x + 28}{(x+3)(x+4)}$

$= \frac{12x + 43}{(x+3)(x+4)}$

$x + 3 \neq 0$ and $x + 4 \neq 0$

$x \neq -3, -4$

d) $\frac{6}{2n-3} - \frac{4}{n-5}$

$= \frac{6(n-5)}{(2n-3)(n-5)} - \frac{4(2n-3)}{(2n-3)(n-5)}$

$= \frac{6(n-5) - 4(2n-3)}{(2n-3)(n-5)}$

$= \frac{6n - 30 - 8n + 12}{(2n-3)(n-5)}$

$= \frac{-2n - 18}{(2n-3)(n-5)}$

$2n - 3 \neq 0$ and $n - 5 \neq 0$

$n \neq \frac{3}{2}, 5$

e) $\frac{7x}{x+4} + \frac{3x}{x-6}$

$= \frac{7x(x-6)}{(x+4)(x-6)} + \frac{3x(x+4)}{(x-6)(x+4)}$

$= \frac{7x(x-6) + 3x(x+4)}{(x+4)(x-6)}$

$= \frac{7x^2 - 42x + 3x^2 + 12x}{(x+4)(x-6)}$

$= \frac{10x^2 - 30x}{(x+4)(x-6)}$

$x + 4 \neq 0$ and $x - 6 \neq 0$

$x \neq -4, 6$

f) $\frac{7}{2x-6} + \frac{4}{10x-15}$

$= \frac{7(10x-15)}{(2x-6)(10x-15)}$

$+ \frac{4(2x-6)}{(10x-15)(2x-6)}$

$= \frac{7(10x-15) + 4(2x-6)}{(2x-6)(10x-15)}$

$= \frac{70x - 105 + 8x - 24}{(2x-6)(10x-15)}$

$= \frac{78x - 129}{(2x-6)(10x-15)}$

$= \frac{78x - 129}{2(x-3)5(2x-3)}$

$= \dfrac{78x - 129}{10(x - 3)(2x - 3)}$

$x - 3 \neq 0$ and $2x - 3 \neq 0$

$x \neq 3, \dfrac{3}{2}$

7. a) $\dfrac{3}{x + 1} + \dfrac{4}{x^2 - 3x - 4}$

$= \dfrac{3}{x + 1} + \dfrac{4}{(x + 1)(x - 4)}$

$= \dfrac{3(x - 4)}{(x + 1)(x - 4)} + \dfrac{4}{(x + 1)(x - 4)}$

$= \dfrac{3(x - 4) + 4}{(x + 1)(x - 4)}$

$= \dfrac{3x - 12 + 4}{(x + 1)(x - 4)}$

$= \dfrac{3x - 8}{(x + 1)(x - 4)}$

$x + 1 \neq 0$ and $x - 4 \neq 0$

$x \neq -1, 4$

b) $\dfrac{2t}{t - 4} - \dfrac{5t}{t^2 - 16}$

$= \dfrac{2t}{t - 4} - \dfrac{5t}{(t + 4)(t - 4)}$

$= \dfrac{2t(t + 4)}{(t - 4)(t + 4)} - \dfrac{5t}{(t + 4)(t - 4)}$

$= \dfrac{2t(t + 4) - 5t}{(t - 4)(t + 4)}$

$= \dfrac{2t^2 + 8t - 5t}{(t - 4)(t + 4)}$

$= \dfrac{2t^2 + 3t}{(t - 4)(t + 4)}$

$t - 4 \neq 0$ and $t + 4 \neq 0$

$t \neq 4, -4$

c) $\dfrac{3}{t^2 + t - 6} + \dfrac{5}{(t + 3)^2}$

$= \dfrac{3}{(t + 3)(t - 2)} + \dfrac{5}{(t + 3)(t + 3)}$

$= \dfrac{3(t + 3)}{(t + 3)(t + 3)(t - 2)}$

$+ \dfrac{5(t - 2)}{(t - 2)(t + 3)(t + 3)}$

$= \dfrac{3(t + 3) + 5(t - 2)}{(t + 3)(t + 3)(t - 2)}$

$= \dfrac{3t + 9 + 5t - 10}{(t + 3)(t + 3)(t - 2)}$

$= \dfrac{8t - 1}{(t + 3)^2(t - 2)}$

$t + 3 \neq 0$ and $t - 2 \neq 0$

$t \neq -3, 2$

d) $\dfrac{4x}{x^2 + 6x + 8} - \dfrac{3x}{x^2 - 3x - 10}$

$= \dfrac{4x}{(x + 2)(x + 4)} - \dfrac{3x}{(x + 2)(x - 5)}$

$= \dfrac{4x(x - 5)}{(x + 2)(x + 4)(x - 5)}$

$- \dfrac{3x(x + 4)}{(x + 4)(x + 2)(x - 5)}$

$= \dfrac{4x^2 - 20x}{(x + 2)(x + 4)(x - 5)}$

$- \dfrac{3x^2 + 12x}{(x + 4)(x + 2)(x - 5)}$

$= \dfrac{4x^2 - 20x - (3x^2 + 12x)}{(x + 2)(x + 4)(x - 5)}$

$= \dfrac{x^2 - 32x}{(x + 2)(x + 4)(x - 5)}$

$x + 2 \neq 0$ and $x + 4 \neq 0$ and $x - 5 \neq 0$

$x \neq -2, -4, 5$

e) $\dfrac{x - 1}{x^2 - 9} + \dfrac{x + 7}{x^2 - 5x + 6}$

$= \dfrac{x - 1}{(x + 3)(x - 3)} + \dfrac{x + 7}{(x - 3)(x - 2)}$

$= \dfrac{(x - 1)(x - 2)}{(x - 2)(x + 3)(x - 3)}$

$+ \dfrac{(x + 7)(x + 3)}{(x + 3)(x - 3)(x - 2)}$

$= \dfrac{x^2 - 3x + 2}{(x - 2)(x + 3)(x - 3)}$

$+ \dfrac{x^2 + 10x + 21}{(x + 3)(x - 3)(x - 2)}$

$= \dfrac{x^2 - 3x + 2 + x^2 + 10x + 21}{(x - 2)(x + 3)(x - 3)}$

$= \dfrac{2x^2 + 7x + 23}{(x - 2)(x + 3)(x - 3)}$

$x - 2 \neq 0$ and $x + 3 \neq 0$ and $x - 3 \neq 0$

$x \neq 2, -3, 3$

f) $\dfrac{2t + 1}{2t^2 - 14t + 24} + \dfrac{5t}{4t^2 - 8t - 12}$

$= \dfrac{2t + 1}{2(t^2 - 7t + 12)} + \dfrac{5t}{4(t^2 - 2t - 3)}$

$= \dfrac{2t + 1}{2(t - 4)(t - 3)} + \dfrac{5t}{4(t + 1)(t - 3)}$

$= \dfrac{(2t + 1)(2)(t + 1)}{2(t - 4)(t - 3)(2)(t + 1)}$

$$+\frac{(5t)(t-4)}{4(t-4)(t+1)(t-3)}$$

$$=\frac{(2t+1)(2t+2)}{2(t-4)(t-3)(2)(t+1)}+\frac{(5t^2-20t)}{4(t-4)(t+1)(t-3)}$$

$$=\frac{4t^2+6t+2}{2(t-4)(t-3)(2)(t+1)}+\frac{(5t^2-20t)}{4(t-4)(t+1)(t-3)}$$

$$=\frac{4t^2+6t+2+5t^2-20t}{4(t-4)(t-3)(t+1)}$$

$$=\frac{9t^2-14t+2}{4(t-4)(t-3)(t+1)}$$

$t-4 \neq 0$ and $t-3 \neq 0$ and $t+1 \neq 0$

$t \neq 4, 3, -1$

8. a) $\frac{3}{4x^2+7x+3}-\frac{5}{16x^2+24x+9}$

$$=\frac{3}{4x^2+4x+3x+3}-\frac{5}{16x^2+12x+12x+9}$$

$$=\frac{3}{4x(1x+1)+3(x+1)}-\frac{5}{4x(4x+3)+3(4x+3)}$$

$$=\frac{3}{(4x+3)(1x+1)}-\frac{5}{(4x+3)(4x+3)}$$

$$=\frac{3(4x+3)}{(4x+3)(4x+3)(x+1)}-\frac{5(x+1)}{(x+1)(4x+3)(4x+3)}$$

$$=\frac{12x+9}{(4x+3)(4x+3)(x+1)}-\frac{5x+5}{(x+1)(4x+3)(4x+3)}$$

$$=\frac{12x+9-(5x+5)}{(4x+3)^2(x+1)}$$

$$=\frac{7x+4}{(4x+3)^2(x+1)}$$

$4x+3 \neq 0$ and $x+1 \neq 0$

$x \neq -\frac{3}{4}, -1$

b) $\frac{a-1}{a^2-8a+15}-\frac{a-2}{2a^2-9a-5}$

$$=\frac{a-1}{(a-3)(a-5)}-\frac{a-2}{2a^2-10a+1a-5}$$

$$=\frac{a-1}{(a-3)(a-5)}-\frac{a-2}{2a(a-5)+1(a-5)}$$

$$=\frac{a-1}{(a-3)(a-5)}-\frac{a-2}{(2a+1)(a-5)}$$

$$=\frac{(a-1)(2a+1)}{(2a+1)(a-3)(a-5)}-\frac{(a-2)(a-3)}{(2a+1)(a-3)(a-5)}$$

$$=\frac{2a^2-a-1}{(2a+1)(a-3)(a-5)}-\frac{a^2-5a+6}{(2a+1)(a-3)(a-5)}$$

$$=\frac{2a^2-a-1-(a^2-5a+6)}{(2a+1)(a-3)(a-5)}$$

$$=\frac{a^2+4a-7}{(2a+1)(a-3)(a-5)}$$

$2a+1 \neq 0$ and $a-3 \neq 0$ and $a-5 \neq 0$

$a \neq -\frac{1}{2}, 3, 5$

c) $\frac{3x+2}{4x^2-1}+\frac{2x-5}{4x^2+4x+1}$

$$=\frac{3x+2}{(2x+1)(2x-1)}+\frac{2x-5}{4x^2+2x+2x+1}$$

$$=\frac{3x+2}{(2x+1)(2x-1)}+\frac{2x-5}{2x(2x+1)+1(2x+1)}$$

$$=\frac{3x+2}{(2x+1)(2x-1)}+\frac{2x-5}{(2x+1)(2x+1)}$$

$$=\frac{(3x+2)(2x+1)}{(2x+1)(2x+1)(2x-1)}+\frac{(2x-5)(2x-1)}{(2x+1)(2x+1)(2x-1)}$$

$$=\frac{(3x+2)(2x+1)+(2x-5)(2x-1)}{(2x+1)(2x+1)(2x-1)}$$

$$=\frac{6x^2+7x+2+4x^2-12x+5}{(2x+1)(2x+1)(2x-1)}$$

$$=\frac{10x^2-5x+7}{(2x+1)(2x+1)(2x-1)}$$

$2x+1 \neq 0$ and $2x-1 \neq 0$

$x \neq -\frac{1}{2}, \frac{1}{2}$

9. a) $\frac{2x^3}{3y^2} \times \frac{9y}{10x} - \frac{2y}{3x}$

$$=\frac{\cancel{2x}(x^2)}{\cancel{3y}(y)} \times \frac{3\cancel{(3y)}}{\cancel{2x}(5)} - \frac{2y}{3x}$$

$= \dfrac{3x^2}{5y} - \dfrac{2y}{3x}$

$= \dfrac{3x^2(3x)}{5y(3x)} - \dfrac{2y(5y)}{3x(5y)}$

$= \dfrac{9x^3}{15xy} - \dfrac{10y^2}{15xy}$

$= \dfrac{9x^3 - 10y^2}{15xy}$

$x \neq 0$ and $y \neq 0$

b) $\dfrac{x+1}{2x-6} \div \dfrac{2(x+1)^2}{2-x} + \dfrac{11}{x-2}$

$= \dfrac{x+1}{2x-6} \times \dfrac{2-x}{2(x+1)^2} + \dfrac{11}{x-2}$

$= \dfrac{\cancel{x+1}}{2(x-3)} \times \dfrac{2-x}{2\cancel{(x+1)}(x+1)} + \dfrac{11}{x-2}$

$= \dfrac{2-x}{4(x+1)(x-3)} + \dfrac{11}{x-2}$

$= \dfrac{(2-x)(x-2)}{4(x+1)(x-2)(x-3)}$

$+ \dfrac{11(4)(x+1)(x-3)}{(4)(x+1)(x-3)(x-2)}$

$= \dfrac{2x - 4 - x^2 + 2x}{4(x+1)(x-2)(x-3)}$

$+ \dfrac{44(x^2 - 2x - 3)}{(4)(x+1)(x-3)(x-2)}$

$= \dfrac{-x^2 + 4x - 4}{4(x+1)(x-2)(x-3)}$

$+ \dfrac{44x^2 - 88x - 132}{(4)(x+1)(x-3)(x-2)}$

$= \dfrac{-x^2 + 4x - 4 + 44x^2 - 88x - 132}{4(x+1)(x-2)(x-3)}$

$= \dfrac{43x^2 - 84x - 136}{4(x+1)(x-2)(x-3)}$

$x + 1 \neq 0$ and $x - 2 \neq 0$ and $x - 3 \neq 0$

$x \neq -1, 2, 3$

c) $\dfrac{p+1}{p^2 + 2p - 35} + \dfrac{p^2 + p - 12}{p^2 - 2p - 24}$

$\times \dfrac{p^2 - 4p - 12}{p^2 + 2p - 15}$

$= \dfrac{p+1}{(p+7)(p-5)} + \dfrac{\cancel{(p+4)}\cancel{(p-3)}}{\cancel{(p+4)}\cancel{(p-6)}}$

$\times \dfrac{(p+2)\cancel{(p-6)}}{(p+5)\cancel{(p-3)}}$

$= \dfrac{p+1}{(p+7)(p-5)} + \dfrac{(p+2)}{(p+5)}$

$= \dfrac{(p+1)(p+5)}{(p+7)(p-5)(p+5)}$

$+ \dfrac{(p+2)(p+7)(p-5)}{(p+5)(p+7)(p-5)}$

$= \dfrac{p^2 + 6p + 5 + (p^2 + 9p + 14)(p-5)}{(p+7)(p-5)(p+5)}$

$= \dfrac{p^2 + 6p + 5 + p^3 + 9p^2 - 5p^2 - 31p - 70}{(p+7)(p-5)(p+5)}$

$= \dfrac{p^3 + 5p^2 - 25p - 65}{(p+7)(p-5)(p+5)}$

$p + 7 \neq 0$ and $p - 5 \neq 0$ and $p + 5 \neq 0$

and $p + 4 \neq 0$ and $p - 6 \neq 0$ and $p - 3 \neq 0$

$p \neq -7, 5, -5, -4, 3, 6$

d) $\dfrac{5m - n}{2m + n} - \dfrac{4m^2 - 4mn + n^2}{4m^2 - n^2}$

$\div \dfrac{6m^2 - mn - n^2}{3m + 15n}$

$= \dfrac{5m - n}{2m + n} - \dfrac{4m^2 - 4mn + n^2}{4m^2 - n^2}$

$\times \dfrac{3m + 15n}{6m^2 - mn - n^2}$

$= \dfrac{5m - n}{2m + n} - \dfrac{4m^2 - 2mn - 2mn + n^2}{(2m + n)(2m - n)}$

$\times \dfrac{3(m + 5n)}{6m^2 - 3mn + 2mn - n^2}$

$= \dfrac{5m - n}{2m + n} - \dfrac{2m(2m - n) - n(2m - n)}{(2m + n)(2m - n)}$

$\times \dfrac{3(m + 5n)}{3m(2m - n) + n(2m - n)}$

$= \dfrac{5m - n}{2m + n} - \dfrac{\cancel{(2m - n)}\cancel{(2m - n)}}{(2m + n)\cancel{(2m - n)}}$

$\times \dfrac{3(m + 5n)}{(3m + n)\cancel{(2m - n)}}$

$= \dfrac{5m - n}{2m + n} - \dfrac{3(m + 5n)}{(2m + n)(3m + n)}$

$= \dfrac{(5m - n)(3m + n)}{(2m + n)(3m + n)} - \dfrac{3m + 15n}{(2m + n)(3m + n)}$

$= \dfrac{15m^2 + 2mn - n^2 - 3m - 15n}{(2m + n)(3m + n)}$

$2m + n \neq 0$ and $3m + n \neq 0$

and $2m - n \neq 0$ and $m + 5n \neq 0$

$m \neq -\dfrac{1}{2}n, \dfrac{1}{2}n, -\dfrac{1}{3}n, -5n$

10. a) $\dfrac{3m + 2}{2} + \dfrac{4m + 5}{5}$

$= \dfrac{(3m + 2)(5)}{2(5)} + \dfrac{(4m + 5)(2)}{5(2)}$

$= \dfrac{15m + 10}{10} + \dfrac{8m + 10}{10}$

$= \frac{15m + 10 + 8m + 10}{10}$

$= \frac{23m + 20}{10}$

b) $\frac{5}{x^2} - \frac{3}{4x^3}$

$= \frac{5(4x)}{x^2(4x)} - \frac{3}{4x^3}$

$= \frac{20x}{4x^3} - \frac{3}{4x^3}$

$= \frac{20x - 3}{4x^3}$

$4x^3 \neq 0$

$x \neq 0$

c) $\frac{2}{y + 1} - \frac{3}{y - 2}$

$= \frac{2(y - 2)}{(y + 1)(y - 2)} - \frac{3(y + 1)}{(y - 2)(y + 1)}$

$= \frac{2y - 4}{(y + 1)(y - 2)} - \frac{3y + 3}{(y - 2)(y + 1)}$

$= \frac{2y - 4 - (3y + 3)}{(y + 1)(y - 2)}$

$= \frac{-y - 7}{(y + 1)(y - 2)}$

$y + 1 \neq 0$ and $y - 2 \neq 0$

$y \neq -1, 2$

d) $\frac{2x}{x^2 + x - 6} + \frac{5}{x^2 + 2x - 8}$

$= \frac{2x}{(x + 3)(x - 2)} + \frac{5}{(x + 4)(x - 2)}$

$= \frac{2x(x + 4)}{(x + 3)(x - 2)(x + 4)}$

$+ \frac{5(x + 3)}{(x + 3)(x + 4)(x - 2)}$

$= \frac{2x^2 + 8x}{(x + 3)(x - 2)(x + 4)}$

$+ \frac{5x + 15}{(x + 3)(x + 4)(x - 2)}$

$= \frac{2x^2 + 8x + (5x + 15)}{(x + 3)(x - 2)(x + 4)}$

$= \frac{2x^2 + 13x + 15}{(x + 3)(x - 2)(x + 4)}$

$x + 3 \neq 0$ and $x - 2 \neq 0$ and $x + 4 \neq 0$

$x \neq -3, 2, -4$

11. Increase s by 1 and decrease t by 1,

$\frac{1}{R} = \frac{1}{s + 1} + \frac{1}{t - 1}$

$= \frac{1(t - 1)}{(s + 1)(t - 1)} + \frac{1(s + 1)}{(t - 1)(s + 1)}$

$= \frac{t - 1 + s + 1}{(s + 1)(t - 1)}$

$\frac{1}{R} = \frac{t + s}{st - s + t - 1}$

Since $\frac{1}{R} = \frac{t + s}{st - s + t - 1}$,

$R = \frac{st - s + t - 1}{t + s}$

$t + s \neq 0$

$t \neq -s$

12. a) To find the difference, subtract the second speed from the first:

$\frac{2x}{3} - \frac{x + 100}{2}$

$= \frac{2x(2)}{3(2)} - \frac{(x + 100)(3)}{2(3)}$

$= \frac{4x}{6} - \frac{3x + 300}{6}$

$= \frac{4x - (3x + 300)}{6}$

$= \frac{x - 300}{6}$

b) To find out when the speeds are equal, set the two equations equal to each other and solve for x:

$\frac{2x}{3} = \frac{x + 100}{2}$

$2(2x) = 3(x + 100)$

$4x = 3x + 300$

$x = 300$

When $x = 300$, the distances are equal. Find each distance if $x < 300$, say $x = 298$.

$\frac{2x}{3} = \frac{2(298)}{3} = \frac{596}{3} = 198\frac{2}{3}$

$\frac{x + 100}{2} = \frac{298 + 100}{2} = \frac{398}{2} = 199$

So, $0 \leq x < 300$.

13. To find the decrease in sound intensity, subtract the intensity when the car is further away from the first intensity:

$I_1 = \frac{k}{d^2}$

$I_2 = \frac{k}{(d + x)^2}$

$I_1 - I_2$

$= \frac{k}{d^2} - \frac{k}{(d + x)^2}$

$= \frac{k(d + x)^2}{d^2(d + x)^2} - \frac{k(d^2)}{(d + x)^2(d^2)}$

$= \frac{k(d^2 + 2dx + x^2)}{d^2(d + x)^2} - \frac{k(d^2)}{(d + k)^2(d^2)}$

$= \frac{kd^2 + 2kdx + kx^2}{d^2(d + x)^2} - \frac{kd^2}{(d + k)^2(d^2)}$

$= \frac{kd^2 + 2kdx + kx^2 - kd^2}{d^2(d + x)^2}$

$= \frac{2kdx + kx^2}{d^2(d + x)^2}$

$d^2 \neq 0$ and $d + x \neq 0$

$d \neq 0, -x$

14. a) i) For example, $\frac{1}{2} + \frac{1}{4}$

$= \frac{1(2)}{2(2)} + \frac{1}{4}$

$= \frac{2}{4} + \frac{1}{4}$

$= \frac{3}{4}$

The denominator is 4.

ii) For example, $\frac{1}{3} + \frac{1}{4}$

$= \frac{1(4)}{3(4)} + \frac{1(3)}{4(3)}$

$= \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

The denominator is 12 which is the product of 3 and 4.

iii) For example, $\frac{1}{4} + \frac{1}{6}$

$= \frac{1(6)}{4(6)} + \frac{1(4)}{6(4)}$

$= \frac{6}{24} + \frac{4}{24}$

$= \frac{10}{24}$

$= \frac{5}{12}$

The denominator is 12, which is none of the above.

b) The LCD of two simplified rational functions with different quadratic denominators by taking the product of the two denominators.

For example, $\frac{1}{(x + 2)} + \frac{1}{(x + 3)}$

$= \frac{1(x + 3)}{(x + 2)(x + 3)} + \frac{1(x + 2)}{(x + 3)(x + 2)}$

$= \frac{x + 3 + x + 2}{(x + 2)(x + 3)}$

$= \frac{2x + 5}{(x + 2)(x + 3)}$

15. a) $\frac{1}{n} - \frac{1}{n + 1}$

$= \frac{1(n + 1)}{n(n + 1)} - \frac{1(n)}{n(n + 1)}$

$= \frac{n + 1 - n}{n(n + 1)}$

$= \frac{1}{n(n + 1)}$

b) For example, 4, 12, 3

$\frac{1}{4} = \frac{1}{3} - \frac{1}{12}$

$\frac{1}{4} = \frac{1(4)}{3(4)} - \frac{1}{12}$

$\frac{1}{4} = \frac{4}{12} - \frac{1}{12}$

$\frac{1}{4} = \frac{3}{12}$

$\frac{1}{4} = \frac{1}{4}$

For example, 5, 20, 4

$\frac{1}{5} = \frac{1}{4} - \frac{1}{20}$

$\frac{1}{5} = \frac{1(5)}{4(5)} - \frac{1}{20}$

$\frac{1}{5} = \frac{5}{20} - \frac{1}{20}$

$\frac{1}{5} = \frac{4}{20}$

$\frac{1}{5} = \frac{1}{5}$

16. a) Let x be the smaller of the two consecutive even or odd numbers. Then $x + 2$ is the larger of the two.

$$\frac{1}{x} + \frac{1}{x+2} = \frac{x + (x+2)}{x(x+2)}$$
$$= \frac{2x+2}{x(x+2)}$$
$(2x + 2)^2 + (x^2 + 2x)^2$
$= 4x^2 + 8x + 4 + x^4 + 4x^3 + 4x^2$
$= x^4 + 4x^3 + 8x^2 + 8x + 4$
$= (x^2 + 2x + 2)^2$
So, x, $x + 2$, and $x^2 + 2x + 2$ are a Pythagorean triple.
b) For example, choose 7 and 9 as the consecutive odd integers. So,
$$\frac{1}{7} + \frac{1}{9} = \frac{16}{63}$$
$16^2 + 63^2 = 4225$
$\sqrt{4225} = 65$
So, 16, 63, and 65 are triples.

Chapter Review, pp. 131–133

1. a) $(7x^2 - 2x + 1) + (9x^2 - 4x + 5) - (4x^2 + 6x - 7)$
$= 7x^2 - 2x + 1 + 9x^2 - 4x + 5 - 4x^2 - 6x + 7$
$= 12x^2 - 12x + 13$
b) $(7a^2 - 4ab + 9b^2) - (-a^2 + 2ab + 6b^2)$
$= 7a^2 - 4ab + 9b^2 + a^2 - 2ab - 6b^2$
$= 8a^2 - 6ab + 3b^2$
2. For example, $f(x) = x^2 + x$; $g(x) = 2x$
$f(0) = (0)^2 + 0 = 0$
$g(0) = 2(0) = 0$
$f(0) = g(0)$
$f(1) = (1)^2 + 1 = 2$
$g(1) = 2(1) = 2$
$f(1) = g(1)$
3) See if $A(n) = B(n) = C(n)$.
$-(n + 30) + (2n + 5)$
$= (7 - n) - (32 - 2n)$
$= (n - 26) - (n + 4) + (n - 3)$
$-n - 30 + 2n + 5$
$= 7 - n - 32 + 2n$
$= n - 26 - n - 4 + n - 3$
$1n - 25 = 1n - 25 = 1n - 33$
No, they are not triplets because their ages are not equal.
b) Astrid and Beatrice are probably twins because their ages are equal.
c) $C(0) = (0 - 26) - (0 + 4)(0 - 3)$
$= -26 - 4 - 3$
$= -33$
When Cassandra was born, Ms. Flanagan was 33 years old.
4. a) $-3(7x - 5)(4x - 7)$
$= -3(28x^2 - 49x - 20x + 35)$
$= -3(28x^2 - 69x + 35)$
$= -84x^2 + 207x - 105$
b) $-(y^2 - 4y + 7)(3y^2 - 5y - 3)$
$= -(3y^4 - 5y^3 - 3y^2 - 12y^3 + 20y^2 + 12y + 21y^2 - 35y - 21)$
$= -(3y^4 - 17y^3 - 38y^2 - 23y - 21)$
$= -3y^4 + 17y^3 + 38y^2 + 23y + 21$
c) $2(a + b)^3$
$= 2(a + b)(a + b)(a + b)$
$= 2(a^2 + 2ab + b^2)(a + b)$
$= 2(a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3)$
$= 2(a^3 + 3a^2b + 3ab^2 + b^3)$
$= 2a^3 + 6a^2b + 6ab^2 + 2b^3$
d) $3(x^2 - 2)^2(2x - 3)^2$
$= 3(x^2 - 2)(x^2 - 2)(2x - 3)(2x - 3)$
$= 3(x^4 - 4x^2 + 4)(4x^2 - 12x + 9)$
$= 3(4x^6 - 12x^5 + 9x^4 - 16x^4 + 48x^3 - 36x^2 + 16x^2 - 48x + 36)$
$= 3(4x^6 - 12x^5 - 7x^4 + 48x^3 - 20x^2 - 48x + 36)$
$= 12x^6 - 36x^5 - 21x^4 + 144x^3 - 60x^2 - 144x + 108$
5. $V = \left(\frac{1}{3}\right)\pi(r + x)^2(h + 2x)$
$V = \left(\frac{1}{3}\right)\pi(r + x)(r + x)(h + 2x)$
$V = \left(\frac{1}{3}\right)\pi(r^2 + 2rx + x^2)(h + 2x)$
$V = \left(\frac{1}{3}\right)\pi(hr^2 + 2hrx + hx^2 + 2r^2x + 4rx^2 + 2x^3)$
6. a) $(2x^4 - 3x^2 - 6) + (6x^4 - x^3 + 4x^2 + 5)$
$= 8x^4 - x^3 + 1x^2 - 1$
b) $(x^2 - 4)(2x^2 + 5x - 2)$
$= 2x^4 + 5x^3 - 2x^2 - 8x^2 - 20x + 8$
$= 2x^4 + 5x^3 - 10x^2 - 20x + 8$
c) $-7x(x^2 + x - 1) - 3x(2x^2 - 5x + 6)$
$= -7x^3 - 7x^2 + 7x - 6x^3 + 15x^2 - 18x$
$= -13x^3 + 8x^2 - 11x$

d) $-2x^2(3x^3 - 7x + 2) - x^3(5x^3 + 2x - 8)$
$= -6x^5 + 14x^3 - 4x^2 - 5x^6 - 2x^4 + 8x^3$
$= -5x^6 - 6x^5 - 2x^4 + 22x^3 - 4x^2$
e) $-2x[5x - (2x - 7)] + 6x[3x - (1 + 2x)]$
$= -2x[3x + 7] + 6x[x - 1]$
$= -6x^2 - 14x + 6x^2 - 6x$
$= -20x$
f) $(x + 2)^2(x - 1)^2 - (x - 4)^2(x + 4)^2$
$= (x + 2)(x + 2)(x - 1)(x - 1)$
$- (x - 4)(x - 4)(x + 4)(x + 4)$
$= (x^2 + 4x + 4)(x^2 - 2x + 1)$
$- (x^2 - 8x + 16)(x^2 + 8x + 16)$
$= (x^4 - 2x^3 + x^2 + 4x^3 - 8x^2 + 4x + 4x^2$
$+ 4x^2 - 8x + 4)$
$- (x^4 + 8x^3 + 16x^2 - 8x^3 - 64x^2 - 128x$
$+ 16x^2 + 128x + 256)$
$= (x^4 + 2x^3 - 3x^2 - 4x + 4)$
$- (x^4 - 32x^2 + 256)$
$= 2x^3 + 29x^2 - 4x - 252$
g) $(x^2 + 5x - 3)^2$
$= (x^2 + 5x - 3)(x^2 + 5x - 3)$
$= x^4 + 5x^3 - 3x^2 + 5x^3 + 25x^2 - 15x$
$- 3x^2 - 15x + 9$
$= x^4 + 10x^3 + 19x^2 - 30x + 9$
7. a) $12m^2n^3 + 18m^3n^2$
$= 6m^2n^2(2n + 3m)$
b) $x^2 - 9x + 20$
$= (x - 5)(x - 4)$
c) $3x^2 + 24x + 45$
$= 3(x^2 + 8x + 15)$
$= 3(x + 3)(x + 5)$
d) $50x^2 - 72$
$= 2(25x^2 - 36)$
$= 2(5x + 6)(5x - 6)$
e) $9x^2 - 6x + 1$
$= 9x^2 - 3x - 3x + 1$
$= 3x(3x - 1) - 1(3x - 1)$
$= (3x - 1)(3x - 1)$
$= (3x - 1)^2$
f) $10a^2 + a - 3$
$= 10a^2 + 6a - 5a - 3$
$= 2a(5a + 3) - 1(5a + 3)$
$= (2a - 1)(5a + 3)$
8. a) $2x^2y^4 - 6x^5y^3 + 8x^3y$
$= 2x^2y(y^3 - 3x^3y + 4x)$
b) $2x(x + 4) + 3(x + 4)$
$= (2x + 3)(x + 4)$
c) $x^2 - 3x - 10 = (x + 2)(x - 5)$
d) $15x^2 - 53x + 42$
$= 15x^2 - 35x - 18x + 42$
$= 5x(3x - 7) - 6(3x - 7)$
$= (5x - 6)(3x - 7)$
e) $a^4 - 16$
$= (a^2 + 4)(a^2 - 4)$
f) $(m - n)^2 - (2m + 3n)^2$
$= (m - n)(m - n) - (2m + 3n)(2m + 3n)$
$= (m^2 - 2mn + n^2) - (4m^2 + 12mn + 9n^2)$
$= -3m^2 - 14mn - 8n^2$
$= -(3m^2 + 14mn + 8n^2)$
$= -(3m^2 + 12mn + 2mn + 8n^2)$
$= -(3m(m + 4n) + 2n(m + 4n))$
$= -(3m + 2n)(m + 4n)$
9. a) $\frac{10a^2b + 15bc^2}{-5b}$
$= \frac{\cancel{-5b}(-2a^2 - 3c^2)}{\cancel{-5b}}$
$= -2a^2 - 3c^2$
$-5b \neq 0$
$b \neq 0$
b) $\frac{30x^2y^3 - 20x^2z^2 + 50x^2}{10x^2}$
$= \frac{\cancel{10x^2}(3y^3 - 2z^2 + 5)}{\cancel{10x^2}}$
$= 3y^3 - 2z^2 + 5$
$10x^2 \neq 0$
$x \neq 0$
c) $\frac{xy - xyz}{xy}$
$= \frac{\cancel{xy}(1 - z)}{\cancel{xy}}$
$= 1 - z$
$x \neq 0$ and $y \neq 0$
d) $\frac{16nmr - 24mnp + 40kmn}{8mn}$
$= \frac{\cancel{8mn}(2r - 3p + 5k)}{\cancel{8mn}}$
$= 2r - 3p + 5k$
$m \neq 0$ and $n \neq 0$
10. a) $8xy^2 + 12x^2y - \frac{6x^3}{2xy}$
$= 8xy^2 + 12x^2y - \frac{\cancel{2x}(3x^2)}{\cancel{2x}(y)}$
$= 8xy^2 + 12x^2y - \frac{3x^2}{y}$

$= \dfrac{8xy^2(y)}{1(y)} + \dfrac{12x^2y(y)}{1(y)} - \dfrac{3x^2}{y}$

$= \dfrac{8xy^3 + 12x^2y^2 - 3x^2}{y}$

$x \neq 0$ and $y \neq 0$

b) $\dfrac{7a - 14b}{2(a - 2b)}$

$= \dfrac{7\cancel{(a - 2b)}}{2\cancel{(a - 2b)}}$

$= \dfrac{7}{2}$

$a - 2b \neq 0$

$a \neq 2b$

c) $\dfrac{m + 3}{m^2 + 10m + 21}$

$= \dfrac{m + 3}{(m + 7)(m + 3)}$

$= \dfrac{\cancel{m + 3}}{(m + 7)\cancel{(m + 3)}}$

$= \dfrac{1}{(m + 7)}$

$m + 7 \neq 0$ and $m + 3 \neq 0$

$m \neq -7, -3$

d) $\dfrac{4x^2 - 4x - 3}{4x^2 - 9}$

$= \dfrac{4x^2 + 2x - 6x - 3}{(2x + 3)(2x - 3)}$

$= \dfrac{2x(2x + 1) - -3(2x + 1)}{(2x + 3)(2x - 3)}$

$= \dfrac{\cancel{(2x + 3)}(2x + 1)}{\cancel{(2x + 3)}(2x - 3)}$

$= \dfrac{(2x + 1)}{(2x - 3)}$

$2x + 3 \neq 0$ and $2x - 3 \neq 0$

$x \neq -\dfrac{3}{2}, \dfrac{3}{2}$

e) $\dfrac{3x^2 - 21x}{7x^2 - 28x + 21}$

$= \dfrac{3x(x - 7)}{7(x^2 - 4x + 3)}$

$= \dfrac{3x(x - 7)}{7(x - 3)(x - 1)}$

$= \dfrac{3x(x - 7)}{7(x - 3)(x - 1)}$

$x - 3 \neq 0$ and $x - 1 \neq 0$

$x \neq 3, 1$

f) $\dfrac{3x^2 - 2xy - y^2}{3x^2 + 4xy + y^2}$

$= \dfrac{3x^2 - 3xy + xy - y^2}{3x^2 + 3xy + xy + y^2}$

$= \dfrac{3x(x - y) + y(x - y)}{3x(x + y) + y(x + y)}$

$= \dfrac{(3x + y)(x - y)}{(3x + y)(x + y)}$

$= \dfrac{\cancel{(3x + y)}(x - y)}{\cancel{(3x + y)}(x + y)}$

$= \dfrac{(x - y)}{(x + y)}$

$3x + y \neq 0$ and $x + y \neq 0$

$x \neq -\dfrac{1}{3}y, -y$

11. Perhaps, but probably not. for example, $\dfrac{x + 1}{x - 1}$ and $\dfrac{x + 2}{x - 1}$ are not equivalent.

12. a) $\dfrac{6x}{8y} \times \dfrac{2y^2}{3x}$

$= \dfrac{\cancel{3x}(2)}{\cancel{2y}(4)} \times \dfrac{\cancel{2y}(y)}{\cancel{3x}}$

$= \dfrac{y}{2}$

$x \neq 0$ and $y \neq 0$

b) $\dfrac{10m^2}{3n} \times \dfrac{6mn}{20m^2}$

$= \dfrac{10m^2}{3n} \times \dfrac{3n(2)(m)}{10m^2(2)}$

$= \dfrac{\cancel{10m^2}}{\cancel{3n}} \times \dfrac{\cancel{3n(2)}(m)}{\cancel{10m^2(2)}}$

$= m$

$3n \neq 0$ and $20m^2 \neq 0$

$n \neq 0$ and $m \neq 0$

c) $\dfrac{2ab}{5bc} \div \dfrac{6ac}{10b}$

$= \dfrac{2ab}{5bc} \times \dfrac{10b}{6ac}$

$= \dfrac{\cancel{2a}(b)}{\cancel{5b}(c)} \times \dfrac{\cancel{5b}(2)}{\cancel{2a}(3c)}$

$= \dfrac{b}{c} \times \dfrac{2}{3c}$

$= \dfrac{2b}{3c^2}$

$a \neq 0, b \neq 0, c \neq 0$

d) $\frac{5p}{8pq} \div \frac{3p}{12q}$

$= \frac{5p}{8pq} \times \frac{12q}{3p}$

$= \frac{5(p)}{4q(2p)} \times \frac{4q(3)}{3(p)}$

$= \frac{5\cancel{(p)}}{\cancel{4q}(2p)} \times \frac{\cancel{4q(3)}}{\cancel{3(p)}}$

$= \frac{5}{2p}$

$p \neq 0$ and $q \neq 0$

13. a) $\frac{x^2}{2xy} \times \frac{x}{2y^2} \div \frac{(3x)^2}{xy^2}$

$= \frac{x^2}{2xy} \times \frac{x}{2y^2} \times \frac{xy^2}{(3x)^2}$

$= \frac{x^2 \times x \times xy^2}{2xy \times 2y^2 \times 9x^2}$

$= \frac{x^4y^2}{36x^3y^3}$

$= \frac{x^3y^2(x)}{x^3y^2(36y)}$

$= \frac{\cancel{x^3y^2}(x)}{\cancel{x^3y^2}(36y)}$

$= \frac{x}{36y}$

$x \neq 0$ and $y \neq 0$

b) $\frac{x^2 - 5x + 6}{x^2 - 1} \times \frac{x^2 - 4x - 5}{x^2 - 4} \div \frac{x - 5}{x^2 + 3x + 2}$

$= \frac{x^2 - 5x + 6}{x^2 - 1} \times \frac{x^2 - 4x - 5}{x^2 - 4} \times \frac{x^2 + 3x + 2}{x - 5}$

$= \frac{(x - 3)(x - 2)}{(x + 1)(x - 1)} \times \frac{(x + 1)(x - 5)}{(x + 2)(x - 2)}$
$\times \frac{(x + 2)(x + 1)}{x - 5}$

$= \frac{(x - 3)\cancel{(x - 2)}}{\cancel{(x + 1)}(x - 1)} \times \frac{\cancel{(x + 1)(x - 5)}}{\cancel{(x + 2)(x - 2)}}$
$\times \frac{\cancel{(x + 2)}(x + 1)}{\cancel{x - 5}}$

$= \frac{(x - 3)(x + 1)}{(x - 1)}$

$x + 1 \neq 0$ and $x - 1 \neq 0$ and $x + 2 \neq 0$ and $x - 2 \neq 0$ and $x - 5 \neq 0$

$x \neq -1, 1, -2, 2, 5$

c) $\frac{1 - x^2}{1 + y} \times \frac{1 - y^2}{x + x^2} \div \frac{y^3 - y}{x^2}$

$= \frac{1 - x^2}{1 + y} \times \frac{1 - y^2}{x + x^2} \times \frac{x^2}{y^3 - y}$

$= \frac{(1 + x)(1 - x)}{1 + y} \times \frac{(1 + y)(1 - y)}{x(1 + x)}$
$\times \frac{x(x)}{y(y^2 - 1)}$

$= \frac{(1 + x)(1 - x)}{1 + y} \times \frac{(1 + y)(1 - y)}{x(1 + x)}$
$\times \frac{x(x)}{y(y + 1)(y - 1)}$

$= \frac{\cancel{(1 + x)}(1 - x)}{\cancel{1 + y}} \times \frac{-\cancel{(1 + y)(y - 1)}}{\cancel{x(1 + x)}}$
$\times \frac{\cancel{x}(x)}{y(y + 1)\cancel{(y - 1)}}$

$= \frac{x(1 - x)}{y(y + 1)}$

$x \neq 0$ and $1 + y \neq 0$ and $1 + x \neq 0$ and $y \neq 0$ and $y - 1 \neq 0$

$x \neq 0, -1$ and $y \neq 0, 1, -1$

d) $\frac{x^2 - y^2}{4x^2 - y^2} \times \frac{4x^2 + 8xy + 3y^2}{x + y} \div \frac{2x + 3y}{2x - y}$

$= \frac{x^2 - y^2}{4x^2 - y^2} \times \frac{4x^2 + 8xy + 3y^2}{x + y} \times \frac{2x - y}{2x + 3y}$

$= \frac{(x + y)(x - y)}{(2x + y)(2x - y)}$
$\times \frac{4x^2 + 6xy + 2xy + 3y^2}{x + y} \times \frac{2x - y}{2x + 3y}$

$= \frac{(x + y)(x - y)}{(2x + y)(2x - y)}$
$\times \frac{2x(2x + 3y) + y(2x + 3y)}{x + y} \times \frac{2x - y}{2x + 3y}$

$= \frac{(x + y)(x - y)}{(2x + y)(2x - y)}$
$\times \frac{(2x + y)(2x + 3y)}{x + y} \times \frac{2x - y}{2x + 3y}$

$= \frac{\cancel{(x + y)}(x - y)}{\cancel{(2x + y)(2x - y)}}$
$\times \frac{\cancel{(2x + y)(2x + 3y)}}{\cancel{x + y}} \times \frac{\cancel{2x - y}}{\cancel{2x + 3y}}$

$= x - y$

$x + y \neq 0$ and $2x + y \neq 0$ and $2x - y \neq 0$ and $2x + 3y \neq 0$

$x \neq -y, -\frac{1}{2}y, \frac{1}{2}y, -\frac{3}{2}y$

14. a) $\frac{4}{5x} - \frac{2}{3x}$

$= \frac{4(3)}{5x(3)} - \frac{2(5)}{3x(5)}$

$= \frac{12}{15x} - \frac{10}{15x}$

$= \frac{2}{15x}$

$x \neq 0$

b) $\frac{5}{x+1} - \frac{2}{x-1}$

$= \frac{5(x-1)}{(x+1)(x-1)} - \frac{2(x+1)}{(x-1)(x+1)}$

$= \frac{5(x-1) - 2(x+1)}{(x+1)(x-1)}$

$= \frac{5x - 5 - 2x - 2}{(x+1)(x-1)}$

$= \frac{3x - 7}{(x+1)(x-1)}$

$x + 1 \neq 0$ and $x - 1 \neq 0$

$x \neq -1, 1$

c) $\frac{1}{x^2 + 3x - 4} + \frac{1}{x^2 + x - 12}$

$= \frac{1}{(x+4)(x-1)} + \frac{1}{(x+4)(x-3)}$

$= \frac{1(x-3)}{(x+4)(x-1)(x-3)}$

$+ \frac{1(x-1)}{(x+4)(x-1)(x-3)}$

$= \frac{(x-3) + (x-1)}{(x+4)(x-1)(x-3)}$

$= \frac{2x - 4}{(x+4)(x-1)(x-3)}$

$= \frac{2(x-2)}{(x+4)(x-1)(x-3)}$

$x + 4 \neq 0$ and $x - 1 \neq 0$ and $x - 3 \neq 0$

$x \neq -4, 1, 3$

d) $\frac{1}{x^2 - 5x + 6} - \frac{1}{x^2 - 9}$

$= \frac{1}{(x-3)(x-2)} - \frac{1}{(x+3)(x-3)}$

$= \frac{1(x+3)}{(x-3)(x-2)(x+3)}$

$- \frac{1(x-2)}{(x+3)(x-3)(x-2)}$

$= \frac{x + 3 - (x-2)}{(x-3)(x-2)(x+3)}$

$= \frac{5}{(x-3)(x-2)(x+3)}$

$x - 3 \neq 0$ and $x - 2 \neq 0$ and $x + 3 \neq 0$

$x \neq 3, 2, -3$

15. a) $\frac{1}{2x} - \frac{7}{3x^2} + \frac{4}{x^3}$

$= \frac{1(3x^2)}{2x(3x^2)} - \frac{7(2x)}{3x^2(2x)} + \frac{4(6)}{x^3(6)}$

$= \frac{3x^2}{6x^3} - \frac{14x}{6x^3} + \frac{24}{6x^3}$

$= \frac{3x^2 - 14x + 24}{6x^3}$

$6x^3 \neq 0$

$x \neq 0$

b) $\frac{3x}{x+2} + \frac{4x}{x-6}$

$= \frac{3x(x-6)}{(x+2)(x-6)} + \frac{4x(x+2)}{(x-6)(x+2)}$

$= \frac{3x(x-6) + 4x(x+2)}{(x+2)(x-6)}$

$= \frac{3x^2 - 18x + 4x^2 + 8x}{(x+2)(x-6)}$

$= \frac{7x^2 - 10x}{(x+2)(x-6)}$

$x + 2 \neq 0$ and $x - 6 \neq 0$

$x \neq -2, 6$

c) $\frac{6x}{x^2 - 5x + 6} - \frac{3x}{x^2 + x - 12}$

$= \frac{6x}{(x-3)(x-2)} - \frac{3x}{(x+4)(x-3)}$

$= \frac{6x(x+4)}{(x-3)(x-2)(x+4)}$

$- \frac{3x(x-2)}{(x+4)(x-2)(x-3)}$

$= \frac{6x(x+4) - 3x(x-2)}{(x-3)(x-2)(x+4)}$

$= \frac{6x^2 + 24x - 3x^2 + 6x}{(x-3)(x-2)(x+4)}$

$= \frac{3x^2 + 30x}{(x-3)(x-2)(x+4)}$

$x - 3 \neq 0$ and $x - 2 \neq 0$ and $x + 4 \neq 0$

$x \neq 3, 2, -4$

d) $\frac{2(x-2)^2}{x^2 + 6x + 5} \times \frac{3x + 15}{(2-x)^2}$

$= \frac{2(x-2)(x-2)}{(x+5)(x+1)} \times \frac{3x + 15}{(2-x)(2-x)}$

$= \frac{2(x-2)(x-2)}{(x+5)(x+1)} \times \frac{3x + 15}{-(x-2) - (x-2)}$

$= \frac{2\cancel{(x-2)}\cancel{(x-2)}}{\cancel{(x+5)}(x+1)} \times \frac{3\cancel{(x+5)}}{\cancel{(x-2)}\cancel{(x-2)}}$

$= \dfrac{6}{x + 1}$

$x + 5 \neq 0$ and $x + 1 \neq 0$ and $x - 2 \neq 0$

$x \neq -5, -1, 2$

e) $\dfrac{(x - 2y)^2}{x^2 - y^2} \div \dfrac{(x - 2y)(x + 3y)}{(x + y)^2}$

$= \dfrac{(x - 2y)^2}{x^2 - y^2} \times \dfrac{(x + y)^2}{(x - 2y)(x + 3y)}$

$= \dfrac{(x - 2y)(x - 2y)}{(x - y)(x + y)} \times \dfrac{(x + y)(x + y)}{(x - 2y)(x + 3y)}$

$= \dfrac{\cancel{(x - 2y)}(x - 2y)}{(x - y)\cancel{(x + y)}} \times \dfrac{\cancel{(x + y)}(x + y)}{\cancel{(x - 2y)}(x + 3y)}$

$= \dfrac{(x - 2y)(x + y)}{(x - y)(x + 3y)}$

$x - y \neq 0$ and $x + y \neq 0$ and $x - 2y \neq 0$ and $x + 3y \neq 0$

$x \neq -y, y, 2y, -3y$

f) $\dfrac{2b - 5}{b^2 - 2b - 15} + \dfrac{3b}{b^2 + b - 30} \times \dfrac{b^2 + 8b + 12}{b + 3}$

$= \dfrac{2b - 5}{(b + 3)(b - 5)} + \dfrac{3b}{(b + 6)(b - 5)} \times \dfrac{(b + 6)(b + 2)}{b + 3}$

$= \dfrac{2b - 5}{(b + 3)(b - 5)} + \dfrac{3b}{\cancel{(b + 6)}(b - 5)} \times \dfrac{\cancel{(b + 6)}(b + 2)}{b + 3}$

$= \dfrac{2b - 5}{(b + 3)(b - 5)} + \dfrac{3b(b + 2)}{(b - 5)(b + 3)}$

$= \dfrac{(2b - 5) + 3b(b + 2)}{(b - 5)(b + 3)}$

$= \dfrac{(2b - 5) + 3b^2 + 6b}{(b - 5)(b + 3)}$

$= \dfrac{3b^2 + 8b - 5}{(b - 5)(b + 3)}$

$b - 5 \neq 0$ and $b + 3 \neq 0$ and $b + 6 \neq 0$

$b \neq 5, -3, -6$

16. $0.25\left(\dfrac{40}{x}\right) + 0.75\left(\dfrac{60}{2x}\right) = 0.50$

$\dfrac{10}{x} + \dfrac{45}{2x} = 0.50$

$\dfrac{10(2)}{x(2)} + \dfrac{45}{2x} = 0.50$

$\dfrac{20}{2x} + \dfrac{45}{2x} = 0.50$

$\dfrac{65}{2x} = 0.50$

$65 = 0.50(2x)$

$65 = x$

17. a) $\dfrac{24}{n^3 - 3n^2 + 2n} \div \dfrac{3}{n}$

$= \dfrac{24}{n^3 - 3n^2 + 2n} \times \dfrac{n}{3}$

$= \dfrac{24}{n(n^2 - 3n + 2)} \times \dfrac{n}{3}$

$= \dfrac{24}{n(n - 2)(n - 1)} \times \dfrac{n}{3}$

$= \dfrac{24}{\cancel{n}(n - 2)(n - 1)} \times \dfrac{\cancel{n}}{3}$

$= \dfrac{8}{(n - 2)(n - 1)}$

$= \dfrac{8}{n^2 - 3n + 2}$

$n - 2 \neq 0$ and $n - 1 \neq 0$ and $n \neq 0$

$n \neq 2, 1, 0$

b) i) $n = 5$

$= \dfrac{6}{(5 - 2)(5 - 1)}$

$= \dfrac{6}{(3)(4)}$

$= \dfrac{6}{12}$

$= \dfrac{1}{2}$

ii) $n = 4$

$= \dfrac{6}{(4 - 2)(4 - 1)}$

$= \dfrac{6}{(2)(3)}$

$= \dfrac{6}{6}$

$= 1$

Chapter Self-Test, p. 134

1. a) $(-x^2 + 2x + 7) + (2x^2 - 7x - 7)$

$= -x^2 + 2x + 7 + 2x^2 - 7x - 7$

$= x^2 - 5x$

b) $(2m^2 - mn + 4n^2) - (5m^2 - n^2) + (7m^2 - 2mn)$

$= 2m^2 - mn + 4n^2 - 5m^2 + n^2 + 7m^2 - 2mn$

$= 4m^2 - 3mn + 5n^2$

2. a) $2(12a - 5)(3a - 7)$
$= 2(36a^2 - 84a - 15a + 35)$
$= 2(36a^2 - 99a + 35)$
$= 72a^2 - 198a + 70$
b) $(2x^2y - 3xy^2)(4xy^2 + 5x^2y)$
$= 8x^3y^3 + 10x^4y^2 - 12x^2y^4 - 15x^3y^3$
$= -7x^3y^3 + 10x^4y^2 - 12x^2y^4$
c) $(4x - 1)(5x + 2)(x - 3)$
$= (20x^2 + 8x - 5x - 2)(x - 3)$
$= (20x^2 + 3x - 2)(x - 3)$
$= 20x^3 - 60x^2 + 3x^2 - 9x - 2x + 6$
$= 20x^3 - 57x^2 - 11x + 6$
d) $(3p^2 + p - 2)^2$
$= (3p^2 + p - 2)(3p^2 + p - 2)$
$= 9p^4 + 3p^3 - 6p^2 + 3p^3 + p^2$
$- 2p - 6p^2 - 2p + 4$
$= 9p^4 + 6p^3 - 11p^2 - 4p + 4$
3. $f(x) = 9x^2 + 4$
$g(x) = (3x - a)^2$
$g(x) = (3x - a)(3x - a)$
$g(x) = 9x^2 - 3xa - 3xa + a^2$
$g(x) = 9x^2 - 6xa + a^2$
$9x^2 - 6xa + a^2 \neq 9x^2 + 4$
$f(x) \neq g(x)$
No, they will never be equal.
4) a) If one day is $(2n + 1)^3$ is day n, then day $n + 1$ is $(2(n + 1) + 1)^3$ or $(2n + 3)^3$. So you need to find the difference between the days.
$= (2n + 3)^3 - (2n + 1)^3$
$= (2n + 3)(2n + 3)(2n + 3)$
$- (2n + 1)(2n + 1)(2n + 1)$
$= (4n^2 + 12n + 9)(2n + 3)$
$- (4n^2 + 4n + 1)(2n + 1)$
$= (8n^3 + 12n^2 + 24n^2 + 36n + 18n + 27)$
$- (8n^3 + 4n^2 + 8n^2 + 4n + 2n + 1)$
$= (8n^3 + 36n^2 + 54n + 27)$
$- (8n^3 + 12n^2 + 6n + 1)$
$= 24n^2 + 48n + 26$
b) $n = 6$
$(2(6) + 1)^3$
$= 13^3$
$= 2197$
$n = 5$
$(2(5) + 1)^3$
$= 11^3$
$= 1331$
The difference is $2197 - 1331 = 866$.
5. a) $3m(m - 1) + 2m(1 - m)$
$= 3m(m - 1) - 2m(m - 1)$
$= (3m - 2m)(m - 1)$
$= m(m - 1)$
b) $x^2 - 27x + 72$
$= (x - 3)(x - 24)$
c) $15x^2 - 7xy - 2y^2$
$= 15x^2 - 10xy + 3xy - 2y^2$
$= 5x(3x - 2y) + y(3x - 2y)$
$= (5x + y)(3x - 2y)$
d) $(2x - y + 1)^2 - (x - y - 2)^2$
To factor this, use the difference of two squares.
$= [(2x - y + 1) - (x - y - 2)]$
$[(2x - y + 1) + (x - y - 2)]$
$= (x + 3)(3x - 2y - 1)$
e) $5xy - 10x - 3y + 6$
$= 5x(y - 2) - 3(y - 2)$
$= (5x - 3)(y - 2)$
f) $p^2 - m^2 + 6m - 9$
$= p^2 - (m^2 - 6m + 9)$
$= p^2 - (m - 3)(m - 3)$
$= (p)^2 - (m - 3)^2$
$= (p - (m - 3))(p + (m - 3))$
$= (p - m + 3)(p + m - 3)$
6. $y = x^3 - 4x^2 - x + 4$
$y = (x^3 - 4x^2) - x + 4$
$y = x^2(x - 4) - 1(x - 4)$
$y = (x^2 - 1)(x - 4)$
$x^2 - 1 = 0$
$x^2 = 1$
$\sqrt{x^2} = \sqrt{1}$
$x = \pm 1$
$x - 4 = 0$
$x = 4$
7. a) $\dfrac{4a^2b}{5ab^3} \div \dfrac{6a^2b}{35ab}$
$= \dfrac{4a^2b}{5ab^3} \times \dfrac{35ab}{6a^2b}$
$= \dfrac{4(a^2b)}{ab(5b^2)} \times \dfrac{35(ab)}{(ab)6a}$
$= \dfrac{4\cancel{(a^2b)}}{\cancel{ab}(5b^2)} \times \dfrac{35\cancel{(ab)}}{\cancel{(a^2b)}6}$
$= \dfrac{140}{30b^2}$
$= \dfrac{14}{3b^2}$
$a \neq 0$ and $b \neq 0$
b) $\dfrac{x - 2}{x^2 - x - 12} \times \dfrac{2x - 8}{x^2 - 4x + 4}$
$= \dfrac{x - 2}{(x + 3)(x - 4)} \times \dfrac{2(x - 4)}{(x - 2)(x - 2)}$

$$= \frac{\cancel{x-2}}{(x+3)\cancel{(x-4)}} \times \frac{2\cancel{(x-4)}}{\cancel{(x-2)}(x-2)}$$

$$= \frac{2}{(x+3)(x-2)}$$

$x - 4 \neq 0$ and $x + 3 \neq 0$ and $x - 2 \neq 0$

$x \neq 4, -3, 2$

c) $$\frac{5}{t^2 - 7t - 18} + \frac{6}{t+2}$$

$$= \frac{5}{(t+2)(t-9)} + \frac{6}{t+2}$$

$$= \frac{5}{(t+2)(t-9)} + \frac{6(t-9)}{(t+2)(t-9)}$$

$$= \frac{5 + 6(t-9)}{(t+2)(t-9)}$$

$$= \frac{5 + 6t - 54}{(t+2)(t-9)}$$

$$= \frac{6t - 49}{(t+2)(t-9)}$$

$t + 2 \neq 0$ and $t - 9 \neq 0$

$t \neq -2, 9$

d) $$\frac{4x}{6x^2 + 13x + 6} - \frac{3x}{4x^2 - 9}$$

$$= \frac{4x}{6x^2 + 9x + 4x + 6} - \frac{3x}{(2x+3)(2x-3)}$$

$$= \frac{4x}{3x(2x+3) + 2(2x+3)} - \frac{3x}{(2x+3)(2x-3)}$$

$$= \frac{4x}{(3x+2)(2x+3)} - \frac{3x}{(2x+3)(2x-3)}$$

$$= \frac{4x(2x-3)}{(2x-3)(3x+2)(2x+3)} - \frac{3x(3x+2)}{(3x+2)(2x+3)(2x-3)}$$

$$= \frac{4x(2x-3) - 3x(3x+2)}{(2x-3)(3x+2)(2x+3)}$$

$$= \frac{(8x^2 - 12x) - (9x^2 + 6x)}{(2x-3)(3x+2)(2x+3)}$$

$$= \frac{-x^2 - 18x}{(2x-3)(3x+2)(2x+3)}$$

$2x - 3 \neq 0$ and $3x + 2 \neq 0$ and $2x + 3 \neq 0$

$x \neq \frac{3}{2}, -\frac{2}{3}, -\frac{3}{2}$

8. Yes, as long as there were no restrictions factored out.

9. If the three consecutive natural numbers are x, $x + 1$, and $x + 2$, you want to show:

$$\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$$

$$\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{1(x+1)(x+2)}{x(x+1)(x+2)} + \frac{1x(x+2)}{x(x+1)(x+2)} + \frac{1x(x+1)}{x(x+1)(x+2)}$$

$$= \frac{(x+1)(x+2) + x(x+2) + x(x+1)}{x(x+1)(x+2)}$$

$$= \frac{x^2 + 3x + 2 + x^2 + 2x + x^2 + x}{x(x+1)(x+2)}$$

$$= \frac{3x^2 + 6x + 2}{x(x+1)(x+2)}$$

$$= \frac{3(x^2 + 2x) + 2}{x(x+1)(x+2)}$$

$$= \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$$

So, $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$.

Yes, it is true.

Chapter 3: Quadratic Functions

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 138

1. $f(x) = -3x^2 + 4x - 1$

a) $f(1) = -3(1)^2 + 4(1) - 1$
$= -3 + 4 - 1$
$= 0$

b) $f(-2) = -3(-2)^2 + 4(-2) - 1$
$= -12 - 8 - 1$
$= -21$

c) $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) - 1$
$= -\frac{1}{3} + \frac{4}{3} - 1$
$= 0$

d) $f(0) = -3(0)^2 + 4(0) - 1$
$= -1$

e) $f(k) = -3(k)^2 + 4(k) - 1$
$= -3k^2 + 4k - 1$

f) $f(-k) = -3(-k)^2 + 4(-k) - 1$
$= -3k^2 - 4k - 1$

2. To express the following quadratics in standard form $f(x) = ax^2 + bx + c$, I will need to do polynomial multiplication and simplify.

a) $f(x) = (x - 3)(x + 5)$
$= x^2 - 3x + 5x - 15$
$= x^2 + 2x - 15$

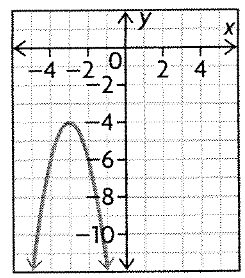
b) $f(x) = 2x(x + 6)$
$= 2x^2 + 12x$

c) $f(x) = -3(x + 2)^2 + 3$
$= -3(x + 2)(x + 2) + 3$
$= -3(x^2 + 2x + 2x + 4) + 3$
$= -3(x^2 + 4x + 4) + 3$
$= -3x^2 - 12x - 12 + 3$
$= -3x^2 - 12x - 9$

d) $f(x) = (x - 1)^2$
$= (x - 1)(x - 1)$
$= x^2 - x - x + 1$
$= x^2 - 2x + 1$

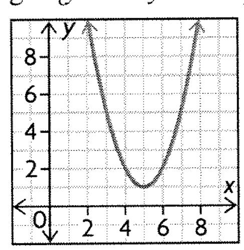
3. The axis of symmetry of a parabola is the vertical line $x = c$, c a fixed number, about which the parabola is symmetric. The vertex of a parabola is the point where the axis of symmetry intersects the parabola. The domain is the set of x-values that I am able to consider, and the range is the set of y-values that are "hit," or attained, by the function in question.

a)



axis of symmetry: $x = -3$
vertex: $(-3, -4)$
domain: $\{x \in \mathbf{R}\}$, "all real numbers"
range: $\{y \in \mathbf{R} | y \leq -4\}$

b)



axis of symmetry: $x = 5$
vertex: (5, 1)
domain: $\{x \in \mathbf{R}\}$, "all real numbers"
range: $\{y \in \mathbf{R} | y \geq 1\}$

4. For a quadratic in standard form $y = ax^2 + bx + c$, where $a \neq 0$, b, c are real numbers, I can manipulate this equation so that it's easier to work with as follows (this is called "completing the square"):

$y = ax^2 + bx + c$

$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$

$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right)$