$= \frac{\cancel{x-2}}{(x+3)\cancel{(x-4)}} \times \frac{2\cancel{(x-4)}}{\cancel{(x-2)}(x-2)}$

$= \frac{2}{(x+3)(x-2)}$

$x - 4 \neq 0$ and $x + 3 \neq 0$ and $x - 2 \neq 0$

$x \neq 4, -3, 2$

c) $\frac{5}{t^2 - 7t - 18} + \frac{6}{t+2}$

$= \frac{5}{(t+2)(t-9)} + \frac{6}{t+2}$

$= \frac{5}{(t+2)(t-9)} + \frac{6(t-9)}{(t+2)(t-9)}$

$= \frac{5 + 6(t-9)}{(t+2)(t-9)}$

$= \frac{5 + 6t - 54}{(t+2)(t-9)}$

$= \frac{6t - 49}{(t+2)(t-9)}$

$t + 2 \neq 0$ and $t - 9 \neq 0$

$t \neq -2, 9$

d) $\frac{4x}{6x^2 + 13x + 6} - \frac{3x}{4x^2 - 9}$

$= \frac{4x}{6x^2 + 9x + 4x + 6} - \frac{3x}{(2x+3)(2x-3)}$

$= \frac{4x}{3x(2x+3) + 2(2x+3)}$
$- \frac{3x}{(2x+3)(2x-3)}$

$= \frac{4x}{(3x+2)(2x+3)} - \frac{3x}{(2x+3)(2x-3)}$

$= \frac{4x(2x-3)}{(2x-3)(3x+2)(2x+3)}$
$- \frac{3x(3x+2)}{(3x+2)(2x+3)(2x-3)}$

$= \frac{4x(2x-3) - 3x(3x+2)}{(2x-3)(3x+2)(2x+3)}$

$= \frac{(8x^2 - 12x) - (9x^2 + 6x)}{(2x-3)(3x+2)(2x+3)}$

$= \frac{-x^2 - 18x}{(2x-3)(3x+2)(2x+3)}$

$2x - 3 \neq 0$ and $3x + 2 \neq 0$ and $2x + 3 \neq 0$

$x \neq \frac{3}{2}, -\frac{2}{3}, -\frac{3}{2}$

8. Yes, as long as there were no restrictions factored out.

9. If the three consecutive natural numbers are x, $x + 1$, and $x + 2$, you want to show:

$\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$

$\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{1(x+1)(x+2)}{x(x+1)(x+2)}$
$+ \frac{1x(x+2)}{x(x+1)(x+2)}$
$+ \frac{1x(x+1)}{x(x+1)(x+2)}$

$= \frac{(x+1)(x+2) + x(x+2) + x(x+1)}{x(x+1)(x+2)}$

$= \frac{x^2 + 3x + 2 + x^2 + 2x + x^2 + x}{x(x+1)(x+2)}$

$= \frac{3x^2 + 6x + 2}{x(x+1)(x+2)}$

$= \frac{3(x^2 + 2x) + 2}{x(x+1)(x+2)}$

$= \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$

So, $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} = \frac{3(x(x+2)) + 2}{x(x+1)(x+2)}$.

Yes, it is true.

CHAPTER 3: Quadratic Functions

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 138

1. $f(x) = -3x^2 + 4x - 1$

a) $f(1) = -3(1)^2 + 4(1) - 1$
$= -3 + 4 - 1$
$= 0$

b) $f(-2) = -3(-2)^2 + 4(-2) - 1$
$= -12 - 8 - 1$
$= -21$

c) $f\left(\frac{1}{3}\right) = -3\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) - 1$
$= -\frac{1}{3} + \frac{4}{3} - 1$
$= 0$

d) $f(0) = -3(0)^2 + 4(0) - 1$
$= -1$

e) $f(k) = -3(k)^2 + 4(k) - 1$
$= -3k^2 + 4k - 1$

f) $f(-k) = -3(-k)^2 + 4(-k) - 1$
$= -3k^2 - 4k - 1$

2. To express the following quadratics in standard form $f(x) = ax^2 + bx + c$, I will need to do polynomial multiplication and simplify.

a) $f(x) = (x - 3)(x + 5)$
$= x^2 - 3x + 5x - 15$
$= x^2 + 2x - 15$

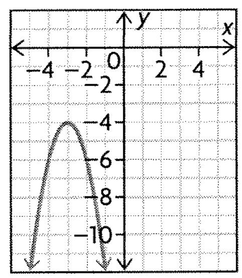
b) $f(x) = 2x(x + 6)$
$= 2x^2 + 12x$

c) $f(x) = -3(x + 2)^2 + 3$
$= -3(x + 2)(x + 2) + 3$
$= -3(x^2 + 2x + 2x + 4) + 3$
$= -3(x^2 + 4x + 4) + 3$
$= -3x^2 - 12x - 12 + 3$
$= -3x^2 - 12x - 9$

d) $f(x) = (x - 1)^2$
$= (x - 1)(x - 1)$
$= x^2 - x - x + 1$
$= x^2 - 2x + 1$

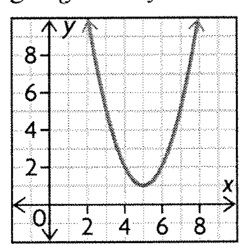
3. The axis of symmetry of a parabola is the vertical line $x = c$, c a fixed number, about which the parabola is symmetric. The vertex of a parabola is the point where the axis of symmetry intersects the parabola. The domain is the set of x-values that I am able to consider, and the range is the set of y-values that are "hit," or attained, by the function in question.

a)



axis of symmetry: $x = -3$
vertex: $(-3, -4)$
domain: $\{x \in \mathbf{R}\}$, "all real numbers"
range: $\{y \in \mathbf{R} | y \leq -4\}$

b)



axis of symmetry: $x = 5$
vertex: $(5, 1)$
domain: $\{x \in \mathbf{R}\}$, "all real numbers"
range: $\{y \in \mathbf{R} | y \geq 1\}$

4. For a quadratic in standard form $y = ax^2 + bx + c$, where $a \neq 0$, b, c are real numbers, I can manipulate this equation so that it's easier to work with as follows (this is called "completing the square"):

$y = ax^2 + bx + c$

$$= a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right)$$

$$= a\left(\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right)$$
$$= a\left(\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a^2}\right)$$
$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

This last expression is quite useful. For instance, I can now find the axis of symmetry quite easily. To do this, notice that the points x and $-\frac{b}{a} - x$ are symmetric about the point $-\frac{b}{2a}$ on the x-axis (for if $x < -\frac{b}{2a}$, then $-x > \frac{b}{2a}$ and so $-\frac{b}{a} - x > -\frac{b}{2a}$. Then $-\frac{b}{2a} - x = \left(-\frac{b}{a} - x\right) - \left(-\frac{b}{2a}\right)$ so that x and $-\frac{b}{a} - x$ are the same distance from, and on opposite sides of, the point $-\frac{b}{2a}$ on the x-axis. A similar argument holds in the case of $x > -\frac{b}{2a}$. So substituting $-\frac{b}{a} - x$ into my modified equation for y, I have

$$y\left(-\frac{b}{a} - x\right) = a\left(\left(-\frac{b}{a} - x\right) + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$
$$= a\left(-\frac{b}{2a} - x\right)^2 + \frac{4ac - b^2}{4a}$$
$$= a\left((-1)\left(x + \frac{b}{2a}\right)\right)^2 + \frac{4ac - b^2}{4a}$$
$$= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$
$$= y(x)$$

That is, the y-values are the same at the two points x and $-\frac{b}{a} - x$, and this shows that the graph of this function is symmetric about the vertical line $x = -\frac{b}{2a}$, and so this is my axis of symmetry. Now I can find the vertex by substituting this point into my equation:

$$y\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a} + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$
$$= \frac{4ac - b^2}{4a}$$

So the vertex is $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$.

a) $y = x^2 + 4$: notice that in this case, $a = 1$, $b = 0$, and $c = 4$ for the standard form of a quadratic. So, from what I just did, this function is symmetric about the vertical line

$$x = -\frac{b}{2a}$$
$$= -\frac{0}{2(1)}$$
$$= 0$$

and this is the axis of symmetry. The vertex is

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right) = \left(0, \frac{4(1)(4)}{4(1)}\right)$$
$$= (0, 4)$$

Finally, the "direction of opening" for the parabola is determined by the sign of the coefficient (number) sitting next to the x^2 term. This is just the number a in standard form, and the parabola will open upward if the sign is $+$, and downward if the sign is $-$. In this case, the sign of $a = 1$ is $+$, and so the parabola opens upward.

b) $y = 3(x - 4)^2 + 1$: This quadratic is conveniently in "completed square" form. So, from what I did above, I know that

$$-\frac{b}{2a} = -(-4)$$
$$= 4$$

and the axis of symmetry is $x = 4$. Also

$$\frac{4ac - b^2}{4a} = 1$$

so the vertex is (4, 1). Finally, $a = 3$ is positive, so this parabola opens upward.

c) $y = -0.5(x + 7)^2 - 3$: From my experience in part b), I know that

$$-\frac{b}{2a} = -7$$
$$\frac{4ac - b^2}{4a} = -3$$
$$a = -0.5$$

So the axis of symmetry is $x = -7$, the vertex is $(-7, -3)$, and since $a = -0.5$ is negative, the parabola opens downward.

d) $y = -3(x + 2)(x - 5)$: We first put this quadratic in standard form:

$$\begin{aligned} y &= -3(x + 2)(x - 5) \\ &= -3(x^2 + 2x - 5x - 10) \\ &= -3(x^2 - 3x - 10) \\ &= -3x^2 + 9x + 30 \end{aligned}$$

So $a = -3$, $b = 9$, $c = 30$, and so

$$-\frac{b}{2a} = \frac{3}{2}$$

$$\frac{4ac - b^2}{4a} = \frac{147}{4}$$

This means that the axis of symmetry is $x = \frac{3}{2}$, the vertex is $\left(\frac{3}{2}, \frac{147}{4}\right)$, and the parabola opens downward (since $a = -3$ is negative).

5. To see how to solve the following equations, I first consider the standard form of the quadratic again, once I have completed the square:

$$\begin{aligned} y &= ax^2 + bx + c \\ &= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} \end{aligned}$$

Setting $y = 0$ gives me

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b\pm\sqrt{b^2 - 4ac}}{2a}$$

This is known as the "quadratic formula," and gives the x-values for roots (zeroes) of the parabola (that is, places where the parabola hits the x-axis). Sometimes, rather than using the quadratic formula, it will be easier to just factor the quadratic to find its roots, like in part a) below, for instance.

a) $x^2 - 11x + 24 = 0$: here, I can factor.

$(x - 3)(x - 8) = 0$

$x - 3 = 0$ or $x - 8 = 0$

So $x = 3$ or $x = 8$.

b) $x^2 - 6x + 3 = 0$: use the quadratic formula.

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6\pm\sqrt{36 - 4(1)(3)}}{2(1)} \\ &= \frac{6\pm\sqrt{24}}{2} \end{aligned}$$

So $x \doteq 0.55$ or $x \doteq 5.45$.

c) $3x^2 - 2x - 5 = 0$: I can factor.

$(x + 1)(3x - 5) = 0$

$x + 1 = 0$ or $3x - 5 = 0$

So $x = -1$ or $x = \frac{5}{3}$.

d) $3x^2 + 2x = x^2 + 9x - 3$: Moving everything to the left-hand side of this equation gives me

$2x^2 - 7x + 3 = 0$

I can factor this quadratic.

$(2x - 1)(x - 3) = 0$

$2x - 1 = 0$ or $x - 3 = 0$

So $x = \frac{1}{2}$ or $x = 3$.

6. The x-intercepts are the points where the "roots" or "zeroes" of the parabola are located. So, if I have a quadratic in standard form $f(x) = ax^2 + bx + c$, finding the x-intercepts is basically the same as solving the equation $f(x) = 0$, I did in problem 5.

a) $x^2 - 9 = 0$: I can factor.

$(x - 3)(x + 3) = 0$

$x - 3 = 0$ or $x + 3 = 0$

So $x = \pm 3$ are the solutions, and the x-intercepts are $(3, 0)$ and $(-3, 0)$.

b) $x^2 - 8x - 18 = 0$: use the quadratic formula.

$$\begin{aligned} x &= \frac{-b\pm\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{8\pm\sqrt{64 - 4(1)(-18)}}{2(1)} \\ &= \frac{8\pm\sqrt{136}}{2} \end{aligned}$$

So $x \doteq -1.83$ and $x \doteq 9.83$ are the solutions, and the x-intercepts are $(-1.83, 0)$ and $(9.83, 0)$.

c) $-3x^2 + 10x - 8 = 0$: I can factor.

$(3x - 4)(2 - x) = 0$

$3x - 4 =$ or $-2 - x = 0$

So $x = \frac{4}{3}$ and $x = 2$ are the solutions, and the x-intercepts are $\left(\frac{4}{3}, 0\right)$ and $(2, 0)$.

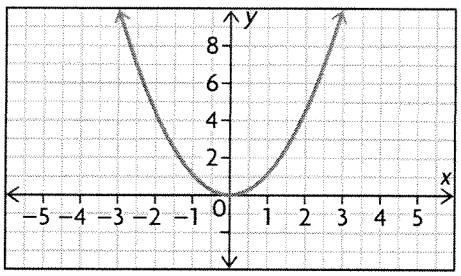
d) $6x - 2x^2 = 0$: I can factor.

$2x(3 - x) = 0$

$2x = 0$ or $3 - x = 0$

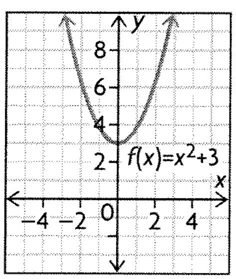
So $x = 0$ and $x = 3$ are the solutions, and the x-intercepts are $(0, 0)$ and $(3, 0)$.

7. The easiest way to graph a parabola is to start with the basic quadratic $y = x^2$. It has the following graph:



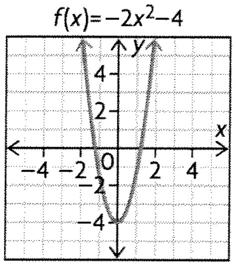
Changing the quadratic to $y = a(x - d)^2 + k$ will have the effect of contracting/stretching this basic parabola by a factor of a (and flipping it if $a < 0$), followed by shifting it horizontally by d units, then finally shifting it vertically by k units. This allows me to graph the following quadratics by hand.

a) $f(x) = x^2 + 3$: shift $y = x^2$ up by 3 units.



b) $f(x) = 2x^2 - 4$: stretch $y = x^2$ by a factor of 2, then shift down by 4 units.

f(x)=−2x²−4

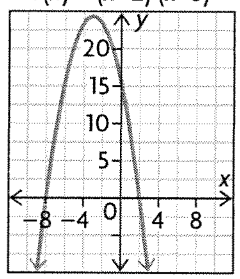


c) $f(x) = -(x - 2)(x + 8)$: multiply out the expression for $f(x)$, simplify, then complete the square.

$$\begin{aligned} f(x) &= -(x - 2)(x + 8) \\ &= -(x^2 - 2x + 8x - 16) \\ &= -(x^2 + 6x - 16) \\ &= -(x^2 + 6x + 9 - 9 - 16) \\ &= -((x + 3)^2 - 9 - 16) \\ &= -(x + 3)^2 + 25 \end{aligned}$$

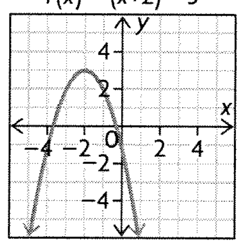
So flip $y = x^2$, shift left 3 units, then up 25 units.

f(x)=−(x−2) (x+8)



d) $f(x) = -(x + 2)^2 + 3$: flip $y = x^2$, shift left 2 units, then up 3 units.

f(x)=−(x+2)²−3



8. Here are some possibilities:

Definition:	Characteristics:
Equation is of form $y = ax^2+bx+c$ of equivalent	graph is a parabola function has two, one or no zero second differences is a constant
Examples: $y = x^2$ $\quad y = -4(x+3)^2-5$	**Non-examples:** $y = 5-4x$ $\quad y = 2\sqrt{x-5}$

Quardratic function

3.1 Properties of Quadratic Functions, pp. 145–147

1. a) Make a table of first and second differences.

x	y	1st differences	2nd differences
-2	15	$11 - 15 = -4$	$-4 - (-4) = 0$
-1	11	$7 - 11 = -4$	$-4 - (-4) = 0$
0	7	$3 - 7 = -4$	$-4 - (-4) = 0$
1	3	$-1 - 3 = -4$	*
2	-1	*	*

Since the first differences are constant and nonzero, and the second differences are zero, the function is linear.

b) Make a table of first and second differences.

x	y	1st differences	2nd differences
-2	1	$3 - 1 = 2$	$3 - 2 = 1$
-1	3	$6 - 3 = 3$	$4 - 3 = 1$
0	6	$10 - 6 = 4$	$5 - 4 = 1$
1	10	$15 - 10 = 5$	*
2	15	*	*

Since the first differences are non-constant, and the second differences are constant and nonzero, the function is quadratic.

c) Make a table of first and second differences.

x	y	1st differences	2nd differences
-2	4	$8 - 4 = 4$	$4 - 4 = 0$
-1	8	$12 - 8 = 4$	$4 - 4 = 0$
0	12	$16 - 12 = 4$	$4 - 4 = 0$
1	16	$20 - 16 = 4$	*
2	20	*	*

Since the first differences are constant and nonzero, and the second differences are zero, the function is linear.

d) Make a table of first and second differences.

x	y	1st differences	2nd differences
-2	7	$4 - 7 = -3$	$-1 - (-3) = 2$
-1	4	$3 - 4 = -1$	$1 - (-1) = 2$
0	3	$4 - 3 = 1$	$3 - 1 = 2$
1	4	$7 - 4 = 3$	*
2	7	*	*

Since the first differences are non-constant, and the second differences are constant and nonzero, the function is quadratic.

2. To determine whether a parabola opens upward or downward, I need only look at the coefficient (number) sitting next to the x^2 term. If this coefficient is positive, the parabola opens upward, and a negative coefficient means the parabola will open downward.

a) $f(x) = 3x^2$: opens upward.

b) $f(x) = -2(x - 3)(x + 1)$
$= -2(x^2 - 2x - 3)$
$= -2x^2 + 4x + 6$

So the parabola opens downward.

c) $f(x) = -(x + 5)^2 - 1$
$= -(x^2 + 10x + 25) - 1$
$= -x^2 - 10x - 26$

So the parabola opens downward.

d) $f(x) = \frac{2}{3}x^2 - 2x - 1$: opens upward.

3. $f(x) = -3(x - 2)(x + 6)$

a) To find the zeros, set the equation equal to zero.

$-3(x - 2)(x + 6) = 0$

$x - 2 = 0$ or $x + 6 = 0$

So the zeros are $x = 2$ and $x = -6$.

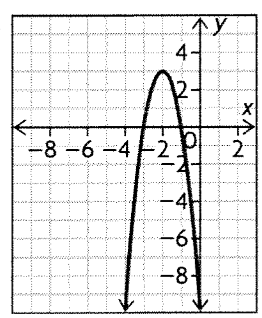
b) $f(x) = -3(x - 2)(x + 6)$
$= -3(x^2 + 4x - 12)$
$= -3x^2 - 12x + 36$

So since the coefficient of x^2 is negative, the parabola will open downward.

c) The axis of symmetry will lie halfway between the zeros. So by part a), the equation for this axis is

$x = \frac{2 - 6}{2}$
$= -2$

4.

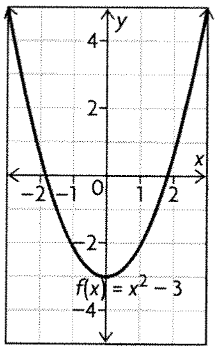


a) I see directly from the graph that the vertex is $(-2, 3)$.

b) The axis of symmetry goes through the vertex, so by part a) the axis of symmetry is $x = -2$. Of course, this agrees with what I see in the graph.

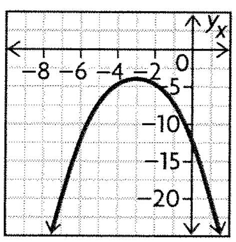
c) The domain is "all real numbers," or $\{x \in \mathbf{R}\}$, and the range is $\{y \in \mathbf{R} | y \leq 3\}$, since all y-values below 3 are "hit" by the graph.

5. a) $f(x) = x^2 - 3$: this will shift the standard parabola $y = x^2$ down by three units, giving the graph



So the axis of symmetry is $x = 0$, the vertex is $(0, -3)$, and the parabola opens upward.

b) $f(x) = -(x + 3)^2 - 4$: this will flip the standard parabola $y = x^2$, then shift it left by 3 units and down by 4 units, giving the graph



So the axis of symmetry is $x = -3$, the vertex is $(-3, -4)$, and the parabola opens downward.

c) $f(x) = 2(x - 4)(x + 2)$: this parabola will have zeros $x = 4$ and $x = -2$, so that the axis of symmetry is

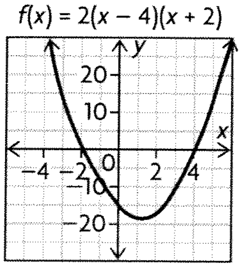
$$x = \frac{4 - 2}{2}$$
$$= 1$$

Then the y-coordinate for the vertex is

$$f(1) = 2(1 - 4)(1 + 2)$$
$$= -18$$

So I have vertex $(1, -18)$. It is clear, from multiplying out the equation, that the coefficient next to x^2 is positive, so the parabola will open upward. All of this information is enough for me to sketch the graph:

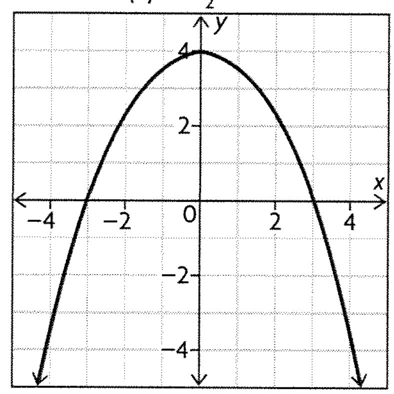
$f(x) = 2(x - 4)(x + 2)$



So the axis of symmetry is $x = -3$, the vertex is $(-3, -4)$, and the parabola opens downward.

d) $f(x) = -\frac{1}{2}x^2 + 4$: this will flip the standard parabola $y = x^2$, then compress the graph by a factor of $\frac{1}{2}$, then shift it up by 4 units, giving the graph

$f(x) = -\frac{1}{2}x^2 + 4$



So the axis of symmetry is $x = 0$, the vertex is $(0, 4)$, and the parabola opens downward.

6. a)
$$f(x) = -3(x - 1)^2 + 6$$
$$= -3(x^2 - 2x + 1) + 6$$
$$= -3x^2 + 6x + 3$$

So the y-coordinate for the y-intercept is

$$f(0) = -3(0)^2 + 6(0) + 3$$
$$= 3$$

That is, I have y-intercept $(0, 3)$.

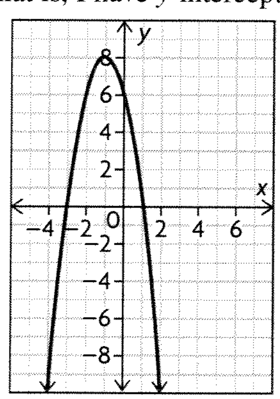
b)
$$f(x) = 4(x - 3)(x + 7)$$
$$= 4(x^2 + 4x - 21)$$
$$= 4x^2 + 16x - 84$$

So the y-coordinate for the y-intercept is

$$f(0) = 4(0)^2 + 16(0) - 84$$
$$= -84$$

That is, I have y-intercept $(0, -84)$.

7.



a) The parabola opens downward.
b) The vertex is $(-1, 8)$.
c) The zeros are $x = -3$ and $x = 1$, so the x-intercepts are $(-3, 0)$ and $(1, 0)$.
d) The domain is $\{x \in \mathbf{R}\}$, and the range is $\{y \in \mathbf{R} | y \leq 8\}$.
e) Since the graph is a parabola, and it opens downward, the second differences would be constant of negative sign.
f) I used the vertex form:
$f(x) = a(x - h)^2 + k$, where (h, k) is the vertex. So since the vertex is $(-1, 8)$, the formula is

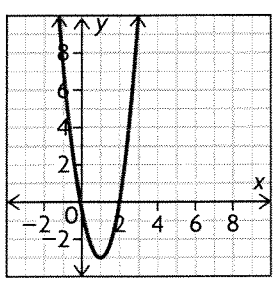
$$\begin{aligned} f(x) &= a(x - h)^2 + k \\ &= a(x + 1)^2 + 8 \end{aligned}$$

I need only find a. Since the point $(1, 0)$ is on the graph (one of the x-intercepts), I have

$$\begin{aligned} 0 &= a(1 + 1)^2 + 8 \\ &= 4a + 8 \\ 4a &= -8 \\ a &= -2 \end{aligned}$$

So the final formula for this parabola (in vertex form) is
$f(x) = -2(x + 1)^2 + 8$
Equivalently, the factored form is
$f(x) = -2(x - 1)(x + 3)$
and the standard form is
$f(x) = -2x^2 - 4x + 6$

8.



a) The parabola opens upward.
b) The vertex is $(1, -3)$.
c) The axis of symmetry is $x = 1$.
d) The domain is $\{x \in \mathbf{R}\}$, and the range is $\{y \in \mathbf{R} | y \geq -3\}$.
e) Since the graph is a parabola, and it opens upward, the second differences would be constant of positive sign.
9. Since in the following parts the pairs of points are equidistant from the vertex, I need only find the average the x-coordinates of these pairs to find the axis of symmetry.

a) $(-2, 2)$, $(2, 2)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{-2 + 2}{2} \\ &= 0 \end{aligned}$$

b) $(-9, 1)$, $(-5, 1)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{-9 - 5}{2} \\ &= -7 \end{aligned}$$

c) $(6, 3)$, $(18, 3)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{6 + 18}{2} \\ &= 12 \end{aligned}$$

d) $(-5, 7)$, $(1, 7)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{-5 + 1}{2} \\ &= -2 \end{aligned}$$

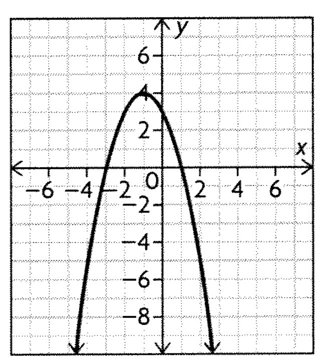
e) $(-6, -1)$, $(3, -1)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{-6 + 3}{2} = 1 \\ &= -1.5 \end{aligned}$$

f) $\left(-\frac{11}{8}, 0\right), \left(\frac{3}{4}, 0\right)$: the axis of symmetry is

$$\begin{aligned} x &= \frac{-\frac{11}{8} + \frac{3}{4}}{2} \\ x &= \frac{-\frac{11}{8} + \frac{6}{8}}{2} \\ &= \frac{-\frac{5}{8}}{2} \\ &= -\frac{5}{16} \end{aligned}$$

10.



a) Here's the completed table:

x	-2	-1	0	1	2
$f(x)$	3	4	3	0	-5

b) Here's a modified table with first and second differences included:

x	$f(x)$	**1st differences**	**2nd differences**
-2	3	$4 - 3 = 1$	$-1 - 1 = -2$
-1	4	$3 - 4 = -1$	$-3 - (-1) = -2$
0	3	$0 - 3 = -3$	$-5 - (-3) = -2$
1	0	$-5 - 0 = -5$	*
2	-5	*	*

I could have predicted the signs of the second differences would be negative and constant, as this graph is a parabola opening downward.

c) I used the factored form:

$f(x) = a(x - r)(x - s)$

where r and s are the zeros of the parabola. By observing the graph, I see that the zeros are $x = -3$ and $x = 1$. So the equation becomes $f(x) = a(x + 3)(x - 1)$ and now I need to find a. For this, notice that $(-1, 4)$ is on the graph (this is the vertex), so

$4 = a(-1 + 3)(-1 - 1)$

$= -4a$

$a = -1$

So the final equation in factored form is

$f(x) = -(x + 3)(x - 1)$

Equivalently, the vertex form is

$f(x) = -(x + 1)^2 + 4$

and the standard form is

$f(x) = -x^2 - 2x + 3$

11. $h(t) = -4t^2 + 32t$, where $h(t)$ is the height of the rocket in metres t seconds after it was launched.

a) Notice that

$h(t) = -4t^2 + 32t$

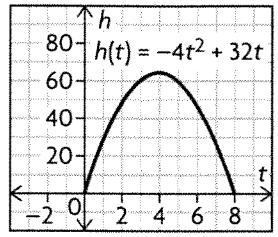
$= -4t(t - 8)$

so that the zeros are $t = 0$ and $t = 8$. The axis of symmetry is halfway between these points, and so $t = 4$. Now

$h(4) = -4(4)(4 - 8)$

$= 64$

so the vertex is $(4, 64)$. From this information, I get the graph



b) I saw in part a) and from the graph that the zeros are $t = 0$ and $t = 8$. This means that the height of the rocket is 0 metres at 0 seconds and 8 seconds after launch. So the rocket is in the air for a total of 8 seconds.

c) After 3 seconds, the rocket will be at height

$h(3) = -4(3)^2 + 32(3)$

$= -36 + 96$

$= 60$ metres

d) The maximum height of the rocket is at the vertex of the graph. I have already seen in part a) that the vertex is $(4, 64)$, so the rocket reaches a maximum height of 64 m.

12. A quadratic function with these characteristics

- $x = -1$ is the axis of symmetry
- $x = 3$ is the location of an x-intercept
- $y = 32$ is the maximum value

has vertex $(-1, 32)$, and must open downward since it has a maximum value. So, using the vertex form of a quadratic, I have the equation

$f(x) = a(x - h)^2 + k$

$= a(x + 1)^2 + 32$

and I know that a must be negative. I also know that I have the x-intercept $(3, 0)$, so

$0 = a(3 + 1)^2 + 32$

$= 16a + 32$

$16a = -32$

$a = -2$

So the final equation for the quadratic (in vertex form) is

$f(x) = -2(x + 1)^2 + 32$

Substituting in $x = 0$, I have

$f(0) = -2(0 + 1)^2 + 32$

$= 30$

so the y-intercept is $(0, 30)$.

13. $f(x) = 2x^2 - 4x$ and $g(x) = -(x - 1)^2 + 2$: first, notice that

$f(x) = 2x^2 - 4x$

$= 2x(x - 2)$

in factored form. So $f(x)$ has zeros $x = 0$ and $x = 2$, and so has axis of symmetry $x = 1$. $g(x)$ is in vertex form, and so has axis of symmetry $x = 1$. Some similarities are that both are quadratic, and both have axis of symmetry $x = 1$. Some differences are that $f(x)$ opens up, while $g(x)$ opens down; $f(x)$ has vertex $(1, f(1)) = (1, -2)$, while $g(x)$ has vertex $(1,2)$.

14. I am given that $f(x)$ is quadratic, and the following incomplete table:

x	-2	-1	0	1	2	3
$f(x)$	19					
First Differences	-10	-6	-2	2	6	
Second Differences	4	4	4	4		

Here is the completed table, with the missing values for $f(x)$ filled in:

x	-2	-1	0	1	2	3
$f(x)$	19	9	3	1	3	9
First Differences	-10	-6	-2	2	6	
Second Differences	4	4	4	4		

This is because $9 - 19 = -10, 3 - 9 = -6$, $1 - 3 = -2, 3 - 1 = 2$ and $9 - 3 = 6$.

15. The computer profit function is modelled by

$$P(x) = -(x - 3)^2 + 50$$

where x is the number of thousands of dollars in computer sales. This quadratic function is maximized at $x = 3$, the axis of symmetry, so I have a maximum profit in computer sales of

$$P(3) = -(3 - 3)^2 + 50$$
$$= 50 \text{ thousand dollars}$$

The stereo system profit function is modelled by $P(x) = -(x - 2)(x - 7)$.

The axis of symmetry is $x = \frac{2 + 7}{2}$, or $x = 4.5$.

So I have a maximum profit in stereo system sales of

$$P(4.5) = -(4.5 - 2)(4.5 - 7)$$
$$= -(2.5)(-2.5)$$
$$= 6.25 \text{ thousand dollars}$$

So the total maximum profit for combined computer and stereo system sales is

$$50 + 6.25 = 56.25 \text{ thousand dollars}$$
$$= \$56\,250$$

16. The units here are metres. The tree is at the origin, and is 20 m high, so this means there is a y-intercept of $(0, 20)$ since the ball is to just clear the top of the tree. Also, the x-intercepts are $(-60, 0)$ and $(35, 0)$, since Jim's ball is 60 m in front of the tree (and so 60 m to the left of the origin), and Jim wants his ball to land 35 m behind the tree, or 5 m before the hole. Using the factored form of a quadratic, the function

$$h(x) = a(x - r)(x - s)$$
$$= a(x + 60)(x - 35)$$

for the height of the ball (in metres) when the ball is x metres away from the tree ($x < 0$ if Jim's ball is between him and the tree, and $x > 0$ if his ball is between the tree and the hole).

Solve for a. Since the y-intercept is $(0, 20)$,

$$20 = a(0 + 60)(0 - 35)$$
$$20 = -2100a$$
$$a = -\frac{20}{2100}$$
$$= -\frac{1}{105}$$

So the final formula in factored form is

$$h(x) = -\frac{1}{105}(x + 60)(x - 35).$$

Rewrite this in standard form.

$$h(x) = -\frac{1}{105}(x + 60)(x - 35)$$
$$= -\frac{1}{105}(x^2 + 25x - 2100)$$
$$= -\frac{1}{105}x^2 - \frac{25}{105}x + \frac{2100}{105}$$
$$= -\frac{1}{105}x^2 - \frac{5}{21}x + 20$$

To get the vertex form, the axis of symmetry is $x = \frac{-60 + 35}{2}$ or $x = -\frac{25}{2}$. So the vertex is

$$\left(-\frac{25}{2}, h\left(-\frac{25}{2}\right)\right)$$

and substituting $x = -\frac{25}{2}$,

$$h\left(-\frac{25}{2}\right) = \frac{1805}{84}.$$

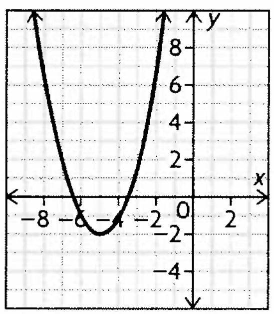
So the vertex form is

$$h(x) = -\frac{1}{105}\left(x + \frac{25}{2}\right)^2 + \frac{1805}{84}.$$

3.2 Determining Maximum and Minimum Values of a Quadratic Function, pp. 153–154

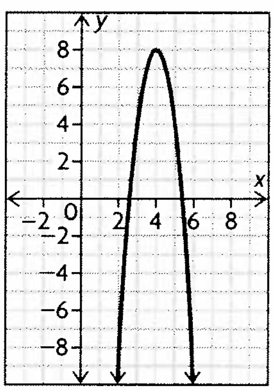
1. a) $y = -x^2 + 7x$: a is negative, so this parabola will open down, and will have a maximum.
b) $f(x) = 3(x - 1)^2 - 4$: a is positive, so this parabola will open up, and will have no maximum.
c) $f(x) = -4(x + 2)(x - 3)$: a is negative, so this parabola will open down, and will have a maximum.
d) $g(x) = 4x^2 + 3x - 5$: a is positive, so this parabola will open up, and will have no maximum.

2. a)



The vertex is $(-5, -2)$, so since the parabola opens up, the minimum value is -2.

b)



The vertex is (4, 8), so since the parabola opens down, the maximum value is 8.

3. a) $y = -4(x + 1)^2 + 6$: this quadratic is in vertex form, and so has vertex $(-1, 6)$. So since a is negative, the parabola opens down, and will have maximum value 6.
b) $f(x) = (x - 5)^2$: this quadratic is in vertex form, and so has vertex (5, 0). So since a is positive, the parabola opens up, and will have a minimum value of 0.
c) $f(x) = -2x(x - 4)$: this quadratic is in factored form, and has zeros $x = 0$ and $x = 4$. So the axis of symmetry is

$$x = \frac{0 + 4}{2}$$
$$= 2$$

Since a is negative, the parabola will open down, and I will have a maximum value. To find this maximum, I substitute the axis of symmetry, and get

$$f(2) = -2(2)(2 - 4)$$
$$= -2(2)(-2)$$
$$= 8$$

for a maximum value.
d) $g(x) = 2x^2 - 7$: this quadratic is in vertex form, and so has vertex $(0, -7)$. So since a is positive, the parabola opens up, and will have a minimum value of -7.

4. a) $y = x^2 - 4x - 1$: here, a is positive, so this quadratic will have a minimum. To find this minimum, first complete the square.

$$y = x^2 - 4x - 1$$
$$= x^2 - 4x + 4 - 4 - 1$$
$$= (x - 2)^2 - 5$$

So, now that it's in vertex form, I see that this quadratic has vertex $(2, -5)$, and so the minimum value is -5.
To see this in another way, I factor the first two terms of y

$$y = x^2 - 4x - 1$$
$$= x(x - 4) - 1$$

So my quadratic has the same axis of symmetry as the quadratic in factored form:
$y = x(x - 4)$
This new quadratic has zeros $x = 0$ and $x = 4$, and so has axis of symmetry $x = \frac{0 + 4}{2}$, or $x = 2$. Substituting this into the original quadratic, I now know that the minimum value is

$$y(2) = (2)^2 - 4(2) - 1$$
$$= 4 - 8 - 1$$
$$= -5$$

which agrees with what I have already found.
b) $f(x) = x^2 - 8x + 12$: My approach will be the same as in part a). Notice that a is positive, so I am looking for a minimum value.

Completing the square, I have

$f(x) = x^2 - 8x + 12$
$= x^2 - 8x + 16 - 16 + 12$
$= (x - 4)^2 - 4$

So the vertex of this quadratic is $(4, -4)$, and the minimum value is -4.

Factoring the first two terms, I have

$f(x) = x^2 - 8x + 12$
$= x(x - 8) + 12$

So this quadratic has the same axis of symmetry as

$y = x(x - 8)$

This new quadratic has zeros $x = 0$ and $x = 8$, so has axis of symmetry $x = 4$. Substituting this into the original quadratic, I get a minimum value of

$f(4) = (4)^2 - 8(4) + 12$
$= 16 - 32 + 12$
$= -4$

just like before.

c) $y = 2x^2 + 12x$: notice that a is positive, so I am looking for a minimum value.

Completing the square, I have

$y = 2x^2 + 12x$
$= 2(x^2 + 6x)$
$= 2(x^2 + 6x + 9) - 18$
$= 2(x + 3)^2 - 18$

So the vertex of this quadratic is $(-3, -18)$, and the minimum value is -18.

Factoring the first two terms, I have

$y = 2x^2 + 12x$
$= 2x(x + 6)$

So this quadratic has zeros $x = 0$ and $x = -6$, so has axis of symmetry $x = -3$. Substituting this into the quadratic, I get a minimum value of

$y(-3) = 2(-3)^2 + 12(-3)$
$= 18 - 36$
$= -18$

just like before.

d) $y = -3x^2 - 12x + 15$: notice that a is negative, so I am looking for a maximum value.

Completing the square, I have

$y = -3x^2 - 12x + 15$
$= -3(x^2 + 4x) + 15$
$= -3(x^2 + 4x + 4) + 12 + 15$
$= -3(x + 2)^2 + 27$

So the vertex of this quadratic is $(-2, 27)$, and the maximum value is 27.

Factoring the first two terms, I have

$y = -3x^2 - 12x + 15$
$= -3x(x + 4) + 15$

So this quadratic has the same axis of symmetry as

$y = -3x(x + 4)$

This new quadratic has zeros $x = 0$ and $x = -4$, so has axis of symmetry $x = -2$.

Substituting this into the original quadratic, I get a maximum value of

$y(-2) = -3(-2)^2 - 12(-2) + 15$
$= -12 + 24 + 15$
$= 27$

just like before.

e) $y = 3x(x - 2) + 5$: notice that a is positive, so I am looking for a minimum value.

First of all, this quadratic has the same axis of symmetry as

$y = 3x(x - 2)$

This new quadratic has zeros $x = 0$ and $x = 2$, so has axis of symmetry $x = 1$. Substituting this into the original quadratic, I get the minimum value

$y(1) = 3(1)(1 - 2) + 5$
$= 2$

To do this in another way, I put the quadratic into standard form.

$y = 3x(x - 2) + 5$
$= 3x^2 - 6x + 5$

Completing the square with this standard form, I get

$y = 3x^2 - 6x + 5$
$= 3(x^2 - 2x) + 5$
$= 3(x^2 - 2x + 1) - 3 + 5$
$= 3(x - 1)^2 + 2$

So the vertex of this quadratic is (1, 2), and the minimum value is 2, which agrees with what I have already found.

f) $g(x) = -2(x + 1)^2 - 5$: notice that a is negative, so I am looking for a maximum value.

This quadratic is already in vertex form, so it has vertex $(-1, -5)$ and a maximum value of -5.

On the other hand, I can put this quadratic in standard form, and factor the first two terms.

$g(x) = -2(x + 1)^2 - 5$
$= -2(x^2 + 2x + 1) - 5$
$= -2x^2 - 4x - 2 - 5$
$= -2x^2 - 4x - 7$
$= -2x(x + 2) - 7$

So this quadratic has the same axis of symmetry as
$y = -2x(x + 2)$
This new quadratic has zeros $x = 0$ and $x = -2$, so has axis of symmetry $x = -1$. Substituting this into the original quadratic, I get a maximum value of

$$g(-1) = -2(-1 + 1)^2 - 5$$
$$= -5$$

just like before.

5. a) i) $p(x) = -x + 5$: here and below, call the revenue function $R(x)$. Then I have

$$R(x) = \text{demand} \times (\text{number sold})$$
$$= p(x) \times x$$
$$= (5 - x)x$$
$$= -x(x - 5)$$

ii) This quadratic is in factored form, and has zeros $x = 5$ and $x = 0$. So $x = 2.5$ is the axis of symmetry. Also, a is negative, so the revenue function has a maximum value of

$$R(2.5) = -2.5(2.5 - 5)$$
$$= 6.25$$
$$= \$6250$$

b) i) $p(x) = -4x + 12$: the revenue function equals

$$R(x) = x(12 - 4x)$$
$$= -4x(x - 3)$$

ii) This quadratic is in factored form, and has zeros $x = 3$ and $x = 0$. So $x = 1.5$ is the axis of symmetry. Also, a is negative, so the revenue function has a maximum value of

$$R(1.5) = -4(1.5)(1.5 - 3)$$
$$= 9$$
$$= \$9000$$

c) i) $p(x) = -0.6x + 15$: the revenue function equals

$$R(x) = x(15 - 0.6x)$$
$$= -0.6x(x - 25)$$

ii) This quadratic is in factored form, and has zeros $x = 25$ and $x = 0$. So $x = 12.5$ is the axis of symmetry. Also, a is negative, so the revenue function has a maximum value of

$$R(12.5) = -0.6(12.5)(12.5 - 25)$$
$$= 93.75$$
$$= \$93\,750$$

d) i) $p(x) = -1.2x + 4.8$: the revenue function equals

$$R(x) = x(4.8 - 1.2x)$$
$$= -1.2x(x - 4)$$

ii) This quadratic is in factored form, and has zeros $x = 4$ and $x = 0$. So $x = 2$ is the axis of symmetry. Also, a is negative, so the revenue function has a maximum value of

$$R(2) = -1.2(2)(2 - 4)$$
$$= 4.8$$
$$= \$4800$$

6. a) $f(x) = 2x^2 - 6.5x + 3.2$: notice that a is positive, so I want to find a minimum value. By graphing this on a calculator, then using the minimum operation, I find the minimum value is about -2.08 and occurs at $x = 1.625$.

b) $f(x) = -3.6x^2 + 4.8x$: notice that a is negative, so I want to find a maximum value. By graphing this on a calculator, then using the maximum operation, I find the maximum value is 1.6, and occurs at around $x = \frac{2}{3}$.

7. a) i) $R(x) = -x^2 + 24x$, $C(x) = 12x + 28$: here and below, call the profit function $P(x)$. Then I have

$$P(x) = R(x) - C(x)$$
$$= (-x^2 + 24x) - (12x + 28)$$
$$= -x^2 + 12x - 28$$
$$= -x(x - 12) - 28$$

ii) Notice that a is negative, so I seek a maximum value. This quadratic is in partial factored form, and since the zeros of $-x(x - 12)$ are $x = 0$ and $x = 12$, the profit function has axis of symmetry $x = 6$. So the maximum profit occurs at $x = 6$.

b) i) $R(x) = -2x^2 + 32x$, $C(x) = 14x + 45$: the profit function equals

$$P(x) = R(x) - C(x)$$
$$= (-2x^2 + 32x) - (14x + 45)$$
$$= -2x^2 + 18x - 45$$
$$= -2x(x - 9) - 45$$

ii) Notice that a is negative, so I seek a maximum value. This quadratic is in partial factored form, and since the zeros of $-2x(x - 9)$ are $x = 0$ and $x = 9$, the profit function has axis of symmetry $x = 4.5$. So the maximum profit occurs at $x = 4.5$.

c) i) $R(x) = -3x^2 + 26x$, $C(x) = 8x + 18$: the profit function equals

$$P(x) = R(x) - C(x)$$
$$= (-3x^2 + 26x) - (8x + 18)$$
$$= -3x^2 + 18x - 18$$
$$= -3x(x - 6) - 18$$

ii) Notice that a is negative, so I seek a maximum value. This quadratic is in partial factored form, and since the zeros of $-3x(x - 6)$ are $x = 0$ and $x = 6$, the profit function has axis of symmetry $x = 3$. So the maximum profit occurs at $x = 3$.

d) i) $R(x) = -2x^2 + 25x$, $C(x) = 3x + 17$: the profit function equals

$$\begin{aligned}P(x) &= R(x) - C(x)\\ &= (-2x^2 + 25x) - (3x + 17)\\ &= -2x^2 + 22x - 17\\ &= -2x(x - 11) - 17\end{aligned}$$

ii) Notice that a is negative, so I seek a maximum value. This quadratic is in partial factored form, and since the zeros of $-2x(x - 11)$ are $x = 0$ and $x = 11$, the profit function has axis of symmetry $x = 5.5$. So the maximum profit occurs at $x = 5.5$.

8. $h(t) = -5t^2 + 20t + 50$

a) Put the height function in partial factored form.

$$\begin{aligned}h(t) &= -5t^2 + 20t + 50\\ &= -5t(t - 4) + 50\end{aligned}$$

Since $-5t(t - 4)$ has zeros $t = 0$ and $t = 4$, the height function has axis of symmetry $t = 2$. Substituting this into the height function, I get a maximum height of

$$\begin{aligned}h(2) &= -5(2)(2 - 4) + 50\\ &= 70 \text{ metres}\end{aligned}$$

b) I saw in part a) that the maximum height is reached after $t = 2$ seconds.

c) To get the height of the rooftop, I substitute $t = 0$ seconds, the time the ball was thrown. So the rooftop has height

$$\begin{aligned}h(0) &= -5(0)^2 + 20(0) + 50\\ &= 50 \text{ metres}\end{aligned}$$

9. $C(x) = 0.28x^2 - 0.7x + 1$: Put the cost function in partial factored form.

$$\begin{aligned}C(x) &= 0.28x^2 - 0.7x + 1\\ &= 0.28x(x - 2.5) + 1\end{aligned}$$

Since $0.28x(x - 2.5)$ has zeros $x = 0$ and $x = 2.5$, the cost function has axis of symmetry $x = 1.25$. So the minimum production cost is

$$\begin{aligned}C(1.25) &= 0.28(1.25)(1.25 - 2.5) + 1\\ &= -0.4375 + 1\\ &= 0.5625\\ &= \$562\,500\end{aligned}$$

10. Notice that the partial factored form of this function is

$$3x^2 - 6x + 5 = 3x(x - 2) + 5$$

So since $3x(x - 2)$ has zeros $x = 0$ and $x = 2$, my quadratic has axis of symmetry $x = 1$. So its minimum value is at $x = 1$, and this minimum value is

$$\begin{aligned}3(1)^2 - 6(1) + 5 &= 3 - 6 + 5\\ &= 2\end{aligned}$$

So, in particular, this quadratic never has a value less than 1 (in fact, it is never less than 2).

11. $P(x) = -5x^2 + 400x - 2550$

a) The profit function has partial factored form

$$\begin{aligned}P(x) &= -5x^2 + 400x - 2550\\ &= -5x(x - 80) - 2550\end{aligned}$$

So since $-5x(x - 80)$ has zeros $x = 0$ and $x = 80$, the axis of symmetry of the profit function is $x = 40$. So the maximum profit is

$$\begin{aligned}P(40) &= -5(40)(40 - 80) - 2550\\ &= 8000 - 2550\\ &= 5450\\ &= \$5\,450\,000\end{aligned}$$

b) I saw in part a) that the maximum profit occurs at $x = 40$, or when I spend \$40 000 on advertising.

c) I need to know when profit equals \$4 000 000, or when $P(x) = 4000$. So I solve this equation

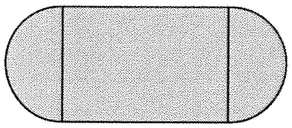
$$\begin{aligned}4000 &= -5x^2 + 400x - 2550\\ 5x^2 - 400x + 6550 &= 0\\ x^2 - 80x + 1310 &= 0\end{aligned}$$

Using the quadratic formula, I get

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{80 \pm \sqrt{6400 - 4(1)(1310)}}{2(1)}\\ &= \frac{80 \pm \sqrt{1160}}{2}\\ &\doteq 22.971 \text{ or } 57.029\end{aligned}$$

So I need to spend at least \$22 971 to guarantee at least \$4 000 000 in profit. However, if I spend more than \$57 029, I will drop below this amount in profit once again.

12. Here is a picture of the track, with the rectangular infield outlined inside:



Call the height of the inside rectangle h, and the width of this rectangle w. Then the distance around the track is given by the formula
2w + distance around two outside half circles
But the two half circles on the outside together form a circle of diameter h, and this circle has circumference ph. So the formula for the distance around the track simplifies to:
$2w + \pi h$
The track coach wants this distance to be 500 metres, so

$$\begin{aligned} 2w + \pi h &= 500 \\ \pi h &= 500 - 2w \\ h &= -\frac{2}{\pi}(w - 250) \end{aligned}$$

The formula for the area of the interior rectangle is:

$$\begin{aligned} A &= wh \\ &= -\frac{2}{\pi}w(w - 250) \end{aligned}$$

This is a quadratic function in the variable w, and it is in factored form. It has zeros $w = 0$ and $w = 250$, so the maximum area for the interior rectangle (maximum since a is negative) occurs at the axis of symmetry $w = 125$. The corresponding height when $w = 125$ metres is

$$\begin{aligned} h &= -\frac{2}{\pi}(125 - 250) \\ &= \frac{250}{\pi} \text{ metres} \end{aligned}$$

So if the high school makes the track have interior rectangle with dimensions

125 m (width) $\times \frac{250}{\pi}$ m (height), then both coaches will have their demands met.

13. $f(x) = 3x^2 - 7x + 2$
One possible response: the function is in standard form, so to find the minimum, I must find the vertex. Completing the square would result in fractions that are more difficult to calculate than whole numbers. Since this function does factor as $(x - 2)(3x - 1)$, putting the function in factored form and averaging the zeros to find the x-coordinate of the vertex would be possible; however, there would still be fractions to work with. Using the graphing calculator to graph the function, then using CALC to find the minimum, would be the easiest method for this function. Alternatively, putting the function in partial factored form:

$3x\left(x - \frac{7}{3}\right) + 2$, and averaging the zeros of

$3x\left(x - \frac{7}{3}\right)$ would also give the x-coordinate of the vertex.

14. $h(t) = -4.9t^2 + v_0t + h_0$: I can put the height function in partial factored form:

$$\begin{aligned} h(t) &= -4.9t^2 + v_0t + h_0 \\ &= -4.9t\left(t - \frac{v_0}{4.9}\right) + h_0 \end{aligned}$$

Then since $-4.9t\left(t - \frac{v_0}{4.9}\right)$ has zeros $t = 0$ and

$t = \frac{v_0}{4.9}$, the height function has axis of

symmetry $t = \frac{v_0}{9.8}$. So the maximum value of

the height function will occur after $t = \frac{v_0}{9.8}$ seconds.

15. Let x be the number of \$1 increases in the price of the ticket. Then the number of students that will attend the dance after x \$1 increases is
$300 - 30x$
The new price of the ticket after x \$1 increases is
$8 + x$
So the total revenue after x \$1 increases in price is

$$\begin{aligned} R(x) &= (\textit{price of ticket}) \\ &\quad \times (\text{\# of students attending}) \\ &= (8 + x)(300 - 30x) \\ &= -30(x - 10)(x + 8) \end{aligned}$$

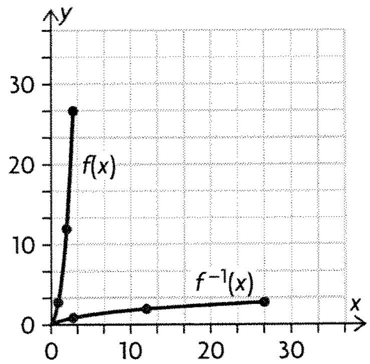
This quadratic function is in factored form, and it has zeros $x = 10$ and $x = -8$. Averaging these two, I get an axis of symmetry for the

revenue function of $x = \frac{10 - 8}{2}$, or $x = 1$. So in

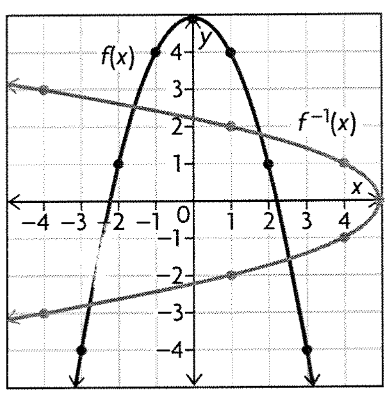
order to maximize revenue, the ticket price should be increased by \$1 to a price of \$9 per ticket.

3.3 The Inverse of a Quadratic Function, pp. 160–162

1. a) Here is the data, and resulting graphs of the function and its inverse:
$\{(0, 0), (1, 3), (2, 12), (3, 27)\}$

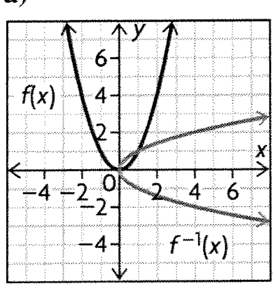


b) Here is the data, and resulting graphs of the function and its inverse:
$\{(-3, -4), (-2, 1), (-1, 4), (0, 5), (1, 4), (2, 1), (3, -4)\}$

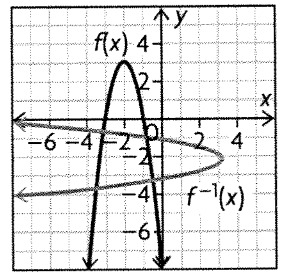


2. I can graph the inverse by reflecting the original graph in the line $y = x$.

a)



b)



3. $f(x) = 2x^2 - 1$: set $y = 2x^2 - 1$, then switch x and y, and solve for y.

$$x = 2y^2 - 1$$
$$x + 1 = 2y^2$$
$$y^2 = \frac{x + 1}{2}$$
$$y = \pm\sqrt{\frac{x + 1}{2}}$$

So I get

$$f^{-1}(x) = \pm\sqrt{\frac{x + 1}{2}}$$

4. $f(x) = 7 - 2(x - 1)^2, x \geq 1$

a) $f(3) = 7 - 2(3 - 1)^2$
$= 7 - 8$
$= -1$

b) Set $x = 7 - 2(y - 1)^2$, and solve for y.

$$x = 7 - 2(y - 1)^2$$
$$2(y - 1)^2 = 7 - x$$
$$(y - 1)^2 = \frac{7 - x}{2}$$
$$y - 1 = \pm\sqrt{\frac{7 - x}{2}}$$
$$y = 1 \pm \sqrt{\frac{7 - x}{2}}$$

So I get

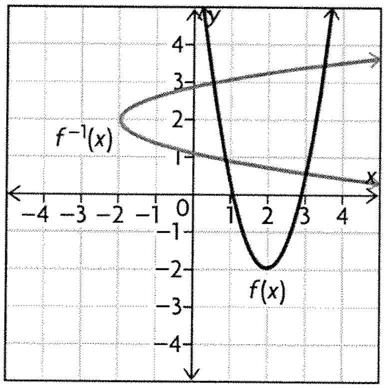
$$f^{-1}(x) = 1 \pm \sqrt{\frac{7 - x}{2}}$$

c) $f^{-1}(5) = 1 \pm \sqrt{\frac{7 - 5}{2}}$
$= 1 \pm 1$
$= 0 \text{ or } 2$

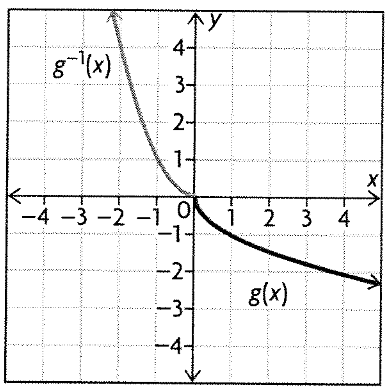
d) $f^{-1}(2a + 7) = 1 \pm \sqrt{\frac{7 - (2a + 7)}{2}}$
$= 1 \pm \sqrt{-a}$

5. a)–b) $f(x) = 3(x - 2)^2 - 2$: this will stretch the graph of $y = x^2$ by a factor of 3, shift it right by 2 units, then down by 2 units.

To get the graph of the inverse, $y = f^{-1}(x)$, I reflect the graph of $y = f(x)$ in the line $y = x$.



6. a)–b) $g(x) = -\sqrt{x}, x \geq 0$: the graph of this is the lower half of the parabola $y = x^2$ after it has been reflected in the line $y = x$. So the graph of $y = g^{-1}(x)$ is just the left half of the parabola $y = x^2$.



c) The domain of $g(x)$ is $\{x \in \mathbf{R} | x \geq 0\}$, and the range of $g(x)$ is $\{y \in \mathbf{R} | y \leq 0\}$. So the domain of $\{g^{-1}(x)$ *is* $x \in \mathbf{R} | x \leq 0\}$, and the range of $g^{-1}(x)$ is $\{y \in \mathbf{R} | y \geq 0\}$.

d) Set $x = -\sqrt{y}$, and solve for y.

$$x = -\sqrt{y}$$
$$x^2 = (-\sqrt{y})^2$$
$$x^2 = y$$

So I get that

$$g^{-1}(x) = x^2, x \leq 0$$

7. $f(x) = -(x + 1)^2 - 3, x \geq -1$: set $x = -(y + 1)^2 - 3$, and solve for y.

$$x = -(y + 1)^2 - 3$$
$$(y + 1)^2 = -x - 3$$
$$y + 1 = \pm\sqrt{-x - 3}$$
$$y = -1 \pm \sqrt{-x - 3}$$

So I get

$$f^{-1}(x) = -1 \pm \sqrt{-x - 3}$$

The function $f(x)$ has maximum value at the vertex, $(-1, -3)$, and all y-values below -3 are achieved by this function for $x \geq -1$. So

domain $f(x) = \{x \in \mathbf{R} \mid x \geq -1\}$
range $f(x) = \{y \in \mathbf{R} \mid y \leq -3\}$
domain $f^{-1}(x) = \{x \in \mathbf{R} \mid x \leq -3\}$
range $f^{-1}(x) = \{y \in \mathbf{R} \mid y \geq -1\}$

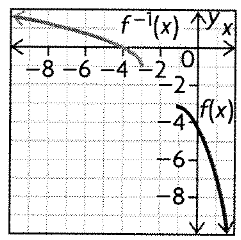
This means that in my case I really have

$$f^{-1}(x) = -1 + \sqrt{-x - 3}, x \leq -3$$

since $-1 - \sqrt{-x - 3} \leq -1$ for all allowable x, and these values are not part of my range for $f^{-1}(x)$.

Finally, the graph of $y = f(x)$ is the graph of the parabola $y = x^2$ flipped, shifted left by 1 unit and down by 3 units. Since $x \geq -1$ in this case, I only graph the right half of this parabola.

For the graph of the inverse, I just reflect in the line $y = x$.

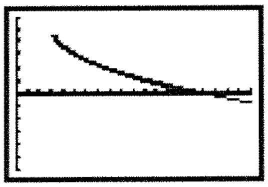


8. $f(x) = \frac{1}{2}(x - 5)^2 + 3, x \leq 5$: first of all, notice that $f(x)$ has vertex (5,3), so has minimum value 3 at $x = 5$. So the graph of $f(x)$ for $x \leq 5$ will be the left half of the parabola $y = x^2$ compressed by a factor of $\frac{1}{2}$, then shifted right by 5 units and up by 3 units. So the range for $f(x)$ is $\{y \in \mathbf{R} | y \geq 3\}$, and so the domain for $f^{-1}(x)$ is $\{x \in \mathbf{R} | x \geq 3\}$. The graph of $f^{-1}(x)$ can then be obtained by reflecting in the line $y = x$, but I will use the calculator to graph this after finding the equation for $f^{-1}(x)$.

To find this equation, set $x = \frac{1}{2}(y - 5)^2 + 3$ and solve for y.

$$x = \frac{1}{2}(y - 5)^2 + 3$$
$$x - 3 = \frac{1}{2}(y - 5)^2$$
$$2(x - 3) = (y - 5)^2$$
$$y - 5 = \pm\sqrt{2(x - 3)}$$
$$y = 5 \pm \sqrt{2(x - 3)}$$

So I get $f^{-1}(x) = 5 \pm\sqrt{2(x - 3)}, x \geq 3$
Since the domain of $f(x)$ is $\{x \in \mathbf{R} \mid x \leq 5\}$, the range of $f^{-1}(x)$ is $\{y \in \mathbf{R} \mid y \leq 5\}$. So I actually have $f^{-1}(x) = 5 - \sqrt{2(x - 3)}, x \leq 3$, $x \leq -3$ since $5 + \sqrt{2(x - 3)} \geq 5$ for all allowable x, and these values are not part of my range for $f^{-1}(x)$. Substituting this equation for $f^{-1}(x)$ into the calculator for $x \geq 3$, I get the graph:



9. $f(x) = 3x^2 - 6x, -2 < x < 3$

a) I am given that the domain of $f(x)$ is $\{x \in \mathbf{R} \mid -2 < x < 3\}$. To find the range, put the function into vertex form.

$$\begin{aligned} f(x) &= 3x^2 - 6x \\ &= 3(x^2 - 2x) \\ &= 3(x^2 - 2x + 1) - 3 \\ &= 3(x - 1)^2 - 3 \end{aligned}$$

So the vertex is $(1, -3)$, and -3 is a minimum since this parabola will open up. Also

$$\begin{aligned} f(-2) &= 3(-2)^2 - 6(-2) \\ &= 24 \end{aligned}$$

and

$$\begin{aligned} f(3) &= 3(3)^2 - 6(3) \\ &= 9 \end{aligned}$$

So $f(-2) > f(3)$, and the range of $f(x)$ on this restricted domain is $\{y \in \mathbf{R} \mid -3 \leq y < 24\}$.

b) I saw in part a) that $f(x)$ has a minimum of -3 at $x = 1$, and that $f(3) = 9$. So if I restrict the domain of $f(x)$ to $\{x \in \mathbf{R} \mid 1 < x < 3\}$, the range of $f(x)$ on this restricted domain is $\{y \in \mathbf{R} \mid -3 < y < 9\}$. So the domain of $f^{-1}(x)$ will be $\{x \in \mathbf{R} \mid -3 < x < 9\}$ and the range will be $\{y \in \mathbf{R} \mid 1 < y < 3\}$. To find the equation for $f^{-1}(x)$ corresponding to this domain and range, set $x = 3(y - 1)^2 - 3$ and solve for y.

$$\begin{aligned} x &= 3(y - 1)^2 - 3 \\ x + 3 &= 3(y - 1)^2 \\ \frac{x + 3}{3} &= (y - 1)^2 \\ y - 1 &= \pm\sqrt{\frac{x + 3}{3}} \\ y &= 1 \pm\sqrt{\frac{x + 3}{3}} \end{aligned}$$

Since the range of $f^{-1}(x)$ is $\{y \in \mathbf{R} \mid 1 < y < 3\}$, the values of $y = 1 - \sqrt{\frac{x + 3}{3}} \leq 1$ are out of this range. So, the equation for $f^{-1}(x)$ on this restricted domain and range is

$$f^{-1}(x) = 1 + \sqrt{\frac{x + 3}{3}}, -3 < x < 9$$

10. $h(t) = -5t^2 + 10t + 35$

a) To find the vertex form, I complete the square:

$$\begin{aligned} h(t) &= -5t^2 + 10t + 35 \\ &= -5(t^2 - 2t) + 35 \\ &= -5(t^2 - 2t + 1) + 5 + 35 \\ &= -5(t - 1)^2 + 40 \end{aligned}$$

b) By the quadratic formula, the zeros of $h(t)$ are

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{100 - 4(-5)(35)}}{2(-5)} \\ &= \frac{-10 \pm \sqrt{800}}{-10} \\ &\doteq -1.83 \text{ or } 3.83 \end{aligned}$$

So, since time is always non-negative, the domain for the height function is $\{t \in \mathbf{R} \mid 0 \leq t \leq 3.83\}$. This means the ball hits the ground after about 3.83 seconds.

From part a), I know that the height function has vertex (1, 40), and so the maximum height is 40 metres. Of course, the minimum height is 0 metres, when the ball hits the ground. So the range of the height function is $\{h \in \mathbf{R} \mid 0 \leq h \leq 40\}$.

c) I need to solve the equation $h = -5(t - 1)^2 + 40$ for t.

$$\begin{aligned} h &= -5(t - 1)^2 + 40 \\ h - 40 &= -5(t - 1)^2 \\ \frac{40 - h}{5} &= (t - 1)^2 \\ t - 1 &= \pm\sqrt{\frac{40 - h}{5}} \\ t &= 1 \pm \sqrt{\frac{40 - h}{5}} \end{aligned}$$

When $t = 0$ s, the height is $h = 35$ m. So at this initial stage, and while the height of the ball rises from 35 m to 40 m (the maximum height), I have

$$t(h) = 1 - \sqrt{\frac{40 - h}{5}}$$

On the other hand, once the ball has reached 40 m in height, the time moves beyond 1 second elapsed. So as the height of the ball decreases from 35 (I think) m down to 0 m, I have

$$t(h) = 1 + \sqrt{\frac{40 - h}{5}}$$

d) The domain and range of $t(h)$ are just the reverse of what they were for $h(t)$. So

domain of $t(h) = \{h \in \mathbf{R} \mid 0 \leq h \leq 40\}$

range of $t(h) = \{t \in \mathbf{R} \mid 0 \leq t \leq 3.83\}$

I suspected the graph would be a downward opening parabola, since the second differences were constant and negative.

c) To graph the inverse, I just need to switch the order of the x and y-values from the table in part a), and then repeat what I did in part b). Or, I can just reflect the graph in part b) across the line $y = x$.

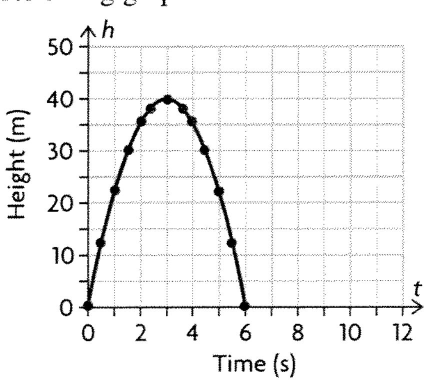
Time (s)	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
Height (m)	0	12.375	22.5	30.375	36.0	39.375	40.5	29.375	36.0	30.375	22.5	12.375	0

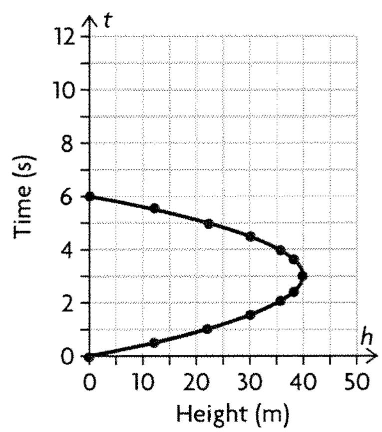
11. a) Here is the extended table:
The is because the table of first and second differences is:

1st differences	2nd differences
12.375 − 0 = 12.375	10.125 − 12.375 = −2.25
22.5 − 12.375 = 10.125	7.875 − 10.125 = −2.25
30.375 − 22.5 = 7.875	5.625 − 7.875 = −2.25
36 − 30.375 = 5.625	3.375 − 5.625 = −2.25
39.375 − 36 = 3.375	*

So the second differences are constantly equal to −2.25, and this allows me to extend the table of first differences, and so also the table of values.

b) Plotting the data from part a), and sketching a curve of good fit for this data delivers the following graph:





d) No, the inverse is not a function since it has multiple y-values for some x-values (that is, it fails the vertical line test).

12. a) I put the function into partial factored form:

$$f(x) = -2x^2 + 3x - 1$$
$$= -2x(x - 1.5) - 1$$

So since the zeros of $-2x(x - 1.5)$ are $x = 0$ and $x = 1.5$, the axis of symmetry for $f(x)$ is $x = 0.75$. This is the x-coordinate for the vertex. The y-coordinate is

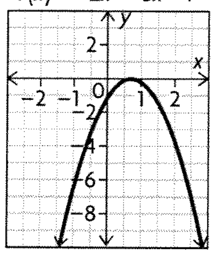
$$f(0.75) = -2(0.75)(0.75 - 1.5) - 1$$
$$= 0.125$$

So the vertex is (0.75, 0.125).

b) The parabola has the vertex I found in part a), and it opens down since a is negative. Also, the point $(0, f(0)) = (0, -1)$ is on the graph.

This information enables me to draw the following graph:

$f(x) = -2x^2 + 3x - 1$



c) To graph $f^{-1}(x)$, I can just reflect the graph from part b) across the line $y = x$, and then restrict to those $y \geq 0.75$. Alternatively, set $x = -2y^2 + 3y - 1$ and solve for y. Since I know the vertex from part a), this equation is the same as

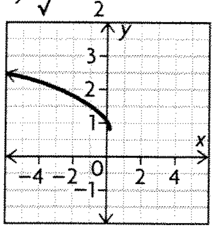
$$x = -2(y - 0.75)^2 + 0.125$$
$$x - 0.125 = -2(y - 0.75)^2$$
$$(y - 0.75)^2 = \frac{0.125 - x}{2}$$
$$y - 0.75 = \pm\sqrt{\frac{0.125 - x}{2}}$$
$$y = 0.75 \pm \sqrt{\frac{0.125 - x}{2}}$$

Since I am restricting to the range $y \geq 0.75$, and $0.75 - \sqrt{\frac{0.125 - x}{2}} \leq 0.75$ for allowable x, I get that

$$f^{-1}(x) = 0.75 + \sqrt{\frac{0.125 - x}{2}}$$

The domain of $f^{-1}(x)$ is $\{x \in \mathbf{R} | x \leq 0.125\}$, since this is necessary to make the number under the square root non-negative, or equivalently since this is the range of $f(x)$ for $x \geq 0.75$ (0.125 is the maximum value of $f(x)$, and this occurs at $x = 0.75$). Putting all of this information together, I get the graph for $f^{-1}(x)$ on this restricted range:

$y = \sqrt{\frac{0.125 - x}{2}} + 0.75$



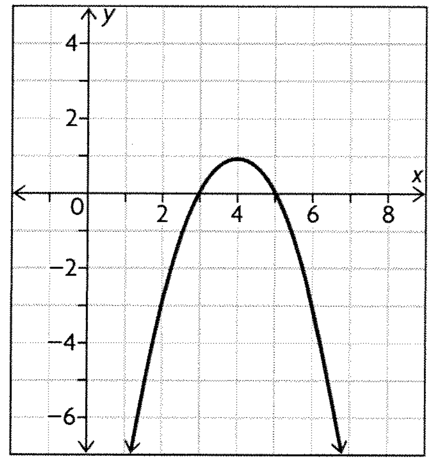
d) In the process of solving part c), I found that

domain of $f^{-1}(x) = \{x \in \mathbf{R} | x \leq 0.125\}$

range of $f^{-1}(x) = \{y \in \mathbf{R} | y \geq 0.75\}$

e) Restricting the y-values in this way guaranteed that the inverse $f^{-1}(x)$ would be a function.

13. a)



i) (4, 1) appears to be the vertex.

$f(x) = a(x - h)^2 + k$

$f(x) = a(x - 4)^2 + 1$

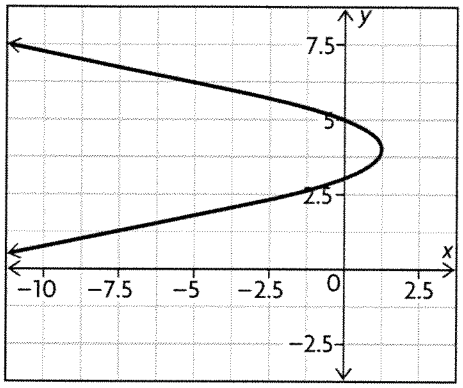
Another point on the graph appears to be (5,0). Substitute this into the function and solve for a.

$$0 = a(5 - 4)^2 + 1$$
$$0 = a(1)^2 + 1$$
$$0 = a + 1$$
$$a = -1$$
$$f(x) = -(x - 4)^2 + 1$$

ii) First, find the inverse of the function. Then, graph the function. To find the inverse, interchange x and y in the equation of the function and solve for y.

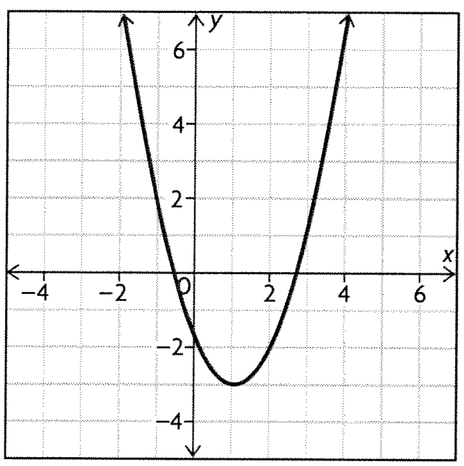
$$y = -(x - 4)^2 + 1$$
$$x = -(y - 4)^2 + 1$$
$$x - 1 = -(y - 4)^2 + 1 - 1$$
$$x - 1 = -(y - 4)^2$$
$$-(x - 1) = (y - 4)^2$$
$$1 - x = (y - 4)^2$$
$$\pm\sqrt{1 - x} = y - 4$$
$$4 \pm \sqrt{1 - x} = y$$
$$4 \pm \sqrt{1 - x} = f^{-1}(x)$$

Graph the inverse on a graphing calculator.



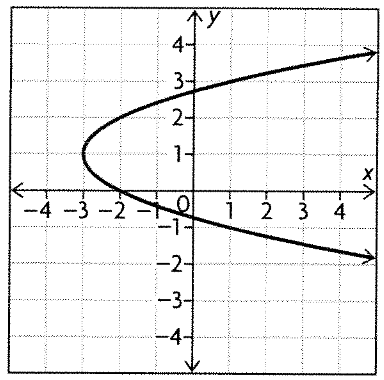
iii) For the inverse to be a function, it needs to pass the vertical line test. So, y needs to be restricted to greater than 4 or less than 4. More specifically, the domain of $f(x)$ would
$= \{x \in \mathbf{R} \mid x \geq 4\}$
OR
$= \{x \in \mathbf{R} \mid x \leq 4\}$.
Under one of these two conditions, the inverse of $f(x)$ would be a function.
iv) In part ii), the equation of the inverse was found to be
$4 \pm \sqrt{1 - x} = f^{-1}(x)$.

b)



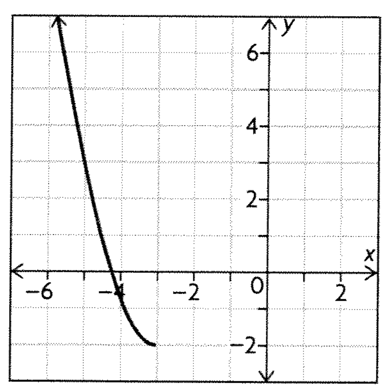
i) $(1, -3)$ appears to be the vertex.
$f(x) = a(x - h)^2 + k$
$f(x) = a(x - 1)^2 - 3$
Another point on the graph appears to be $(0, -2)$. Substitute this into the function and solve for a.
$-2 = a(0 - 1)^2 - 3$
$-2 = a(-1)^2 - 3$
$-2 = a(1) - 3$
$-2 = a - 3$
$1 = a$
$f(x) = (x - 1)^2 - 3$

ii) First, find the inverse of the function. Then, graph the function. To find the inverse, interchange x and y in the equation of the function and solve for y.
$y = (x - 1)^2 - 3$
$x = (y - 1)^2 - 3$
$x + 3 = (y - 1)^2 - 3 + 3$
$x + 3 = (y - 1)^2$
$\pm\sqrt{x + 3} = y - 1$
$1 \pm \sqrt{x + 3} = y$
$1 \pm \sqrt{x + 3} = f^{-1}(x)$
Graph the inverse on a graphing calculator.



iii) For the inverse to be a function, it needs to pass the vertical line test. So, y needs to be restricted to greater than 1 or less than 1. More specifically, the domain of $f(x)$ would
$= \{x \in \mathbf{R} \mid x \geq 1\}$
OR
$= \{x \in \mathbf{R} \mid x \leq 1\}$.
Under one of these two conditions, the inverse of $f(x)$ would be a function.
iv) In part ii), the equation of the inverse was found to be
$1 \pm \sqrt{x + 3} = f^{-1}(x)$

c)

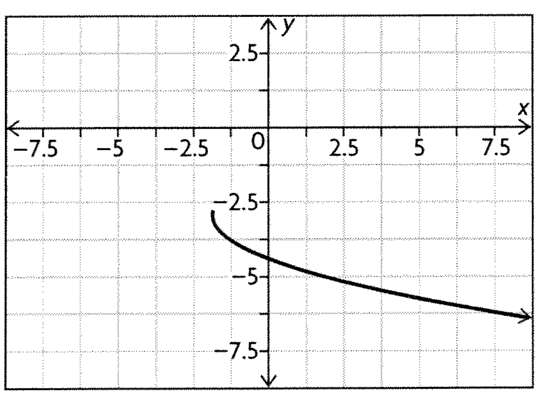


i) The function appears to be a quadratic function with a restriction. If there was no restriction, the vertex would be at $(-3, -2)$.

$f(x) = a(x - h)^2 + k$
$f(x) = a(x - (-3))^2 - 2$
$f(x) = a(x + 3)^2 - 2$
Another point on the graph appears to be $(-4, -1)$. Substitute this into the function and solve for a.
$-1 = a(-4 + 3)^2 - 2$
$-1 = a(-1)^2 - 2$
$-1 = a(1) - 2$
$-1 = a - 2$
$1 = a$
$f(x) = (x + 3)^2 - 2$
My graph is this function restricted to the left of $x = -3$.
$f(x) = (x + 3)^2 - 2; x \leq -3$
ii) First, find the inverse of the function. Then, graph the function. To find the inverse, interchange x and y in the equation of the function and solve for y.

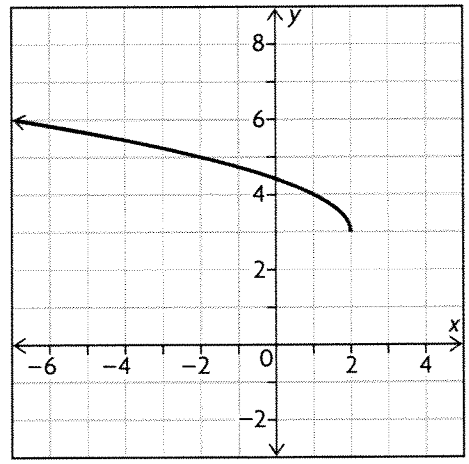
$$y = (x + 3)^2 - 2$$
$$x = (y + 3)^2 - 2$$
$$x + 2 = (y + 3)^2 - 2 + 2$$
$$x + 2 = (y + 3)^2$$
$$\pm\sqrt{x + 2} = y + 3$$
$$-3 \pm \sqrt{x + 2} = y$$
$$-3 \pm \sqrt{x + 2} = f^{-1}(x)$$

$-3 - \sqrt{x + 2} = f^{-1}(x)$ is the only inverse because of the restriction on $f(x)$ that $x \leq -3$ and so $y \leq -3$ for $f^{-1}(x)$.
Graph the inverse on a graphing calculator.



iii) The restriction on the domain of $f(x)$ is $x \leq -3$.
iv) In part ii), the equation of the inverse was found to be
$-3 \pm \sqrt{x + 2} = f^{-1}(x)$

d)

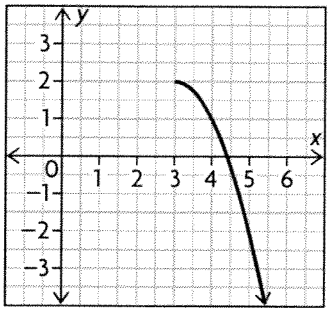


i) The easiest way to find the equation of this function is to compare this graph with a similar graph. Look at the graph of part a) ii). The top half of the graph looks very similar to this graph. The equation of the graph in part a) ii) is
$4 \pm \sqrt{1 - x} = f^{-1}(x)$
The graph reflects $\sqrt{x}$ with a shift up 4 units and to the right 1 unit.
So, this graph should reflect $\sqrt{x}$ with a shift up 3 units and to the right 2 units.
$3 \pm \sqrt{2 - x} = y$
Also, this graph should only be the top half of this equation. So, the graph of the function is
$f(x) = 3 + \sqrt{2 - x}$
Graph this function on a graphing calculator to check.
ii) First, find the inverse of the function. Then, graph the function. To find the inverse, interchange x and y in the equation of the function and solve for y.

$$y = 3 + \sqrt{2 - x}$$
$$x = 3 + \sqrt{2 - y}$$
$$x - 3 = \sqrt{2 - y}$$
$$(x - 3)^2 = 2 - y$$
$$-2 + (x - 3)^2 = -y$$
$$2 - (x - 3)^2 = y$$
$$-(x - 3)^2 + 2 = y$$
$$-(x - 3)^2 + 2 = f^{-1}(x)$$

However, the range of $f(x)$ is
$= \{y \in \mathbf{R} \mid y \geq 3\}$,
So, the domain of $f^{-1}(x)$ is
$= \{x \in \mathbf{R} \mid x \geq 3\}$.
So, the equation of $f^{-1}(x)$ is
$= -(x - 3)^2 + 2; x \geq 3$

Graph the inverse on a graphing calculator.



The graph of $f(x)$ in part a) reflects this graph.

iii) The range of $f(x)$ is

$= \{y \in \mathbf{R} \mid y \geq 3\}$

iv) In part ii), the equation of the inverse was found to be

$-(x - 3)^2 + 2 ; x \geq 3.$

14. The original function must be restricted so that only one branch of the quadratic function is admissible. For example, if $f(x) = x^2$ had its domain restricted to $x \geq 0$, the inverse of $f(x)$ would be a function.

15. a) Possible response: Switch x and y and solve the resulting quadratic equation for y, either by completing the square or by using the quadratic formula.

b) No, because the original function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values.

16. $m(x) = 14\,700 - 3040x$

a) For every kilogram of beef she sells, she makes a total of $x - 3.21$ dollars of profit. So the profit function for one week is

$$P(x) = (\text{profit per kg}) \times (\text{\# kg of beef sold})$$
$$= (x - 3.21) \times m(x)$$
$$= (x - 3.21)(14\,700 - 3040x)$$

b) The zeros of the profit function are $x = 3.21$ and $x = \frac{14\,700}{3040}$, so the axis of symmetry for this quadratic is

$$x = \frac{3.21 + \frac{14\,700}{3040}}{2} = \frac{\frac{244\,584}{30\,400}}{2} = \frac{244\,584}{60\,800} \doteq 4.02$$

So, since I can easily see that $a = -3040$ for this quadratic, I can write the profit function in vertex form

$$P(x) = -3040\left(x - \frac{244\,584}{60\,800}\right)^2 + k$$

To find k, notice that

$$P(0) = (-3.21)(14\,700)$$
$$= -47\,187$$

So, I have

$$-47\,187 = -3040\left(\frac{244\,584}{60\,800}\right)^2 + k$$
$$k = 3040\left(\frac{244\,584}{60\,800}\right)^2 - 47\,187$$
$$\doteq 2008.2$$

To make things easier, write the vertex form as $P(x) = -3040(x - h)^2 + k$ even though I know the expressions for h and k. Then setting $x = -3040(y - h)^2 + k$, I now solve for y.

$$x = -3040(y - h)^2 + k$$
$$(y - h)^2 = \frac{k - x}{3040}$$
$$y = h \pm \sqrt{\frac{k - x}{3040}}$$

So I get

$$P^{-1}(x) = h \pm \sqrt{\frac{k - x}{3040}}$$
$$\doteq 4.02 \pm \sqrt{\frac{2008.2 - x}{3040}}$$

This function will take as an input the total profit I want to have per week, and output what I need to charge per kilogram to obtain this amount of profit.

c) From what I observed in part b), I should charge $P^{-1}(1900)$ dollars per kilogram to earn \$1900 in profit. So I need to charge

$$P^{-1}(1900) \doteq 4.02 \pm \sqrt{\frac{2008.2 - 1900}{3040}}$$
$$\doteq \$3.83 \text{ or } \$4.21 \text{ per kilogram}$$

That is, if the meat manager charges either \$3.83 or \$4.21 per kilogram, she will earn \$1900 in profit for the week.

d) I found the vertex for the profit function in part b), (h, k). So $k \doteq \$2008.18$ is the maximum profit, and this occurs when she charges $h \doteq \$4.02$ per kilogram of meat.

e) The axis of symmetry from part b) would change to

$$x = \frac{3.10 + \frac{14\,700}{3040}}{2} = \frac{\frac{241\,240}{30\,400}}{2} = \frac{241\,240}{60\,800} \doteq 3.97$$

So the x-coordinate for the vertex changes to roughly 3.97, and so the meat manager should charge \$3.97/kg is she wishes to maximize profit. Of course, the new profit function is $P(x) = (x - 3.10)(14\,700 - 3040x)$

So, substituting the axis of symmetry, the maximum profit will be

$$P\left(\frac{241\,240}{60\,800}\right) \doteq (3.97 - 3.10) \times (14\,700 - 3040(3.97)) = (0.87)(2631.2) = \$2289.10$$

17. $x = 4 - 4y + y^2$

a) Notice that

$x = 4 - 4y + y^2$
$= (y - 2)^2$

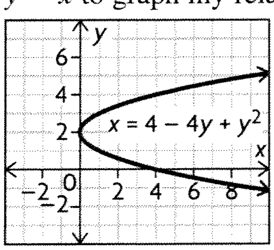
So, solving for y,

$y = 2 \pm \sqrt{x}$

So if $f(x) = (x - 2)^2$, then the given relation is describing the inverse function, and I just found that

$f^{-1}(x) = 2 \pm \sqrt{x}$

$f(x)$ is just the parabola $y = x^2$ shifted right by 2 units, so I can reflect this through the line $y = x$ to graph my relation.



b) The domain of $f(x) = (x - 2)^2$ is $\{x \in \mathbf{R}\}$, so the range of the given relation, which is the same as the domain of $f(x)$, is $\{y \in \mathbf{R}\}$. The range of $f(x)$ is $\{y \in \mathbf{R} \mid y \geq 0\}$, so the domain of the given relation is $\{x \in \mathbf{R} \mid x \geq 0\}$.

c) I found the inverse of the given relation in part a) to be $y = (x - 2)^2$.

d) Yes, the inverse of the given relation describes the parabola $y = x^2$ shifted right by 2 units, and so is a function (that is, it passes the vertical line test).

3.4 Operations with Radicals, pp. 167–168

1. a) $\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

b) $\sqrt{50} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

c) $\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$

d) $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

2. a) $\sqrt{5} \times \sqrt{7} = \sqrt{35}$

b) $\sqrt{11} \times \sqrt{6} = \sqrt{66}$

c) $2\sqrt{3} \times 5\sqrt{2} = (2 \times 5) \times (\sqrt{3} \times \sqrt{2}) = 10\sqrt{6}$

d) $-4\sqrt{3} \times 8\sqrt{13} = (-4 \times 8) \times (\sqrt{3} \times \sqrt{13}) = -32\sqrt{39}$

3. a) $4\sqrt{5} + 3\sqrt{5} = (4 + 3)\sqrt{5} = 7\sqrt{5}$

b) $9\sqrt{7} - 4\sqrt{7} = (9 - 4)\sqrt{7} = 5\sqrt{7}$

c) $3\sqrt{3} + 8\sqrt{2} - 4\sqrt{3} + 11\sqrt{2}$
$= (3 - 4)\sqrt{3} + (8 + 11)\sqrt{2}$
$= -\sqrt{3} + 19\sqrt{2}$

d) $\sqrt{8} - \sqrt{18} = \sqrt{4} \times \sqrt{2} - \sqrt{9} \times \sqrt{2} = 2\sqrt{2} - 3\sqrt{2} = (2 - 3)\sqrt{2} = -\sqrt{2}$

4. a) $3\sqrt{12} = 3 \times \sqrt{4} \times \sqrt{3} = 3 \times 2 \times \sqrt{3} = 6\sqrt{3}$

b) $-5\sqrt{125} = -5 \times \sqrt{25} \times \sqrt{5} = -5 \times 5 \times \sqrt{5} = -25\sqrt{5}$

c) $10\sqrt{40} = 10 \times \sqrt{4} \times \sqrt{10}$
$= 10 \times 2 \times \sqrt{10}$
$= 20\sqrt{10}$

d) $-\frac{1}{2}\sqrt{60} = -\frac{1}{2} \times \sqrt{4} \times \sqrt{15}$
$= -\frac{1}{2} \times 2 \times \sqrt{15}$
$= -\sqrt{15}$

e) $\frac{2}{3}\sqrt{45} = \frac{2}{3} \times \sqrt{9} \times \sqrt{5}$
$= \frac{2}{3} \times 3 \times \sqrt{5}$
$= 2\sqrt{5}$

f) $-\frac{9}{10}\sqrt{1200} = -\frac{9}{10} \times \sqrt{400} \times \sqrt{3}$
$= -\frac{9}{10} \times 20 \times \sqrt{3}$
$= -18\sqrt{3}$

5. a) $\sqrt{3}(2 - \sqrt{5}) = \sqrt{3} \times 2 - \sqrt{3} \times \sqrt{5}$
$= 2\sqrt{3} - \sqrt{15}$

b) $2\sqrt{2}(\sqrt{7} + 3\sqrt{3})$
$= 2 \times \sqrt{2} \times \sqrt{7} + 2 \times \sqrt{2} \times 3 \times \sqrt{3}$
$= 2\sqrt{14} + (2 \times 3) \times (\sqrt{2} \times \sqrt{3})$
$= 2\sqrt{14} + 6\sqrt{6}$

c) $(4\sqrt{2})^2 = (4\sqrt{2}) \times (4\sqrt{2})$
$= (4 \times 4) \times (\sqrt{2} \times \sqrt{2})$
$= 16 \times \sqrt{4}$
$= 16 \times 2$
$= 32$

d) $(-2\sqrt{3})^3$
$= (-2\sqrt{3}) \times (-2\sqrt{3}) \times (-2\sqrt{3})$
$= ((-2) \times (-2) \times (-2))$
$\times (\sqrt{3} \times \sqrt{3} \times \sqrt{3})$
$= -8 \times (\sqrt{9} \times \sqrt{3})$
$= -8 \times 3 \times \sqrt{3}$
$= -24\sqrt{3}$

e) $4\sqrt{3} \times 3\sqrt{6} = (4 \times 3) \times (\sqrt{3} \times \sqrt{6})$
$= 12 \times (\sqrt{3} \times \sqrt{3} \times \sqrt{2})$
$= 12 \times (\sqrt{9} \times \sqrt{2})$
$= 12 \times 3 \times \sqrt{2}$
$= 36\sqrt{2}$

f) $-7\sqrt{2} \times 5\sqrt{8} = (-7 \times 5) \times (\sqrt{2} \times \sqrt{8})$
$= -35 \times \sqrt{16}$
$= -35 \times 4$
$= -140$

6. a) $\sqrt{8} - \sqrt{32} = \sqrt{4} \times \sqrt{2} - \sqrt{16} \times \sqrt{2}$
$= 2\sqrt{2} - 4\sqrt{2}$
$= -2\sqrt{2}$

b) $\sqrt{12} + \sqrt{18} - \sqrt{27} + \sqrt{50}$
$= \sqrt{4} \times \sqrt{3} + \sqrt{9} \times \sqrt{2} - \sqrt{9} \times \sqrt{3}$
$+ \sqrt{25} \times \sqrt{2}$
$= 2\sqrt{3} + 3\sqrt{2} - 3\sqrt{3} + 5\sqrt{2}$
$= -\sqrt{3} + 8\sqrt{2}$

c) $3\sqrt{98} - 5\sqrt{72}$
$= 3 \times \sqrt{49} \times \sqrt{2} - 5 \times \sqrt{36} \times \sqrt{2}$
$= 3 \times 7 \times \sqrt{2} - 5 \times 6 \times \sqrt{2}$
$= 21\sqrt{2} - 30\sqrt{2}$
$= -9\sqrt{2}$

d) $-4\sqrt{200} + 5\sqrt{242}$
$= -4 \times \sqrt{100} \times \sqrt{2} + 5 \times \sqrt{121} \times \sqrt{2}$
$= -4 \times 10 \times \sqrt{2} + 5 \times 11 \times \sqrt{2}$
$= -40\sqrt{2} + 55\sqrt{2}$
$= 15\sqrt{2}$

e) $-5\sqrt{45} + \sqrt{52} + 3\sqrt{125}$
$= -5 \times \sqrt{9} \times \sqrt{5} + \sqrt{4} \times \sqrt{13} + 3$
$\times \sqrt{25} \times \sqrt{5}$
$= -5 \times 3 \times \sqrt{5} + 2 \times \sqrt{13}$
$+ 3 \times 5 \times \sqrt{5}$
$= -15\sqrt{5} + 2\sqrt{13} + 15\sqrt{5}$
$= 2\sqrt{13}$

f) $7\sqrt{12} - 3\sqrt{28} + \frac{1}{2}\sqrt{48} + \frac{2}{3}\sqrt{63}$
$= 7 \times \sqrt{4} \times \sqrt{3} - 3 \times \sqrt{4} \times \sqrt{7}$
$+ \frac{1}{2} \times \sqrt{16} \times \sqrt{3} + \frac{2}{3} \times \sqrt{9} \times \sqrt{7}$
$= 7 \times 2 \times \sqrt{3} - 3 \times 2 \times \sqrt{7} + \frac{1}{2} \times 4$
$\times \sqrt{3} + \frac{2}{3} \times 3 \times \sqrt{7}$
$= 14\sqrt{3} - 6\sqrt{7} + 2\sqrt{3} + 2\sqrt{7}$
$= 16\sqrt{3} - 4\sqrt{7}$

7. a) $(6 - \sqrt{5})(3 + 2\sqrt{10})$
$= 6 \times 3 + 6 \times 2 \times \sqrt{10}$
$-3 \times \sqrt{5} - \sqrt{5} \times 2 \times \sqrt{10}$
$= 18 + 12\sqrt{10} - 3\sqrt{5} - 2\sqrt{50}$
$= 18 + 12\sqrt{10} - 3\sqrt{5}$
$-2 \times \sqrt{25} \times \sqrt{2}$
$= 18 + 12\sqrt{10} - 3\sqrt{5} - 2 \times 5 \times \sqrt{2}$
$= 18 + 12\sqrt{10} - 3\sqrt{5} - 10\sqrt{2}$

b) $(2 + 3\sqrt{3})^2 = (2 + 3\sqrt{3})(2 + 3\sqrt{3})$
$= 2 \times 2 + 2 \times 3\sqrt{3} + 2 \times 3\sqrt{3}$
$+ 3\sqrt{3} \times 3\sqrt{3}$
$= 4 + 6\sqrt{3} + 6\sqrt{3} + 9\sqrt{9}$
$= 4 + 12\sqrt{3} + 9 \times 3$
$= 31 + 12\sqrt{3}$

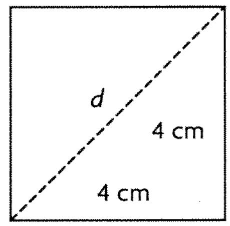
c) $(\sqrt{2} + \sqrt{5})(\sqrt{2} - \sqrt{5})$
$= \sqrt{2} \times \sqrt{2} + \sqrt{2} \times \sqrt{5} - \sqrt{2}$
$\times \sqrt{5} - \sqrt{5} \times \sqrt{5}$
$= 2 - 5$
$= -3$

d) $(3\sqrt{3} + 4\sqrt{2})(\sqrt{3} - 2\sqrt{2})$
$= 3\sqrt{3} \times \sqrt{3} + 4\sqrt{2} \times \sqrt{3}$
$-2\sqrt{2} \times 3\sqrt{3} - 4\sqrt{2} \times 2\sqrt{2}$
$= 9 + 4\sqrt{6} - 6\sqrt{6} - 16$
$= -7 - 2\sqrt{6}$

e) $(2\sqrt{5} - 3\sqrt{7})^2$
$= (2\sqrt{5} - 3\sqrt{7})(2\sqrt{5} - 3\sqrt{7})$
$= 2\sqrt{5} \times 2\sqrt{5} - 3\sqrt{7} \times 2\sqrt{5}$
$-3\sqrt{7} \times 2\sqrt{5} + 3\sqrt{7} \times 3\sqrt{7}$
$= 20 - 6\sqrt{35} - 6\sqrt{35} + 63$
$= 83 - 12\sqrt{35}$

f) $(1 - \sqrt{3})(2 + \sqrt{6})(5 + \sqrt{2})$
$= (1 \times 2 + 1 \times \sqrt{6} - \sqrt{3} \times 2$
$- \sqrt{3} \times \sqrt{6})(5 + \sqrt{2})$
$= (2 + \sqrt{6} - 2\sqrt{3} - \sqrt{18})(5 + \sqrt{2})$
$= (2 + \sqrt{6} - 2\sqrt{3} - \sqrt{9} \times \sqrt{2})(5 + \sqrt{2})$
$= (2 + \sqrt{6} - 2\sqrt{3} - 3\sqrt{2})(5 + \sqrt{2})$
$= 2 \times 5 + \sqrt{6} \times 5 - 2\sqrt{3} \times 5$
$-3\sqrt{2} \times 5 + 2 \times \sqrt{2} + \sqrt{6} \times \sqrt{2}$
$-2\sqrt{3} \times \sqrt{2} - 3\sqrt{2} \times \sqrt{2}$
$= 10 + 5\sqrt{6} - 10\sqrt{3} - 15\sqrt{2}$
$+2\sqrt{2} + \sqrt{12} - 2\sqrt{6} - 6$
$= 4 + 3\sqrt{6} - 13\sqrt{2} - 10\sqrt{3} + \sqrt{4} \times \sqrt{3}$
$= 4 + 3\sqrt{6} - 13\sqrt{2} - 10\sqrt{3} + 2\sqrt{3}$
$= 4 + 3\sqrt{6} - 13\sqrt{2} - 8\sqrt{3}$

8. The diagonal and two sides of the square form a right triangle, and the legs of this right triangle have length 4 cm.



Call the length of the diagonal d. Then by the Pythagorean theorem
$d^2 = 4^2 + 4^2$
$= 32$
$d = \sqrt{32}$
$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$ cm

9. Call the square's side length s. Then since this square has area 450 cm^2, and the area of the square is given by s^2, I get the equation
$s^2 = 450$
$s = \sqrt{450}$
$= \sqrt{225} \times \sqrt{2}$
$= 15\sqrt{2}$ cm

10. A side of length 3 cm and a side of length 9 cm, together with the diagonal of the rectangle form a right triangle (like in problem 8). Let the length of the diagonal be d. Then by the Pythagorean theorem
$d^2 = 3^2 + 9^2$
$= 90$
$d = \sqrt{90}$
$= \sqrt{9} \times \sqrt{10}$
$= 3\sqrt{10}$ cm

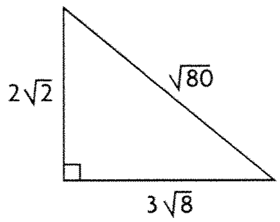
11. I form a right triangle using the points $A(-2,7)$, $B(4,1)$, and $C(-2,1)$. Then the line segment from A to B is the hypotenuse of this right triangle. Let AB be the length of this hypotenuse, and similarly for the other two legs of the right triangle. Then by the Pythagorean theorem
$(AB)^2 = (BC)^2 + (AC)^2$
$= 6^2 + 6^2$
$= 72$

$AB = \sqrt{72}$

$= \sqrt{36} \times \sqrt{2}$

$= 6\sqrt{2}$ units

12.



The perimeter is

$2\sqrt{2} + 3\sqrt{8} + \sqrt{80}$

$= 2\sqrt{2} + 3 \times \sqrt{4} \times \sqrt{2} + \sqrt{16} \times \sqrt{5}$

$= 2\sqrt{2} + 3 \times 2 \times \sqrt{2} + 4 \times \sqrt{5}$

$= 2\sqrt{2} + 6\sqrt{2} + 4\sqrt{5}$

$= 8\sqrt{2} + 4\sqrt{5}$ units

The area is

$\frac{1}{2} \times 3\sqrt{8} \times 2\sqrt{2} = 3 \times \sqrt{16}$

$= 3 \times 4$

$= 12$ units^2

13. Notice that

$\sqrt{a^2} + \sqrt{b^2} = a + b$

On the other hand,

$(\sqrt{a} + \sqrt{b})^2 = \sqrt{a} \times \sqrt{a} + 2 \times \sqrt{a} \times \sqrt{b} + \sqrt{b} \times \sqrt{b}$

$= a + b + 2\sqrt{ab}$

So I see that $(\sqrt{a} + \sqrt{b})^2$ is larger, since it includes the extra positive term $2\sqrt{ab}$.

14. I have

$\sqrt{200} = \sqrt{100} \times \sqrt{2}$

$= 10\sqrt{2}$

$\sqrt{200} = \sqrt{25} \times \sqrt{8}$

$= 5\sqrt{8}$

$\sqrt{200} = \sqrt{4} \times \sqrt{50}$

$= 2\sqrt{50}$

So one possible response is:

$2\sqrt{50}$, $5\sqrt{8}$, $10\sqrt{2}$; the last one is in simplest radical form because the number under the radical sign cannot be simplified.

15. a) $\sqrt{a^3} = \sqrt{a^2} \times \sqrt{a}$

$= a\sqrt{a}$

b) $\sqrt{x^5y^6} = \sqrt{x^5} \times \sqrt{y^6}$

$= \sqrt{x^4} \times \sqrt{x} \times y^3$

$= x^2 \times \sqrt{x} \times y^3$

$= x^2y^3\sqrt{x}$

c) $5\sqrt{n^7} - 2n\sqrt{n^5}$

$= 5 \times \sqrt{n^6} \times \sqrt{n} - 2n \times \sqrt{n^4} \times \sqrt{n}$

$= 5 \times n^3 \times \sqrt{n} - 2n \times n^2 \times \sqrt{n}$

$= 5n^3\sqrt{n} - 2n^3\sqrt{n}$

$= 3n^3\sqrt{n}$

d) $(\sqrt{p} + 2\sqrt{q})(\sqrt{q} - \sqrt{p})$

$= \sqrt{p} \times \sqrt{q} + 2\sqrt{q} \times \sqrt{q} - \sqrt{p} \times \sqrt{p} - 2\sqrt{q} \times \sqrt{p}$

$= \sqrt{pq} + 2q - p - 2\sqrt{pq}$

$= 2q - p - \sqrt{pq}$

16. $\sqrt{\sqrt{\sqrt{4096}}} = \sqrt{\sqrt{\sqrt{2^{12}}}}$

$= \sqrt{\sqrt{2^6}}$

$= \sqrt{2^3}$

$= \sqrt{2^2 \times 2}$

$= \sqrt{2^2} \times \sqrt{2}$

$= 2\sqrt{2}$

17. $(\sqrt{2})^x = 256$

$((\sqrt{2})^x)^2 = 256^2$

$((\sqrt{2})^2)^x = 65\,536$

$2^x = 65\,536$

$x = 16$

Mid-Chapter Review, p. 1[illegible]0

1. a)

x	y	1st differences	2nd differences
−2	−8	$-2 - (-8) = 6$	$2 - 6 = -4$
−1	−2	$0 - (-2) = 2$	$-2 - 2 = -4$
0	0	$-2 - 0 = -2$	$-6 - (-2) = -4$
1	−2	$-8 - (-2) = -6$	*
2	−8	*	*

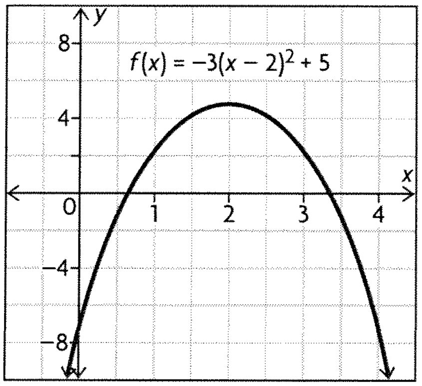
Since the second differences are constant and negative (so non-zero), the function is quadratic.

b)

x	y	1st differences	2nd differences
−2	0	$1 - 0 = 1$	$3 - 1 = 2$
−1	1	$4 - 1 = 3$	$5 - 3 = 2$
0	4	$9 - 4 = 5$	$7 - 5 = 2$
1	9	$16 - 9 = 7$	*
2	16	*	*

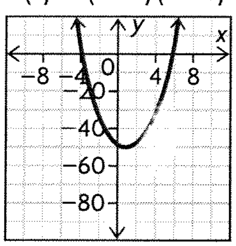
Since the second differences are constant and positive (so non-zero), the function is quadratic.

2. a) $f(x) = -3(x - 2)^2 + 5$: the graph of this quadratic will be the parabola $y = x^2$ flipped, stretched by a factor of 3, shifted right by 2 units and then up by 5 units.



b) $f(x) = 2(x + 4)(x - 6)$: the graph of this quadratic will be a parabola opening up with zeros $x = 6$ and $x = -4$. To sketch the graph precisely, I only need one other point, for instance $(0, f(0)) = (0, -48)$. With this information, I can now sketch the graph.

$f(x) = 2(x + 4)(x - 6)$



3. a) $f(x) = -3(x - 2)^2 + 5$: this quadratic is in vertex form, so has vertex (2, 5) and axis of symmetry $x = 2$. Since a is negative, the parabola opens down, and 5 is a maximum value. So the domain of $f(x)$ is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} | y \leq 5\}$.

b) $f(x) = 2(x + 4)(x - 6)$: since the zeros are $x = 6$ and $x = -4$, the axis of symmetry is

$$x = \frac{6 - 4}{2}$$
$$= 1$$

So the vertex is $(1, f(1)) = (1, 2(5)(-5))$, or $(1, -50)$. Since a is positive, the parabola opens up, and so -50 is a minimum for this quadratic. So the domain of $f(x)$ is $\{x \in \mathbf{R}\}$ and the range is $\{y \in \mathbf{R} | y \geq -50\}$.

4. a)
$$\begin{aligned} f(x) &= -3(x - 2)^2 + 5 \\ &= -3(x^2 - 4x + 4) + 5 \\ &= -3x^2 + 12x - 12 + 5 \\ &= -3x^2 + 12x - 7 \end{aligned}$$

b)
$$\begin{aligned} f(x) &= 2(x + 4)(x - 6) \\ &= 2(x^2 - 2x - 24) \\ &= 2x^2 - 4x - 48 \end{aligned}$$

5. a) $f(x) = x^2 - 6x + 2$: the partial factored form of this quadratic is

$$\begin{aligned} f(x) &= x^2 - 6x + 2 \\ &= x(x - 6) + 2 \end{aligned}$$

Since the zeros of $x(x - 6)$ are $x = 0$ and $x = 6$, I know that $f(x)$ has axis of symmetry $x = 3$. So the vertex is $(3, f(3)) = (3, 3(-3) + 2)$, or $(3, -7)$. Since a is positive for this quadratic, this means that -7 is the minimum value for $f(x)$.

b) $f(x) = 2(x - 4)(x + 6)$: since the zeros of this quadratic are $x = 4$ and $x = -6$, I know that $f(x)$ has axis of symmetry $x = \frac{4 - 6}{2}$, or $x = -1$. So the vertex is $(-1, f(-1)) = (-1, 2(-5)(5))$, or $(-1, -50)$. Since a is positive for this quadratic, this means that -50 is the minimum value for $f(x)$.

c) $f(x) = -2x^2 + 10x$: the factored form of this quadratic is

$$\begin{aligned} f(x) &= -2x^2 + 10x \\ &= -2x(x - 5) \end{aligned}$$

Since the zeros are $x = 0$ and $x = 5$, I know that $f(x)$ has axis of symmetry $x = 2.5$. So the vertex is $(2.5, f(2.5)) = (2.5, -2(2.5)(-2.5))$, or $(2.5, 12.5)$. Since a is negative for this quadratic, this means that 12.5 is the maximum value for $f(x)$.

d) $f(x) = 3.2x^2 + 15x - 7$: the partial factored form of this quadratic is

$$\begin{aligned} f(x) &= 3.2x^2 + 15x - 7 \\ &= 3.2x(x + 4.6875) - 7 \end{aligned}$$

Since the zeros of $3.2x(x + 4.6875)$ are $x = 0$ and $x = -4.6875$, I know that $f(x)$ has axis of symmetry $x = -\frac{4.6875}{2}$, or $x = -2.343\,75$. So the vertex is $(-2.343\,75, f(-2.343\,75))$, and

$$\begin{aligned} f(-2.343\,75) &= 3.2(-2.343\,75)(2.343\,75) - 7 \\ &= -24.578\,125 \end{aligned}$$

So the vertex is $(-2.343\,75, -24.578\,125)$. Since a is positive for this quadratic, this means that $-24.578\,125$ is the minimum value for $f(x)$.

6. $P(x) = -4x^2 + 16x - 7$: the partial factored form for this quadratic is

$P(x) = -4x^2 + 16x - 7$
$= -4x(x - 4) - 7$

So since $-4x(x - 4)$ has zeros $x = 0$ and $x = 4$, $P(x)$ has axis of symmetry $x = 2$. So the vertex of $P(x)$ is $(2, P(2)) = (2, -4(2)(-2) - 7)$, or $(2, 9)$. Since a is negative for this quadratic, this means that the maximum value of the profit function is 9, or \$9000, and that I obtain this profit at $x = 2$, or when I sell 2000 items.

7. $C(x) = 0.3x^2 - 1.2x + 2$: the partial factored form for this quadratic is

$C(x) = 0.3x^2 - 1.2x + 2$
$= 0.3x(x - 4) + 2$

So since the zeros of $0.3x(x - 4)$ are $x = 0$ and $x = 4$, the axis of symmetry for $C(x)$ is $x = 2$. So, since a is positive for this quadratic, this means that the minimum value for $C(x)$ occurs when $x = 2$, or that the most economical production level is when I produce 2000 items per hour.

8. Call the two numbers x and y. Then I know that

$x + y = 16$
$y = 16 - x$

The product of these two numbers is given by

$P = xy$

Substituting for y, I get

$P(x) = x(16 - x)$
$= -x(x - 16)$

This quadratic is in factored form, and has zeros $x = 0$ and $x = 16$. So $P(x)$ has axis of symmetry $x = 8$, and so has vertex $(8, P(8)) = (8, 8(8))$, or $(8, 64)$. Since a is negative for this quadratic, this means that $P(x)$ has maximum value 64 when $x = 8$. That is, the largest possible product for these two numbers is 64, when $x = 8$ and $y = 8$.

9. $f(x) = x^2 - 4x + 3$

a) First put this quadratic in vertex form by completing the square.

$f(x) = x^2 - 4x + 3$
$= x^2 - 4x + 4 - 4 + 3$
$= (x - 2)^2 - 1$

Now set $x = (y - 2)^2 - 1$, and solve for y.

$x = (y - 2)^2 - 1$
$x + 1 = (y - 2)^2$
$y = 2 \pm \sqrt{x + 1}$

So I get

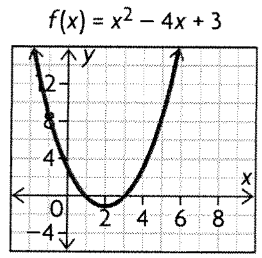
$f^{-1}(x) = 2 \pm \sqrt{x + 1}$

b) The vertex form of $f(x)$ is

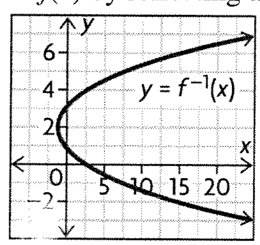
$f(x) = (x - 2)^2 - 1$

So $f(x)$ has vertex $(2, -1)$. Since a is positive for this quadratic, this means that $f(x)$ has minimum value -1. So $f(x)$ has domain $\{x \in \mathbf{R}\}$ and range $\{y \in \mathbf{R} \mid y \geq -1\}$. This means that $f^{-1}(x)$ has domain $\{x \in \mathbf{R} \mid x \geq -1\}$ and range $\{y \in \mathbf{R}\}$.

c) The graph of $f(x)$ is the parabola $y = x^2$ shifted right by 2 units and down by 1 unit.



The graph of $f^{-1}(x)$ is obtained from the graph of $f(x)$ by reflecting across the line $y = x$.



10. Set $R = -2.8(x - 10)^2 + 15$, and solve for x.

$$R = -2.8(x - 10)^2 + 15$$
$$R - 15 = -2.8(x - 10)^2$$
$$(x - 10)^2 = \frac{15 - R}{2.8}$$
$$x = 10 \pm \sqrt{\frac{15 - R}{2.8}}$$

This is an expression giving the number sold in terms of revenue. For instance, if the revenue i $R = 5$ thousand dollars, then the number of items sold is either

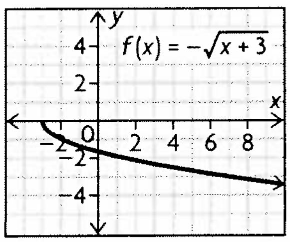
$$x = 10 - \sqrt{\frac{10}{2.8}}$$
$\doteq 8.11$ thousand items

or

$$x = 10 + \sqrt{\frac{10}{2.8}}$$
$\doteq 11.89$ thousand items

11. Usually, the original quadratic function assigns some y-values to two x-values, so the inverse assigns two y-values to some x-values, making it fail the vertical line test and therefore not a function.

12. $f(x) = -\sqrt{x+3}$: to graph this function, notice that this is the negative branch of the square root function $y = -\sqrt{x}$, shifted left by 3 units.



a) The domain of $f(x)$ is $\{x \in \mathbf{R} | x \geq -3\}$, since the number under the square root must not be negative. The range of $f(x)$ is all negative numbers and zero, or $\{y \in \mathbf{R} | y \leq 0\}$.

b) Set $x = -\sqrt{y+3}$, and solve for y.

$x = -\sqrt{y+3}$

$x^2 = (-\sqrt{y+3})^2$

$x^2 = y + 3$

$y = x^2 - 3$

So $f^{-1}(x) = x^2 - 3, x \leq 0$ since the range for $f(x)$ was $\{y \in \mathbf{R} | y \leq 0\}$.

13. a) $\sqrt{48} = \sqrt{16} \times \sqrt{3}$
$= 4\sqrt{3}$

b) $\sqrt{68} = \sqrt{4} \times \sqrt{17}$
$= 2\sqrt{17}$

c) $\sqrt{180} = \sqrt{36} \times \sqrt{5}$
$= 6\sqrt{5}$

d) $-3\sqrt{75} = -3 \times \sqrt{25} \times \sqrt{3}$
$= -3 \times 5 \times \sqrt{3}$
$= -15\sqrt{3}$

e) $5\sqrt{98} = 5 \times \sqrt{49} \times \sqrt{2}$
$= 5 \times 7 \times \sqrt{2}$
$= 35\sqrt{2}$

f) $-8\sqrt{12} = -8 \times \sqrt{4} \times \sqrt{3}$
$= -8 \times 2 \times \sqrt{3}$
$= -16\sqrt{3}$

14. a) $\sqrt{7} \times \sqrt{14} = \sqrt{98}$
$= \sqrt{49} \times \sqrt{2}$
$= 7\sqrt{2}$

b) $3\sqrt{5} \times 2\sqrt{15} = (3 \times 2) \times (\sqrt{5} \times \sqrt{15})$
$= 6 \times \sqrt{75}$
$= 6 \times \sqrt{25} \times \sqrt{3}$
$= 6 \times 5 \times \sqrt{3}$
$= 30\sqrt{3}$

c) $\sqrt{12} + 2\sqrt{48} - 5\sqrt{27}$
$= \sqrt{4} \times \sqrt{3} + 2 \times \sqrt{16} \times \sqrt{3}$
$- 5 \times \sqrt{9} \times \sqrt{3}$
$= 2\sqrt{3} + 2 \times 4 \times \sqrt{3} - 5 \times 3 \times \sqrt{3}$
$= 2\sqrt{3} + 8\sqrt{3} - 15\sqrt{3}$
$= -5\sqrt{3}$

d) $3\sqrt{28} - 2\sqrt{50} + \sqrt{63} - 3\sqrt{18}$
$= 3 \times \sqrt{4} \times \sqrt{7} - 2 \times \sqrt{25} \times \sqrt{2}$
$+ \sqrt{9} \times \sqrt{7} - 3 \times \sqrt{9} \times \sqrt{2}$
$= 3 \times 2 \times \sqrt{7} - 2 \times 5 \times \sqrt{2}$
$+ 3 \times \sqrt{7} - 3 \times 3 \times \sqrt{2}$
$= 6\sqrt{7} - 10\sqrt{2} + 3\sqrt{7} - 9\sqrt{2}$
$= 9\sqrt{7} - 19\sqrt{2}$

e) $(4 - \sqrt{3})(5 + 2\sqrt{3})$
$= 4 \times 5 + 4 \times 2\sqrt{3} - 5 \times \sqrt{3}$
$- 2\sqrt{3} \times \sqrt{3}$
$= 20 + 8\sqrt{3} - 5\sqrt{3} - 6$
$= 14 + 3\sqrt{3}$

f) $(3\sqrt{5} + 2\sqrt{10})(-2\sqrt{5} + 5\sqrt{10})$
$= -2\sqrt{5} \times 3\sqrt{5} - 2\sqrt{5} \times 2\sqrt{10}$

3.5 Quadratic Function Models: Solving Quadratic Equations, pp. 177–178

1. The roots of the equation are the values of x that satisfy the equation.

a) $x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$

$x + 4 = 0$ or $x + 1 = 0$

$x = -4$ or -1

Therefore, the roots of the equation are $x = -4$ or -1.

b) $x^2 - 11x + 18 = 0$

$(x - 9)(x - 2) = 0$

$x - 9 = 0$ or $x - 2 = 0$

$x = 9$ or $x = 2$

Therefore, the roots of the equation are $x = 9$ or 2.

c) $4x^2 - 9 = 0$

$4x^2 = 9$

$x^2 = \frac{9}{4}$

$x = \pm\sqrt{\frac{9}{4}}$

$x = \pm\frac{3}{2}$

Therefore, the roots of the equation are $x = \frac{3}{2}$ or $-\frac{3}{2}$.

d) $2x^2 - 7x - 4 = 0$

$(2x + 1)(x - 4) = 0$

$2x + 1 = 0$ or $x - 4 = 0$

$2x = -1$ or $x = 4$

$x = -\frac{1}{2}$ or $x = 4$

Therefore, the roots of the equation are $x = -\frac{1}{2}$ or 4.

2. The quadratic formula is

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; where a is the constant term of the value for x^2, b is the constant term of the value for x, and c is the remaining constant term.

a) $x^2 - 4x - 9 = 0$

Using the quadratic formula with $a = 1, b = -4,$ and $c = -9$:

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-9)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16 - (-36)}}{2}$

$x = \frac{4 \pm \sqrt{52}}{2}$

$x \doteq \frac{4 \pm 7.21}{2}$

$x \doteq \frac{4 + 7.21}{2}$ or $x \doteq \frac{4 - 7.21}{2}$

$x \doteq 5.61$ or $x \doteq -1.61$

b) $3x^2 + 2x - 8 = 0$

Using the quadratic formula with $a = 3, b = 2,$ and $c = -8$:

$x = \frac{-(2) \pm \sqrt{2^2 - 4(3)(-8)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{4 - (-96)}}{6}$

$x = \frac{-2 \pm \sqrt{100}}{6}$

$x = \frac{-2 \pm 10}{6}$

$x = \frac{-2 + 10}{6}$ or $x = \frac{-2 - 10}{6}$

$x = \frac{8}{6}$ or $x = \frac{-12}{6}$

$x \doteq 1.33$ or $x = -2$

c) $-2x^2 + 3x - 6 = 0$

Using the quadratic formula with $a = -2, b = 3,$ and $c = -6$:

$x = \frac{-(3) \pm \sqrt{3^2 - 4(-2)(-6)}}{2(-2)}$

$x = \frac{-3 \pm \sqrt{9 - 48}}{-4}$

$x = \frac{-3 \pm \sqrt{-39}}{-4}$

Because the radical has a negative number and hence a nonreal (imaginary) number, there are no real roots.

d) $0.5x^2 - 2.2x - 4.7 = 0$

Using the quadratic formula with $a = 0.5, b = -2.2,$ and $c = -4.7$:

$x = \frac{-(-2.2) \pm \sqrt{(-2.2)^2 - 4(0.5)(-4.7)}}{2(0.5)}$

$x = \frac{2.2 \pm \sqrt{4.84 - (-9.4)}}{1}$

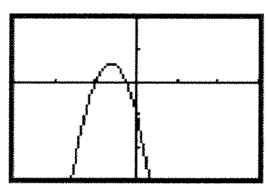
$x = 2.2 \pm \sqrt{14.24}$

$x \doteq 2.2 \pm 3.77$

$x \doteq 2.2 + 3.77$ or $x \doteq 2.2 - 3.77$

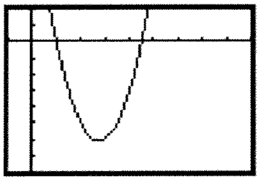
$x \doteq 5.97$ or $x \doteq -1.57$

3. a)



Use the zero function on the calculator to find that $x = -1$ or $x = -0.25$.

b)



Use the zero function on the calculator to find that $x = 1$ or $x = 4.5$.

4. a) i) $2x^2 - 3x = x^2 + 7x$

Group all terms on left side and simplify

$$2x^2 - x^2 - 3x - 7x = 0$$
$$x^2 - 10x = 0$$

This function factors, so solving by factoring is chosen because it is quicker and easier.

ii) $x^2 - 10x = 0$

$x(x - 10) = 0$

$x = 0$ or $x = 10$

Therefore, $x = 0$ or 10

b) i) $4x^2 + 6x + 1 = 0$

The function does not appear to be factorable. Therefore, the quadratic formula is the chosen method.

ii) Using the quadratic formula with $a = 4$, $b = 6$, and $c = 1$:

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(4)(1)}}{2(4)}$$
$$x = \frac{-6 \pm \sqrt{36 - 16}}{8}$$
$$x = \frac{-6 \pm \sqrt{20}}{8}$$
$$x = \frac{-6 \pm \sqrt{(4)(5)}}{8}$$
$$x = \frac{-6 \pm 2\sqrt{5}}{8}$$
$$x = \frac{-3 \pm \sqrt{5}}{4}$$

c) i) $x^2 + 4x - 3 = 0$

The function does not appear to be factorable. Therefore, the quadratic formula is the chosen method.

ii) Using the quadratic formula with $a = 1$, $b = 4$, and $c = -3$:

$$x = \frac{(-4) \pm \sqrt{(4) - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-4 \pm \sqrt{16 + 12}}{2}$$
$$x = \frac{-4 \pm \sqrt{28}}{2}$$
$$x = \frac{-4 \pm \sqrt{(4)(7)}}{2}$$
$$x = \frac{-4 \pm 2\sqrt{7}}{2}$$
$$x = -2 \pm \sqrt{7}$$

d) i) $(x + 3)^2 = -2x$

Simplify and group all terms on the left side.

$$(x + 3)(x + 3) = -2x$$
$$x^2 + 3x + 3x + 9 = -2x$$
$$x^2 + 6x + 9 = -2x$$
$$x^2 + 8x + 9 = 0$$

The function does not appear to be factorable. Therefore, the quadratic formula is the chosen method.

ii) Using the quadratic formula with $a = 1$, $b = 8$, and $c = 9$:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(9)}}{2(1)}$$
$$x = \frac{-8 \pm \sqrt{64 - 36}}{2}$$
$$x = \frac{-8 \pm \sqrt{28}}{2}$$
$$x = \frac{-8 \pm \sqrt{(4)(7)}}{2}$$
$$x = \frac{-8 \pm 2\sqrt{7}}{2}$$
$$x = -4 \pm \sqrt{7}$$

e) i) $3x^2 - 5x = 2x^2 + 4x + 10$

Simplify and group all terms on the left side.

$$3x^2 - 2x^2 - 5x - 4x - 10 = 0$$
$$x^2 - 9x - 10 = 0$$

This function factors, so solving by factoring is chosen because it is quicker and easier.

ii) $x^2 - 9x - 10 = 0$

$(x - 10)(x + 1) = 0$

$x - 10 = 0$ or $x + 1 = 0$

$x = 10$ or $x = -1$

Therefore, $x = 10$ or -1.

f) i) $2(x + 3)(x - 4) = 6x + 6$

Simplify and group all terms on the left side.

$$2(x^2 - 4x + 3x - 12) = 6x + 6$$
$$2(x^2 - x - 12) = 6x + 6$$
$$2x^2 - 2x - 24 = 6x + 6$$
$$2x^2 - 2x - 6x - 24 - 6 = 0$$
$$2x^2 - 8x - 30 = 0$$

The function does not appear to be factorable. Therefore, the quadratic formula is the chosen method.

ii) Using the quadratic formula with $a = 2, b = -8$, and $c = -30$:

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(-30)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 + 240}}{4}$$

$$x = \frac{8 \pm \sqrt{304}}{4}$$

$$x = \frac{8 \pm \sqrt{(16)(19)}}{4}$$

$$x = \frac{8 \pm 4\sqrt{19}}{4}$$

$$x = 2 \pm \sqrt{19}$$

5. The x intercepts are found by setting the function, $f(x)$, equal to 0.

a) $f(x) = 3x^2 - 7x - 2$

$0 = 3x^2 - 7x - 2$

The function does not appear to be easily factorable. Therefore, use the quadratic formula to determine the roots. Using the quadratic formula with $a = 3, b = -7$, and $c = -2$:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{7 \pm \sqrt{49 + 24}}{6}$$

$$x = \frac{7 \pm \sqrt{73}}{6}$$

$$x \doteq \frac{7 \pm 8.54}{6}$$

$$x \doteq \frac{7 + 8.54}{6} \text{ or } x \doteq \frac{7 - 8.54}{6}$$

$x \doteq 2.59$ or $x \doteq -0.26$

Therefore, the x intercepts are the points $(2.59, 0)$ and $(-0.26, 0)$.

b) $f(x) = -4x^2 + 25x - 21$

$0 = -4x^2 + 25x - 21$

Use the factoring method:

$0 = -(4x^2 - 25x + 21)$

$0 = -(4x - 21)(x - 1)$

$4x - 21 = 0$ or $x - 1 = 0$

$4x = 21$ or $x = 1$

$x = \frac{21}{4}$ or $x = 1$

Therefore, the x intercepts are the points $\left(\frac{21}{4}, 0\right)$ and $(1, 0)$.

6. Referring to Example 1, breaking even means that the profit is zero. This is the x-intercept. This is where the profit function, $P(x)$, is equal to 0.

a) $P(x) = -x^2 + 12x + 28$

$0 = -x^2 + 12x + 28$

Use the factoring method:

$0 = -(x^2 - 12x - 28)$

$0 = -(x - 14)(x + 2)$

$x - 14 = 0$ or $x = -2$

$x = 14$ or $x = -2$

Since x is measured in thousands, $x = 14\,000$ or -2000. However, there cannot be -2000 objects sold. Therefore, there is one break-even quantity of 14 000.

b) $P(x) = -2x^2 + 18x - 40$

$0 = -2x^2 + 18x - 40$

Use the factoring method:

$0 = -2(x^2 - 9x + 20)$

$0 = -2(x - 5)(x - 4)$

$x - 5 = 0$ or $x - 5 = 0$

$x = 5$ or $x = 4$

Since x is measured in thousands, $x = 5000$ or 4000. Therefore, the break-even quantity is 5000 or 4000.

c) $P(x) = -2x^2 + 22x - 17$

$0 = -2x^2 + 22x - 17$

The function does not appear to be easily factorable. Therefore, use the quadratic formula to determine the roots. Using the quadratic formula with $a = -2, b = 22$, and $c = -17$:

$$x = \frac{-(22) \pm \sqrt{(22)^2 - 4(-2)(-17)}}{2(-2)}$$

$$x = \frac{-22 \pm \sqrt{484 - 136}}{-4}$$

$$x = \frac{-22 \pm \sqrt{348}}{-4}$$

$$x \doteq \frac{-22 \pm 18.655}{-4}$$

$$x \doteq \frac{-22 + 18.655}{-4} \text{ or } x \doteq \frac{-22 - 18.655}{-4}$$

$x \doteq 0.836$ or $x \doteq 10.164$

Since x is measured in thousands, $x \doteq 836$ or $x \doteq 10\,164$.

d) $P(x) = -0.5x^2 + 6x - 5$

$0 = -0.5x^2 + 6x - 5$

The function does not appear to be easily factorable. Therefore, use the quadratic formula to determine the roots. Using the quadratic formula with $a = -0.5$, $b = 6$, and $c = -5$:

$$x = \frac{(-6) \pm \sqrt{(6)^2 - 4(-0.5)(-5)}}{2(-0.5)}$$

$$x = \frac{-6 \pm \sqrt{36 - 10}}{-1}$$

$$x = \frac{-6 \pm \sqrt{26}}{-1}$$

$$x \doteq \frac{6 + 5.099}{1} \text{ or } x \doteq \frac{-6 - 5.099}{-1}$$

$x \doteq 0.901$ or $x \doteq 11.099$

Since x is measured in thousands,

$x \doteq 901$ or $x \doteq 11\ 099$.

7. $h(t) = -4.9t^2 + 6t + 0.6$

The ball will hit the ground when $h(t)$, the height of the ball in metres, is 0. Hence, set $h(t)$ equal to 0 and solve the equation for t.

$0 = -4.9t^2 + 6t + 0.6$

The function does not appear to be easily factorable. Therefore, use the quadratic formula to determine the roots. Using the quadratic formula with $a = -4.9$, $b = 6$, and $c = 0.6$:

$$x = \frac{(-6) \pm \sqrt{(6)^2 - 4(-4.9)(0.6)}}{2(-4.9)}$$

$$x = \frac{-6 \pm \sqrt{36 + 11.76}}{-9.8}$$

$$x = \frac{-6 \pm \sqrt{47.76}}{-9.8}$$

$$x \doteq \frac{-6 \pm 6.9109}{-9.8}$$

$$x \doteq \frac{-6 + 6.9109}{-9.8} \text{ or } x \doteq \frac{-6 - 6.9109}{-9.8}$$

$x \doteq -0.09$ or $x \doteq 1.32$

A value of -0.09 seconds does not make sense. Therefore, it will take 1.32 seconds for the ball to hit the ground.

8. $P(t) = 0.4t^2 + 10t + 50$

a) In 1995, 0 years have passed since the year 1995. Hence, substitute $t = 0$ in to the function and solve for $P(t)$, the population in thousands.

$P(t) = 0.4t^2 + 10t + 50$

$P(t) = 0.4(0)2 + 10(0) + 50$

$P(t) = 0 + 0 + 50$

$P(t) = 50$

Since $P(t)$ is the population in thousands, the population in 1995 was 50 000.

b) In 2010, $2010 - 1995 = 15$ years have passed since the year 1995. Hence, substitute $t = 15$ in to the function and solve for $P(t)$, the population in thousands.

$P(t) = 0.4t^2 + 10t + 50$

$P(t) = 0.4(15)^2 + 10(15) + 50$

$P(t) = 90 + 150 + 50$

$P(t) = 290$

Since $P(t)$ is the population in thousands, the population in 2010 will be 290 000.

c) $P(t)$ is the population in thousands. Hence, the year the population will be at least 450 000 can be found by setting $P(t) = 450$ and solving for t, then computing the number of years passed since 1995.

$P(t) = 0.4t^2 + 10t + 50$

$450 = 0.4t^2 + 10t + 50$

$0 = 0.4t^2 + 10t + 50 - 450$

$0 = 0.4t^2 + 10t - 400$

Using the quadratic formula with $a = 0.4$, $b = 10$, and $c = -400$:

$$t = \frac{-(10) \pm \sqrt{(10)^2 - 4(0.4)(-400)}}{2(0.4)}$$

$$t = \frac{-10 \pm \sqrt{100 + 640}}{0.8}$$

$$t = \frac{-10 \pm \sqrt{740}}{0.8}$$

$$t = \frac{-10 + \sqrt{740}}{0.8} \text{ or } t = \frac{-10 - \sqrt{740}}{0.8}$$

$$t \doteq \frac{-10 + 27.203}{0.8} \text{ or } t \doteq \frac{-10 - 27.203}{.8}$$

$t \doteq 21.50$ or $t \doteq -46.50$

A value of -46.50 years does not make sense. Hence, the time in years since 1995 is 21.50. Now, 21 years later in 2016, the population will not be at least 450 000. Therefore, round up to 22 years later and the year 2017.

9. Let x be the length in metres of one side of the rectangle. Then, the other side of the rectangle is $(x + 7)$ metres long. Because the area is 330 m^2, the area equation is:

$330 = (x)(x + 7)$
$330 = x^2 + 7x$
$0 = x^2 + 7x - 330$

Now, solve for x to find the length of the two sides of the rectangle. Although the quadratic formula might be the most obvious method for solving, factoring does work as well.

$0 = (x - 15)(x + 22)$
$x - 15 = 0$ or $x + 22 = 0$
$x = 15$ or $x = -22$

Since a length of -22 metres does not make sense, $x = 15$ metres. Therefore, the other side of the rectangle is $15 + 7 = 22$ metres long. The dimensions of the rectangle are 15 metres by 22 metres.

10. Let x be one integer. Because the two integers are consecutive numbers, let the other integer be $(x + 1)$. Then, the sum of the squares of the integers is:

$$(x^2) + (x + 1)^2 = 685$$
$$(x^2) + (x^2 + x + x + 1) = 685$$
$$(x^2) + (x^2 + 2x + 1) = 685$$
$$(x^2 + x^2 + 2x + 1) = 685$$
$$2x^2 + 2x + 1 = 685$$
$$2x^2 + 2x + 1 - 685 = 685 - 685$$
$$2x^2 + 2x - 684 = 0$$

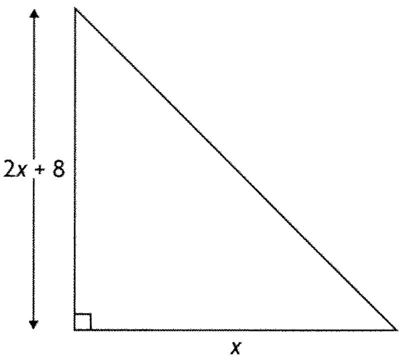
Using the quadratic formula with $a = 2, b = 2$, and $c = -68$:

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(2)(-684)}}{2(2)}$$
$$x = \frac{-2 \pm \sqrt{4 + 5472}}{4}$$
$$x = \frac{-2 \pm \sqrt{5476}}{4}$$
$$x = \frac{-2 \pm 74}{4}$$
$$x = \frac{-2 + 74}{4} \text{ or } x = \frac{-2 - 74}{4}$$
$$x = \frac{72}{4} \text{ or } x = \frac{-76}{4}$$

$x = 18$ or $x = -19$

If one integer is 18, then the second integer is $(18 + 1) = 19$.
If one integer is -19, then the second integer is $(-19 + 1) = -18$.
The two integers can be 18 and 19, or they can be -19 and -18.

11.



Area A = 96 cm²

Let x be the length of the base of the right triangle in centimetres. Then, the height of the right triangle in centimetres is $2x + 8$. If the area is 96 cm^2 and the area of a triangle is

$$\text{area} = \frac{1}{2}(\text{base})(\text{height}),$$

then the area equation is

$$96 = \frac{1}{2}(x)(2x + 8).$$

Now, solve for x to find the dimensions of the triangle.

$$96 = \frac{1}{2}(x)(2x + 8)$$
$$192 = (x)(2x + 8)$$
$$192 = 2x^2 + 8x$$
$$0 = 2x^2 + 8x - 192$$

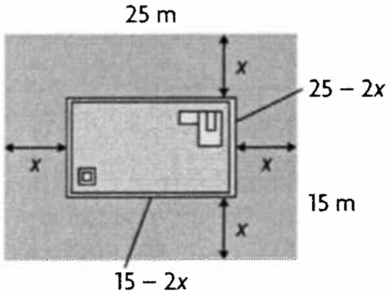
Using the quadratic formula with $a = 2, b = 8$, and $c = -192$:

$$x = \frac{-(8) \pm \sqrt{(8)^2 - 4(2)(-192)}}{2(2)}$$
$$x = \frac{-8 \pm \sqrt{64 + 1536}}{4}$$
$$x = \frac{-8 \pm \sqrt{1600}}{4}$$
$$x = \frac{-8 \pm 40}{4}$$
$$x = \frac{-8 + 40}{4} \text{ or } x = \frac{-8 - 40}{4}$$
$$x = \frac{32}{4} \text{ or } x = \frac{-48}{4}$$

$x = 8$ or $x = -12$

Since a length of -12 cm does not make sense, the length of the base is 8 cm. Hence, the length of the height is $2x + 8 = 2(8) + 8 = 16 + 8 = 24$ cm.
Therefore, the right triangle has a base of 8 cm and a height of 24 cm.

12. This problem closely follows Example 3.



The lawn is 25 m by 15 m. Therefore, the area of the lawn is 375 m^2. After Jackie mows a strip around the lawn, the new lawn is 60% of the original area or 60% of 375, or 225 m^2.
Let x be the width in m of the strip that Jackie mows. Then the length of the new area is $(25 - 2x)$ and the width of the new area is $(15 - 2x)$. The equation of the area of the new patch of lawn is
$225 = (25 - 2x)(15 - 2x)$.
Now, solve for x to determine the width of the strip.

$$225 = 375 - 50x - 30x + 4x^2$$
$$225 = 375 - 80x + 4x^2$$
$$225 - 225 = 4x^2 - 80x + 375 - 225$$
$$0 = 4x^2 - 80x + 150$$

The function does not appear to be easily factorable. Therefore, the quadratic formula is the chosen method for factoring. Using the quadratic formula with $a = 4$, $b = -80$, and $c = 150$:

$$x = \frac{-(-80) \pm \sqrt{(-80)^2 - 4(4)(150)}}{2(4)}$$
$$x = \frac{80 \pm \sqrt{6400 - 2400}}{8}$$
$$x = \frac{80 \pm \sqrt{4000}}{8}$$
$$x \doteq \frac{80 \pm 63.2456}{8}$$
$$x \doteq \frac{80 + 63.2456}{8} \text{ or } x \doteq \frac{80 - 63.2456}{8}$$
$$x \doteq \frac{143.2456}{8} \text{ or } x \doteq \frac{16.7544}{8}$$
$x \doteq 17.9$ or $x \doteq 2.1$

If the strip is 17.9 m long, then the width of the new area is
$= (15 - 2x)$
$= (15 - 2(17.9))$
$= (15 - 35.8)$
$= -20.8$
This clearly does not make any sense.
Therefore $x = 2.1$ m. In this case, the width of the new area is
$= (15 - 2x)$
$= (15 - 2(2.1))$
$= (15 - 4.2)$
$= 10.8$
This is an acceptable answer.
The width of the strip is 2.1 m.

13. $h(t) = -4.9t^2 + 92t + 9$

a) To determine the time t at which the flare's height will be 150 m, substitute 150 for the height $h(t)$ and solve for t.

$$h(t) = -4.9t^2 + 92t + 9$$
$$150 = -4.9t^2 + 92t + 9$$
$$150 - 150 = -4.9t^2 + 92t + 9 - 150$$
$$0 = -4.9t^2 + 92t - 141$$

The function does not appear to be easily factorable. Therefore, the quadratic formula is the chosen method for factoring. Using the quadratic formula with $a = -4.9$, $b = 92$, and $c = -141$:

$$x = \frac{-(92) \pm \sqrt{(92)^2 - 4(-4.9)(-141)}}{2(-4.9)}$$
$$x = \frac{-92 \pm \sqrt{8464 - 2763.6}}{-9.8}$$
$$x = \frac{-92 \pm \sqrt{5700.4}}{-9.8}$$
$$x \doteq \frac{-92 \pm 75.501}{-9.8}$$
$$x \doteq \frac{-92 + 75.501}{-9.8} \text{ or } x \doteq \frac{-92 - 75.501}{-9.8}$$
$$x \doteq \frac{-16.499}{-9.8} \text{ or } x \doteq \frac{-167.501}{-9.8}$$
$x \doteq 1.68$ or $x \doteq 17.09$

The flare's height will be 150 m at 1.68 seconds and again at 17.09 seconds. In this problem, two answers make sense. The first time is when the flare is shooting upward and the second time is when the flare is returning to the ground.

b) The flare's height will be above 150 m between 1.68 seconds and 17.09 seconds. Therefore, the flare is above 150 m for $(17.09 - 1.68) = 15.41$ seconds.

14. Let x be the number of \$0.15 increases. The total fare will be $(2 + 0.15x)$ and the total number of people will be $(4000 - 40x)$. If the company needs to take in \$10 450, then the equation is

$$10\,450 = (2 + 0.15x)(4000 - 40x)$$
$$10\,450 = 8000 - 80x + 600x - 6x^2$$
$$2450 = -6x^2 + 540x$$
$$6x^2 - 540x + 2450 = 0$$

Using the quadratic formula with $a = 6, b = -540$, and $c = 2450$:

$$x = \frac{(540) \pm \sqrt{-(540)^2 - 4(6)(2450)}}{2(6)}$$
$$x = \frac{540 \pm \sqrt{291\,600 - 58\,800}}{12}$$
$$x = \frac{540 \pm \sqrt{232\,800}}{12}$$
$$x \doteq \frac{540 \pm 482.494}{12}$$
$$x \doteq \frac{540 + 482.494}{12} \text{ or } x \doteq \frac{540 - 482.494}{12}$$
$$x \doteq \frac{1022.494}{12} \text{ or } x \doteq \frac{57.506}{12}$$

$x \doteq 85.21$ or $x \doteq 4.79$.

The number of \$0.15 increases has to be a whole number, so round these numbers to $x = 85$ or $x = 5$.

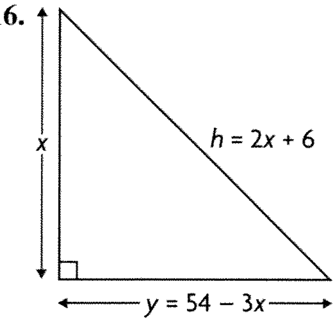
The solution of 85 does not work and so 5 is the only answer. Five \$0.15 increases is \$0.75. The 2 dollar fare plus \$0.75 is a \$2.75 fare.

15. First method: Graph the function on a graphing calculator and use the 'zero' operation to find the zeros of the function.

Second method: Use the factoring method. Factor a -2 out of the function and then break the remaining value into two brackets. Then, find the values of x that would give a zero in each bracket.

Third method: Use the quadratic formula with $a = -2, b = 14$, and $c = -24$.

16.



Let x be one side of the triangle, y be another side of the triangle, and h be the hypotenuse of the triangle. From the description of the triangle, $h = 2x + 6$. I also know that the perimeter is 60 and so $x + y + h = 60$. Substitute h in the equation and simplify.

$$x + y + (2x + 6) = 60$$
$$x + y + 2x + 6 = 60$$
$$3x + y + 6 = 60$$

Solve for y.

$$y = 60 - 3x - 6$$
$$y = 54 - 3x$$

Now, h and y are in terms of x.

Substitute h and y in the Pythagorean theorem and solve for x.

$$x^2 + y^2 = h^2$$
$$x^2 + (54 - 3x)^2 = (2x + 6)^2$$
$$x^2 + 2916 - 162x - 162x + 9x^2 = 4x^2 + 12x + 12x + 36$$
$$10x^2 - 324x + 2916 = 4x^2 + 24x + 36$$
$$6x^2 - 348x + 2880 = 0$$
$$x^2 - 58x + 480 = 0$$
$$(x - 10)(x - 48) = 0$$
$$x - 10 = 0 \text{ or } x - 48 = 0$$
$$x = 10 \text{ or } x = 48$$

If 48 is substituted for x in the equation for h, then $h = 2(48) + 6 = 96 + 6 = 102$. This is not possible because the hypotenuse length would be longer than the entire perimeter.

If 10 is substituted for x in the equation for h, then $h = 2(10) + 6 = 20 + 6 = 26$.

If 10 is substituted for x in the equation for y, then $y = 54 - 3(10) = 54 - 30 = 24$.

The lengths of the sides of the triangle are 10 cm, 24 cm, and 26 cm.

17. $f(x) = 3x - 1 + \frac{1}{x + 1}$

To find the zeros, set the function $f(x)$ equal to zero and solve for x.

$$0 = 3x - 1 + \frac{1}{x + 1}$$
$$0(x + 1) = 3x\,(x + 1) - 1(x + 1) + \frac{1}{x + 1}(x + 1)$$
$$0 = 3x^2 + 3x - x - 1 + 1$$
$$0 = 3x^2 + 2x$$
$$0 = x\,(3x + 2)$$
$$x = 0 \text{ or } 3x + 2 = 0$$
$$x = 0 \text{ or } 3x = -2$$
$$x = 0 \text{ or } x = -\frac{2}{3}$$

The zeros of the function are $x = 0$ or $x = -\frac{2}{3}$.

3.6 The Zeros of a Quadratic Function, pp. 185–186

1. a) $f(x) = 3x^2 - 5$

$= 3(x - 0)^2 + (-5)$

Therefore, the vertex is $(0, -5)$.

Since $a < 0$, the parabola opens up.

Since $a < 0$ and the vertex is below the x-axis, there are 2 zeros.

b) $f(x) = -4x^2 + 7$

$= -4(x - 0)^2 + 7$

Therefore, the vertex is $(0, 7)$.

Since $a > 0$, the parabola opens down.

Since $a > 0$ and the vertex is above the x-axis, there are 2 zeros.

c) $f(x) = 5x^2 + 3$

$= 5(x - 0)^2 + 3$

Therefore, the vertex is $(0, 3)$.

Since $a > 0$, the parabola opens up.

Since $a > 0$ and the vertex is above the x-axis, there are no zeros.

d) $f(x) = 3(x + 2)^2$

The vertex is $(-2, 0)$.

Since $a > 0$, the parabola opens up.

Since the vertex is on the x-axis, there is 1 zero.

e) $f(x) = -4(x + 3)^2 - 5$

The vertex is $(-3, -5)$.

Since $a < 0$, the parabola opens down.

Since $a < 0$ and the vertex is below the x-axis, there are no zeros.

f) $f(x) = 0.5(x - 4)^2 - 2$

The vertex is $(4, -2)$.

Since $a > 0$, the parabola opens up.

Since $a > 0$ and the vertex is below the x-axis, there are 2 zeros.

2. a) $f(x) = x^2 - 6x - 16$

$= (x - 8)(x + 2)$

This function has 2 zeros.

b) $f(x) = 2x^2 - 6x$

$= 2x(x - 3)$

This function has 2 zeros.

c) $f(x) = 4x^2 - 1$

$= (2x - 1)(2x + 1)$

This function has 2 zeros.

d) $f(x) = 9x^2 + 6x + 1$

$= (3x + 1)(3x + 1)$

This function has 1 zero.

3. a) $f(x) = 2x^2 - 6x - 7$

$b^2 - 4ac = (-6)^2 - 4(2)(-7)$

$= 36 + 56$

$= 92$

Since the discriminant is greater than zero, there are 2 zeros.

b) $f(x) = 3x^2 + 2x + 7$

$b^2 - 4ac = (2)^2 - 4(3)(7)$

$= 4 - 84$

$= -80$

Since the discriminant is less than zero, there are no zeros.

c) $f(x) = x^2 + 8x + 16$

$b^2 - 4ac = (8)^2 - 4(1)(16)$

$= 64 - 64$

$= 0$

Since the discriminant is zero, there is 1 zero.

d) $f(x) = 9x^2 - 14.4x + 5.76$

$b^2 - 4ac = (-14.4)^2 - 4(9)(5.76)$

$= 207.36 - 207.36$

$= 0$

Since the discriminant is zero, there is 1 zero.

4. a) $f(x) = -3(x - 2)^2 + 4$

The vertex is $(2, 4)$.

Since $a < 0$ and the vertex is above the x-axis, there are 2 zeros.

b) $f(x) = 5(x - 3)(x + 4)$

This function is already factored.

This function has 2 zeros.

c) $f(x) = 4x^2 - 2x$

$= 2x(2x - 1)$

This function has 2 zeros.

d) $f(x) = 3x^2 - x + 5$

$$\begin{aligned} b^2 - 4ac &= (-1)^2 - 4(3)(5) \\ &= 1 - 60 \\ &= -59 \end{aligned}$$

Since the discriminant is less than zero, there are no zeros.

5. a) At a break-even point, the profit is zero. Hence, $P(x) = 0$.

$$\begin{aligned} P(x) &= -2.1x^2 + 9.06x - 5.4 \\ 0 &= -2.1x^2 + 9.06x - 5.4 \\ b^2 - 4ac &= (9.06)^2 - 4(-2.1)(-5.4) \\ &= 82.0836 - 45.36 \\ &= 36.7236 \end{aligned}$$

Since the discriminant is greater than zero, there are 2 zeros for this function. Therefore, it is possible for the company to break even in 2 ways.

b) At a break-even point, the profit is zero. Hence, $P(x) = 0$.

$$\begin{aligned} P(x) &= -0.3x^2 + 2x - 7.8 \\ 0 &= -0.3x^2 + 2x - 7.8 \\ b^2 - 4ac &= (2)^2 - 4(-0.3)(-7.8) \\ &= 4 - 9.36 \\ &= -5.36 \end{aligned}$$

Since the discriminant is less than zero, there are no zeros.
Therefore, it is not possible for the company to break even.

c) At a break-even point, the profit is zero. Hence, $P(x) = 0$.

$$\begin{aligned} P(x) &= -2x^2 + 6.4x - 5.12 \\ 0 &= -2x^2 + 6.4x - 5.12 \\ b^2 - 4ac &= (6.4)^2 - 4(-2)(-5.12) \\ &= 40.96 - 40.96 \\ &= 0 \end{aligned}$$

Since the discriminant is zero, there is 1 zero. Therefore, it is possible for the company to break even in 1 way.

d) At a break-even point, the profit is zero. Hence, $P(x) = 0$.

$$\begin{aligned} P(x) &= -2.4x^2 + x - 1.2 \\ 0 &= -2.4x^2 + x - 1.2 \\ b^2 - 4ac &= (1)^2 - 4(-2.4)(-1.2) \\ &= 1 - 11.52 \\ &= -10.52 \end{aligned}$$

Since the discriminant is less than zero, there are no zeros.
Therefore, it is not possible for the company to break even.

6. $f(x) = 3x^2 - 4x + k$

The function will have one x-intercept when the discriminant is zero. Hence, set $b^2 - 4ac$ equal to zero, substitute values for a, b, and c, and solve for k.

$$\begin{aligned} 0 &= b^2 - 4ac \\ 0 &= (-4)^2 - 4(3)(k) \\ 0 &= 16 - 12k \\ 12k &= 16 \\ k &= \frac{16}{12} \\ k &= \frac{4}{3} \end{aligned}$$

The function will have one x-intercept when $k = \frac{4}{3}$.

7. $f(x) = kx^2 - 4x + k$

The function will have no zeros when the discriminant is less than zero.

$$\begin{aligned} b^2 - 4ac &< 0 \\ (-4)^2 - 4(k)(k) &< 0 \\ 16 - 4k^2 &< 0 \\ 16 &< 4k^2 \\ 4 &< k^2 \\ k^2 &> 4 \\ k > 2 &\text{ or } k < -2 \end{aligned}$$

The function will have no zeros when $k > 2$ or $k < -2$.

8. $f(x) = 3x^2 + 4x + k = 0$

The function will have no zeros when the discriminant is less than zero.

$$\begin{aligned} b^2 - 4ac &> 0 \\ (4)^2 - 4(3)(k) &> 0 \\ 16 - 12k &> 0 \\ 16 &> 12k \\ 12k &< 16 \\ k &< \frac{16}{12} \\ k &< \frac{4}{3} \end{aligned}$$

The function will have no zeros when $k < \frac{4}{3}$.

The function will have one zero when the discriminant is equal to zero.

$$\begin{aligned} b^2 - 4ac &= 0 \\ (4)^2 - 4(3)(k) &= 0 \\ 16 - 12k &= 0 \\ 16 &= 12k \end{aligned}$$

$$12k = 16$$
$$k = \frac{16}{12}$$
$$k = \frac{4}{3}$$

The function will have one zero when $k = \frac{4}{3}$.

The function will have two zeros when the discriminant is greater than zero.

$$b^2 - 4ac > 0$$
$$(4)^2 - 4(3)(k) > 0$$
$$16 - 12k > 0$$
$$16 > 12k$$
$$12k < 16$$
$$k < \frac{16}{12}$$
$$k < \frac{4}{3}$$

The function will have two zeros when $k < \frac{4}{3}$.

9. $f(x) = x^2 - kx + k + 8$

Since the function touches the x-axis at one point, the discriminant is equal to zero.

$$b^2 - 4ac = 0$$
$$(-k)^2 - 4(1)(k + 8) = 0$$
$$k^2 - 4(k + 8) = 0$$
$$k^2 - 4k - 32 = 0$$
$$(k - 8)(k + 4) = 0$$
$$k - 8 = 0 \text{ or } k + 4 = 0$$
$$k = 8 \text{ or } k = -4$$

The possible values of k are 8 and -4.

10. Set $n^2 + 25$ equal to $-8n$ and solve for n to see if there are any solutions.

$$n^2 + 25 = -8n$$
$$n^2 + 8n + 25 = -8n + 8n$$
$$n^2 + 8n + 25 = 0$$
$$b^2 - 4ac = (8)^2 - 4(1)(25)$$
$$= 64 - 100$$
$$= -36$$

Since the discriminant is less than zero, the function has no zeros and hence no values of n for which the equation is true. Therefore, it is not possible for $n^2 + 25$ to equal $-8n$.

11. Answers may vary.

a) $y = -2(x + 1)(x + 2)$

b) $y = 2x^2 + 1$

c) $y = -2(x - 2)^2$

12. The income is the price times the number of items sold. In this case, it is $p(x)$ times x which is $-4x^2 + 42.5x$. To find the break even quantities for each machine, set the profit equal to a machine and solve for x.

Machine A: $-4x^2 + 42.5x = 4.1x + 92.16$

$0 = 4x^2 - 38.4x + 92.16$

Using the quadratic formula with $a = 4, b = -38.4$, and $c = 92.16$:

$$x = \frac{-(-38.4) \pm \sqrt{(-38.4)2 - 4(4)(92.16)}}{2(4)}$$
$$x = \frac{38.4 \pm \sqrt{1474.56 - 1474.56}}{8}$$
$$x = \frac{38.4 \pm \sqrt{0}}{8}$$
$$x = \frac{38.4}{8}$$
$$x = 4.8$$

Machine B: $-4x^2 + 42.5x = 17.9x + 19.36$

$0 = 4x^2 - 24.6x + 19.36$

Using the quadratic formula with $a = 4, b = -24.6$, and $c = 19.36$:

$$x = \frac{-(-24.6) \pm \sqrt{(-24.6)2 - 4(4)(19.36)}}{2(4)}$$
$$x = \frac{24.6 \pm \sqrt{605.16 - 309.76}}{8}$$
$$x = \frac{24.6 \pm \sqrt{295.4}}{8}$$
$$x \doteq \frac{24.6 \pm 17.187}{8}$$
$$x \doteq \frac{24.6 + 17.187}{8} \text{ or } x \doteq \frac{24.6 - 17.187}{8}$$
$$x \doteq \frac{41.787}{8} \text{ or } x \doteq \frac{7.413}{8}$$
$$x \doteq 5.2 \text{ or } x \doteq 0.93$$

The first break-even point is when $x = 0.93$.

Machine C: $-4x^2 + 42.5x = 8.8x + 55.4$

$0 = 4x^2 - 33.7x + 55.4$

Using the quadratic formula with $a = 4, b = -33.7$, and $c = 55.4$:

$$x = \frac{-(-33.7) \pm \sqrt{(-33.7)2 - 4(4)(55.4)}}{2(4)}$$
$$x = \frac{33.7 \pm \sqrt{1135.69 - 886.4}}{8}$$
$$x = \frac{33.7 \pm \sqrt{249.29}}{8}$$

$x \doteq \frac{33.7 \pm 15.7889}{8}$

$x \doteq \frac{33.7 + 15.7889}{8}$ or

$x \doteq \frac{33.7 - 15.7889}{8}$

$x \doteq \frac{49.4889}{8}$ or $x \doteq \frac{17.9111}{8}$

$x \doteq 6.2$ or $x \doteq 2.2$

The first break-even point is when $x = 2.2$. Since Machine B has the earliest break-even point, it is recommended to the company.

13. $f(x) = 3x^2$

The vertex of the function is on the x-axis and the function has 1 zero.

a) The vertex is still on the x-axis. There is no effect.

b) The vertex is still on the x-axis. There is no effect.

c) The vertex is still on the x-axis and even though the function is now opening downward, there is still 1 zero. There is no effect.

d) The vertex is now below the x-axis and the function is opening upward. It now has 2 zeros.

e) The vertex is now above the x-axis and the function is opening upward. It now has no zeros.

f) The vertex is now above the x-axis and the function is opening downward. It now has 2 zeros.

14. Points of intersection between two equations are found by setting the equations equal to each other.

$$f(x) = g(x)$$
$$x^2 - 6x + 14 = -x^2 - 20x - k$$
$$2x^2 + 14x + 14 + k = 0$$

If there is exactly one point of intersection, then the discriminant is equal to 0. Therefore, set the discriminant equal to 0 and solve for k.

$$b^2 - 4ac = 0$$
$$(14)^2 - 4(2)(14 + k) = 0$$
$$196 - 8(14 + k) = 0$$
$$196 - 112 - 8k = 0$$
$$84 - 8k = 0$$
$$84 = 8k$$
$$10.5 = k$$

There is exactly one point of intersection between the two parabolas when $k = 10.5$.

15. $f(x) = -(x - 3)(3x + 1) + 4$ is a vertical translation of 4 units up of the function $g(x) = -(x - 3)(3x + 1)$. The function $g(x)$ opens down and has 2 zeros. Translating this function 4 units up will have no effect on the number of zeros, so $f(x)$ has 2 zeros.

In particular, $f(x)$ has a vertex above the x-axis with $a < 0$ and hence 2 zeros.

16. a) If the vertex is above the x-axis, the function will have 2 zeros if it opens down and no zeros if it opens up. If the vertex is below the x-axis, there will be 2 zeros if the function opens up and no zeros if it opens down. If the vertex is on the x-axis, there is only 1 zero.

b) If the linear factors are equal or multiples of each other, there is 1 zero; otherwise, there are 2 zeros.

c) If possible, factor and determine the number of zeros as in part b). If not, use the value of $b^2 - 4ac$. If $b^2 - 4ac > 0$, there are 2 zeros, if $b^2 - 4ac = 0$, 1 zero, and if $b^2 - 4ac < 0$, no zeros.

17. $(x^2 - 1)k = (x - 1)^2$

$$kx^2 - k = x^2 - x - x + 1$$
$$kx^2 - k = x^2 - 2x + 1$$
$$0 = x^2 - kx^2 - 2x + 1 + k$$
$$0 = x^2(1 - k) - 2x + (1 + k)$$

The equation will have one solution when the discriminant is equal to zero.

$$b^2 - 4ac = 0$$
$$(-2)^2 - 4(1 - k)(1 + k) = 0$$
$$4 - 4(1 - k^2) = 0$$
$$4 - 4 + 4k^2 = 0$$
$$4k^2 = 0$$
$$k^2 = 0$$
$$k = 0$$

Therefore, the function will have one solution when $k = 0$.

18. $f(x) = (k + 1)x^2 + 2kx + k - 1$

The function will have no zeros when the discriminant is less than zero.

$$b^2 - 4ac < 0$$
$$(2k)^2 - 4(k + 1)(k - 1) < 0$$
$$4k^2 - 4(k^2 - 1) < 0$$
$$4k^2 - 4k^2 + 4 < 0$$
$$4 < 0$$

This inequality is never true. Hence, there are no values of k such that the function has no zeros.
The function will have one zero when the discriminant is equal to zero.

$$b^2 - 4ac = 0$$
$$(2k)^2 - 4(k + 1)(k - 1) = 0$$
$$4k^2 - 4(k^2 - 1) = 0$$
$$4k^2 - 4k^2 + 4 = 0$$
$$4 = 0$$

This equality is never true. Hence, there are no values of k such that the function has 1 zero.
The function will have two zeros when the discriminant is greater than zero.

$$b^2 - 4ac > 0$$
$$(2k)^2 - 4(k + 1)(k - 1) > 0$$
$$4k^2 - 4(k^2 - 1) > 0$$
$$4k^2 - 4k^2 + 4 > 0$$
$$4 > 0$$

This inequality is always true. Hence, for all values of k the function will have 2 zeros.

3.7 Families of Quadratic Functions, pp. 192–193

1. $f(x) = a(x - 3)(x + 4)$
Any two parabolas in this family will have zeros at $x = 3$ and $x = -4$.

2. The two parabolas $f(x)$ and $g(x)$ are similar in that they both have the same vertex of $(2, -4)$. However, $f(x)$ opens downward and $g(x)$ opens upward. Furthermore, the two parabolas are stretched vertically by different factors.

3. At the point where $x = 0$ (the y-intercept), both parabolas have a y value of -7. This can be seen by substituting 0 for x and solving for $f(x)$ and $g(x)$. This means that both parabolas have a y-intercept at the point $(0, -7)$.

4. Start with a quadratic function in factored form.
$f(x) = a(x - r)(x - s)$
Substitute the zeros and then substitute the values of x and $f(x)$ from the given point to find the value of a.

a) $f(x) = a(x - (-4))(x - (3))$
$$f(x) = a(x + 4)(x - 3)$$
$$7 = a(2 + 4)(2 - 3)$$
$$7 = a(6)(-1)$$
$$7 = -6a$$
$$-\frac{7}{6} = a$$
$$f(x) = -\frac{7}{6}(x + 4)(x - 3)$$

b) $f(x) = a(x - (0))(x - (8))$
$$f(x) = a(x)(x - 8)$$
$$-6 = a(-3)(-3 - 8)$$
$$-6 = a(-3)(-11)$$
$$-6 = a(33)$$
$$-\frac{6}{33} = a$$
$$f(x) = -\frac{6}{33}x(x - 8)$$

c) $f(x) = a\left(x - \left(\sqrt{7}\right)\right)\left(x - \left(-\sqrt{7}\right)\right)$
$$f(x) = a\left(x - \sqrt{7}\right)\left(x + \sqrt{7}\right)$$
$$f(x) = a\left(x^2 + \sqrt{7}x - \sqrt{7}x - \left(\sqrt{7}\right)\left(\sqrt{7}\right)\right)$$
$$f(x) = a(x^2 - 7)$$
$$3 = a((-5)^2 - 7)$$
$$3 = a(25 - 7)$$
$$3 = a(18)$$
$$\frac{3}{18} = a$$
$$\frac{1}{6} = a$$
$$f(x) = \frac{1}{6}(x^2 - 7)$$

d) $f(x) = a\left(x - \left(1 - \sqrt{2}\right)\right)\left(x - \left(1 + \sqrt{2}\right)\right)$
$$f(x) = a\left(x^2 - x\left(1 + \sqrt{2}\right) - x\left(1 - \sqrt{2}\right) + \left(1 - \sqrt{2}\right)\left(1 + \sqrt{2}\right)\right)$$
$$f(x) = a(x^2 - x - \sqrt{2} - x + \sqrt{2} + \left(1 + \sqrt{2} - \sqrt{2} - \sqrt{4}\right))$$
$$f(x) = a(x^2 - 2x + (1 - 2))$$
$$f(x) = a(x^2 - 2x - 1)$$
$$4 = a((2)^2 - 2(2) - 1)$$
$$4 = a(4 - 4 - 1)$$
$$4 = a(-1)$$
$$-4 = a$$
$$f(x) = -4(x^2 - 2x - 1)$$

5. Start with a quadratic function in vertex form. $f(x) = a(x - h)^2 + k$
Substitute the vertex point and then substitute the values of x and $f(x)$ from the given point to find the value of a.

a) $f(x) = a(x - (-2))^2 + (5)$
$f(x) = a(x + 2)^2 + 5$
$-8 = a(4 + 2)^2 + 5$
$-8 = a(6)^2 + 5$
$-13 = 36a$
$-\frac{13}{36} = a$
$f(x) = -\frac{13}{36}(x + 2)^2 + 5$

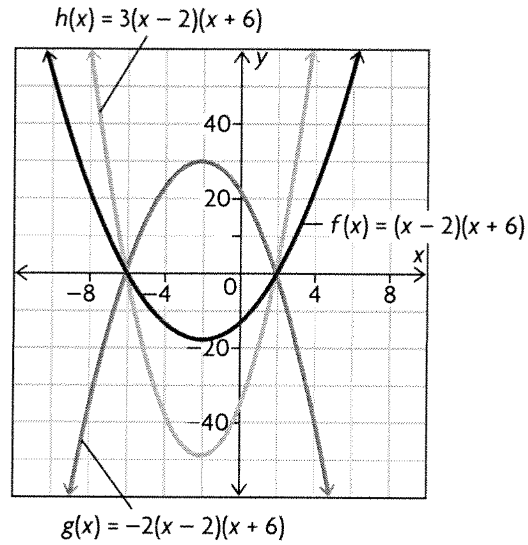
b) $f(x) = a(x - (1))^2 + (6)$
$f(x) = a(x - 1)^2 + 6$
$-7 = a(0 - 1)^2 + 6$
$-7 = a(-1)^2 + 6$
$-13 = a$
$f(x) = -13(x - 1)^2 + 6$

c) $f(x) = a(x - (4))^2 + (-5)$
$f(x) = a(x - 4)^2 - 5$
$-3 = a(-1 - 4)^2 - 5$
$-3 = a(-5)^2 - 5$
$2 = 25a$
$\frac{2}{25} = a$
$f(x) = \frac{2}{25}(x - 4)^2 - 5$

d) $f(x) = a(x - (4))^2 + (0)$
$f(x) = a(x - 4)^2$
$8 = a(11 - 4)^2$
$8 = a(7)^2$
$8 = 49a$
$\frac{8}{49} = a$
$f(x) = \frac{8}{49}(x - 4)^2$

6. $f(2) = 3$ means when $x = 2$, then $y = 3$.
$f(x) = ax^2 - 6x - 7$
$3 = a(2)^2 - 6(2) - 7$
$3 = 4a - 12 - 7$
$3 = 4a - 19$
$22 = 4a$
$\frac{22}{4} = a$
$\frac{11}{2} = a$
$f(x) = \frac{11}{2}x^2 - 6x - 7$

7. a)–c)



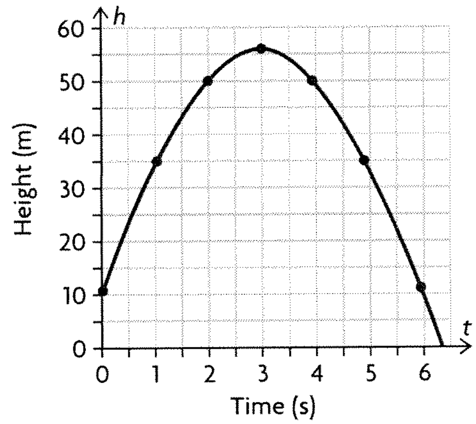
8. Start with a quadratic function in factored form. $f(x) = a(x - r)(x - s)$
Substitute the x-intercepts (zeros) and then substitute the values of x and $f(x)$ from the given point to find the value of a.
$f(x) = a(x - (4))(x - (-4))$
$f(x) = a(x - 4)(x + 4)$
$6 = a(3 - 4)(3 + 4)$
$6 = a(-1)(7)$
$6 = -7a$
$-\frac{6}{7} = a$
$f(x) = -\frac{6}{7}(x - 4)(x + 4)$

9. Start with a quadratic function in factored form.
$f(x) = a(x - r)(x - s)$
Substitute the zeros and then substitute the values of x and $f(x)$ from the given point to find the value of a.
$f(x) = a\left(x - \left(2 + \sqrt{3}\right)\right)\left(x - \left(2 - \sqrt{3}\right)\right)$
$f(x) = a\left(x^2 - x\left(2 - \sqrt{3}\right) - x\left(2 + \sqrt{3}\right) + \left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)\right)$
$f(x) = a\left(x^2 - 2x + x\sqrt{3} - 2x - x\sqrt{3} + \left(4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}\right)\right)$

$f(x) = a(x^2 - 4x + (4 - 3))$
$f(x) = a(x^2 - 4x + 1)$
$5 = a((-4)^2 - 4(-4) + 1)$
$5 = a(16 + 16 + 1)$
$5 = a(33)$
$\frac{5}{33} = a$
$f(x) = \frac{5}{33}(x^2 - 4x + 1)$

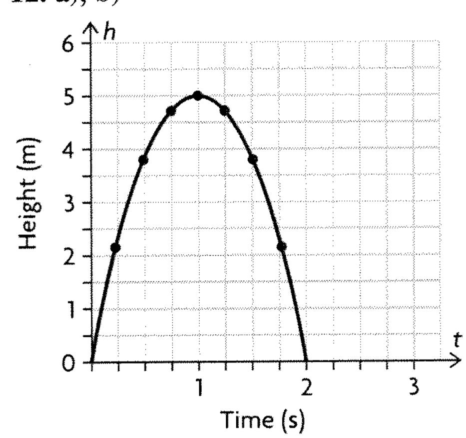
10. This problem is similar to Example 3. If the edge of the tunnel is at the origin, then one of the zeros is 0. If the tunnel is 12 m wide, then the other zero is (12, 0). Now, write the equation in factored form.
$h = ax(x - 12)$
The height of the arch 4 m from the left edge is 6 m. Hence, when $x = 4$, $h = 6$. This is the point (4, 6). Substitute this point into the equation and solve for a.
$6 = a(4)(4 - 12)$
$6 = a(4)(-8)$
$6 = -32a$
$-\frac{6}{32} = a$
$-\frac{3}{16} = a$
$h = -\frac{3}{16}x(x - 12)$
Substitute the given height, h, of 5 m in the equation and solve for x.
$5 = -\frac{3}{16}x(x - 12)$
$80 = -3x(x - 12)$
$80 = -3x^2 + 36x$
$0 = -3x^2 + 36x - 80$
Using the quadratic formula with $a = -3$, $b = 36$, and $c = -80$:
$$x = \frac{-(36) \pm \sqrt{(36)2 - 4(-3)(-80)}}{2(-3)}$$
$$x = \frac{-36 \pm \sqrt{1296 - 960}}{-6}$$
$$x = \frac{-36 \pm \sqrt{336}}{-6}$$
$$x \doteq \frac{-36 \pm 18.33}{-6}$$
$$x \doteq \frac{-36 + 18.33}{-6} \text{ or } x \doteq \frac{-36 - 18.33}{-6}$$
$$x \doteq \frac{-17.67}{-6} \text{ or } x \doteq \frac{-54.33}{-6}$$
$x \doteq 2.94$ or $x \doteq 9.05$
These are the zeros at a height of 5 m. Hence, at a height of 5 m the bridge is (9.05 − 2.94) = 6.11 m wide. Therefore, a truck that is 5 m tall and 3.5 m wide can pass through the tunnel.

11. a), b)



c) The vertex is at (3, 56).
Use the quadratic equation in vertex form.
$h(t) = a(t - h)^2 + k$
$h(t) = a(t - 3)^2 + 56$
Now, choose another point on the parabola, Substitute the values in, and solve for a. (6, 11) is a point on the parabola.
$11 = a(6 - 3)^2 + 56$
$11 = a(3)^2 + 56$
$-45 = 9a$
$-5 = a$
$h(t) = -5(t - 3)^2 + 56$

12. a), b)



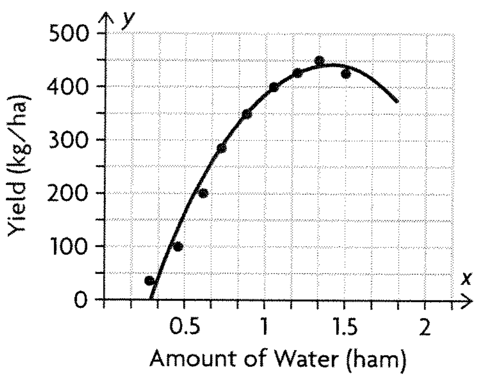
c) The vertex is at (1, 5).
Use the quadratic equation in vertex form.

$h(t) = a(t - h)^2 + k$
$h(t) = a(t - 1)^2 + 5$

Now, choose another point on the parabola, substitute the values in, and solve for a. (2, 0) is a point on the parabola.

$0 = a(2 - 1)^2 + 5$
$0 = a(1)^2 + 5$
$-5 = a$
$h(t) = -5(t - 1)^2 + 5$
$h(t) = -5(t^2 - t - t + 1) + 5$
$h(t) = -5(t^2 - 2t + 1) + 5$
$h(t) = -5t^2 + 10t - 5 + 5$
$h(t) = -5t^2 + 10t$

13. a)



b) The vertex is at approximately (1.35, 442).
c) Use the quadratic equation in vertex form.

$f(x) = a(x - h)^2 + k$

Now, choose another point on the parabola, substitute the values in, and solve for a. (0.60, 198) is a point on the parabola.

$198 = a(0.60 - 1.35)^2 + 442$
$198 = a(-0.75)^2 + 442$
$-244 = 0.5625a$
$\frac{-244}{0.5625} = a$
$-433.78 \doteq a$
$f(x) = -433.78(x - 1.35)^2 + 442$

14. By looking at the graph, one can see that the function is quadratic, the vertex is at $(-2, 3)$, and there are two zeros at $x = -1$ and $x = -3$. An equation of the quadratic function in factored form is:

$f(x) = a(x - r)(x - s)$
$f(x) = a(x - (-1))(x - (-3))$
$f(x) = a(x + 1)(x + 3)$

Substitute the given point $(-4, -9)$ in the equation for x and $f(x)$ and solve for a.

$-9 = a(-4 + 1)(-4 + 3)$
$-9 = a(-3)(-1)$
$-9 = 3a$
$-3 = a$
$f(x) = -3(x + 1)(x + 3)$

15. Sample response:

Definition:	**Characteristics:**
A group of parabolas with a common characteristic	Family may share zeros, a vertex, or a y-intercept
Families of Parabolas	
Examples:	**Non-examples:**
$f(x) = x^2$ $g(x) = -2x^2$ $h(x) = 5x^2$ $p(x) = 3x^2 - x + 5$ $q(x) = -4x^2 + 3x + 5$	$f(x) = 2(x - 3)^2 + 1$ $g(x) = 2(x + 1)^2 - 3$

16. This problem is similar to Example 3. If the edge of the bridge is at the origin, then one of the zeros is $x = 0$. If the tunnel is 40 m wide, then the other zero is $x = 40$. Now, write the equation in factored form.

$h = a(x - r)(x - s)$
$h = a(x - 0)(x - 40)$
$h = ax(x - 40)$

Substitute the given point (5, 8) into the equation for x and $f(x)$ and solve for a.

$8 = a(5)(5 - 40)$
$8 = 5a(-35)$
$8 = -175a$
$-\frac{8}{175} = a$
$-0.0457 \doteq a$
$h \doteq -0.0457x(x - 40)$

The height of the bridge is wanted at 12 m in from the outside edge. Substitute 12 in for x and solve for h.

$h \doteq -0.0457(12)(12 - 40)$
$h \doteq -0.5484(-28)$
$h \doteq 15.36$

Therefore, the height of the bridge 12 m in from the outside edge is 15.36 m.

17. Substitute the given point (3, 6) into the equation for x and $f(x)$ and solve for a.

$f(x) = a(x + 3)(x - 1)(x - 5)$

$6 = a(3 + 3)(3 - 1)(3 - 5)$

$6 = a(6)(2)(-2)$

$6 = -24a$

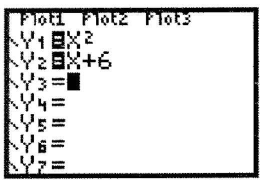
$-\frac{6}{24} = a$

$-\frac{1}{4} = a$

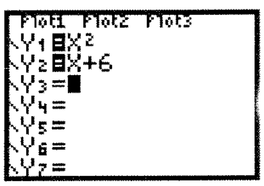
$f(x) = -\frac{1}{4}(x + 3)(x - 1)(x - 5)$

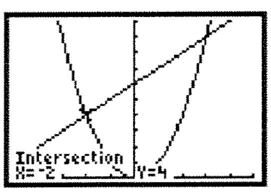
3.8 Linear-Quadratic Systems, pp. 198–199

1. a) Graph both functions on a graphing calculator.

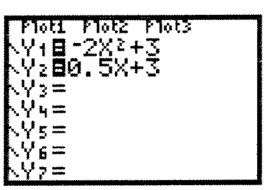


Adjust the window, and use the intersection operation to locate the points (3, 9) and (−2, 4).

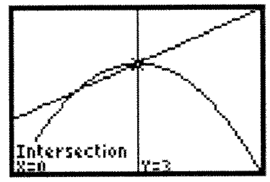


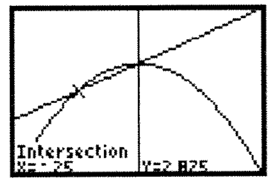


b) Graph both functions on a graphing calculator.

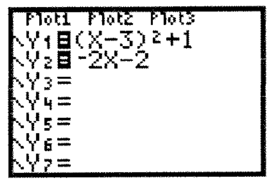


Adjust the window, and use the intersection operation to locate the points (0, 3 and (−0.25, 2.875.

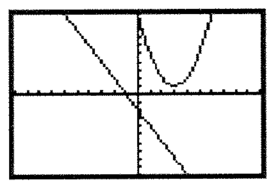




c) Graph both functions on a graphing calculator.



Adjust the window, and notice that there are no points of intersection.



2. a) $f(x) = g(x)$

$-x^2 + 6x - 5 = -4x + 19$

$0 = x^2 - 4x - 6x + 19 + 5$

$0 = x^2 - 10x + 24$

$0 = (x - 4)(x - 6)$

$x = 4$ or $x = 6$

Substitute 4 and 6 back into one of the equations to find the y-coordinates.

$g(x) = -4x + 19$

$g(x) = -4(4) + 19$

$g(x) = -16 + 19$

$g(x) = 3$

When $x = 4$, $g(x) = 3$. Hence, one point of intersection is (4, 3).

$g(x) = -4(6) + 19$

$g(x) = -24 + 19$

$g(x) = -5$

When $x = 6$, $g(x) = -5$. Hence, another point of intersection is (6, −5).

b) $f(x) = g(x)$

$2x^2 - 1 = 3x + 1$

$2x^2 - 3x - 2 = 0$

Using the quadratic formula with
$a = 2, b = -3$, and $c = -2$:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{4}$$

$$x = \frac{3 \pm \sqrt{25}}{4}$$

$$x = \frac{3 \pm 5}{4}$$

$$x = \frac{3 + 5}{4} \text{ or } x = \frac{35}{4}$$

$$x = \frac{8}{4} \text{ or } x = \frac{2}{4}$$

$$x = 2 \text{ or } x = \frac{1}{2}$$

Substitute 2 and $\frac{1}{2}$ back into one of the equations to find the y-coordinates.

$g(x) = 3x + 1$
$g(x) = 3(2) + 1$
$g(x) = 6 + 1$
$g(x) = 7$

When $x = 2$, $g(x) = 7$. Hence, one point of intersection is (2, 7).

$$g(x) = 3\left(\frac{-1}{2}\right) + 1$$

$$g(x) = \frac{-3}{2} + \frac{2}{2}$$

$$g(x) = -\frac{1}{2}$$

When $x = -\frac{1}{2}$, $g(x) = -\frac{1}{2}$. Hence, another point of intersection is $\left(-\frac{1}{2}, -\frac{1}{2}\right)$.

c) $f(x) = g(x)$
$3x^2 - 2x - 1 = -x - 6$
$3x^2 - x + 5 = 0$

Calculate the value of the discriminant.
$b^2 - 4ac$
$= (-1)^2 - 4(3)(5)$
$= 1 - 60$
$= -59$

Since the value of the discriminant is less than zero, there are no real solutions and hence no points of intersection.

3. $f(x) = g(x)$
$4x^2 + x - 3 = 5x - 4$
$4x^2 - 4x + 1 = 0$

Calculate the value of the discriminant.
$b^2 - 4ac$
$= (-4)^2 - 4(4)(1)$
$= 16 - 16$
$= 0$

Since the value of the discriminant is equal to zero, there is one solution and hence one point of intersection.

4. a) $f(x) = g(x)$
$-2x^2 - 5x + 20 = 6x - 1$
$0 = 2x^2 + 11x - 21$

Using the quadratic formula with
$a = 2, b = 11$, and $c = -21$:

$$x = \frac{-(11) \pm \sqrt{(11)2 - 4(2)(-21)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121 + 168}}{4}$$

$$x = \frac{-11 \pm \sqrt{289}}{4}$$

$$x = \frac{-11 \pm 17}{4}$$

$$x = \frac{-11 + 17}{4} \text{ or } x = \frac{-11 - 17}{4}$$

$$x = \frac{6}{4} \text{ or } x = \frac{-28}{4}$$

$$x = \frac{3}{2} \text{ or } x = -7$$

Substitute $\frac{3}{2}$ and -7 back into one of the equations to find the y-coordinates.

$g(x) = 6x - 1$

$$g(x) = 6\left(\frac{3}{2}\right) - 1$$

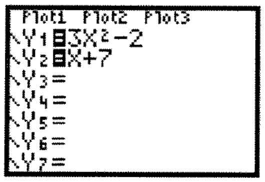
$g(x) = 9 - 1$
$g(x) = 8$

When $x = \frac{3}{2}$, $g(x) = 8$. Hence, one point of intersection is $\left(\frac{3}{2}, 8\right)$.

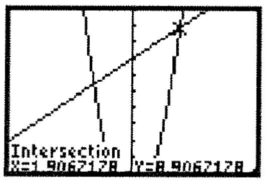
$g(x) = 6(-7) - 1$
$g(x) = -42 - 1$
$g(x) = -43$

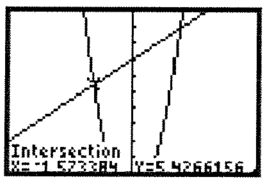
When $x = -7$, $g(x) = -43$. Hence, another point of intersection is $(-7, -43)$.

b) Graph both functions on a graphing calculator.



Adjust the window, and use the intersection operation to locate the points (1.91, 8.91) and (−1.57, 5.43).





c) $f(x) = g(x)$

$5x^2 + x - 2 = -3x - 6$

$5x^2 + 4x + 4 = 0$

Calculate the value of the discriminant.

$b^2 - 4ac$

$= (4)^2 - 4(5)(4)$

$= 16 - 80$

$= -64$

Since the value of the discriminant is less than zero, there are no real solutions and hence no points of intersection.

d) $f(x) = g(x)$

$-4x^2 - 2x + 3 = 5x + 4$

$0 = 4x^2 + 7x + 1$

Using the quadratic formula with $a = 4, b = 7,$ and $c = 1$:

$$x = \frac{-(7) \pm \sqrt{(7)2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 - 16}}{8}$$

$$x = \frac{-7 \pm \sqrt{33}}{8}$$

$$x \doteq \frac{-7 \pm 5.7446}{8}$$

$$x \doteq \frac{-7 + 5.7446}{8} \text{ or } x \doteq \frac{-7 - 5.7446}{8}$$

$$x \doteq \frac{-1.2554}{8} \text{ or } x \doteq \frac{-12.7446}{8}$$

$x \doteq -0.16$ or $x \doteq -1.59$

Substitute −0.16 and −1.59 back into one of the equations to find the y-coordinates.

$g(x) = 5x + 4$

$g(x) = 5(-0.16) + 4$

$g(x) = -0.8 + 4$

$g(x) = 3.2$

When $x = -0.16, g(x) = 3.2$. Hence, one point of intersection is $(-0.16, 3.2)$.

$g(x) = 5(-1.59) + 4$

$g(x) = -7.95 + 4$

$g(x) = -3.95$

When $x = -1.59, g(x) = -3.95$. Hence, another point of intersection is $(-1.59, -3.95)$.

5. Let x be the first integer. Also, let y be the second integer.

$x = 2 + y$

$2x = 1 + y^2$

Solve these equations by substituting one equation into the other and finding x and y.

$2x = 1 + y^2$

$2(2 + y) = 1 + y^2$

$4 + y = 1 + y^2$

$0 = y^2 - y - 3$

$0 = (y - 3)(y + 1)$

$y = 3$ or $y = -1$

If $y = 3$, then $x = 2 + 3 = 5$.

If $y = -1$, then $x = 2 + (-1) = 1$.

Therefore, the two integers can be 5 and 3 or 1 and −1.

6. $R(t) = C(t)$

$-50t^2 + 300t = 600 - 50t$

$0 = 50t^2 - 350t + 600$

Using the quadratic formula with $a = 50, b = -350,$ and $c = 600$:

$$t = \frac{-(-350) \pm \sqrt{(-350)2 - 4(50)(600)}}{2(50)}$$

$$t = \frac{350 \pm \sqrt{122\,500 - 120\,000}}{100}$$

$$t = \frac{350 \pm \sqrt{2500}}{100}$$

$$t = \frac{350 \pm 50}{100}$$

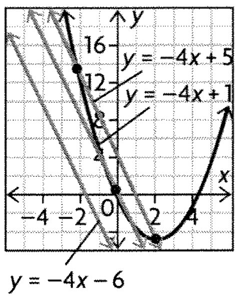
$$t = \frac{350 + 50}{100} \text{ or } t = \frac{350 - 50}{100}$$

$t = \frac{400}{100}$ or $t = \frac{300}{100}$

$t = 4$ or $t = 3$

Production will break even when the ticket price is \$3.00 or \$4.00.

7. a)



b) $y = -4x - 6$

$y = -4x + 1$

$y = -4x + 5$

c) They all have y-intercepts that are less than 1.

8. $f(x) = g(x)$

$2x^2 - 5x + 3 = 3x + k$

$2x^2 - 8x + 3 - k = 0$

Set the value of the discriminant equal to 0 and solve for k.

$$b^2 - 4ac = 0$$
$$(-8)^2 - 4(2)(3 - k) = 0$$
$$64 - 8(3 - k) = 0$$
$$64 - 24 + 8k = 0$$
$$40 = -8k$$
$$-5 = k$$

9. $f(x) = g(x)$

$-3x^2 - x + 4 = 4x + k$

$0 = 3x^2 + 5x + k - 4$

Set the value of the discriminant as less than 0 and solve for k.

$b^2 - 4ac < 0$

$(5)^2 - 4(3)(k - 4) < 0$

$25 - 12(k - 4) < 0$

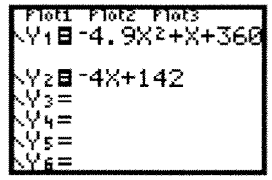
$25 - 12k + 48 < 0$

$73 < 12k$

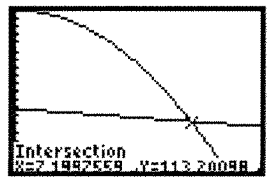
$\frac{73}{12} < k$

$k > \frac{73}{12}$

10. This problem is similar to Example 1. One method is to graph both functions on a graphing calculator.



Adjust the window, and use the intersection operation to locate the point (7.20, 113).



That means that the daredevil released his parachute 7.20 s after jumping.

11. The line will be tangent to the parabola when it intersects the parabola in only one spot. Set the two equations equal to each other and then set the discriminant equal to 0 and solve for m.

$$f(x) = g(x)$$
$$3x^2 + 4x - 2 = mx - 5$$
$$3x^2 + 4x - mx + 3 = 0$$
$$3x^2 + (4 - m)x + 3 = 0$$
$$b^2 - 4ac = 0$$
$$(4 - m)^2 - 4(3)(3) = 0$$
$$16 - 4m - 4m + m^2 - 36 = 0$$
$$m^2 - 8m - 20 = 0$$
$$(m - 10)(m + 2) = 0$$
$$m = 10 \text{ or } m = -2$$

Therefore, the line will be tangent to the parabola when the slope of the line is 10 or −2.

12. Set $h(t)$ and $g(t)$ equal to each other and solve for t.

$$h(t) = g(t)$$
$$-4.9t^2 + 18.24t + 0.8 = -1.43t + 4.26$$
$$0 = 4.9t^2 - 19.67t + 3.46$$

Using the quadratic formula with $a = 4.9$, $b = -19.67$, and $c = 3.46$:

$$t = \frac{-(-19.67) \pm \sqrt{(-19.67)2 - 4(4.9)(3.46)}}{2(4.9)}$$
$$t = \frac{19.67 \pm \sqrt{386.9089 - 67.816}}{9.8}$$
$$t = \frac{19.67 \pm \sqrt{319.0929}}{9.8}$$
$$t \doteq \frac{19.67 \pm 17.863}{9.8}$$

$$t \doteq \frac{19.67 + 17.863}{9.8} \text{ or } t \doteq \frac{19.67 - 17.863}{9.8}$$

$$t \doteq \frac{37.533}{9.8} \text{ or } t \doteq \frac{1.807}{9.8}$$

$t \doteq 3.83$ or $t \doteq 0.18$

Because of the context of the problem, 0.18 is the wanted time.

Substitute this value back into the function for the blocker's hand and solve for the height, $g(t)$.

$g(t) = -1.43t + 4.26$

$g(t) = -1.43(0.18) + 4.26$

$g(t) = -0.2574 + 4.26$

$g(t) = 4.0$

Therefore, the blocker can knock down the punt 0.18 s after the kick at the point (0.18, 4.0).

13. One way to determine the number of points of intersection is to graph the two functions and count the points of intersection. Another method is to calculate the value of the discriminant. If $b^2 - 4ac > 0$, there are two points of intersection. If $b^2 - 4ac = 0$, there is one point of intersection. If $b^2 - 4ac < 0$, there are no points of intersection.

14. $2x + 3y + 6 = 0$

$3y = -6 - 2x$

Substitute this into the other equation and solve for x.

$x^2 - 2x + (-6 - 2x) + 6 = 0$

$x^2 - 2x - 6 - 2x + 6 = 0$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0$ or $x = 4$

Now, find the values of y when $x = 0$ and $x = 4$ by substituting them back into one of the original equations.

$2(0) + 3y + 6 = 0$

$3y = -6$

$y = -2$

One point of intersection is $(0, -2)$.

$2(4) + 3y + 6 = 0$

$8 + 3y + 6 = 0$

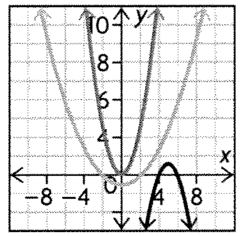
$14 + 3y = 0$

$3y = -14$

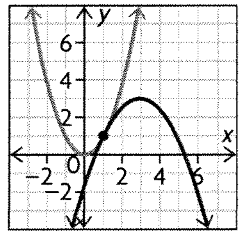
$$y = -\frac{14}{3}$$

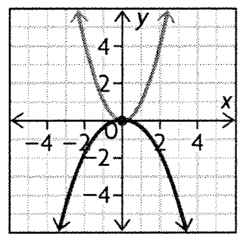
Another point of intersection is $\left(4, -\frac{14}{3}\right)$.

15. Zero points of intersection:

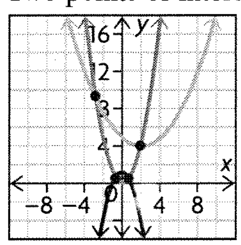


One point of intersection:





Two points of intersection:



16. $f(x) = g(x)$

$x^2 - 4 = -3x^2 + 2x + 8$

$4x^2 - 2x - 12 = 0$

$(2x - 4)(2x + 3) = 0$

$2x = 4$ or $2x = -3$

$$x = 2 \text{ or } x = -\frac{3}{2}$$

$f(x) = (2)^2 - 4$

$f(x) = 4 - 4$

$f(x) = 0$

When $x = 2$, $f(x) = 0$. One point of intersection is (2, 0).

$$f(x) = \left(-\frac{3}{2}\right)2 - 4$$

$$f(x) = \frac{9}{4} - \frac{16}{4}$$

$$f(x) = -\frac{7}{4}$$

When $x = -\frac{3}{2}$, $f(x) = -\frac{7}{4}$. Another point of intersection is $\left(-\frac{3}{2}, -\frac{7}{4}\right)$.

These points are 2 points on the line of intersection of the two parabolas. Use these points to calculate slope, y-intercept, and an equation.

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{slope} = \frac{0 - \left(-\frac{7}{4}\right)}{2 - \left(-\frac{3}{2}\right)}$$

$$\text{slope} = \frac{\frac{7}{4}}{\frac{4}{2} + \frac{3}{2}}$$

$$\text{slope} = \frac{\frac{7}{4}}{\frac{7}{2}}$$

$$\text{slope} = \frac{7}{4} \cdot \frac{2}{7}$$

$$\text{slope} = \frac{2}{4}$$

$$\text{slope} = \frac{1}{2}$$

Use the point (2, 0) and rise over run to determine that another point is 1 down and 2 left of (2, 0). This is the point (0 −1). This is the y-intercept. Hence, an equation of the line that passes through the points of intersection of the two quadratic functions is $y = \frac{1}{2}x - 1$.

Chapter Review, pp. 202–203

1. $f(x) = -(x - 2)^2 + 5$

a) This quadratic is in vertex form. So the vertex is (2, 5). Also, since a is negative, the parabola will open down. Since the vertex is (2, 5), the axis of symmetry is $x = 2$.

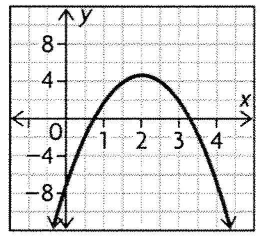
b) The domain of $f(x)$ is all real numbers, $\{x \in \mathbf{R}\}$. Since the parabola opens down and the vertex is (2, 5), $f(x)$ has maximum value 5. So the range of $f(x)$ is $\{y \in \mathbf{R} | y \le 5\}$.

c) Since

$$f(0) = -4 + 5$$
$$= 1$$

the point (0, 1) is on the graph. So, since the vertex is (2, 5), I can now graph the parabola.

$f(x) = -3(x - 2)^2 + 5$



2. $f(x) = 4(x - 2)(x + 6)$

a) This quadratic is in factored form. Since a is positive, this parabola will open up. The zeros are $x = 2$ and $x = -6$.

b) Since the zeros are $x = 2$ and $x = -6$, the axis of symmetry is

$$x = \frac{2 - 6}{2}$$
$$= -2$$

So since

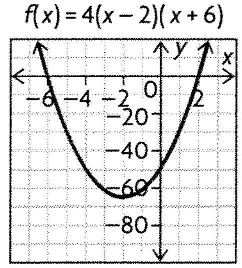
$$f(-2) = 4(-2 - 2)(-2 + 6)$$
$$= 4(-4)(4)$$
$$= -64$$

the vertex is $(-2, -64)$.

c) The parabola opens up, and has vertex $(-2, -64)$, so $f(x)$ has minimum value -64. So the domain of $f(x)$ is $\{x \in \mathbf{R}\}$, and the range is $\{y \in \mathbf{R} | y \ge -64\}$.

d) Since I have vertex $(-2, -64)$, and the x-intercepts are (2, 0) and $(-6, 0)$, I can now graph the parabola.

$f(x) = 4(x - 2)(x + 6)$



3. Since the points $(-5, 3)$ and $(3, 3)$ are equidistant from, and on opposite sides of, the vertex, I can average the x-coordinates to get the axis of symmetry.

$$x = \frac{-5 + 3}{2}$$
$$= -1$$

4. a) $f(x) = -3(x - 4)^2 + 7$: this quadratic is in vertex form, so has vertex (4, 7). Since a is negative, the parabola opens down, so the y-coordinate of the vertex, 7, is a maximum value for this quadratic, and it occurs at $x = 4$.
b) $f(x) = 4x(x + 6)$: this quadratic is in factored form, and has zeros $x = 0$ and $x = -6$. So the axis of symmetry is

$$x = \frac{0 - 6}{2}$$
$$= -3$$

So since

$$f(-3) = 4(-3)(-3 + 6)$$
$$= 4(-3)(3)$$
$$= -36$$

the vertex of this quadratic is $(-3, -36)$. Since a is positive, this means that $f(x)$ has a minimum value of -36, and it occurs at $x = -3$.
5. $h(t) = 2 + 28t - 4.9t^2$: the partial factored form of this quadratic is

$$h(t) = 2 + 28t - 4.9t^2$$
$$= -4.9t\left(t - \frac{28}{4.9}\right) + 2$$

So, since the zeros of $-4.9t\left(t - \frac{28}{4.9}\right)$ are $t = 0$ and $t = \frac{28}{4.9}$, this means that the axis of symmetry for $h(t)$ is

$$t = \frac{0 + \frac{28}{4.9}}{2}$$
$$= \frac{28}{9.8}$$
$$\doteq 2.9$$

So, since a is negative for this quadratic, the maximum value of $h(t)$ occurs at $t \doteq 2.9$ seconds after the ball is thrown. The value of this maximum height is

$$h\left(\frac{28}{9.8}\right) = -4.9\left(\frac{28}{9.8}\right)\left(\frac{28}{9.8} - \frac{28}{4.9}\right) + 2$$
$$= -4.9\left(\frac{28}{9.8}\right)\left(-\frac{28}{9.8}\right) + 2$$
$$= 42 \text{ metres}$$

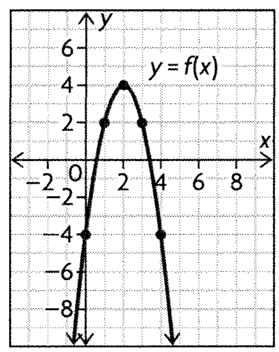
6. If $f(x) = x^2$, and I set $x = y^2$ and solve for y, I get

$$y = \pm\sqrt{x}$$

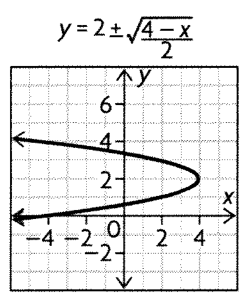
This means that

$$f^{-1}(x) = \pm\sqrt{x}$$

So $g(x) = \sqrt{x}$ and $h(x) = -\sqrt{x}$ are the two branches of the inverse of $f(x) = x^2$.
7. The inverse of a quadratic function is not a function, because it has two y-values for some x-values (for if I reflect a parabola across $y = x$, the resulting graph will fail the vertical line test). It can be a function only if the domain of the original function has been restricted to a single branch of the parabola.
8. a) I am given the following graph.



To get the graph of $y = f^{-1}(x)$, I need to reflect this graph in the line $y = x$. So, for instance, the point $(0, -4)$ on the original graph goes to $(-4, 0)$, the vertex (2, 4) goes to (4, 2), and the x-intercepts (0.5, 0) and (3.5, 0) go to (0, 0.5) and (0, 3.5). This is enough information for me to graph the inverse relation.



b) By observing the graph of $y = f^{-1}(x)$ I see that its domain is $\{x \in \mathbf{R} | x \leq 4\}$ and its range is $\{y \in \mathbf{R}\}$.
c) The inverse relation is not a function; it does not pass the vertical line test (some x-values have more than one y-value).
9. a) $\sqrt{98} = \sqrt{49} \times \sqrt{2}$

$$= 7\sqrt{2}$$

b) $-5\sqrt{32} = -5 \times \sqrt{16} \times \sqrt{2}$
$= -5 \times 4 \times \sqrt{2}$
$= -20\sqrt{2}$

c) $4\sqrt{12} - 3\sqrt{48}$
$= 4 \times \sqrt{4} \times \sqrt{3} - 3 \times \sqrt{16} \times \sqrt{3}$
$= 4 \times 2 \times \sqrt{3} - 3 \times 4 \times \sqrt{3}$
$= 8\sqrt{3} - 12\sqrt{3}$
$= -4\sqrt{3}$

d) $\left(3 - 2\sqrt{7}\right)^2$
$= \left(3 - 2\sqrt{7}\right)\left(3 - 2\sqrt{7}\right)$
$= 3 \times 3 - 3 \times 2\sqrt{7}$
$-3 \times 2\sqrt{7} + 2\sqrt{7} \times 2\sqrt{7}$
$= 9 - 12\sqrt{7} + 28$
$= 37 - 12\sqrt{7}$

10. Let $a = 5, b = 7$ and $c = 10$. Then

$$s = \frac{a + b + c}{2} = \frac{5 + 7 + 10}{2} = \frac{22}{2} = 11$$

So, applying Heron's formula, a triangle with sides of length 5, 7 and 10 will have area

$A = \sqrt{s(s - a)(s - b)(s - c)}$
$= \sqrt{11(11 - 5)(11 - 7)(11 - 10)}$
$= \sqrt{11(6)(4)(1)}$
$= \sqrt{4 \times 66}$
$= \sqrt{4} \times \sqrt{66}$
$= 2\sqrt{66}$ units^2

11. If the legs of a right triangle have length 6 cm and 3 cm, then the length of the third side, or the hypotenuse, can be found using the Pythagorean theorem. Call the length of the hypotenuse h. Then

$h^2 = 6^2 + 3^2$
$= 45$
$h = \sqrt{45}$
$= \sqrt{9} \times \sqrt{5}$
$= 3\sqrt{5}$ cm

So to find the perimeter of the triangle, I just need to add these side lengths together.

perimeter $= 6 + 3 + 3\sqrt{5}$
$= \left(9 + 3\sqrt{5}\right)$ cm

12. $f(x) = 2x^2 + x - 15$
$0 = (2x - 5)(x + 3)$
$2x - 5 = 0$ or $x + 3 = 0$
$2x = 5$ or $x = -3$
$x = \frac{5}{2}$ or $x = -3$

So, the x-intercepts of the quadratic function are $\left(\frac{5}{2}, 0\right)$ and $(-3, 0)$.

13. a) In 2020, t will be $(2020 - 2007) = 13$. Substitute $t = 13$ in to the function and solve for the population, $P(t)$.

$P(t) = 12t^2 + 800t + 40\,000$
$P(t) = 12(13)^2 + 800(13) + 40\,000$
$P(t) = 2028 + 10\,400 + 40\,000$
$P(t) = 52\,428$

The population in 2010 will be 52 428.

b) Substitute 30 000 in for $P(t)$ and solve for t.

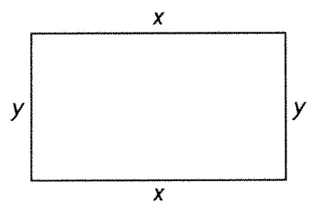
$30\,000 = 12t^2 + 800t + 40\,000$
$0 = 12t^2 + 800t + 10\,000$

Using the quadratic formula with $a = 12, b = 800$, and $c = 10\,000$:

$$t = \frac{-(800) \pm \sqrt{(800)^2 - 4(12)(10\,000)}}{2(12)}$$
$$t = \frac{-800 \pm \sqrt{640\,000 - 480\,000}}{24}$$
$$t = \frac{-800 \pm \sqrt{160\,000}}{24}$$
$$t = \frac{-800 \pm 400}{24}$$
$$t = \frac{-800 + 400}{24} \text{ or } t = \frac{-800 - 400}{24}$$
$$t = \frac{-400}{24} \text{ or } t = \frac{-1200}{24}$$
$$t \doteq -16.67 \text{ or } t = -50$$

Now, t is the number of years after 2007. So, I use -16.67 and round up to -17, because I am counting in whole years, and find that the year is $2007 + (-17) = 1990$. In 1990 the population is 30 000.

14.



Primeter: $x + y + x + y = 400$
Perimeter: $2x + 2y = 400$
Area: $xy = 8000$
Area: $y = \frac{8000}{x}$

$$2x + 2\left(\frac{8000}{x}\right) = 400$$
$$2x^2 + 16\,000 = 400x$$
$$2x^2 - 400x + 16\,000 = 0$$

Using the quadratic formula with $a = 2, b = -400$, and $c = 16\,000$:

$$x = \frac{-(-400) \pm \sqrt{(-400)^2 - 4(2)(16\,000)}}{2(2)}$$
$$x = \frac{400 \pm \sqrt{160\,000 - 128\,000}}{4}$$
$$x = \frac{400 \pm \sqrt{32\,000}}{4}$$
$$x \doteq \frac{400 \pm 178.8854}{4}$$
$$x \doteq \frac{400 + 178.8854}{4} \text{ or } x \doteq \frac{400 - 178.8854}{4}$$
$$x \doteq \frac{578.8854}{4} \text{ or } x \doteq \frac{221.1146}{4}$$
$$x \doteq 144.72 \text{ or } x \doteq 55.28$$

These values are the dimensions of the rectangular field. To be sure, substitute these values in to an equation and solve for y.

$$xy = 8000$$
$$(144.72)y = 8000$$
$$y = 55.28$$

When $x = 144.72, y = 55.28$.

$$xy = 8000$$
$$(55.28)y = 8000$$
$$y = 144.72$$

When $x = 55.28, y = 144.72$.

To the nearest tenth of a metre, the dimensions of the field are 55.3 m by 144.7 m.

15. $h(t) = 14t - 5t^2$

$$9 = 14t - 5t^2$$
$$5t^2 - 14t + 9 = 0$$
$$b^2 - 4ac$$
$$= (-14)^2 - 4(5)(9)$$
$$= 196 - 180$$
$$= 16$$

Because the value of the discriminant is greater than zero, there are two roots. The parabola opens downward and is above the t-axis for small, positive values of t. So, at least one of the roots is positive. Yes, the projectile can reach a height of 9 m.

16. $f(x) = 4x^2 - 3x + 2kx + 1$

$$f(x) = 4x^2 - (3 - 2k)x + 1$$

The function will have two zeros when the value of the discriminant is greater than zero.

$$b^2 - 4ac > 0$$
$$(3 - 2k)^2 - 4(4)(1) > 0$$
$$9 - 6k - 6k + 4k^2 - 16 > 0$$
$$4k^2 - 12k - 7 > 0$$

Using the quadratic formula with $a = 4, b = -12$, and $c = -7$:

$$k = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(-7)}}{2(4)}$$
$$k = \frac{12 \pm \sqrt{144 + 112}}{8}$$
$$k = \frac{12 \pm \sqrt{256}}{8}$$
$$k = \frac{12 \pm 16}{8}$$
$$k = \frac{12 + 16}{8} \text{ or } k = \frac{12 - 16}{8}$$
$$k = \frac{28}{8} \text{ or } k = \frac{-4}{8}$$
$$k = \frac{7}{2} \text{ or } k = -\frac{1}{2}$$

These are the zeros. Now, substitute points to determine where k is true.

$k = -1$:

$$4(-1)^2 - 12(-1) - 7 > 0$$
$$4(1) + 12 - 7 > 0$$
$$4 + 5 > 0$$
$$9 > 0$$

This is true. So, $k < -\frac{1}{2}$.

$k = 0$:

$$4(0)2 - 12(0) - 7 > 0$$
$$0 - 0 - 7 > 0$$
$$-7 > 0$$

This is never true. So k is not between $-\frac{1}{2}$ and $\frac{7}{2}$.

$k = 4$:

$$4(4)^2 - 12(4) - 7 > 0$$
$$4(16) - 48 - 7 > 0$$
$$64 - 55 > 0$$
$$9 > 0$$

This is true. So, $k > \frac{7}{2}$.

Therefore, the function will have two zeros when $k < -\frac{1}{2}$ or $k > \frac{7}{2}$.

Now, I must check these values in the original equation.

$$k = -\frac{1}{2}$$
$$f(x) = 4x^2 - 3x + 2\left(-\frac{1}{2}\right)x + 1$$
$$f(x) = 4x^2 - 3x - x + 1$$
$$f(x) = 4x^2 - 4x + 1$$
$$b^2 - 4ac$$
$$= (-4)^2 - 4(4)(1)$$
$$= 16 - 16$$
$$= 0$$
$$k = \frac{7}{2}$$
$$f(x) = 4x^2 - 3x + 2\left(\frac{7}{2}\right)x + 1$$
$$f(x) = 4x^2 - 3x + 7x + 1$$
$$f(x) = 4x^2 + 4x + 1$$
$$b^2 - 4ac$$
$$= (4)^2 - 4(4)(1)$$
$$= 16 - 16$$
$$= 0$$

So, when $k = -\frac{1}{2}$ or $k = \frac{7}{2}$, the discriminant is zero and the function will have 1 zero. When $k < -\frac{1}{2}$ or $k > \frac{7}{2}$, the discriminant is positive and the function will have 2 zeros. When $-\frac{1}{2} < k < \frac{7}{2}$, the discriminant is negative and the function will have no zeros.

17. The break-even points are where $P(x) = 0$.

$$P(x) = -2x^2 + 7x + 8$$
$$0 = -2x^2 + 7x + 8$$

Using the quadratic formula with $a = -2$, $b = 7$, and $c = 8$:

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(8)}}{2(-2)}$$
$$x = \frac{-7 \pm \sqrt{49 + 64}}{-4}$$
$$x = \frac{-7 \pm \sqrt{113}}{-4}$$
$$x \doteq \frac{7 \pm 10.63}{-4}$$
$$x \doteq \frac{7 + 10.63}{-4} \text{ or } x \doteq \frac{-7 - 0.63}{-4}$$
$$x \doteq \frac{3.63}{-4} \text{ or } x \doteq \frac{17.63}{4}$$
$$x \doteq -0.91 \text{ or } x \doteq 4.408$$

There cannot be a negative number of bikes, so -0.91 is not accepted.

Since x is the number of dirt bikes produced in thousands, the break-even point of the function is when $x = 4408$ bikes.

18. Start with a quadratic function in factored form.

$$f(x) = a(x - r)(x - s)$$

Substitute the zeros and then substitute the values of x and $f(x)$ from the given point to find the value of a.

$$f(x) = a\left(x - \left(2 + \sqrt{3}\right)\right)\left(x - \left(2 - \sqrt{3}\right)\right)$$
$$f(x) = a\left(x^2 - x\left(2 - \sqrt{3}\right) - x\left(2 + \sqrt{3}\right) + \left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)\right)$$
$$f(x) = a\left(x^2 - 2x + x\sqrt{3} - 2x - x\sqrt{3} + \left(4 - 2\sqrt{3} + 2\sqrt{3} - \sqrt{9}\right)\right)$$
$$f(x) = a(x^2 - 4x + (4 - 3))$$
$$f(x) = a(x^2 - 4x + 1)$$
$$5 = a((2)^2 - 4(2) + 1)$$
$$5 = a(4 - 8 + 1)$$
$$5 = a(-3)$$
$$-\frac{5}{3} = a$$
$$f(x) = -\frac{5}{3}(x^2 - 4x + 1)$$

19. The family of parabolas will all have vertex $(-3, -4)$.

$f(x) = a(x + 3)^2 - 4$
$6 = a(-2 + 3)^2 - 4$
$6 = a(1)^2 - 4$
$6 = a - 4$
$10 = a$
$f(x) = 10(x + 3)^2 - 4$

20. a) Because the arch is symmetric, the point at which the arch is 15 m high will be at the center of the arch. So, the vertex of an equation modeling the design of the arch is (0, 15). Substitute this point into the vertex form of a quadratic equation.

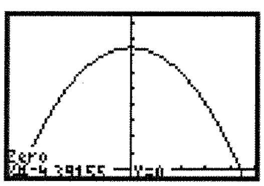
$f(x) = a(x - h)^2 + k$
$f(x) = a(x - 0)^2 + 15$
$f(x) = ax^2 + 15$

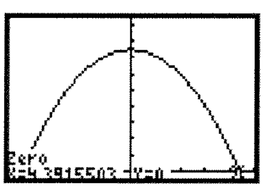
Now, since the arch is 6 m wide at a height of 8 m, points $6 \div 2 = 3$ m to each side of the origin will be 8 m in height. So, the point (3, 8) is on the graph. Substitute to find the value of *a*.

$8 = a(3)^2 + 15$
$-7 = 9a$
$-\frac{7}{9} = a$

$f(x) = -\frac{7}{9}x^2 + 15$

b)





Graph the function and use the zero function on the calculator to find that the *x*-intercepts are $x = -4.391$ and $x = 4.391$. The distance between these two points is $(4.391 - (-4.391)) = 8.8$. This is the length of the base of the arch. So, the width is 8.8 m.

21. $f(x) = g(x)$
$2x^2 + 4x - 11 = -3x + 4$
$2x^2 + 7x - 15 = 0$

Using the quadratic formula with $a = 2, b = 7$, and $c = -15$:

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-15)}}{2(2)}$$
$$x = \frac{-7 \pm \sqrt{49 + 120}}{4}$$
$$x = \frac{-7 \pm \sqrt{169}}{4}$$
$$x = \frac{-7 \pm 13}{4}$$
$$x = \frac{-7 + 13}{4} \text{ or } x = \frac{-7 - 13}{4}$$
$$x = \frac{6}{4} \text{ or } x = \frac{-20}{4}$$
$$x = \frac{3}{2} \text{ or } x = -5$$

$$g(x) = -3\left(\frac{3}{2}\right) + 4$$
$$g(x) = \frac{-9}{2} + \frac{8}{2}$$
$$g(x) = -\frac{1}{2}$$

One point of intersection is $\left(\frac{3}{2}, -\frac{1}{2}\right)$.

$g(x) = -3(-5) + 4$
$g(x) = 15 + 4$
$g(x) = 19$

Another point of intersection is $(-5, 19)$.

22. $h(t) = g(t)$
$-5t^2 + 20t + 15 = 3t + 3$
$0 = 5t^2 - 17t - 12$
$0 = (5t + 3)(t - 4)$
$5t + 3 = 0$ or $t - 4 = 0$
$5t = -3$ or $t = 4$
$t = -\frac{3}{5}$ or $t = 4$

A time of $-\frac{3}{5}$ seconds does not make sense. So, $t = 4$ seconds is the only acceptable answer. Yes, the paintball will hit the baseball at a time, *t*, of 4 seconds.

$h(t) = -5(4)^2 + 20(4) + 15$
$h(t) = -5(16) + 80 + 15$
$h(t) = -80 + 80 + 15$
$h(t) = 15$

The ball will be at a height of 15 m when it is hit by the paintball.

23. a) $f(x) = g(x)$

$x^2 - 6x + 9 = -3x - 5$

$x^2 - 3x + 14 = 0$

The parabola will intersect the line if the value of the discriminant is equal to 0 or greater than 0.

$b^2 - 4ac$
$= (-3)^2 - 4(1)(14)$
$= 9 - 56$
$= -47$

This is not equal to 0 or greater than 0. So, the parabola will not intersect the line.

b) Answers will vary.
Change the slope of the line from -3 to 3.

$g(x) = 3x - 5$
$f(x) = g(x)$
$x^2 - 6x + 9 = 3x - 5$
$x^2 - 9x + 14 = 0$
$b^2 - 4ac$
$= (-9)^2 - 4(1)(14)$
$= 81 - 56$
$= 25$

The line will now intersect the parabola in 2 locations because the value of the discriminant is greater than 0.

Chapter Self-Test, p. 204

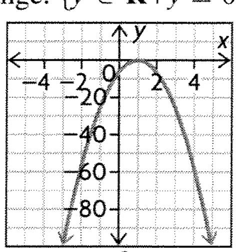
1. $f(x) = -5x^2 + 10x - 5$

a) $f(x) = -5(x^2 - 2x + 1)$
$f(x) = -5(x - 1)(x - 1)$
$f(x) = -5(x - 1)^2$

The vertex is the point (1, 0).

b) The zero is at $x = 1$.
The axis of symmetry is the line $x = 1$.
Since a is negative, the function opens down.

c) Domain: $\{x \in \mathbf{R}\}$
Range: $\{y \in \mathbf{R} \mid y \le 0\}$

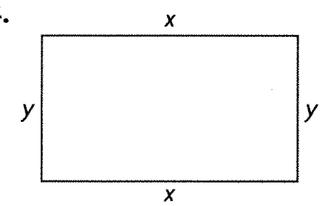
d)


$f(x) = -5x^2 + 10x - 5$

2. a) $f(x) = -2x^2 - 8x + 3$
Because $a = -2$ is negative, the function will have a maximum value. To find the maximum value, complete the square to get the function in vertex form.

b) $f(x) = 3(x - 1)(x + 5)$
Because a is positive, the function will have a minimum value. To find the minimum value, average the zeros.

3. a) Choose the vertex form because the vertex is visible in the equation.
b) Choose the standard form because the y-intercept is visible in the equation as the value of c.
c) Choose the factored form because the zeros are visible in the equation.
d) Choose the vertex form. Use the x-coordinate of the vertex.
e) Choose the vertex form. Use the vertex and direction of opening.

4.


Let x be the length of one side of the field and y be the length of an adjacent side. So,

Perimeter: $x + y + x + y = 2400$
Perimeter: $2x + 2y = 2400$
Perimeter: $x + y = 1200$
Perimeter: $y = 1200 - x$

Let the maximum area be defined by the value A.
$A = xy$
$A = x(1200 - x)$
$A = x(1200 - x)$

The intercepts are $x = 0$ and $x = 1200$. To find the x-coordinate of the vertex, average the two intercepts.

x-coordinate:
$$= \frac{0 + 1200}{2}$$
$$= \frac{1200}{2}$$
$$= 600$$

$x + y = 1200$
$(600) + y = 1200$
$y = 600$

When $x = 600$, $y = 600$.
The maximum area of a rectangular field that can be enclosed by 2400 m of fencing is then $(600) \cdot (600) = 360\,000 \text{ m}^2$.

5. $f(x) = 2(x - 1)^2 - 3$
To determine the equation of the inverse, interchange x and y in the equation and solve for y.

$$y = 2(x - 1)^2 - 3$$
$$x = 2(y - 1)^2 - 3$$
$$x + 3 = 2(y - 1)^2$$
$$\frac{x + 3}{2} = (y - 1)^2$$
$$\pm\sqrt{\frac{x + 3}{2}} = y - 1$$
$$1 \pm \sqrt{\frac{x + 3}{2}} = y$$
$$1 \pm \sqrt{\frac{x + 3}{2}} = f^{-1}(x)$$

6. a) $(2 - \sqrt{8})(3 + \sqrt{2})$
$= 6 + 2\sqrt{2} - 3\sqrt{8} - \sqrt{16}$
$= 6 + 2\sqrt{2} - 3\sqrt{4}\sqrt{2} - 4$
$= 2 + 2\sqrt{2} - 6\sqrt{2}$
$= 2 - 4\sqrt{2}$

b) $(3 + \sqrt{5})(5 - \sqrt{10})$
$= 15 - 3\sqrt{10} + 5\sqrt{5} - \sqrt{50}$
$= 15 - 3\sqrt{10} + 5\sqrt{5} - \sqrt{25}\sqrt{2}$
$= 15 - 3\sqrt{10} + 5\sqrt{5} - 5\sqrt{2}$

c) $\sqrt{8}$ can be simplified to $2\sqrt{2}$. This resulted in like radicals that could be combined.

7. $kx^2 - 4x + k = 0$
The function will have one root when the discriminant is equal to zero.

$$b^2 - 4ac = 0$$
$$(-4)^2 - 4(k)(k) = 0$$
$$16 - 4k^2 = 0$$
$$16 = 4k^2$$
$$4 = k^2$$
$$k = \pm 2$$
$$k = 2 \text{ or } k = -2$$

The function will have one root when $k = 2$ or $k = -2$.

8. Set the two equations equal to each other and then calculate the discriminant. The linear function will intersect the quadratic function if the value of the discriminant is equal to 0 or greater than 0.

$$f(x) = g(x)$$
$$2x^2 - 3x + 2 = 6x - 5$$
$$2x^2 - 9x + 7 = 0$$

$b^2 - 4ac$
$= (-9)^2 - 4(2)(7)$
$= 81 - 56$
$= 25$

Since the value of the discriminant is greater than 0, the linear function intersects the quadratic function in two places.
Using the quadratic formula with $a = 2, b = -9$, and $c = 7$:

$$x = \frac{-(-9) \pm \sqrt{25}}{2(2)}$$
$$x = \frac{9 \pm 5}{4}$$
$$x = \frac{9 + 5}{4} \text{ or } x = \frac{9 - 5}{4}$$
$$x = \frac{14}{4} \text{ or } x = \frac{4}{4}$$
$$x = \frac{7}{2} \text{ or } x = 1$$

$$g(x) = 6\left(\frac{7}{2}\right) - 5$$
$$g(x) = 21 - 5$$
$$g(x) = 16$$

When $x = \frac{7}{2}, g(x) = 16$.

One point of intersection is $\left(\frac{7}{2}, 16\right)$.

$$g(x) = 6(1) - 5$$
$$g(x) = 6 - 5$$
$$g(x) = 1$$

When $x = 1, g(x) = 1$.
Another point of intersection is $(1, 1)$.
So, the points of intersection of the function are $\left(\frac{7}{2}, 16\right)$ and $(1, 1)$.

Chapters 1-3 Cumulative Review, pp. 206–209

1. The correct answer is **c)**. This is not a function. This is because the value 3 is sent to both 12 and 31. A function is only sent to one value.
2. The correct answer is **b)**. A reflection in the x-axis would not be applied to $f(x)$.
3. $f(x) = 2x + 30$
To determine the equation of the inverse, interchange x and y in the equation and solve for y.

$$y = 2x + 30$$
$$x = 2y + 30$$
$$x - 30 = 2y$$
$$\frac{x - 30}{2} = y$$
$$\frac{x - 30}{2} = f^{-1}(x)$$

The correct answer is **b)**.

4. The vertex of the function is (2, 3) and the equation opens down. So, the range is any value equal to 3 or below 3. This can also be checked by graphing the function. The correct answer is **a)**.

5. The correct answer is **b)**.

In i), $b(x)$ can be simplified to $(x + 3)(x - 6)(x + 6)$, which is the same as $h(x)$.

In ii), $b(t) = (3t + 2)^3$

$= (9t^2 + 12t + 4)(3t + 2)$

$= 27t^3 + 18t^2 + 36t^2 + 24t + 12t + 8$

$= 27t^3 + 54t^2 + 36t + 8$

This is the same equation as $c(t)$.

6. The correct answer is **c)**.

$\frac{(3y + 1)}{(2y - 1)} \div \frac{3y(y + 1)}{2y - 1}$ is the same as

$$\frac{(3y + 1)}{(2y - 1)} \cdot \frac{2y - 1}{3y(y + 1)}$$

In the first expression, y cannot equal $\frac{1}{2}$ or the denominator would be zero. In the second expression, y cannot equal 0 or -1 or the denominator would be zero. So, the correct answer is c).

7. $\frac{x^2 - 5x + 6}{x^2 - 1} \cdot \frac{x^2 - 4x - 5}{x^2 - 4}$

$$\frac{(x - 3)(x - 2)}{(x - 1)(x + 1)} \cdot \frac{(x - 5)(x + 1)}{(x - 2)(x + 2)}$$

Cancel $(x - 2)$ from the numerator of the first expression and the denominator of the second expression. Also, cancel $(x + 1)$ from the numerator of the first expression and the denominator of the second expression. Now, multiply the two terms together.

$$\frac{(x - 3)(x - 5)}{(x - 1)(x + 2)}$$

The correct answer is **d)**.

8. $\frac{5x - 6}{x + 1} + \frac{3x}{x - 4}$

First, form a common denominator.

$$\left(\frac{5x - 6}{x + 1} \cdot \frac{x - 4}{x - 4}\right) + \left(\frac{3x}{x - 4} \cdot \frac{x + 1}{x + 1}\right)$$

Now, add the two expressions together and multiply the numerators.

$$\frac{(5x^2 - 20x - 6x + 24) + (3x^2 + 3x)}{(x + 1)(x - 4)}$$

$$\frac{8x^2 - 23x + 24}{(x + 1)(x - 4)}$$

The correct answer is **b)**.

9. Put the function into vertex form by completing the square.

$f(x) = 3x^2 - 6x + 15$

$f(x) = 3(x^2 - 2x + 5)$

$f(x) = 3(x^2 - 2x + 1 - 1 + 5)$

$f(x) = 3(x^2 - 2x + 1) - 3 + 15$

$f(x) = 3(x - 1)^2 + 12$

The vertex is at (1, 12).

The correct answer is **a)**.

10. The correct answer is **b)**. The x-intercepts are visible in factored form.

11. Find the vertex and the maximum height is the y-coordinate of the vertex. First find the x-intercepts and then average those to find the vertex.

$h(t) = 0.8 + 2.9t - 4.9t^2$

Using the quadratic formula with $a = -4.9$, $b = 2.9$, and $c = 0.8$:

$$t = \frac{-(2.9) \pm \sqrt{(2.9)^2 - 4(-4.9)(0.8)}}{2(-4.9)}$$

$$t = \frac{-2.9 \pm \sqrt{8.41 + 15.68}}{-9.8}$$

$$t = \frac{-2.9 \pm \sqrt{24.09}}{-9.8}$$

$$t \doteq \frac{-2.9 \pm 4.908}{-9.8}$$

$$t \doteq \frac{-2.9 + 4.908}{-9.8} \text{ or } t \doteq \frac{-2.9 - 4.908}{-9.8}$$

$$t \doteq \frac{2.008}{-9.8} \text{ or } t \doteq \frac{-7.808}{-9.8}$$

$t \doteq -0.205$ or $t \doteq 0.7967$

t-coordinate of vertex:

$$= \frac{-0.205 + 0.7967}{2}$$

$$= \frac{0.5917}{2}$$

$$= 0.29585$$

$h(0.29585) = 0.8 + 2.9(0.29585) - 4.9(0.29585)^2$

$h(0.29585) = 1.2290816095$ The correct answer is **c)**.

12. Let x be the number of empty seats.

profit = income − cost

profit = (fare price)(number of people) − (cost)

$\text{profit} = (60 + 5x)(22 - x) - (225 + 30(22 - x))$

$\text{profit} = 1320 - 60x + 110x - 5x^2 - 225 - (660 - 30x)$

$\text{profit} = -5x^2 + 50x + 1095 - 660 + 30x$

$\text{profit} = -5x^2 + 80x + 435$

$\text{profit} = -5(x^2 - 16x - 87)$

$\text{profit} = -5(x^2 - 16x + 64 - 64 - 87)$

$\text{profit} = -5(x - 8)^2 + 320 + 435$

$\text{profit} = -5(x - 8)^2 + 755$

The vertex is at the point (8, 755).

So, the bus should run with 8 empty seats to maximize profit from the trip.

The correct answer is **a)**.

13. $g(x) = -2(x + 3.6)^2 + 4.1$

This function is above the x-axis because 4.1, the y-intercept, is positive. Also, $a = -2$ is negative. So, the function is above the x-axis and opening down. This function will have two zeros.

The correct answer is **d)**.

14. $f(x) = x^2 - kx + k + 8$

A function touches the x-axis at one point when the discriminant is equal to 0.

$$b^2 - 4ac = 0$$
$$(-k)^2 - 4(1)(k + 8) = 0$$
$$k^2 - 4(k + 8) = 0$$
$$k^2 - 4k - 32 = 0$$
$$(k - 8)(k + 4) = 0$$
$$k = 8 \text{ or } k = -4$$

The correct answer is **b)**.

15. $f(x) = 2(x - 3)^2 + 5; x \geq 3$

To determine the equation of the inverse, interchange x and y in the equation and solve for y.

$$y = 2(x - 3)^2 + 5$$
$$x = 2(y - 3)^2 + 5$$
$$x - 5 = 2(y - 3)^2$$
$$\frac{x - 5}{2} = (y - 3)^2$$
$$\pm\sqrt{\frac{x - 5}{2}} = y - 3$$
$$3 \pm \sqrt{\frac{x - 5}{2}} = y$$
$$3 + \sqrt{\frac{x - 5}{2}} = y$$ The correct answer is **a)**.

16. The correct answer is **c)**.

17. $f(x) = x^2 - 5x + 3$

$$f(-1) = (-1)^2 - 5(-1) + 3$$
$$f(-1) = 1 + 5 + 3$$
$$f(-1) = 9$$

The correct answer is **d)**.

18. The vertical line test, not the horizontal line test, can be used to show that a relation is a function.

The correct answer is **a)**.

19. y cannot be zero. So, **d)** is the best choice.

The correct answer is **d)**.

20. $f(x) = 5x - 7$

To determine the equation of the inverse, interchange x and y in the equation and solve for y.

$$y = 5x - 7$$
$$x = 5y - 7$$
$$x + 7 = 5y$$
$$\frac{x + 7}{5} = y$$
$$\frac{x + 7}{5} = f^{-1}(x)$$

The correct answer is **d)**.

21. $g(x) = x^2 - 5x - 6$

To determine the equation of the inverse, interchange x and y in the equation and solve for y.

$$y = x^2 - 5x - 6$$
$$x = y^2 - 5y - 6$$
$$x = y^2 - 5y + \frac{25}{4} - \frac{25}{4} - 6$$
$$x = \left(y - \frac{5}{2}\right)^2 - \frac{25}{4} - \frac{24}{4}$$
$$x = \left(y - \frac{5}{2}\right)^2 - \frac{49}{4}$$
$$x + \frac{49}{4} = \left(y - \frac{5}{2}\right)^2$$
$$\pm\sqrt{x + \frac{49}{4}} = y - \frac{5}{2}$$
$$\frac{5}{2} \pm \sqrt{x + \frac{49}{4}} = y$$ The correct answer is **c)**.

22. To determine the equation of the inverse, interchange x and y in the equation and solve for y, not x.

The correct answer is **d)**.

23.

$$f(x) = 3(x + 2)^2 - 5$$
$$y = 3(x + 2)^2 - 5$$
$$x = 3(y + 2)^2 - 5$$
$$x + 5 = 3(y + 2)^2$$
$$\frac{x + 5}{3} = (y + 2)^2$$
$$\pm\sqrt{\frac{x + 5}{3}} = y + 2$$
$$-2 \pm \sqrt{\frac{x + 5}{3}} = y$$
$$-2 \pm \sqrt{\frac{x + 5}{3}} = f^{-1}(x)$$

Look at the graph of this function. It shows that y must be greater than or equal to -2 and y must be greater than or equal to 2. The later is covered in the former. So, $y \geq -2$ for $f^{-1}(x)$. So, $x \geq -2$ for $f(x)$. The correct answer is **b)**.

24. $f(x) = \sqrt{x - 1}; x \geq 1$

$$y = \sqrt{x - 1}$$
$$x = \sqrt{y - 1}$$
$$x^2 = y - 1$$
$$x^2 + 1 = y; x \geq 1$$

The correct answer is **c)**.

25. The correct answer is **b)**.

26. $y = -2x^2 - 12x - 19$

$$y = -2(x^2 + 6x) - 19$$
$$y = -2(x^2 + 6x + 9 - 9) - 19$$
$$y = -2(x^2 + 6x + 9) + 18 - 19$$
$$y = -2(x + 3)2 - 1$$

The correct answer is **c)**.

27. $y = (x + 2)(x - 3)$

$$y = x^2 - x - 6$$
$$y = x^2 - x + \frac{1}{4} - \frac{1}{4} - 6$$
$$y = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - \frac{24}{4}$$
$$y = \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$$

The vertex is $\left(\frac{1}{2}, -\frac{25}{4}\right)$.

The correct answer is **d)**.

28. The company will break even when the profit is 0.

$$0 = -4x^2 + 28x - 40$$
$$0 = -4(x^2 - 7x + 10)$$
$$0 = -4(x - 5)(x - 2)$$
$$x = 5 \text{ or } x = 2$$

Since x is the number sold in thousands, the company must sell 5000 or 2000 items to break even. The correct answer is **a)**.

29. The correct answer is **b)**.

30. $7x^2 + 12x + 6 = 0$

$$D = b^2 - 4ac$$
$$D = (12)^2 - 4(7)(6)$$
$$D = 144 - 168$$
$$D = -24$$

Since the value of the discriminant is less than 0, there are no roots. $n = 0$

The correct answer is **d)**.

31. $\left(\frac{7}{ab} \cdot \frac{3a}{3a}\right) - \left(\frac{2}{b} \cdot \frac{3a^2}{3a^2}\right) + \left(\frac{1}{3a^2} \cdot \frac{b}{b}\right)$

$$= \frac{21a}{3a^2b} - \frac{6a^2}{3a^2b} + \frac{b}{3a^2b}$$
$$= \frac{21a - 6a^2 + b}{3a^2b}; a, b \neq 0$$

The correct answer is **b)**.

32. $\frac{x^2 - 4}{x + 3} \div \frac{2x + 4}{x^2 - 9}$

$$= \frac{(x - 2)(x + 2)}{x + 3} \cdot \frac{(x - 3)(x + 3)}{2(x + 2)}$$

Cancel $(x + 2)$ from the numerator of the first expression and the denominator of the second expression. Also, cancel $(x + 3)$ from the denominator of the first expression and the numerator of the second expression.

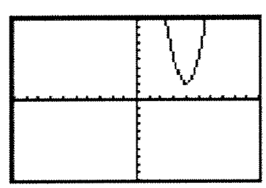
$$= \frac{(x - 2)(x - 3)}{2}$$

The correct answer is **c)**.

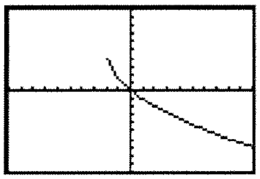
33. a) $f(x) = 3x^2 - 24x + 50$

$$= 3(x^2 - 8x) + 50$$
$$= 3(x^2 - 8x + 16) + 50 - 48$$
$$= 3(x - 4)^2 + 2$$

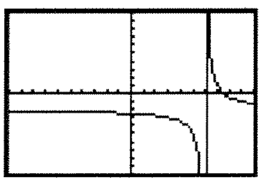
Domain: $\{x \in \mathbf{R}\}$, Range: $\{y \in \mathbf{R} \mid y \geq 2\}$; Parent function: $y = x^2$; Transformations: Vertical stretch by a factor of 3, horizontal translation 4 right, vertical translation 2 up; Graph:



b) Domain: $\{x \in \mathbf{R} \mid x \geq -2\}$, Range: $\{y \in \mathbf{R} \mid y \geq 5\}$; Parent function: $y = \sqrt{x}$; Transformations: Vertical stretch by a factor of 2, reflection in the x-axis, horizontal compression by a factor of , horizontal translation 2 left, vertical translation 5 up; Graph:



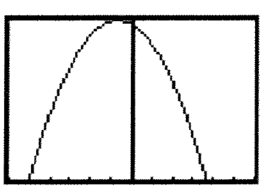
c) Domain: $\{x \in \mathbf{R} \mid x \neq 6\}$, Range: $\{y \in \mathbf{R} \mid y \neq -2\}$; Parent function: $y = \frac{1}{x^2}$; Transformations: Horizontal stretch by a factor of 3, horizontal translation 6 to the right, vertical translation 2 down; Graph:



34. Let x be Sacha's speed and let y be Jill's speed.

$$x = y + 1.4$$
$$\frac{30}{x} + \frac{1}{3} = \frac{30}{y} - 2$$
$$\frac{30}{(y + 1.4)} + \frac{1}{3} = \frac{30}{y} - 2$$
$$\frac{30}{(y + 1.4)} = \frac{30}{y} - \frac{7}{3}$$
$$30 = \frac{30(y + 1.4)}{y} - \frac{7y + 9.8}{3}$$
$$90y = 90y + 126 - 7y^2 - 9.8y$$
$$0 = -7y^2 - 9.8y + 126$$

Graph the function and use the zero function on the calculator to see that the $y = -5$ or $y = 3.6$



A speed of -5 km/h does not make sense. So, 3.6 km/h is Jill's speed.

$x = (3.6) + 1.4$
$x = 5.0$

Sacha's speed is 5.0 km/h.

$$\frac{30 \text{ km}}{5.0 \text{ km/h}} = 6 \text{ hours}$$

Sacha also stopped for 20 minutes. So, Sacha finished the walk in 6 h 20 min.

$$\frac{30 \text{ km}}{3.6 \text{ km/h}} = 8\frac{1}{3}$$

Jill finished the walk in 8 h 20 min.

35. a) Let x be the number of \$50 increases. So, there will be $(25 - 2x)$ people going on the trip.

profit = income − cost

$$0 = (25 - 2x)(550 + 50x) - (5500 + (240(25 - 2x)))$$
$$0 = 13\,750 + 1250x - 1100x - 100x^2 - 5500 - (6000 - 480x)$$
$$0 = 8250 + 150x - 100x^2 - 6000 + 480x$$
$$0 = -100x^2 + 630x + 2250$$

Using the quadratic formula with $a = -100$, $b = 630$, and $c = 2250$:

$$x = \frac{-(630) \pm \sqrt{(630)^2 - 4(-100)(2250)}}{2(-100)}$$
$$x = \frac{-630 \pm \sqrt{396\,900 + 900\,000}}{-200}$$
$$x = \frac{-630 \pm \sqrt{1\,296\,900}}{-200}$$
$$x \doteq \frac{-630 + 1138.82}{-200}$$
$$x \doteq \frac{-630 + 1138.82}{-200} \text{ or } x \doteq \frac{-630 - 1138.82}{-200}$$
$$x \doteq \frac{508.82}{-200} \text{ or } x \doteq \frac{-1768.82}{-200}$$
$$x \doteq -2.54 \text{ or } x \doteq 8.84$$

-2.54 \$50 increases does not make sense.

The number of students who must go for Josh to break even is

$= (25 - 2x)$
$= 25 - 2(8.84)$
$= 25 - 17.68$

But, 17.68 people is not possible. So, round down to 17 people.

$= 25 - 17$
$= 8$

8 students

b) To find the maximum profit, locate the x-coordinate of the vertex by averaging the two zeros.

$$\frac{(8.84 + (2.54))}{2}$$
$$= \frac{6.3}{2}$$
$$= 3.15$$

So, there must be 3.15 \$50 increases. Because there cannot be a 0.15 \$50 increase, round down to 3. There must be 3 \$50 increases. This is \$150.

\$550 + \$150 = \$700

The cost of the trip that will maximize the profit is \$700.

CHAPTER 4: Exponential Functions

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 212

1. a) There are 2 factors of 7.

$7^2 = (7)(7)$
$= 49$

b) There are 5 factors of 2.

$2^5 = (2)(2)(2)(2)(2)$
$= 32$

c) An integer raised to a negative exponent is equivalent to the reciprocal of the power with a positive exponent.

$5^{-1} = \frac{1}{5^1} \text{ or } \frac{1}{5}$

d) When an exponent is zero, the value of the power is 1. $10^0 = 1$

e) There are 2 factors of 100.

$100^2 = (100)(100)$
$= 10\ 000$

f) An integer raised to a negative exponent is equivalent to the reciprocal of the power with a positive exponent.

$2^{-3} = \frac{1}{2^3}$
$= \frac{1}{(2)(2)(2)}$
$= \frac{1}{8}$

2. a) There are 2 factors of -3.

$(-3)^2 = (-3)(-3)$
$= 9$

b) There are 3 factors of -3.

$(-3)^3 = (-3)(-3)(-3)$
$= -27$

c) There are 2 factors of 4. The negative sign is not inside brackets, so the entire power is negative.

$-4^2 = -(4)(4)$
$= -16$

d) There are 2 factors of -4.

$(-4)^2 = (-4)(-4)$
$= 16$

e) There are 3 factors of -5.

$(-5)^3 = (-5)(-5)(-5)$
$= -125$

f) There are 3 factors of 5. The negative sign is not inside brackets, so the entire power is negative.

$-5^3 = -(5)(5)(5)$
$= -125$

3. $(-5)^{120}$ will result in a positive answer since the exponent is an even number.

4. a) To raise a power to a power, keep the base and multiply the exponents.

$(3^2)^2 = 3^{2\times2}$
$= 3^4$
$= (3)(3)(3)(3)$
$= 81$

b) To raise a power to a power, keep the base and multiply the exponents.

$(7^2)^4 = 7^{2\times4}$
$= 7^8$
$= (7)(7)(7)(7)(7)(7)(7)(7)$
$= 5\ 764\ 801$

c) To raise a power to a power, keep the base and multiply the exponents.

$[(-4)^2]^3 = (-4)^{2\times3}$
$= (-4)^6$
$= (-4)(-4)(-4)(-4)(-4)(-4)$
$= 4096$

d) $[-(10^2)]^3$ is equivalent to $[(-1)(10^2)]^3$. Raise each factor to the third power. To raise a power to a power, keep the base and multiply the exponents.

$[-(10^2)]^3 = [(-1)(10^2)]^3$
$= (-1)^3(10^2)^3$
$= (-1)(10^{2\times3})$
$= -10^6$
$= -(10)(10)(10)(10)(10)(10)$
$= -1\ 000\ 000$