b) To find the maximum profit, locate the x-coordinate of the vertex by averaging the two zeros.

$$\frac{(8.84 + (2.54))}{2}$$
$$= \frac{6.3}{2}$$
$$= 3.15$$

So, there must be 3.15 \$50 increases. Because there cannot be a 0.15 \$50 increase, round down to 3. There must be 3 \$50 increases. This is \$150.

\$550 + \$150 = \$700

The cost of the trip that will maximize the profit is \$700.

CHAPTER 4: Exponential Functions

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 212

1. a) There are 2 factors of 7.

$7^2 = (7)(7)$
$= 49$

b) There are 5 factors of 2.

$2^5 = (2)(2)(2)(2)(2)$
$= 32$

c) An integer raised to a negative exponent is equivalent to the reciprocal of the power with a positive exponent.

$5^{-1} = \frac{1}{5^1}$ or $\frac{1}{5}$

d) When an exponent is zero, the value of the power is 1. $10^0 = 1$

e) There are 2 factors of 100.

$100^2 = (100)(100)$
$= 10\,000$

f) An integer raised to a negative exponent is equivalent to the reciprocal of the power with a positive exponent.

$$2^{-3} = \frac{1}{2^3}$$
$$= \frac{1}{(2)(2)(2)}$$
$$= \frac{1}{8}$$

2. a) There are 2 factors of -3.

$(-3)^2 = (-3)(-3)$
$= 9$

b) There are 3 factors of -3.

$(-3)^3 = (-3)(-3)(-3)$
$= -27$

c) There are 2 factors of 4. The negative sign is not inside brackets, so the entire power is negative.

$-4^2 = -(4)(4)$
$= -16$

d) There are 2 factors of -4.

$(-4)^2 = (-4)(-4)$
$= 16$

e) There are 3 factors of -5.

$(-5)^3 = (-5)(-5)(-5)$
$= -125$

f) There are 3 factors of 5. The negative sign is not inside brackets, so the entire power is negative.

$-5^3 = -(5)(5)(5)$
$= -125$

3. $(-5)^{120}$ will result in a positive answer since the exponent is an even number.

4. a) To raise a power to a power, keep the base and multiply the exponents.

$(3^2)^2 = 3^{2\times2}$
$= 3^4$
$= (3)(3)(3)(3)$
$= 81$

b) To raise a power to a power, keep the base and multiply the exponents.

$(7^2)^4 = 7^{2\times4}$
$= 7^8$
$= (7)(7)(7)(7)(7)(7)(7)(7)$
$= 5\,764\,801$

c) To raise a power to a power, keep the base and multiply the exponents.

$[(-4)^2]^3 = (-4)^{2\times3}$
$= (-4)^6$
$= (-4)(-4)(-4)(-4)(-4)(-4)$
$= 4096$

d) $[-(10^2)]^3$ is equivalent to $[(-1)(10^2)]^3$. Raise each factor to the third power. To raise a power to a power, keep the base and multiply the exponents.

$[-(10^2)]^3 = [(-1)(10^2)]^3$
$= (-1)^3(10^2)^3$
$= (-1)(10^{2\times3})$
$= -10^6$
$= -(10)(10)(10)(10)(10)(10)$
$= -1\,000\,000$

e) To raise a power to a power, keep the base and multiply the exponents.

$$\begin{aligned}[(2^2)^2]^2 &= (2^{2\times2})^2\\ &= (2^4)^2\\ &= 2^{4\times2}\\ &= 2^8\\ &= (2)(2)(2)(2)(2)(2)(2)(2)\\ &= 256\end{aligned}$$

f) To raise a power to a power, keep the base and multiply the exponents. When an exponent is zero, the value of the power is 1. The negative sign is not inside the brackets, so the entire power is negative.

$$\begin{aligned}-[(2^2)^2]^0 &= -(2^{2\times2})^0\\ &= -(2^4)^0\\ &= -(2)^{4\times0}\\ &= -(2)^0\\ &= -1\end{aligned}$$

5. a) 49 is a perfect square.

$$\begin{aligned}(\sqrt{49})^2 &= (7)^2\\ &= 49\end{aligned}$$

b) 64 is a perfect square.

$$\begin{aligned}3\sqrt{64} &= 3(8)\\ &= 24\end{aligned}$$

c) 4 and 16 are perfect squares.

$$\begin{aligned}\sqrt{4}\sqrt{16} &= (2)(4)\\ &= 8\end{aligned}$$

d) 9 and 81 are perfect squares.

$$\frac{\sqrt{9}}{\sqrt{81}} = \frac{3}{9} \text{ or } \frac{1}{3}$$

6. a) To add the fractions, first find a common denominator. A common denominator of $\frac{5}{8}$ and $\frac{5}{3}$ is 24. Write the fractions using the comon denominator. Then add the numerators and place the sum over the denominator.

$$\begin{aligned}\frac{5}{8}+\frac{5}{3} &= \frac{15}{24}+\frac{40}{24}\\ &= \frac{15+40}{24}\\ &= \frac{55}{24}\end{aligned}$$

b) To subtract the fractions, first find a common denominator. A common denominator of $\frac{5}{8}$ and $\frac{5}{3}$ is 24. Write the fractions using the common denominator. Then subtract the numerators and place the difference over the denominator.

$$\begin{aligned}\frac{5}{8}-\frac{5}{3} &= \frac{15}{24}-\frac{40}{24}\\ &= \frac{15-40}{24}\\ &= \frac{-25}{24} \text{ or } -\frac{25}{24}\end{aligned}$$

c) To divide by a fraction, multiply by the reciprocal.

$$\begin{aligned}\frac{7}{8}\div\frac{2}{3} &= \frac{7}{8}\times\frac{3}{2}\\ &= \frac{(7)(3)}{(8)(2)}\\ &= \frac{21}{16}\end{aligned}$$

d) First, multiply the fractions. Factor the numerators and denominators by dividing out their GCF. Next, find a common denominator and do the subtraction.

$$\begin{aligned}\frac{1}{5}-\frac{3}{8}\left(\frac{4}{3}\right) &= \frac{1}{5}-\frac{\cancel{3}^{1}}{\cancel{8}_{2}}\left(\frac{\cancel{4}^{1}}{\cancel{3}_{1}}\right)\\ &= \frac{1}{5}-\frac{1}{2}\\ &= \frac{2}{10}-\frac{5}{10}\\ &= -\frac{3}{10}\end{aligned}$$

e) First, divide the fractions by multiplying by the reciprocal. Factor the numerators and denominators by dividing out their GCF. Next, do the addition.

$$\begin{aligned}-\frac{4}{3}+\left(\frac{9}{10}\div\frac{5}{12}\right) &= -\frac{4}{3}+\left(\frac{9}{10}\times\frac{12}{5}\right)\\ &= -\frac{4}{3}+\left(\frac{9}{\cancel{10}_{5}}\times\frac{\cancel{12}^{6}}{5}\right)\\ &= -\frac{4}{3}+\frac{54}{25}\\ &= -\frac{100}{75}+\frac{162}{75}\\ &= \frac{62}{75}\end{aligned}$$

f) First, add the fractions by writing them with a common denominator. A common denominator of $\frac{3}{8}$ and $\frac{7}{3}$ is 24. Next, do the multiplication. Factor the numerators and denominators by dividing out their GCF.

$$-\frac{9}{10}\left(\frac{3}{8}+\frac{7}{3}\right) = -\frac{9}{10}\left(\frac{9}{24}+\frac{56}{24}\right)$$
$$= -\frac{9}{10}\left(\frac{65}{24}\right)$$
$$= -\frac{\cancel{9}^{3}}{\cancel{10}_{2}}\left(\frac{\cancel{65}^{13}}{\cancel{24}_{8}}\right)$$
$$= -\frac{39}{16}$$

7. a) To multiply powers with the same base, keep the base the same and add exponents.

$$a^2(a^5) = a^{2+5}$$
$$= a^7$$

b) To divide powers with the same base, keep the base the same and subtract exponents.

$$b^{12} \div b^8 = b^{12-8}$$
$$= b^4$$

c) To raise a power to a power, keep the base and multiply the exponents.

$$(c^3)^4 = c^{3\times 4}$$
$$= c^{12}$$

d) To multiply powers with the same base, keep the base the same and add exponents.

$$d(d^6)d^3 = d^{1+6+3}$$
$$= d^{10}$$

8. a) $9^x = 81$

$9^2 = 81$

$9^x = 9^2$, so $x = 2$.

b) $8^m = 256$

$(2^3)^m = 256$

$2^{3m} = 256$

$2^8 = 256$

$2^{3m} = 2^8$, so $3m = 8$, or $m = \frac{8}{3}$.

c) $(-5)^a = -125$

$(-5)^3 = -125$

$(-5)^a = (-5)^3$, so $a = 3$.

d) $-10^r = -100\,000\,000$

$-10^8 = -100\,000\,000$

$-10^r = -10^8$, so $r = 8$.

9. a) Substitute 2.5 for r and 4.8 for h in the formula.

$$V = \pi r^2 h$$
$$= \pi(2.5)^2(4.8)$$
$$= \pi(6.25)(4.8)$$
$$\doteq 94.248 \text{ cm}^3$$

b) Substitute 2.5 for r in the formula.

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(2.5)^3$$
$$= \frac{4}{3}\pi(15.625)$$
$$\doteq 65.45 \text{ cm}^3$$

10. a) To find the first differences, subtract the consecutive y-values.

x	y	First Differences
−4	12	
		−5
−2	7	
		−5
0	2	
		−5
2	−3	
		−5
4	−8	
		−5
6	−13	

The first differences are all −5. Since the first differences are constant, the data represent a linear function.

b) To find the first differences, subtract the consecutive y-values. To find the second differences, subtract the consecutive first differences.

x	y	First differences	Second differences
−3	9		
		1	1
−2	10		
		2	1
−1	12		
		3	1
0	15		
		4	1
1	19		
		5	1
2	24		

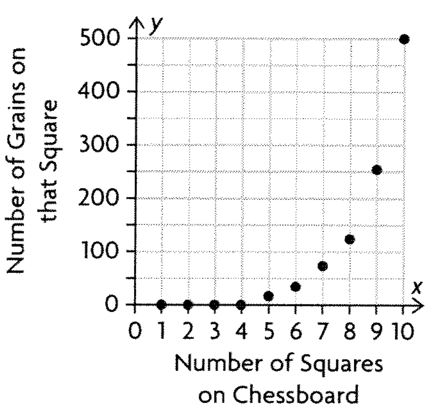
The first differences are 1, 2, 3, 4, 5; the second differences are all 1. Since the first differences are not constant, the function is nonlinear. The second differences are all equal. So the data represent a quadratic function.

4.1 Exploring Growth and Decay, p. 216

1. a) Both graphs decrease rapidly at the beginning, then continue to decrease less rapidly before levelling off.
b) At the start of the experiment, Time = 0. At this time, the temperature is 85°C.
c) At the end of the experiment, the cocoa had cooled to room temperature. The temperature of the classroom was 20°C.
2. a)

Number of Squares on the Chessboard	Number of Grains on that Square	First Differences
1	1	
2	2	1
3	4	2
4	8	4
5	16	8
6	32	16
7	64	32
8	128	64
9	256	128
10	512	256

b)



c) They are similar in their shape; that is both decrease rapidly at the beginning and then level off. They are different in that they have different y-intercepts and asymptotes.

4.2 Working with Integer Exponents, pp. 221–223

1. a) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$5^{-4} = \frac{1}{5^4}$$

b) A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\left(-\frac{1}{10}\right)^{-3} = \left(-\frac{10}{1}\right)^3$$
$$= (-10)^3$$

c) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{1}{2^{-4}} = \frac{1}{\frac{1}{2^4}}$$
$$= 1 \times \frac{2^4}{1}$$
$$= 2^4$$

d) A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. The negative sign is not inside the brackets, so the entire power is negative.

$$-\left(\frac{6}{5}\right)^{-3} = -\left(\frac{5}{6}\right)^3$$

e) A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\left(\frac{3}{11}\right)^{-1} = \left(\frac{11}{3}\right)^1$$
$$= \frac{11}{3}$$

f) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{7^{-2}}{8^{-1}} = \frac{\frac{1}{7^2}}{\frac{1}{8^1}}$$
$$= \frac{1}{7^2} \times \frac{8^1}{1}$$
$$= \frac{8}{7^2}$$

2. a) To multiply powers with the same base, keep the base the same and add exponents. When an exponent is zero, the value of the power is 1.

$$(-10)^8(-10)^{-8} = (-10)^{8+(-8)}$$
$$= (-10)^0$$
$$= 1$$

b) To multiply powers with the same base, keep the base the same and add exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$6^{-7} \times 6^5 = 6^{-7+5}$$
$$= 6^{-2}$$
$$= \frac{1}{6^2}$$

c) To divide powers with the same base, keep the base the same and subtract exponents.

$$\frac{2^8}{2^{-5}} = 2^{8-(-5)}$$
$$= 2^{13}$$

d) To divide powers with the same base, keep the base the same and subtract exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{11^{-3}}{11^5} = 11^{-3-5}$$
$$= 11^{-8}$$
$$= \frac{1}{11^8}$$

e) To raise a power to a power, keep the base and multiply the exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$(-9^4)^{-1} = -9^{4\times(-1)}$$
$$= -9^{-4}$$
$$= -\frac{1}{9^4}$$

f) To raise a power to a power, keep the base and multiply the exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$[(7^{-3})^{-2}]^{-2} = (7^{-3\times-2)})^{-2}$$
$$= (7^6)^{-2}$$
$$= 7^{6\times(-2)}$$
$$= 7^{-12}$$
$$= \frac{1}{7^{12}}$$

3. $2^{-5} = \frac{1}{2^5}$ and $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5$ or 2^5. Since $\frac{1}{2^5}$ is less than 2^5, $\left(\frac{1}{2}\right)^{-5}$ is the greater power.

4. a) To multiply powers with the same base, keep the base the same and add exponents.

$$2^{-3}(2^7) = 2^{-3+7}$$
$$= 2^4$$
$$= 16$$

b) To multiply powers with the same base, keep the base the same and add exponents. When an exponent is zero, the value of the power is 1.

$$(-8)^3(-8)^{-3} = (-8)^{3+(-3)}$$
$$= (-8)^0$$
$$= 1$$

c) To divide powers with the same base, keep the base the same and subtract exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{5^4}{5^6} = 5^{4-6}$$
$$= 5^{-2}$$
$$= \frac{1}{5^2}$$
$$= \frac{1}{25}$$

d) To divide powers with the same base, keep the base the same and subtract exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{3^{-8}}{3^{-6}} = 3^{-8-(-6)}$$
$$= 3^{-2}$$
$$= \frac{1}{3^2}$$
$$= \frac{1}{9}$$

e) To raise a power to a power, keep the base and multiply the exponents.

$$\begin{aligned}(4^{-3})^{-1} &= 4^{-3\times(-1)}\\ &= 4^3\\ &= 64\end{aligned}$$

f) To raise a power to a power, keep the base and multiply the exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(7^{-1})^2 &= 7^{-1\times2}\\ &= 7^{-2}\\ &= \frac{1}{7^2}\\ &= \frac{1}{49}\end{aligned}$$

5. a) First find the power raised to a power by multiplying exponents. Then multiply the powers with the same base by adding exponents.

$$\begin{aligned}3^3(3^2)^{-1} &= 3^3(3^{2\times(-1)})\\ &= 3^3(3^{-2})\\ &= 3^{3+(-2)}\\ &= 3^1\\ &= 3\end{aligned}$$

b) First multiply the powers with the same base by adding exponents. Then find the power raised to a power by multiplying exponents. When an exponent is zero, the value of the power is 1.

$$\begin{aligned}(9\times9^{-1})^{-2} &= (9^{1+(-1)})^{-2}\\ &= (9^0)^{-2}\\ &= 9^{0\times(-2)}\\ &= 9^0\\ &= 1\end{aligned}$$

c) First simplify the numerator by multiplying exponents. Then simplify the resulting fraction.

$$\begin{aligned}\frac{(12^{-1})^3}{12^{-3}} &= \frac{12^{-1\times3}}{12^{-3}}\\ &= \frac{12^{-3}}{12^{-3}}\\ &= 12^{-3-(-3)}\\ &= 12^0\\ &= 1\end{aligned}$$

d) First simplify the numerator by multiplying exponents. Then simplify the resulting fraction.

$$\begin{aligned}\frac{(5^3)^{-2}}{5^{-6}} &= \frac{5^{3\times(-2)}}{5^{-6}}\\ &= \frac{5^{-6}}{5^{-6}}\\ &= 5^{-6-(-6)}\\ &= 5^0\\ &= 1\end{aligned}$$

e) First multiply the powers with the same base by adding exponents. Then find the power raised to a power by multiplying exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(3^{-2}(3^3))^{-2} &= (3^{-2+3})^{-2}\\ &= (3^1)^{-2}\\ &= 3^{1\times(-2)}\\ &= 3^{-2}\\ &= \frac{1}{3^2}\\ &= \frac{1}{9}\end{aligned}$$

f) First find the power raised to a power by multiplying exponents. Then multiply the powers with the same base by adding exponents.

$$\begin{aligned}9^7(9^3)^{-2} &= 9^7(9^{3\times(-2)})\\ &= 9^7(9^{-6})\\ &= 9^{7+(-6)}\\ &= 9^1\\ &= 9\end{aligned}$$

6. a) First multiply the powers in the brackets by adding exponents. Then multiply by 10 by adding exponents.

$$\begin{aligned}10(10^4(10^{-2})) &= 10(10^{4+(-2)})\\ &= 10(10^2)\\ &= 10^{1+2}\\ &= 10^3\\ &= 1000\end{aligned}$$

b) Multiply the powers with the same base by adding exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}8(8^2)(8^{-4}) &= 8^{1+2+(-4)}\\ &= 8^{-1}\\ &= \frac{1}{8}\end{aligned}$$

c) First simplify the denominator by multiplying exponents. Then divide the numerator by the denominator by subtracting exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{6^{-5}}{(6^2)^{-2}} = \frac{6^{-5}}{6^{2\times(-2)}}$$
$$= \frac{6^{-5}}{6^{-4}}$$
$$= 6^{-5-(-4)}$$
$$= 6^{-1}$$
$$= \frac{1}{6}$$

d) First simplify the denominator by multiplying exponents. Then divide the numerator by the denominator by subtracting exponents.

$$\frac{4^{-10}}{(4^{-4})^3} = \frac{4^{-10}}{4^{-4\times3}}$$
$$= \frac{4^{-10}}{4^{-12}}$$
$$= 4^{-10-(-12)}$$
$$= 4^2$$
$$= 16$$

e) First simplify the fraction by subtracting exponents. Then multiply the powers with the same base by adding exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$2^8 \times \left(\frac{2^{-5}}{2^6}\right) = 2^8 \times 2^{-5-6}$$
$$= 2^8 \times 2^{-11}$$
$$= 2^{8+(-11)}$$
$$= 2^{-3}$$
$$= \frac{1}{2^3}$$
$$= \frac{1}{8}$$

f) First simplify the fraction by subtracting exponents. Then find the power raised to a power by multiplying exponents. Finally, multiply the powers with the same base by adding exponents.

$$13^{-5} \times \left(\frac{13^2}{13^8}\right)^{-1} = 13^{-5} \times (13^{2-8})^{-1}$$
$$= 13^{-5} \times (13^{-6})^{-1}$$
$$= 13^{-5} \times 13^{-6\times(-1)}$$
$$= 13^{-5} \times 13^6$$
$$= 13^{-5+6}$$
$$= 13^1 \text{ or } 13$$

7. a) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$16^{-1} - 2^{-2} = \frac{1}{16^1} - \frac{1}{2^2}$$
$$= \frac{1}{16} - \frac{1}{4}$$
$$= \frac{1}{16} - \frac{4}{16}$$
$$= -\frac{3}{16}$$

b) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. When an exponent is zero, the value of the power is 1.

$$(-3)^{-1} + 4^0 - 6^{-1} = \frac{1}{(-3)^1} + 1 - \frac{1}{6^1}$$
$$= -\frac{1}{3} + 1 - \frac{1}{6}$$
$$= -\frac{2}{6} + \frac{6}{6} - \frac{1}{6}$$
$$= \frac{3}{6} \text{ or } \frac{1}{2}$$

c) A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\left(-\frac{2}{3}\right)^{-1} + \left(\frac{2}{5}\right)^{-1} = \left(-\frac{3}{2}\right)^1 + \left(\frac{5}{2}\right)^1$$
$$= -\frac{3}{2} + \frac{5}{2}$$
$$= \frac{2}{2} \text{ or } 1$$

d) A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\left(\frac{1}{5}\right)^{-1} + \left(-\frac{1}{2}\right)^{-2} = \left(\frac{5}{1}\right)^1 + \left(-\frac{2}{1}\right)^2$$
$$= 5 + 4$$
$$= 9$$

e) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$5^{-3} + 10^{-3} - 8(1000^{-1}) = \frac{1}{5^3} + \frac{1}{10^3}$$
$$-8\left(\frac{1}{1000^1}\right)$$

$$= \frac{1}{125} + \frac{1}{1000} - \frac{8}{1000}$$
$$= \frac{8}{1000} + \frac{1}{1000} - \frac{8}{1000}$$
$$= \frac{1}{1000}$$

f) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$3^{-2} - 6^{-2} + \frac{3}{2}(-9)^{-1} = \frac{1}{3^2} - \frac{1}{6^2} + \frac{3}{2}\left(\frac{1}{(-9)^1}\right)$$
$$= \frac{1}{9} - \frac{1}{36} + \frac{3}{2}\left(-\frac{1}{9}\right)$$
$$= \frac{1}{9} - \frac{1}{36} - \frac{3}{18}$$
$$= \frac{4}{36} - \frac{1}{36} - \frac{6}{36}$$
$$= -\frac{3}{36} \text{ or } -\frac{1}{12}$$

8. a) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$5^2(-10)^{-4} = 5^2\left(-\frac{1}{10}\right)^4$$
$$= 25\left(\frac{1}{10\ 000}\right)$$
$$= \frac{1}{400}$$

b) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$16^{-1}(2)^5 = \frac{1}{16^1}(2)^5$$
$$= \frac{1}{16}(32)$$
$$= 2$$

c) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{12^{-1}}{(-4)^{-1}} = \frac{\frac{1}{12^1}}{\frac{1}{(-4)^1}}$$
$$= \frac{1}{12^1} \times \frac{(-4)^1}{1}$$
$$= -\frac{1}{3}$$

d) First simplify the denominator by multiplying exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\frac{(-9)^{-2}}{(3^{-1})^2} = \frac{(-9)^{-2}}{3^{-1\times 2}}$$
$$= \frac{(-9)^{-2}}{3^{-2}}$$
$$= \frac{\frac{1}{(-9)^2}}{\frac{1}{3^2}}$$
$$= \frac{1}{(-9)^2} \times \frac{3^2}{1}$$
$$= \frac{1}{81} \times 9$$
$$= \frac{1}{9}$$

e) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$(8^{-1})\left(\frac{2^{-3}}{4^{-1}}\right) = \left(\frac{1}{8^1}\right)\frac{\frac{1}{2^3}}{\frac{1}{4^1}}$$
$$= \left(\frac{1}{8^1}\right)\left(\frac{1}{2^3} \times \frac{4^1}{1}\right)$$
$$= \left(\frac{1}{8}\right)\left(\frac{1}{8}\right)(4)$$
$$= \frac{1}{16}$$

f) To divide powers with the same base, keep the base the same and subtract exponents. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}\frac{(-5)^3(-25)^{-1}}{(-5)^{-2}} &= (-5)^{3-(-2)}(-25)^{-1}\\ &= (-5)^5(-25)^{-1}\\ &= (-5)^5\left(\frac{1}{-25^1}\right)\\ &= \frac{-3125}{-25}\\ &= 125\end{aligned}$$

9. a) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(-4)^{-3} &= \frac{1}{(-4)^3}\\ &= \frac{1}{-64} \text{ or } -\frac{1}{64}\end{aligned}$$

b) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(-4)^{-2} &= \frac{1}{(-4)^2}\\ &= \frac{1}{16}\end{aligned}$$

c) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. The negative sign is not inside brackets, so the entire power is negative.

$$\begin{aligned}-(5)^{-3} &= -\frac{1}{5^3}\\ &= -\frac{1}{125}\end{aligned}$$

d) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. The negative sign is not inside brackets, so the entire power is negative.

$$\begin{aligned}-(5)^{-2} &= -\frac{1}{5^2}\\ &= -\frac{1}{25}\end{aligned}$$

e) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(-6)^{-3} &= \frac{1}{(-6)^3}\\ &= \frac{1}{-216} \text{ or } -\frac{1}{216}\end{aligned}$$

f) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. The negative sign is not inside brackets, so the entire power is negative.

$$\begin{aligned}-(6)^{-2} &= -\frac{1}{6^2}\\ &= -\frac{1}{36}\end{aligned}$$

10. $5^{-2} = \frac{1}{25}$, $10^{-1} = \frac{1}{10}$, $3^{-2} = \frac{1}{9}$, $2^{-3} = \frac{1}{8}$, $4^{-1} = \frac{1}{4}$, and $(0.1)^{-1} = \frac{1}{0.1}$. If the numerators of the numbers are all the same (1), then the larger the denominator, the smaller the number. Since $25 > 10 > 9 > 8 > 4 > 0.1$, the numbers in order from least to greatest are 5^{-2}, 10^{-1}, 3^{-2}, 2^{-3}, 4^{-1}, $(0.1)^{-1}$.

11. a) In the expression, substitute -2 for x, 3 for y, and -1 for n. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(x^n + y^n)^{-2n} &= [(-2)^{-1} + 3^{-1}]^{-2(-1)}\\ &= [(-2)^{-1} + 3^{-1}]^2\\ &= \left(\frac{1}{(-2)^1} + \frac{1}{3^1}\right)^2\\ &= \left(-\frac{1}{2} + \frac{1}{3}\right)^2\\ &= \left(-\frac{3}{6} + \frac{2}{6}\right)^2\\ &= \left(\frac{1}{6}\right)^2\\ &= \frac{1}{36}\end{aligned}$$

b) In the expression, substitute -2 for x, 3 for y, and -1 for n. Find the power raised to a power by multiplying exponents. To multiply powers with the same base, add exponents.

$$\begin{aligned}(x^2)^n(y^{-2n})x^{-n} &= (-2^2)^{-1}(3^{-2(-1)})(-2)^{-(-1)}\\ &= (-2^{2\times(-1)})(3^2)(-2)^1\\ &= (-2^{-2})(9)(-2)^1\\ &= (-2^{-2+1})(9)\\ &= (-2^{-1})(9)\\ &= \frac{1}{(-2)^1}(9)\\ &= -\frac{9}{2}\end{aligned}$$

c) In the expression, substitute -2 for x, 3 for y, and -1 for n. A fractional base or an integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\begin{aligned}\left(\frac{x^n}{y^n}\right)^n &= \left(\frac{(-2)^{-1}}{3^{-1}}\right)^{-1}\\ &= \left(\frac{3^{-1}}{(-2)^{-1}}\right)^1\\ &= \frac{\frac{1}{3^1}}{\frac{1}{(-2)^1}}\\ &= \frac{1}{3}\times\frac{-2}{1}\\ &= -\frac{2}{3}\end{aligned}$$

d) In the expression, substitute -2 for x, 3 for y, and -1 for n. A fractional base or an integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\begin{aligned}\left(\frac{xy^n}{(xy)^{2n}}\right)^{2n} &= \left(\frac{(-2)(3)^{-1}}{(-2\times 3)^{2(-1)}}\right)^{2(-1)}\\ &= \left(\frac{(-2)(3)^{-1}}{(-6)^{-2}}\right)^{-2}\\ &= \left(\frac{(-6)^{-2}}{(-2)(3)^{-1}}\right)^{2}\\ &= \left(\frac{(3)^1}{(-2)(-6)^2}\right)^{2}\\ &= \left(\frac{3}{72}\right)^2\\ &= \frac{1}{576}\end{aligned}$$

12. a) Erik: $3^{-1} \neq -\frac{1}{3}$ (negative exponents do not make numbers negative)
Vinn: $3 = 3^1$ and he did not add the exponents correctly.

b) Correct solution:

$$\begin{aligned}&3^{-2}\times 3\\ &= \frac{1}{3^2}\times 3\\ &= \frac{1}{9}\times 3\\ &= \frac{3}{9}\\ &= \frac{1}{3}\end{aligned}$$

13. a) Multiply and divide from left to right. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}2^3\times 4^{-2}\div 2^2 &= 8\times\frac{1}{4^2}\div 2^2\\ &= \frac{8}{16}\div 4\\ &= \frac{1}{8}\end{aligned}$$

b) First simplify inside the brackets. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\begin{aligned}(2\times 3)^{-1} &= 6^{-1}\\ &= \frac{1}{6^1}\text{ or }\frac{1}{6}\end{aligned}$$

c) A fractional base or an integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. Dividing by a fraction is the same as multiplying by its reciprocal.

$$\begin{aligned}\left(\frac{3^{-1}}{2^{-1}}\right)^{-2} &= \left(\frac{2^{-1}}{3^{-1}}\right)^2\\ &= \left(\frac{\frac{1}{2^1}}{\frac{1}{3^1}}\right)^2\\ &= \left(\frac{1}{2}\times\frac{3}{1}\right)^2\end{aligned}$$

$$= \left(\frac{3}{2}\right)^2$$

$$= \frac{9}{4}$$

d) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. When an exponent is zero, the value of the power is 1.

$$4^{-1}(4^2 + 4^0) = \frac{1}{4^1}(16 + 1)$$
$$= \frac{1}{4}(17)$$
$$= \frac{17}{4}$$

e) To divide powers with the same base, keep the base the same and subtract exponents.

$$\frac{2^5}{3^{-2}} \times \frac{3^{-1}}{2^4} = \frac{2^5}{2^4} \times \frac{3^{-1}}{3^{-2}}$$
$$= 2^{5-4} \times 3^{-1-(-2)}$$
$$= 2^1 \times 3^1$$
$$= 6$$

f) When an exponent is zero, the value of the power is 1. An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$(5^0 + 5^2)^{-1} = (1 + 25)^{-1}$$
$$= 26^{-1}$$
$$= \frac{1}{26^1} \text{ or } \frac{1}{26}$$

g) To divide powers with the same base, keep the base the same and subtract exponents.

$$\frac{3^{-2} \times 2^{-3}}{3^{-1} \times 2^{-2}} = 3^{-2-(-1)} \times 2^{-3-(-2)}$$
$$= 3^{-1} \times 2^{-1}$$
$$= \frac{1}{3} \times \frac{1}{2}$$
$$= \frac{1}{6}$$

h) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{4^{-2} + 3^{-1}}{3^{-2} + 2^{-3}} = \frac{\frac{1}{4^2} + \frac{1}{3^1}}{\frac{1}{3^2} + \frac{1}{2^3}}$$
$$= \frac{\frac{1}{16} + \frac{1}{3}}{\frac{1}{9} + \frac{1}{8}}$$
$$= \frac{\frac{19}{48}}{\frac{17}{72}}$$
$$= \frac{19}{48} \times \frac{72}{17}$$
$$= \frac{57}{34}$$

i) An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

$$\frac{5^{-1} - 2^{-2}}{5^{-1} + 2^{-2}} = \frac{\frac{1}{5^1} - \frac{1}{2^2}}{\frac{1}{5^1} + \frac{1}{2^2}}$$
$$= \frac{\frac{1}{5} - \frac{1}{4}}{\frac{1}{5} + \frac{1}{4}}$$
$$= \frac{-\frac{1}{20}}{\frac{9}{20}}$$
$$= -\frac{1}{20} \times \frac{20}{9}$$
$$= -\frac{1}{9}$$

14. a) In the expression, substitute 1 for a, and 2 for c.

$$ac^c = (1)(2^{2)}$$
$$= (1)(4)$$
$$= 4$$

b) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$a^cb^c = 1^2(3^2)$$
$$= 1(9)$$
$$= 9$$

c) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}(ab)^{-c} &= [(1)(3)]^{-2}\\ &= 3^{-2}\\ &= \frac{1}{3^2}\\ &= \frac{1}{9}\end{aligned}$$

d) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}(b \div c)^{-a} &= (3 \div 2)^{-1}\\ &= \left(\frac{3}{2}\right)^{-1}\\ &= \left(\frac{2}{3}\right)^{1} \text{ or } \frac{2}{3}\end{aligned}$$

e) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}(-a \div b)^{-c} &= (-1 \div 3)^{-2}\\ &= \left(\frac{-1}{3}\right)^{-2}\\ &= \left(\frac{3}{-1}\right)^{2}\\ &= 9\end{aligned}$$

f) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}(a^{-1}b^{-2})^{c} &= (1^{-1} \times 3^{-2})^2\\ &= \left(\frac{1}{1^1} \times \frac{1}{3^2}\right)^2\\ &= \left(1 \times \frac{1}{9}\right)^2\\ &= \frac{1}{81}\end{aligned}$$

g) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}(a^b b^a)^c &= (1^3 \times 3^1)^2\\ &= (1 \times 3)^2\\ &= 3^2\\ &= 9\end{aligned}$$

h) In the expression, substitute 1 for a, 3 for b, and 2 for c.

$$\begin{aligned}[(b)^{-a}]^{-c} &= [(3)^{-1}]^{-2}\\ &= 3^{-1\times(-2)}\\ &= 3^2\\ &= 9\end{aligned}$$

15. a) $(-10)^3$ is multiplied by itself three times. 10^{-3} is the reciprocal of 10 cubed.

b) $(-10)^4$ is multiplied by itself four times. -10^4 is the negative of 10^4.

16. a) $16^{-1} = \frac{1}{16}$, so $x = -1$.

b)

$$\begin{aligned}0.01 &= \frac{1}{100}\\ &= \frac{1}{10^2}\\ &= 10^{-2}\end{aligned}$$

$10^{-2} = 0.01$, so $x = -2$.

c) When an exponent is zero, the value of the power is 1.

$2^0 = 1$, so $x = 0$.

d)

$$\begin{aligned}0.25 &= 0.5^2\\ &= \left(\frac{1}{2}\right)^2\\ &= 2^{-2}\end{aligned}$$

$2^{-2} = 0.25$, so $n = -2$.

e)

$$\begin{aligned}\frac{1}{625} &= \frac{1}{25^2}\\ &= 25^{-2}\end{aligned}$$

$25^{-2} = \frac{1}{625}$, so $n = -2$.

f)

$$\begin{aligned}\frac{1}{144} &= \frac{1}{12^2}\\ &= 12^{-2}\end{aligned}$$

$12^{-2} = \frac{1}{144}$, so $n = -2$.

17. Take the square root of each side of the equation.

$$\begin{aligned}10^{2y} &= 25\\ (10^y)^2 &= 25\\ \sqrt{(10^y)^2} &= \sqrt{25}\\ 10^y &= 5\\ \frac{1}{10^y} &= \frac{1}{5}\\ 10^{-y} &= \frac{1}{5}\end{aligned}$$

18. a) To find a power raised to a power, multiply exponents.

$$\begin{aligned}(x^2)^{5-r} &= x^{2(5-r)}\\ &= x^{10-2r}\end{aligned}$$

b) To divide powers with the same base, keep the base the same and subtract exponents.

$$\begin{aligned}(b^{2m+3m}) \div (b^{m-n}) &= (b^{5m}) \div (b^{m-n})\\ &= b^{5m-(m-n)}\\ &= b^{4m+n}\end{aligned}$$

c) To divide powers with the same base, keep the base the same and subtract exponents.

$$(b^{2m+3n}) \div (b^{m-n}) = b^{2m+3n-(m-n)}$$
$$= b^{m+4n}$$

d) To multiply powers with the same base, keep the base the same and add exponents.

$$x^{3(7-r)}x^{r} = x^{3(7-r)+r}$$
$$= x^{21-3r+r}$$
$$= x^{21-2r}$$

e) To divide powers with the same base, keep the base the same and subtract exponents.

$$(a^{10-p})\left(\frac{1}{a}\right)^{p} = (a^{10-p})\left(\frac{1}{a^{p}}\right)$$
$$= a^{10-p-p}$$
$$= a^{10-2p}$$

f) To find a power raised to a power, multiply exponents. To divide powers with the same base, keep the base the same and subtract exponents.

$$[(3x^{4})^{6-m}]\left(\frac{1}{x}\right)^{m} = \left(3^{1\times(6-m)}x^{4\times(6-m)}\left(\frac{1}{x^{m}}\right)\right)$$
$$= 3^{6-m}x^{24-4m}\left(\frac{1}{x^{m}}\right)$$
$$= 3^{6-m}x^{24-4m-m}$$
$$= 3^{6-m}x^{24-5m}$$

4.3 Working with Rational Exponents, pp. 229–230

1. a) An exponent of $\frac{1}{2}$ means square root.

$$49^{\frac{1}{2}} = \sqrt{49}$$
$$= 7$$

b) An exponent of $\frac{1}{2}$ means square root.

$$100^{\frac{1}{2}} = \sqrt{100}$$
$$= 10$$

c) An exponent of $\frac{1}{3}$ means cube root.

$$(-125)^{\frac{1}{3}} = \sqrt[3]{-125}$$
$$= -5$$

d) Change the exponent to its equivalent fraction. An exponent of $\frac{1}{4}$ means fourth root.

$$16^{0.25} = 16^{\frac{1}{4}}$$
$$= \sqrt[4]{16}$$
$$= 2$$

e) An exponent of $\frac{1}{4}$ means fourth root.

$$81^{\frac{1}{4}} = \sqrt[4]{81}$$
$$= 3$$

f) Change the exponent to its equivalent fraction. An exponent of $\frac{1}{2}$ means square root.

$$-(144)^{0.5} = -(144)^{\frac{1}{2}}$$
$$= -\sqrt{144}$$
$$= -12$$

2. a) The radical is the ninth root of 512. An exponent of $\frac{1}{9}$ indicates the ninth root.

$$\sqrt[9]{512} = 512^{\frac{1}{9}}$$
$$= 2$$

b) The radical is the cube root of -27. An exponent of $\frac{1}{3}$ indicates the cube root.

$$\sqrt[3]{-27} = (-27)^{\frac{1}{3}}$$
$$= -3$$

c) The radical is the cube root of 27^2. An exponent of $\frac{1}{3}$ indicates the cube root. Use the power-of-a-power rule to combine exponents.

$$\sqrt[3]{27^2} = (27^2)^{\frac{1}{3}}$$
$$= 27^{\frac{2}{3}}$$

To evaluate, square 27 and then find the cube root of the result.

$$\sqrt[3]{27^2} = \sqrt[3]{729}$$
$$= 9$$

d) The radical is the fifth power of the cube root of -216. An exponent of $\frac{1}{3}$ indicates the cube root. Use the power-of-a-power rule to combine exponents.

$$\left(\sqrt[3]{-216}\right)^{5} = ((-216)^{\frac{1}{3}})^{5}$$
$$= (-216)^{\frac{5}{3}}$$

To evaluate, find the cube root of -216 and then take the fifth power of the result.

$$\left(\sqrt[3]{-216}\right)^{5} = (-6)^{5}$$
$$= -7776$$

e) The radical is the fifth root of $\frac{-32}{243}$. An exponent of $\frac{1}{5}$ indicates the fifth root.

$$\sqrt[5]{\frac{-32}{243}} = \left(\frac{-32}{243}\right)^{\frac{1}{5}}$$
$$= \frac{-2}{3}$$

f) The radical is the fourth root of $\left(\frac{16}{81}\right)^{-1}$. The exponent $\frac{1}{4}$ indicates the fourth root. Use the power-of-a-power rule to combine exponents.

$$\sqrt[4]{\left(\frac{16}{81}\right)^{-1}} = \left(\left(\frac{16}{81}\right)^{-1}\right)^{\frac{1}{4}}$$
$$= \left(\frac{16}{81}\right)^{-\frac{1}{4}}$$

A fractional base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent. To evaluate, write $\frac{16}{81}$ with a positive exponent and then take the fourth root of the result.

$$\sqrt[4]{\left(\frac{16}{81}\right)^{-1}} = \sqrt[4]{\left(\frac{81}{16}\right)^{1}}$$
$$= \sqrt[4]{\frac{81}{16}}$$
$$= \frac{3}{2}$$

3. a) To multiply powers with the same base, keep the base the same and add exponents.

$$8^{\frac{2}{3}}(8^{\frac{1}{3}}) = 8^{\frac{2}{3}+\frac{1}{3}}$$
$$= 8^{\frac{3}{3}}$$
$$= 8^1$$

b) To divide powers with the same base, keep the base the same and subtract exponents.

$$8^{\frac{2}{3}} \div 8^{\frac{1}{3}} = 8^{\frac{2}{3}-\frac{1}{3}}$$
$$= 8^{\frac{1}{3}}$$

c) To multiply powers with the same base, keep the base the same and add exponents.

$$(-11)^2(-11)^{\frac{3}{4}} = (-11)^{2+\frac{3}{4}}$$
$$= (-11)^{\frac{11}{4}}$$

d) To find a power raised to a power, multiply exponents.

$$(7^{\frac{5}{6}})^{-\frac{6}{5}} = 7^{\frac{5}{6}\times-(\frac{6}{5})}$$
$$= 7^{-1}$$

e) To divide powers with the same base, keep the base the same and subtract exponents.

$$\frac{9^{-\frac{1}{5}}}{9^{\frac{2}{3}}} = 9^{-\frac{1}{5}-\frac{2}{3}}$$
$$= 9^{-\frac{13}{15}}$$

f) To multiply powers with the same base, keep the base the same and add exponents. To divide powers with the same base, keep the base the same and subtract exponents.

$$10^{-\frac{4}{5}}(10^{\frac{1}{15}}) \div 10^{\frac{2}{3}} = 10^{-\frac{4}{5}+\frac{1}{15}} \div 10^{\frac{2}{3}}$$
$$= 10^{-\frac{11}{15}} \div 10^{\frac{2}{3}}$$
$$= 10^{-\frac{11}{15}-\frac{2}{3}}$$
$$= 10^{-\frac{7}{5}}$$

4. a) Convert the radicals into exponent form. Then multiply the powers by adding exponents.

$$\sqrt{5}\sqrt{5} = 5^{\frac{1}{2}} \times 5^{\frac{1}{2}}$$
$$= 5^1 \text{ or } 5$$

b) Convert the radicals into exponent form. Then use the quotient of powers rule to write the expression as a single power.

$$\frac{\sqrt[3]{-16}}{\sqrt[3]{2}} = \frac{-16^{\frac{1}{3}}}{2^{\frac{1}{3}}}$$
$$= \left(\frac{-16}{2}\right)^{\frac{1}{3}}$$
$$= (-8)^{\frac{1}{3}}$$
$$= \sqrt[3]{-8}$$
$$= -2$$

c) Convert the radicals into exponent form. Then use the product of powers rule and the quotient of powers rule to write the expression as a single power.

$$\frac{\sqrt{28}\sqrt{4}}{\sqrt{7}} = \frac{28^{\frac{1}{2}}4^{\frac{1}{2}}}{7^{\frac{1}{2}}}$$
$$= \frac{(28 \times 4)^{\frac{1}{2}}}{7^{\frac{1}{2}}}$$
$$= \frac{112^{\frac{1}{2}}}{7^{\frac{1}{2}}}$$
$$= \left(\frac{112}{7}\right)^{\frac{1}{2}}$$
$$= 16^{\frac{1}{2}}$$
$$= \sqrt{16}$$
$$= 4$$

d) Convert the radicals into exponent form. Then use the product of powers rule and the quotient of powers rule to write the expression as a single power.

$$\frac{\sqrt[4]{18}\sqrt[4]{9}}{\sqrt[4]{2}} = \frac{18^{\frac{1}{4}}9^{\frac{1}{4}}}{2^{\frac{1}{4}}}$$
$$= \frac{(18 \times 9)^{\frac{1}{4}}}{2^{\frac{1}{4}}}$$
$$= \frac{162^{\frac{1}{4}}}{2^{\frac{1}{4}}}$$
$$= \left(\frac{162}{2}\right)^{\frac{1}{4}}$$
$$= 81^{\frac{1}{4}}$$
$$= \sqrt[4]{81}$$
$$= 3$$

5. a) Write each term in radical form and then evaluate.

$$49^{\frac{1}{2}} + 16^{\frac{1}{2}} = \sqrt{49} + \sqrt{16}$$
$$= 7 + 4$$
$$= 11$$

b) Write each term in radical form and then evaluate.

$$27^{\frac{2}{3}} - 81^{\frac{3}{4}} = (27^{\frac{1}{3}})^2 - (81^{\frac{1}{4}})^3$$
$$= (\sqrt[3]{27})^2 - (\sqrt[4]{81})^3$$
$$= 3^2 - 3^3$$
$$= 9 - 27$$
$$= -18$$

c) Write each term in radical form and then evaluate. When a power has a negative exponent, write the power using the reciprocal of its base and its opposite exponent.

$$16^{\frac{3}{4}} + 16^{\frac{3}{4}} - 81^{-\frac{1}{4}} = (16^{\frac{1}{4}})^3 + (16^{\frac{1}{4}})^3 - \frac{1}{81^{\frac{1}{4}}}$$
$$= \left(\sqrt[4]{16}\right)^3 + \left(\sqrt[4]{16}\right)^3 - \frac{1}{\sqrt[4]{81}}$$
$$= 2^3 + 2^3 - \frac{1}{3}$$
$$= 8 + 8 - \frac{1}{3}$$
$$= \frac{47}{3}$$

d) Write each term in radical form and then evaluate. Rewrite the decimal exponent using an equivalent fraction. When a power has a negative exponent, write the power using the reciprocal of its base and its opposite exponent.

$$128^{-\frac{5}{7}} - 16^{0.75} = \frac{1}{128^{\frac{5}{7}}} - 16^{\frac{3}{4}}$$
$$= \frac{1}{(128^{\frac{1}{7}})^5} - (16^{\frac{1}{4}})^3$$
$$= \frac{1}{(\sqrt[7]{128})^5} - \left(\sqrt[4]{16}\right)^3$$
$$= \frac{1}{2^5} - 2^3$$
$$= \frac{1}{32} - 8$$
$$= -\frac{255}{32}$$

e) Write each term in radical form and then evaluate. Rewrite the decimal exponent using an equivalent fraction. When a power has a negative exponent, write the power using the reciprocal of its base and its opposite exponent.

$$16^{\frac{3}{2}} + 16^{-0.5} + 8 - 27^{\frac{2}{3}}$$
$$= 16^{\frac{3}{2}} + 16^{-\frac{1}{2}} + 8 - 27^{\frac{2}{3}}$$
$$= (16^{\frac{1}{2}})^3 + \frac{1}{16^{\frac{1}{2}}} + 8 - (27^{\frac{1}{3}})^2$$
$$= \left(\sqrt{16}\right)^3 + \frac{1}{\sqrt{16}} + 8 - \left(\sqrt[3]{27}\right)^2$$
$$= 4^3 + \frac{1}{4} + 8 - 3^2$$
$$= 64 + \frac{1}{4} + 8 - 9$$
$$= \frac{253}{4}$$

f) Write each term in radical form and then evaluate.

$$81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{4}{5}} + 16^{\frac{3}{4}}$$
$$= \sqrt{81} + \sqrt[3]{8} - (32^{\frac{1}{5}})^4 + (16^{\frac{1}{4}})^3$$
$$= \sqrt{81} + \sqrt[3]{8} - \left(\sqrt[5]{32}\right)^4 + \left(\sqrt[4]{16}\right)^3$$
$$= 9 + 2 - 2^4 + 2^3$$
$$= 9 + 2 - 16 + 8$$
$$= 3$$

6. a) Rewrite the decimal exponent using an equivalent fraction. To multiply powers with the same base, keep the base the same and add exponents.

$$4^{\frac{1}{5}}(4^{0.3}) = 4^{\frac{1}{5}+\frac{3}{10}}$$
$$= 4^{\frac{1}{2}}$$
$$= \sqrt{4}$$
$$= 2$$

b) Rewrite the decimal exponent using an equivalent fraction. To multiply powers with the same base, keep the base the same and add exponents. When a power has a negative exponent, write the power using the reciprocal of its base and its opposite exponent.

$$\begin{aligned}100^{0.2}(100^{-\frac{7}{10}}) &= 100^{\frac{2}{10}+(-\frac{7}{10})}\\ &= 100^{-\frac{1}{2}}\\ &= \frac{1}{100^{\frac{1}{2}}}\\ &= \frac{1}{\sqrt{100}}\\ &= \frac{1}{10}\end{aligned}$$

c) To divide powers with the same base, keep the base the same and subtract exponents.

$$\begin{aligned}\frac{64^{\frac{4}{3}}}{64} &= 64^{\frac{4}{3}-1}\\ &= 64^{\frac{1}{3}}\\ &= \sqrt[3]{64}\\ &= 4\end{aligned}$$

d) To divide powers with the same base, keep the base the same and subtract exponents. When a power has a negative exponent, write the power using the reciprocal of its base and its opposite exponent.

$$\begin{aligned}\frac{27^{-1}}{27^{-\frac{2}{3}}} &= 27^{-1-(-\frac{2}{3})}\\ &= 27^{-\frac{1}{3}}\\ &= \frac{1}{27^{\frac{1}{3}}}\\ &= \frac{1}{\sqrt[3]{27}}\\ &= \frac{1}{3}\end{aligned}$$

e) First find the power raised to a power by multiplying exponents. Then rewrite the decimal exponent using an equivalent fraction and subtract exponents to simplify the whole expression.

$$\begin{aligned}\frac{(16^{-2.5})^{-0.2}}{16^{\frac{3}{4}}} &= \frac{16^{-2.5\times(-0.2)}}{16^{\frac{3}{4}}}\\ &= \frac{16^{0.5}}{16^{\frac{3}{4}}}\\ &= 16^{\frac{1}{2}-\frac{3}{4}}\\ &= 16^{-\frac{1}{4}}\\ &= \frac{1}{16^{\frac{1}{4}}}\\ &= \frac{1}{\sqrt[4]{16}}\\ &= \frac{1}{2}\end{aligned}$$

f) Multiply powers with the same base by keeping the base the same and adding exponents. Find the power raised to a power by multiplying the exponents. Finally, subtract exponents to simplify the whole expression.

$$\begin{aligned}\frac{(8^{-2})(8^{2.5})}{(8^{6})^{-0.25}} &= \frac{8^{-2+2.5}}{8^{6\times(-0.25)}}\\ &= \frac{8^{0.5}}{8^{-1.5}}\\ &= 8^{0.5-(-1.5)}\\ &= 8^{2}\\ &= 64\end{aligned}$$

7. Prediction: c, d, a, e, b, f

a) $\sqrt[4]{623} \doteq 4.996$

b) $125^{\frac{2}{5}} \doteq 6.899$

c) $\sqrt[10]{10.24} \doteq 1.262$

d) $80.9^{\frac{1}{4}} \doteq 2.999$

e) $17.5^{\frac{5}{8}} \doteq 5.983$

f) $21.4^{\frac{3}{2}} \doteq 98.997$

8. The volume formula for a cube is $V = x^3$, where V is the volume and x is the side length. The length of each side is $\sqrt[3]{V}$.

$$\begin{aligned}x &= \sqrt[3]{V}\\ &= \sqrt[3]{0.015\,625}\\ &= 0.25\end{aligned}$$

The length of each side is 0.25 m.

9. $27^{\frac{4}{3}} = 81$

$27^{1.3333} \doteq 80.99\,110\,173$

The values are not equal as $\frac{4}{3} \neq 1.3333$.

10. $0.2 = \frac{1}{5}$, an odd root, $0.5 = \frac{1}{2}$, an even root. Even root of a negative number is not real.

11. Write the power using the reciprocal of its base and its opposite exponent. Use the power-of-a-power rule to separate the exponents. Then evaluate the cube root and square the result.

$$\begin{aligned}125^{-\frac{2}{3}} &= \frac{1}{125^{\frac{2}{3}}}\\ &= \frac{1}{(125^{\frac{1}{3}})^2}\\ &= \frac{1}{\left(\sqrt[3]{125}\right)^2} = \frac{1}{(\sqrt[3]{125})^2}\\ &= \frac{1}{5^2}\\ &= \frac{1}{25}\end{aligned}$$

12. a) Rewrite the decimal exponent using an equivalent fraction and write the power in radical form. Then find the eighth root of 256 and cube the result. The negative sign is not inside brackets, so the entire power is negative.

$$\begin{aligned} -256^{0.375} &= -256^{\frac{3}{8}} \\ &= -(256^{\frac{1}{8}})^3 \\ &= -\left(\sqrt[8]{256}\right)^3 \\ &= -2^3 \\ &= -8 \end{aligned}$$

b) Write the power in radical form. Then find the cube root of 15.625 and take the fourth power of the result.

$$\begin{aligned} 15.625^{\frac{4}{3}} &= (15.625^{\frac{1}{3}})^4 \\ &= \left(\sqrt[3]{15.625}\right)^4 \\ &= 2.5^4 \\ &= 39.0625 \end{aligned}$$

c) Find the cube root of -0.027 and take the fourth power of the result.

$$\begin{aligned} \sqrt[3]{-0.027^4} &= (\sqrt[3]{-0.027})^4 \\ &= (-0.3)^4 \\ &= 0.0081 \end{aligned}$$

d) Write the power in radical form. Then find the cube root of -3.375 and square the result.

$$\begin{aligned} (-3.375)^{\frac{2}{3}} &= (-3.375^{\frac{1}{3}})^2 \\ &= \left(\sqrt[3]{-3.375}\right)^2 \\ &= (-1.5)^2 \\ &= 2.25 \end{aligned}$$

e) Find the fourth root of 0.0016 and cube the result.

$$\begin{aligned} \sqrt[4]{(0.0016)^3} &= \left(\sqrt[4]{0.0016}\right)^3 \\ &= 0.2^3 \\ &= 0.008 \end{aligned}$$

f) Rewrite the decimal exponent using an equivalent fraction and write the power in radical form. Then find the fifth root of -7776 and cube the result.

$$\begin{aligned} (-7776)^{1.6} &= (-7776)^{\frac{8}{5}} \\ &= \left(\sqrt[5]{-7776}\right)^8 \\ &= (-6)^8 \\ &= 1\,679\,616 \end{aligned}$$

13. $4^{2.5} = 4^{\frac{5}{2}} = \left(\sqrt{4}\right)^5$. Change 2.5 to a fraction as $\frac{5}{2}$. This is the same as taking the square root of the four and then taking the fifth power of the result.

14. a)

$$\begin{aligned} 9^{\frac{1}{2}} + 4^{\frac{1}{2}} &= \sqrt{9} + \sqrt{4} \\ &= 3 + 2 \\ &= 5 \\ (9 + 4)^{\frac{1}{2}} &= 13^{\frac{1}{2}} \\ &= \sqrt{13} \end{aligned}$$

Since $5 \neq \sqrt{13}$, the expression is false.

b)

$$\begin{aligned} 9^{\frac{1}{2}} + 4^{\frac{1}{2}} &= \sqrt{9} + \sqrt{4} \\ &= 3 + 2 \\ &= 5 \\ (9 \times 4)^{\frac{1}{2}} &= 36^{\frac{1}{2}} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

Since $5 \neq 6$, the expression is false.

c)

$$\begin{aligned} \left(\frac{1}{a} + \frac{1}{b}\right)^{-1} &= \left(\frac{b}{ab} + \frac{a}{ab}\right)^{-1} \\ &= \left(\frac{b + a}{ab}\right)^{-1} \\ &= \frac{ab}{b + a} \end{aligned}$$

Since $\frac{ab}{b + a} \neq a + b$, the expression is false.

d)

$$\begin{aligned} \left(\frac{1}{a} \times \frac{1}{b}\right)^{-1} &= \left(\frac{1}{ab}\right)^{-1} \\ &= \frac{ab}{1} \\ &= ab \end{aligned}$$

Since $\left(\frac{1}{a} \times \frac{1}{b}\right)^{-1} = ab$, the expression is true.

e) $(x^{\frac{1}{3}} + y^{\frac{1}{3}})^6$ cannot be simplified since the powers are added. So $(x^{\frac{1}{3}} + y^{\frac{1}{3}})^6 \neq x^2 + y^2$. The expression is false.

f) $[(x^{\frac{1}{3}})(y^{\frac{1}{3}})]^6 = [(xy)^{\frac{1}{3}}]^6$
$= (xy)^{\frac{1}{3}\times 6}$
$= (xy)^2$
$= x^2y^2$

Since $[(x^{\frac{1}{3}})(y^{\frac{1}{3}})]^6 = x^2y^2$, the expression is true.

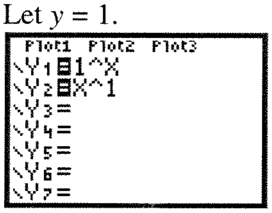
15. a) n cannot be zero because a fraction cannot have a denominator of zero. n cannot be an even number because an even root cannot have a negative base. There are no restrictions on m.

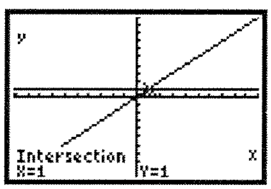
b) n cannot be zero because a fraction cannot have a denominator of zero. There are no restrictions on m.

16. There are an infinite number of solutions. The most obvious solutions are of the form where $x = y$. For example (1, 1), (2, 2), (3, 3), … are all solutions. This is also true for any real number other than 0. $\{(x, y) \in \mathbf{R} \mid x = y, x \neq 0, y \neq 0\}$.

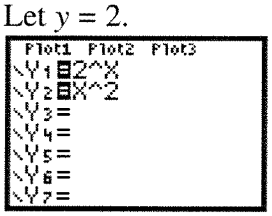
Others can be found by graphing a system of equations and determining the points of intersection. To do this let y equal a specific number. For example:

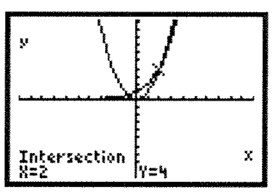
Let $y = 1$.

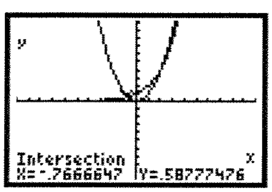




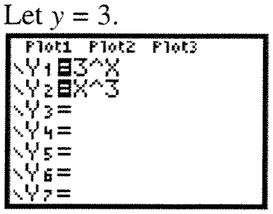
Let $y = 2$.

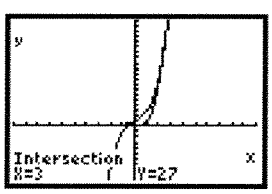


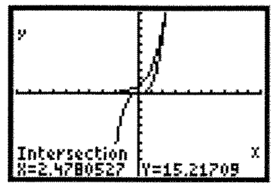




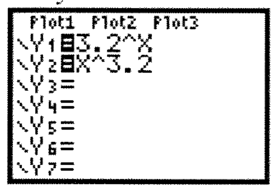
Let $y = 3$.

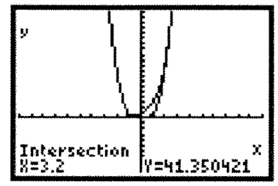


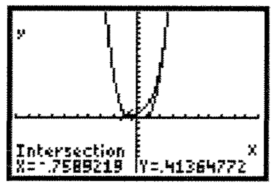




Let $y = 3.2$.







17. Take the 12th root of each side of the equation.

$$1.225 = (1 + i)^{12}$$
$$\sqrt[12]{1.225} = \sqrt[12]{(1 + i)^{12}}$$
$$1.01705555 = 1 + i$$
$$0.01705555 = i$$

Yes this works; taking the 12th root of each side undoes the operation of raising the right side to the 12th power. The value of i is approximately 0.017.

18. a) Write $\left(\frac{1}{16}\right)^{\frac{1}{4}}$ in radical form. Then evaluate each term and solve for x.

$$\left(\frac{1}{16}\right)^{\frac{1}{4}} - \sqrt[3]{\frac{8}{27}} = \sqrt{x^2}$$
$$\sqrt[4]{\frac{1}{16}} - \sqrt[3]{\frac{8}{27}} = \sqrt{x^2}$$
$$\frac{1}{2} - \frac{2}{3} = x$$
$$-\frac{1}{6} = x$$

b) Evaluate each term. Then solve for x.

$$\sqrt[3]{\frac{1}{8}} - \sqrt[4]{x^4} + 15 = \sqrt[4]{16}$$
$$\frac{1}{2} - x + 15 = 2$$
$$-x = -\frac{27}{2}$$
$$x = \frac{27}{2}$$

4.4 Simplifying Algebraic Expressions Involving Exponents, pp. 235–237

1. a) $x^4(x^3) = x^{4+3}$
$= x^7$

b) $(p^{-3})(p)^5 = p^{-3}p^5$
$= p^{-3+5}$
$= p^2$

c) $\frac{m^5}{m^{-3}} = m^{5-(-3)}$
$= m^8$

d) $\frac{a^{-4}}{a^{-2}} = a^{-4-(-2)}$
$= a^{-2}$
$= \frac{1}{a^2}$

e) $(y^3)^2 = y^6$

f) $(k^6)^{-2} = k^{-12}$
$= \frac{1}{k^{12}}$

2. a) $y^{10}(y^4)^{-3} = y^{10}y^{-12}$
$= y^{10-12}$
$= y^{-2}$
$= \frac{1}{y^2}$

b) $(x^{-3})^{-3}(x^{-1})^5 = x^9x^{-5}$
$= x^{9-5}$
$= x^4$

c) $\frac{(n^{-4})^3}{(n^{-3})^{-4}} = \frac{n^{-12}}{n^{12}}$
$= n^{-12-12}$
$= n^{-24}$
$= \frac{1}{n^{24}}$

d) $\frac{w^4(w^{-3})}{(w^{-2})^{-1}} = \frac{w^{4-3}}{w^2}$
$= \frac{w}{w^2}$
$= \frac{1}{w}$

e) $\frac{(x^{-1})^4x}{x^{-3}} = \frac{x^{-4}x}{x^{-3}}$
$= \frac{x^{-4+1}}{x^{-3}}$
$= \frac{x^{-3}}{x^{-3}}$
$= 1$

f) $\frac{(b^{-7})^2}{b(b^{-5})b^9} = \frac{b^{-14}}{b^{1-5+9}}$
$= \frac{b^{-14}}{b^5}$
$= b^{-14-5}$
$= b^{-19}$
$= \frac{1}{b^{19}}$

3. a) $\frac{(-2)^7(3^2)^3}{(-2)^5(3)^4} = \frac{-128(9)^3}{(-32)(81)}$
$= \frac{(-128)(729)}{(-32)(81)}$
$= \frac{-93\,312}{-2\,592}$
$= 36$

b) $\frac{(x)^7(y^2)^3}{(x)^5(y)^4} = \frac{x^7y^6}{x^5x^4}$
$= x^{7-5}y^{6-4}$
$= x^2y^2$

For $x = -2$ and $y = 3$, x^2y^2 is $(-2)^2(3)^2$, which is equal to (4)(9), or 36

c) Usually it is faster to substitute numbers into the simplified form of the expression.

4. a) $(pq^2)^{-1}(p^3q^3) = (p^{-1}q^{-2})(p^3q^3)$
$= p^{-1+3}q^{-2+3}$
$= p^2q$

b) $\left(\frac{x^3}{y}\right)^{-2} = \frac{x^{-6}}{y^{-2}}$
$= \frac{y^2}{x^6}$

c) $\frac{(ab)^{-2}}{b^5} = \frac{a^{-2}b^{-2}}{b^5}$
$= a^{-2}b^{-2-5}$
$= a^{-2}b^{-7}$
$= \frac{1}{a^2b^7}$

d) $\frac{m^2n^2}{(m^3n^{-2})^2} = \frac{m^2n^2}{m^6n^{-4}}$
$= m^{2-6}n^{2-(-4)}$
$= m^{-4}n^6$
$= \frac{n^6}{m^4}$

e) $\frac{(w^2x)^2}{(x^{-1})^2w^3} = \frac{w^4x^2}{x^{-2}w^3}$
$= w^{4-3}x^{2-(-2)}$
$= wx^4$

f) $\left(\dfrac{(ab)^{-1}}{a^2b^{-3}}\right)^{-2} = \left(\dfrac{a^{-1}b^{-1}}{a^2b^3}\right)^{-2}$

$= (a^{-1-2}b^{-1-(-3)})^{-2}$

$= (a^{-3}b^2)^{-2}$

$= a^6b^{-4}$

$= \dfrac{a^6}{b^4}$

5. a) $(3xy^4)^2(2x^2y)^3 = (3^2x^2y^8)(2^3x^6y^3)$

$= (9)(8)(x^{2+6})(y^{8+3})$

$= 72x^8y^{11}$

b) $\dfrac{(2a^3)^2}{4ab^2} = \dfrac{4a^6}{4ab^2}$

$= \dfrac{a^{6-1}}{b^2}$

$= \dfrac{a^5}{b^2}$

c) $\dfrac{(10x)^{-1}y^3}{15x^3y^{-3}} = \dfrac{10^{-1}x^{-1}y^3}{15x^3y^{-3}}$

$= \dfrac{10^{-1}x^{-1-3}y^{3-(-3)}}{15}$

$= \dfrac{10^{-1}x^{-4}y^6}{15}$

$= \dfrac{y^6}{150x^4}$

d) $\dfrac{(3m^4n^2)^2}{12m^{-2}n^6} = \dfrac{9m^8n^4}{12m^{-2}n^6}$

$= \dfrac{3}{4}(m^{8-(-2)}n^{4-6})$

$= \dfrac{3}{4}(m^{10}n^{-2})$

$= \dfrac{3m^{10}}{4n^2}$

e) $\dfrac{p^{-5}(r^3)^2}{(p^2r)^2(p^{-1})^2} = \dfrac{p^{-5}r^6}{p^4r^2p^{-2}}$

$= \dfrac{p^{-5}r^6}{p^{4-2}r^2}$

$= \dfrac{p^{-5}r^6}{p^2r^2}$

$= p^{-5-2}r^{6-2}$

$= p^{-7}r^4$

$= \dfrac{r^4}{p^7}$

f) $\left(\dfrac{(x^3y)^{-1}(x^4y^3)}{(x^2y^{-3})^{-2}}\right)^{-1} = \left(\dfrac{(x^{-3}y^{-1})(x^4y^3)}{x^{-4}y^6}\right)^{-1}$

$= \left(\dfrac{x^{-3+4}y^{-1+3}}{x^{-4}y^6}\right)^{-1}$

$= \left(\dfrac{xy^2}{x^{-4}y^6}\right)^{-1}$

$= (x^{1-(-4)}y^{2-6})^{-1}$

$= (x^5y^{-4})^{-1}$

$= x^{-5}y^4$

$= \dfrac{y^4}{x^5}$

6. a) $(x^4)^{\frac{1}{2}}$

$(x^6)^{\frac{-1}{3}} = x^2x^{-2}$

$= x^{2-2}$

$= x^0$

$= 1$

b) $\dfrac{9(c^8)^{0.5}}{(16c^{12})^{0.25}} = \dfrac{9(c^4)}{16^{0.25}c^3}$

$= \dfrac{9(c^4)}{2c^3}$

$= \dfrac{9}{2}c^{4-3}$

$= \dfrac{9}{2}c$

c) $\dfrac{\sqrt{25m^{-12}}}{\sqrt{36m^{10}}} = \dfrac{(25m^{-12})^{\frac{1}{2}}}{(36m^{10})^{\frac{1}{2}}}$

$= \dfrac{5m^{-6}}{6m^5}$

$= \dfrac{5}{6}m^{-6-5}$

$= \dfrac{5}{6}m^{-11}$

$= \dfrac{5}{6m^{11}}$

d) $\sqrt[3]{\dfrac{(10x^3)^2}{(10x^6)^{-1}}} = \dfrac{((10x^3)^2)^{\frac{1}{3}}}{((10x^6)^{-1})^{\frac{1}{3}}}$

$= \dfrac{(10x^3)^{\frac{2}{3}}}{(10x^6)^{\frac{-1}{3}}}$

$= \dfrac{10^{\frac{2}{3}}x^2}{10^{\frac{-1}{3}}x^{-2}}$

$= 10^{\frac{2}{3}-\frac{-1}{3}}x^{2-(-2)}$

$= 10x^4$

e) $\left(\frac{(32x^5)^{-2}}{(x^{-1})^{10}}\right)^{0.2} = \left(\frac{32^{-2}x^{-10}}{x^{-10}}\right)^{0.2}$

$= \frac{32^{\frac{-2}{5}}x^{-2}}{x^{-2}}$

$= \sqrt[5]{32^{-2}}$

$= \sqrt[5]{\frac{1}{1024}}$

$= \frac{1}{4}$

f) $\frac{\sqrt[10]{1024x^{20}}}{\sqrt[9]{512x^{27}}} = \frac{1024^{\frac{1}{10}}x^2}{512^{\frac{1}{9}}x^3}$

$= \frac{2x^2}{2x^3}$

$= x^{2-3}$

$= x^{-1}$

$= \frac{1}{x}$

7. a) $(16x^6y^4)^{\frac{1}{2}} = 16^{\frac{1}{2}}x^3y^2$

$= 4x^3y^2$

For $x = 2$, $y = 1$, $4x^3y^2$ is $4(2)^3(1)^2$, which is equal to $4(8)$, or 32.

b) $\frac{(9p^{-2})^{\frac{1}{2}}}{6p^2} = \frac{9^{\frac{1}{2}}p^{-1}}{6p^2}$

$= \frac{3p^{-1}}{6p^2}$

$= \frac{1}{2}p^{-1-2}$

$= \frac{1}{2}p^{-3}$

$= \frac{1}{2p^3}$

For $p = 3$, $\frac{1}{2p^3}$ is $\frac{1}{2(3)^3}$, which is equal to $\frac{1}{2(27)}$, or $\frac{1}{54}$.

c) $\frac{(81x^4y^6)^{\frac{1}{2}}}{8(x^9y^3)^{\frac{1}{3}}} = \frac{81^{\frac{1}{2}}x^2y^3}{8(x^3y)}$

$= \frac{9}{8}x^{2-3}y^{3-1}$

For $x = 10$, $y = 5$, $\frac{9y^2}{8x}$ is $\frac{9(5)^2}{8(10)}$, or $\frac{9(25)}{80}$, which is equal to $\frac{225}{80}$, or $\frac{45}{16}$.

d) $\left(\frac{(25a^4)^{-1}}{(7a^{-2}b)^2}\right)^{\frac{1}{2}} = \left(\frac{25^{-1}a^{-4}}{7^2a^{-4}b^2}\right)^{\frac{1}{2}}$

$= \left(\frac{a^{-4-(-4)}}{(25)(49)b^2}\right)^{\frac{1}{2}}$

$= \left(\frac{a^0}{(25)(49)b^2}\right)^{\frac{1}{2}}$

$= \frac{1}{25^{\frac{1}{2}}49^{\frac{1}{2}}b}$

$= \frac{1}{35b}$

For $a = 11$, $b = 10$, $\frac{1}{35b}$ is $\frac{1}{35(10)}$, or $\frac{1}{350}$.

8. a) $(\sqrt{10\,000x})^{\frac{3}{2}} = ((10\,000x)^{\frac{1}{2}})^{\frac{3}{2}}$

$= (10\,000x)^{\frac{3}{4}}$

$= \sqrt[4]{10\,000^3}\,x^{\frac{3}{4}}$

$= 1000x^{\frac{3}{4}}$

For $x = 16$, $1000x^{\frac{3}{4}}$ is $1000(16)^{\frac{3}{4}}$, or $1000\sqrt[4]{16^3}$, which is equal to $1000\sqrt[4]{4096}$, or 8000.

b) $\left(\frac{(4x^3)^4}{(x^3)^6}\right)^{-0.5} = \left(\frac{4^4x^{12}}{x^{18}}\right)^{-0.5}$

$= (256x^{12-18})^{-0.5}$

$= (256x^{-6})^{-0.5}$

$= 256^{\frac{-1}{2}}x^3$

$= \frac{x^3}{16}$

For $x = 5$, $\frac{x^3}{16}$ is $\frac{5^3}{16}$, or $\frac{125}{16}$.

c) $(-2a^2b)^{-3}\sqrt{25a^4b^6} = (-2^{-3}a^{-6}b^{-3})(25^{\frac{1}{2}}a^2b^3)$

$= \frac{-5}{8}a^{-6+2}b^{-3+3}$

$= \frac{-5}{8}a^{-4}b^0$

$= \frac{-5}{8a^4}$

For $a = 1$ and $b = 2$, $\frac{-5}{8a^4}$ is $\frac{-5}{8(1)^4}$, or $\frac{-5}{8}$.

d) $\sqrt{\frac{(18m^{-5}n^2)(32m^2n)}{4mn^{-3}}}$

$= \sqrt{\frac{(18)(32)m^{-5+2}n^{2+1}}{4mn^{-3}}}$

$= \sqrt{\frac{576m^{-3}n^3}{4mn^{-3}}}$

$= (144m^{-3-1}n^{3-(-3)})^{\frac{1}{2}}$
$= (144m^{-4}n^{6})^{\frac{1}{2}}$
$= 12m^{-2}n^{3}$
$= \frac{12n^3}{m^2}$

For $m = 10$ and $n = 1$, $\frac{12n^3}{m^2}$ is $\frac{12(1)^3}{(10)^2}$, which is equal to $\frac{12}{100}$, or $\frac{3}{25}$

9. a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$
$= (36^{0.5}m^2n^3)(81^{0.25}m^3n^2)$
$= (6^5m^2n^3)(3m^3n^2)$
$= 18m^{2+3}n^{3+2}$
$= 18m^5n^5$

b) $\left(\frac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{\frac{-1}{3}} = \left(\frac{(6^2x^6)(6y^3)}{9^6x^6y^6}\right)^{\frac{-1}{3}}$
$= \left(\frac{36(6)x^6y^3}{9^6x^6y^6}\right)^{\frac{-1}{3}}$
$= \left(\frac{216}{9^6}(x^{6-6}y^{3-6}\right)^{\frac{-1}{3}}$
$= \left(\frac{216}{9^6}(x^0y^{-3}\right)^{\frac{-1}{3}}$
$= \left(\frac{216}{9^6}\right)^{\frac{-1}{3}}(y)$
$= \frac{y}{\left(\frac{216}{9^6}\right)^{\frac{-1}{3}}}$
$= \frac{y}{\left(\frac{6}{9^2}\right)}$
$= \frac{y}{\left(\frac{6}{81}\right)}$
$= \frac{81y}{6}$
$= \frac{27y}{2}$

c) $\left(\frac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}} = \left(\frac{(64a^{12})^{\frac{1}{2}}}{a^{-9}}\right)^{\frac{2}{3}}$
$= \left(\frac{8a^6}{a^{-9}}\right)^{\frac{2}{3}}$
$= (8a^{6-(-9)})^{\frac{2}{3}}$
$= (8a^{15})^{\frac{2}{3}}$
$= 8^{\frac{2}{3}}a^{10}$
$= \sqrt[3]{8^2}a^{10}$
$= \sqrt[3]{64}a^{10}$
$= 4a^{10}$

d) $\left(\frac{(x^{18})^{\frac{-1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5} = \left(\frac{x^{-3}}{243^{\frac{1}{5}}x^2}\right)^{0.5}$
$= \left(\frac{x^{-3-2}}{3}\right)^{0.5}$
$= \left(\frac{x^{-5}}{3}\right)^{0.5}$
$= \frac{x^{-2.5}}{\sqrt{3}}$
$= \frac{1}{\sqrt{3}x^{\frac{5}{2}}}$

10. $M = \frac{(16x^8y^{-4})^{\frac{1}{4}}}{32x^{-2}y^8}$
$= \frac{16^{\frac{1}{4}}x^2y^{-1}}{32x^{-2}y^8}$
$= \frac{2}{32}(x^{2-(-2)}y^{-1-8})$
$= \frac{1}{16}(x^4y^{-9})$
$= \frac{x^4}{16y^9}$

a) For $x = 2$, $y = 1$, $\frac{x^4}{16y^9}$ is $\frac{16}{16}$, or 1.

b) For $x = 3$, $y = 1$, $\frac{x^4}{16y^9}$ is $\frac{81}{16}$, which is greater than 1.

c) For $x = 1$, $y = 1$, $\frac{x^4}{16y^9}$ is $\frac{1}{16}$, which is greater than 0 and less than 1.

d) For $x = 1$, $y = -1$, $\frac{x^4}{16y^9}$ is $\frac{-1}{16}$, which is less than 0.

11. a) $\frac{SA}{V} = \frac{2\pi rh + 2\pi r^2}{\pi r^2 h}$
$= \frac{\pi r(2h + 2r)}{\pi r \quad rh}$
$= \frac{2h + 2r}{rh}$

b) For $r = 0.8$ cm and $h = 12$ cm, $\frac{SA}{V}$ is $\frac{2(12) + 2(0.8)}{(0.8)(12)}$, or $\frac{24 + 1.6}{9.6}$, which is equal to $\frac{8\text{ cm}}{3\text{ cm}^2}$, or about 2.67 cm^{-1}.

12. $$\frac{y^{-4}(x^2)^{-3}y^{-3}}{x^{-5}(y^{-4})^2} = \frac{y^{-4}x^{-6}y^{-3}}{x^{-5}y^{-8}}$$
$$= \frac{y^{-7}x^{-6}}{x^{-5}y^{-8}}$$
$$= y^{-7-(-8)}x^{-6-(-5)}$$
$$= yx^{-1}$$
$$= \frac{y}{x}$$

For $x = 2$, $y = 3$, $\frac{y}{x}$ is $\frac{-3}{2}$.

$$\frac{x^{-3}(y^{-1})^{-2}}{(x^{-5})(y^4)} = \frac{x^{-3}y^2}{x^{-5}y^4}$$
$$= x^{-3-(-5)}y^{2-4}$$
$$= x^2y^{-2}$$
$$= \frac{x^2}{y^2}$$

For $x = 2$, $y = 3$, $\frac{x^2}{y^2}$ is $\frac{2^2}{3^2} = \frac{4}{9}$.

$$(y^{-5})(x^5)^{-2}(y^2)(x^{-3})^{-4}$$
$$= (y^{-5})(y^2)(x^{-10})(x^{12})$$
$$= y^{-5+2}x^{-10+12}$$
$$= y^{-3}x^2$$
$$= \frac{x^2}{y^3}$$

For $x = 2$, $y = 3$, $\frac{x^2}{y^3}$ is $\frac{2^2}{3^3}$, or $\frac{4}{27}$.

The proper order from least to greatest is $\frac{-3}{2}, \frac{4}{27}, \frac{4}{9}$.

13. Algebraic and numerical expressions are similar in the following way: when simplifying algebraic or numerical expressions, you have to follow the order of operations. When simplifying algebraic expressions, you can only add or subtract like terms, while unlike terms may be multiplied or divided. In this way algebraic expressions are different than numerical expressions.

14. a) $$V = \frac{4}{3}\pi r^3$$
$$r^3 = \frac{V}{\frac{4}{3}\pi}$$
$$= \frac{3V}{4\pi}$$
$$r = \sqrt[3]{\frac{3V}{4\pi}}, \text{ or } r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$$

b) $$V = \frac{256\pi}{3}\text{ m}^3$$
$$r = \left(\frac{3\left(\frac{256\pi}{3}\right)}{4\pi}\right)^{\frac{1}{3}}$$
$$= \left(\frac{256}{4}\right)^{\frac{1}{3}}$$
$$= (64)^{\frac{1}{3}}$$
$$= 4\text{ m}$$

15. $$\frac{\sqrt{x(x^{2n+1})}}{\sqrt[3]{x^{3n}}} = \frac{x(x^{2n+1})^{\frac{1}{2}}}{(x^{3n})^{\frac{1}{3}}}$$
$$= \frac{(x^{2n+1+1})^{\frac{1}{2}}}{x^n}$$
$$= \frac{(x^{2n+2})^{\frac{1}{2}}}{x^n}$$
$$= \frac{x^{\frac{1}{2}(2n+2)}}{x^n}$$
$$= \frac{x^{n+1}}{x^n}$$
$$= x^{n+1-n}$$
$$= x$$

Mid-Chapter Review, p. 239

1. a) $$5(5^4) = 5^{1+4}$$
$$= 5^5$$

b) $$\frac{(-8)^4}{(-8)^5} = (-8)^{4-5}$$
$$= (-8)^{-1}$$
$$= \frac{-1}{8}$$

c) $$(9^3)^6 = 9^{(3)(6)}$$
$$= 9^{18}$$

d) $\frac{3(3)^6}{3^5} = \frac{3^{1+6}}{3^5}$
$= \frac{3^7}{3^5}$
$= 3^{7-5}$
$= 3^2$

e) $\left(\frac{1}{10}\right)^6\left(\frac{1}{10}\right)^{-4} = \left(\frac{1}{10}\right)^{6-4}$
$= \left(\frac{1}{10}\right)^2$

f) $\left(\frac{(7)^2}{(7)^4}\right)^{-1} = ((7)^{2-4})^{-1}$
$= (7^{-2})^{-1}$
$= 7^2$

2. a) $4^{-2} - 8^{-1} = \frac{1}{4^2} - \frac{1}{8}$
$= \frac{1}{16} - \frac{1}{8}$
$= \frac{1}{16} - \frac{2}{16}$
$= \frac{-1}{16}$

b) $(4 + 8)^0 - 5^{-2} = (-4)^0 - 5^{-2}$
$= 1 - \frac{1}{5^2}$
$= 1 - \frac{1}{25}$
$= \frac{25}{25} - \frac{1}{25}$
$= \frac{24}{25}$

c) $25^{-1} + 3(5^{-1})^2 = 25^{-1} + 3(5^{-2})$
$= \frac{1}{25} + \frac{3}{5^2}$
$= \frac{1}{25} + \frac{3}{25}$
$= \frac{4}{25}$

d) $\left(\frac{-1}{2}\right)^3 + 4^{-3} = \frac{-1^3}{2^3} + \frac{1}{4^3}$
$= \frac{-1}{8} + \frac{1}{64}$
$= \frac{-8}{64} + \frac{1}{64}$
$= \frac{-7}{64}$

3. a) $\left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2}$
$= \frac{16}{49}$

b) $\left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{5^3}$
$= \frac{-8}{125}$

c) $\left(\frac{-2}{3}\right)^{-3} = \frac{(-2)^{-3}}{3^{-3}}$
$= \frac{3^3}{(-2)^3}$
$= \frac{-27}{8}$

d) $\frac{(-3)^{-2}}{(-3)^{-5}} = (-3)^{-2-(-5)}$
$= (-3)^3$
$= -27$

4. x cannot be 0 or a negative number for $x^{\frac{-1}{2}}$.
x can be 0 for $x^{\frac{1}{2}}$, but not negative.

5. a) $\left(\frac{49}{81}\right)^{\frac{1}{2}} = \frac{49^{\frac{1}{2}}}{81^{\frac{1}{2}}}$
$= \frac{7}{9}$

b) $\sqrt{\frac{100}{121}} = \frac{\sqrt{100}}{\sqrt{121}}$
$= \frac{10}{11}$

c) $\left(\frac{16}{9}\right)^{-0.5} = \frac{16^{-0.5}}{9^{-0.5}}$
$= \frac{9^{0.5}}{16^{0.5}}$
$= \frac{3}{4}$

d) $((-125)^{\frac{1}{3}})^{-3} = (-125)^{-1}$
$= \frac{-1}{125}$

e) $\sqrt[4]{(-9)^{-2}} = \sqrt[4]{\frac{1}{(-9)^2}}$
$= \sqrt[4]{\frac{1}{81}}$
$= \frac{1}{3}$

f) $\frac{-\sqrt[3]{512}}{\sqrt[5]{-1\,024}} = \frac{-8}{-4}$
$= 2$

6.

	Exponential Form	Radical Form	Evaluation of Expression
a)	$100^{\frac{1}{2}}$	$\sqrt{100}$	10
b)	$16^{0.25}$	$\sqrt[4]{16}$	2
c)	$121^{\frac{1}{2}}$	$\sqrt{121}$	11
d)	$(-27)^{\frac{5}{3}}$	$\sqrt[3]{(-27)^5}$	-243
e)	$49^{2.5}$	$\sqrt{49^5}$	16 807
f)	$1024^{\frac{1}{10}}$	$\sqrt[10]{1024}$	2

7. a) $-456^{\frac{4}{7}} = -33.068$
b) $98^{0.75} = 31.147$
c) $\left(\frac{5}{8}\right)^{\frac{5}{8}} = 0.745$
d) $\left(\sqrt[5]{-1000}\right)^3 = (-1000)^{\frac{3}{5}}$
$= -63.096$

8. $-8^{\frac{4}{3}} = -\left(8^{\frac{4}{3}}\right)$
$= -\left(\sqrt[3]{8}\right)^4$
$= -(2^4)$
$= -16$
$(-8)^{\frac{4}{3}} = \left(\sqrt[3]{-8}\right)^4$
$= (-2^4)$
$= 16$

The second expression has an even root so the negative sign is eliminated.

9. a) $\frac{(x^{-3})x^5}{x^7} = \frac{x^{-3+5}}{x^7}$
$= \frac{x^2}{x^7}$
$= x^{2-7}$
$= x^{-5}$
$= \frac{1}{x^5}$

b) $\frac{(n^{-4})n^{-6}}{(n^{-2})^7} = \frac{n^{-4-6}}{n^{-14}}$
$= \frac{n^{-10}}{n^{-14}}$
$= n^{-10-(-14)}$
$= n^4$

c) $\left(\frac{(y^2)^6}{y^9}\right)^{-2} = \left(\frac{y^{12}}{y^9}\right)^{-2}$
$= (y^{12-9})^{-2}$
$= (y^3)^{-2}$
$= y^{-6}$
$= \frac{1}{y^6}$

d) $\frac{(-2x^5)^3}{8x^{10}} = \frac{-8x^{15}}{8x^{10}}$
$= -x^{15-10}$
$= -x^5$

e) $(3a^2)^{-3}(9a^{-1})^2 = (3^{-3}a^{-6})(9^2a^{-2})$
$= (3^{-3}(3^2)^2)(a^{-6-2})$
$= (3^{-3}(3^4))(a^{-8})$
$= 3^{-3+4}a^{-8}$
$= \frac{3}{a^8}$

f) $\frac{(4r^{-6})(-2r^2)^5}{(-2r)^4} = \frac{(4r^{-6})((-2)^5r^{10})}{(-2)^4r^4}$
$= \frac{(4r^{-6})(-32r^{10})}{16r^4}$
$= \frac{-128r^{-6+10}}{16r^4}$
$= \frac{-8r^4}{r^4}$
$= -8$

10. a) $\frac{x^{0.5}y^{1.8}}{x^{0.3}y^{2.5}} = x^{0.5-0.3}y^{1.8-2.5}$
$= x^{0.2}y^{-0.7}$
$= \frac{x^{0.2}}{y^{0.7}}$

b) $\frac{(mn^3)^{\frac{-1}{2}}}{m^{\frac{1}{2}}n^{\frac{-5}{2}}} = \frac{m^{\frac{-1}{2}}n^{\frac{-3}{2}}}{m^{\frac{1}{2}}n^{\frac{-5}{2}}}$
$= m^{\frac{-1}{2}-\frac{1}{2}}n^{\frac{-3}{2}-\frac{-5}{2}}$
$= m^{-1}n^{\frac{2}{2}}$
$= \frac{n}{m}$

c) $\frac{\sqrt{x^2y^4}}{(x^{-2}y^3)^{-1}} = \frac{\sqrt{x^2}\sqrt{y^4}}{x^2y^{-3}}$
$= \frac{xy^2}{x^2y^{-3}}$
$= x^{1-2}y^{2-(-3)}$
$= x^{-1}y^5$
$= \frac{y^5}{x}$

d) $\left(\frac{2abc^3}{(2a^2b^3c)^2}\right)^{-2} = \left(\frac{2abc^3}{4a^4b^6c^2}\right)^{-2}$
$= \left(\frac{1}{2}a^{1-4}b^{1-6}c^{3-2}\right)^{-2}$
$= \left(\frac{1}{2}a^{-3}b^{-5}c\right)^{-2}$

$$= \left(\left(\frac{1}{2}\right)^{-2} a^6b^{10}c^{-2}\right)$$
$$= \frac{4a^6b^{10}}{c^2}$$

e) $\dfrac{\sqrt[4]{81p^8}}{\sqrt{9p^4}} = \dfrac{\sqrt[4]{81}\sqrt[4]{p^8}}{\sqrt{9}\sqrt{p^4}}$
$$= \frac{3p^2}{3p^2}$$
$$= 1$$

f) $\dfrac{\sqrt[6]{(8x^6)^2}}{\sqrt[4]{625x^8}} = \dfrac{\sqrt[6]{64x^{12}}}{\sqrt[4]{625x^8}}$
$$= \frac{\sqrt[6]{64}\sqrt[6]{x^{12}}}{\sqrt[4]{625}\sqrt[4]{x^8}}$$
$$= \frac{2x^2}{5x^2}$$
$$= \frac{2}{5}$$

11. a) $\left(\dfrac{b^3}{a^{\frac{5}{2}}}\right)^2\left(\dfrac{2a^4}{b^5}\right) = \left(\dfrac{b^6}{a^5}\right)\left(\dfrac{2a^4}{b^5}\right)$
$$= \frac{2a^4b^6}{a^5b^5}$$
$$= 2(a^{4-5})(b^{6-5})$$
$$= 2a^{-1}b$$
$$= \frac{2b}{a}$$

For $a = 2$ and $b = 3$, $\dfrac{2b}{a}$ is $\dfrac{2(3)}{2}$, or 3.

b) $\sqrt{\dfrac{9b^3(ab)^2}{(a^2b^3)^3}} = \sqrt{\dfrac{9b^3a^2b^2}{a^6b^9}}$
$$= \sqrt{\frac{9b^{3+2}a^2}{a^6b^9}}$$
$$= \sqrt{\frac{9b^5a^2}{a^6b^9}}$$
$$= (9a^{2-6}b^{5-9})^{\frac{1}{2}}$$
$$= (9a^{-4}b^{-4})^{\frac{1}{2}}$$
$$= 9^{\frac{1}{2}}a^{-2}b^{-2}$$
$$= \frac{3}{a^2b^2}$$

For $a = 2$ and $b = 3$, $\dfrac{3}{a^2b^2}$ is $\dfrac{3}{2^23^2}$, or $\dfrac{3}{(4)(9)}$, which is equal to $\dfrac{3}{36}$, or $\dfrac{1}{12}$.

12. a) $(a^{10+2p})(a^{-p-8}) = a^{10-8+2p-p}$
$$= a^{2+p}$$

b) $(2x^2)^{3-2m}\left(\dfrac{1}{x}\right)^{2m} = (2^{3-2m}x^{2(3-2m)})(x^{-1})^{2m}$
$$= (2^{3-2m}x^{6-4m})(x^{-2m})$$
$$= 2^{3-2m}(x^{6-4m-2m})$$
$$= 2^{3-2m}x^{6-6m}$$

c) $[(c)^{2n-3m}](c^3)^m \div (c^2)^n$
$= [(c)^{2n-3m}](c^{3m}) \div (c^{2n})$
$= (c)^{2n-3m+3m} \div (c^{2n})$
$= (c)^{2n} \div (c)^{2n}$
$= c^{2n-2n}$
$= c^0$
$= 1$

d) $(x^{4n-m})\left(\dfrac{1}{x^3}\right)^{m+n} = (x^{4n-m})(x^{-3})^{m+n}$
$$= (x^{4n-m})(x^{-3(m+n)})$$
$$= (x^{4n-m})(x^{-3m-3n})$$
$$= x^{4n-m-3m-3n}$$
$$= x^{n-4m}$$

4.5 Exploring the Properties of Exponential Functions, p. 243

x	y
−4	5
−3	8
−2	13
−1	20
0	29
1	40

1. a)

x	y	First differences	Second differences
−4	5		
		3	
−3	8		2
		5	
−2	13		2
		7	
−1	20		2
		9	
0	29		2
		11	
1	40		

Quadratic; since the second differences of the function are constant (all are equal to 2), the function is quadratic.

x	y
−5	32
−4	16
−3	8
−2	4
−1	2
0	1

b)

x	y	First differences	Second differences
−5	32		
−4	16	16	8
−3	8	8	4
−2	4	4	2
−1	2	2	1
0	1	1	

Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

x	y
−2	−2.75
0	−2
2	1
4	13
6	61
8	253

c)

x	y	First differences	Second differences
−2	−2.75		
0	−2	.75	2.75
2	1	3	9
4	13	12	36
6	61	48	144
8	253	192	

Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

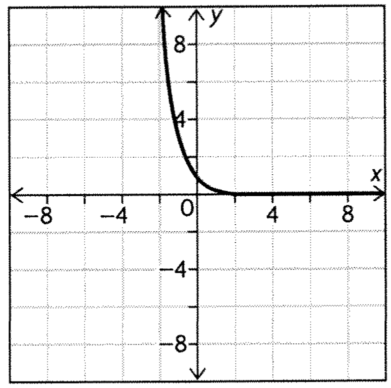
x	y
0.5	0.9
0.75	1.1
1	1.3
1.25	1.5
1.5	1.7
1 .75	1.9

d)

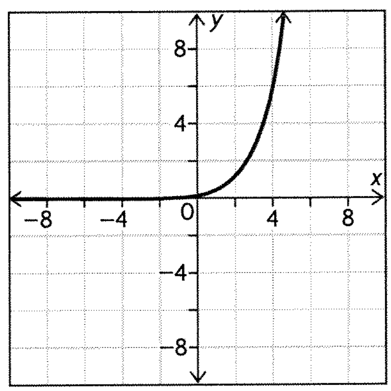
x	y	First differences
0.5	0.9	
0.75	1.1	.2
1	1.3	.2
1.25	1.5	.2
1.5	1.7	.2
1.75	1.9	

Linear; since the first differences of the function are constant (all are equal to .2), the function is linear.

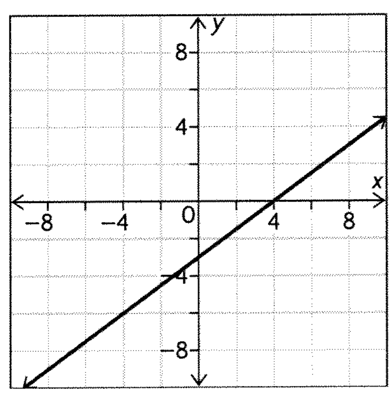
2. a) Exponential; the function is exponential since its graph is a quickly decreasing curve with a horizontal asymptote.



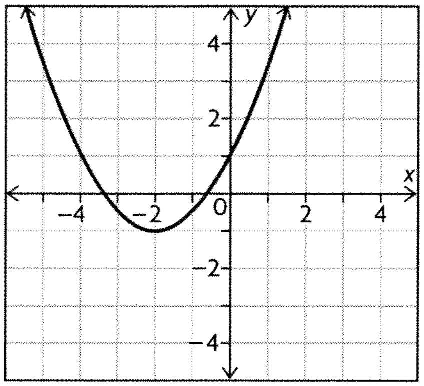
b) Exponential; the function is exponential since its graph is a quickly increasing curve with a horizontal asymptote.



c) Linear; the function is linear since its graph is a straight line.



d) Quadratic: the function is quadratic since its graph is a parabola.



4.6 Transformations of Exponential Functions, pp. 251–253

1. a) There is a vertical translation of 3, moving the function up 3 units.
b) There is a horizontal translation of 3, moving the function 3 units to the left.
c) There is a vertical compression, decreasing the function's values by a factor of $\frac{1}{3}$.
d) There is a horizontal stretch, increasing the function's x-values by a factor of 3.
2. a) The base function is $y = 4^x$. There is a horizontal translation to the left 1 unit, a vertical stretch of factor 3, and a reflection about the x-axis.
b) The base function is $y = \left(\frac{1}{2}\right)^x$. There is a vertical stretch of factor 2, a horizontal compression of factor $\frac{1}{2}$, and a vertical translation of 3 units up.
c) The base function is $y = \left(\frac{1}{2}\right)^x$. There is a vertical stretch of factor 7, a vertical translation of 1 unit down, and a horizontal translation of 4 units to the right.
d) The base function is $y = 5^x$. There is a horizontal compression by a factor of $\frac{1}{3}$ and a horizontal translation of 2 units to the right.
3. See table below.
4. a) There is a horizontal compression of factor $\frac{1}{2}$ and a reflection in the x-axis.
b) There is a vertical stretch of factor 5, a reflection in the y-axis, and a horizontal translation 3 units to the right.
c) There is a vertical stretch of factor 4, a horizontal compression of factor $\frac{1}{3}$, a reflection in the x-axis, a horizontal translation of 3 units to the left, and a vertical translation of 6 units down.

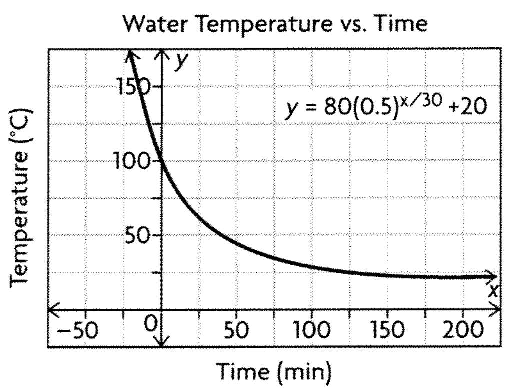
Function	y-intercept	Asymptote	Domain	Range
$y = 3^x + 3$	4	$y = 3$	$x \in \mathbf{R}$	$y > 3, y \in \mathbf{R}$
$y = 3^{x+3}$	27	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$
$y = \frac{1}{3}(3^x)$	$\frac{1}{3}$	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$
$y = 3^{\frac{x}{3}}$	1	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$
$y = -3(4^{x+1})$	-12	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$
$y = 2\left(\frac{1}{2}\right)^{2x} + 3$	5	$y = 3$	$x \in \mathbf{R}$	$y > 3, y \in \mathbf{R}$
$y = 7(0.5^{x-4}) - 1$	111	$y = -1$	$x \in \mathbf{R}$	$y > -1, y \in \mathbf{R}$
$y = 5^{3x-6}$	6.4×10^{-5}	$y = 0$	$x \in \mathbf{R}$	$y > 0, y \in \mathbf{R}$

5.

Function	Transformations	y-intercept	Asymptote	Domain	Range
$0.5f(-x) + 2$	• vertical compression by a factor of $\frac{1}{2}$ • reflection in the y-axis • translation of 2 units up	2.5	$y = 2$	$x \in \mathrm{R}$	$y > 2, y \in \mathrm{R}$
$-f(0.25x + 1) - 1$	• reflection in the x-axis • horizontal stretch of 4 • translation 1 down and 4 left	-5	$y = -1$	$x \in \mathrm{R}$	$y < -1, y \in \mathrm{R}$
$-2f(2x - 6)$	• reflection in the x-axis • vertical stretch of 2 • horizontal compression by factor of $\frac{1}{2}$ • translation 3 units right	$\frac{-2}{4^6}$	$y = 0$	$x \in \mathrm{R}$	$y < 0, y \in \mathrm{R}$
$f(-0.5x + 1)$	• reflection in the y-axis • horizontal stretch of 2 • horizontal translation of 2 units right	4	$y = 0$	$x \in \mathrm{R}$	$y > 0, y \in \mathrm{R}$

6. Both functions have the y-intercept of 1 and the horizontal asymptote of $y = 0$. Their domains are all real numbers and their ranges are $y > 0$. $g(x) = 3^{2x}$ will increase at a faster rate.

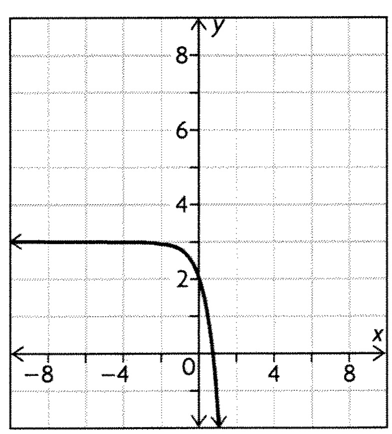
7. The y-intercept represents the initial temperature of 100°C. The asymptote is the room temperature and the lower limit on the temperature of the water.



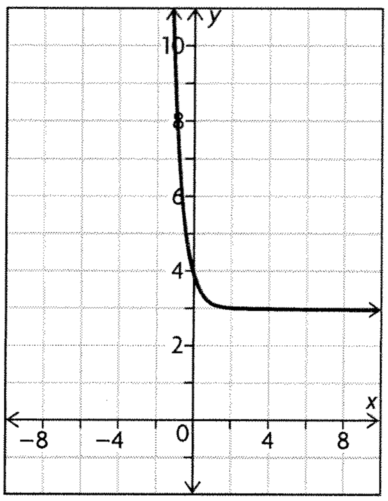
8. a) If the doubling time were changed to 9 hours, the exponent would change from $\frac{t}{3}$ to $\frac{t}{9}$. The graph would not rise as fast.

b) The domain is $t \geq 0, t \in \mathrm{R}$; the range is $N > N_0 \in \mathrm{R}$.

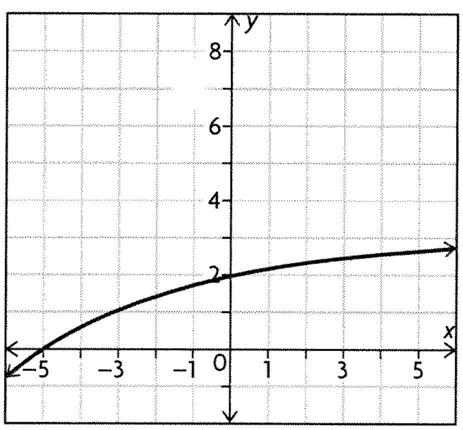
9. a) iii; since the base function is $y = \left(\frac{1}{4}\right)^x$, the graph will be quickly decreasing. There will also be a reflection in both the x-axis, and the y-axis, making (iii) the correct choice.



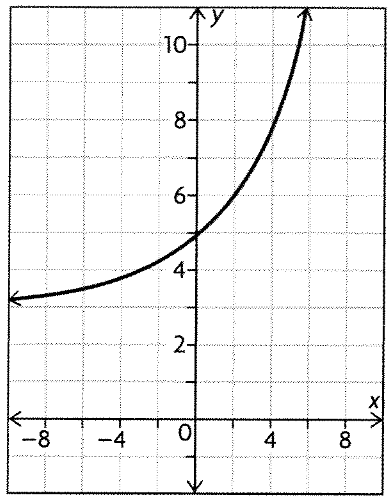
b) ii; since the base function is $y = \left(\frac{1}{4}\right)^x$, the graph will be quickly decreasing, but there will not be any reflection, so (ii) is the correct choice.



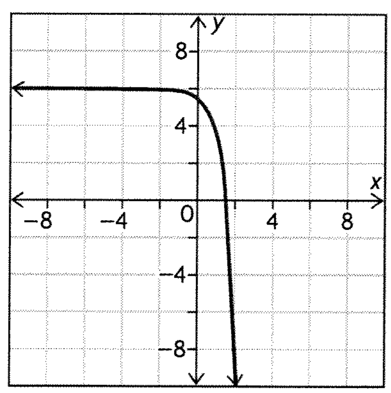
c) iv; since the base function is $y = \left(\frac{5}{4}\right)^x$, the graph will be quickly increasing. There will also be a reflection in both the x-axis and the y-axis, making (iv) the correct choice.



d) i; since the base function is $y = \left(\frac{5}{4}\right)^x$, the graph will be quickly increasing, but there will not be any reflection, so (i) is the correct choice.

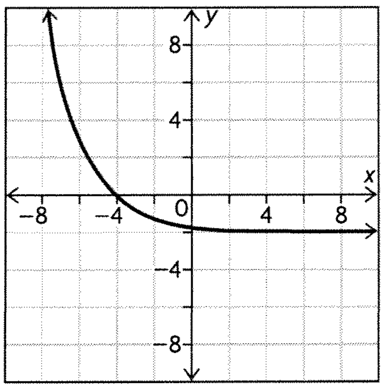


10. a) From an original graph of $y = 2^x$, there is a reflection in the x-axis, a horizontal compression of factor 2, and a vertical translation of 6 up (since the original y-intercept, after the reflection, would be -1). Therefore, a possible equation for the transformation is $y = -2^{2x} + 6$.

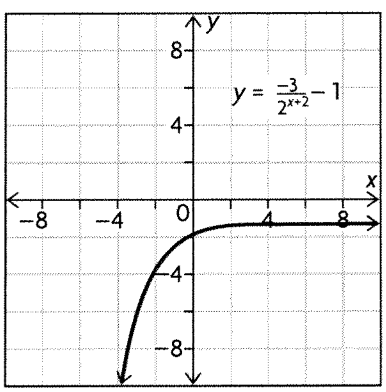


b) From an original graph of $y = 2^x$, there is a reflection in the y-axis, a horizontal translation of 3 units to the right, and a vertical translation of 2 down (since the original y-intercept would be 1). Therefore, a possible equation for the transformation is $y = 2^{-x-3} - 2$.

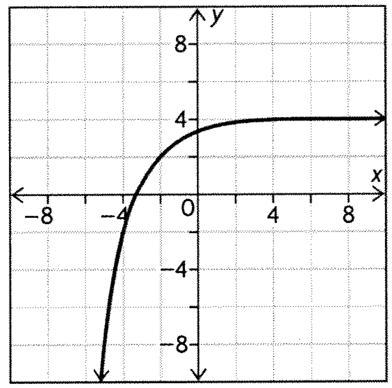
11. To transform $f(x) = 2^{x+1} + 5$ to $g(x) = \frac{1}{4}(2^x)$, translate down 5 units, translate 1 unit to the right, and vertically compress by a factor of $\frac{1}{4}$.



12. The function $f(x) = \frac{-3}{2^{x+2}} - 1$ is equal to the function $f(x) = -3(2^{-(x+2)}) - 1$. The base of this function is $f(x) = 2^x$, and it has the following transformations: a reflection in the x-axis, a vertical stretch of factor 3, a reflection in the y-axis, a horizontal translation 2 units to the left, and a vertical translation 1 unit down.



13. The function $g(x) = 4 - 2\left(\frac{1}{3}\right)^{-0.5x+1}$ is equal to the function $g(x) = -2\left(\frac{1}{3}\right)^{-0.5x+1} + 4$. The base of this function is $g(x) = \left(\frac{1}{3}\right)^x$, and it has the following transformations: a reflection in the x-axis, a vertical stretch of factor 2, a reflection in the y-axis, a horizontal translation of 2 units to the right, and a vertical translation of 4 units up.



14. Since $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$, $\left(\frac{3}{2}\right)^{2x-2}$, or $\left(\frac{3}{2}\right)^{2(x-1)}$ is equal to $\left(\frac{9}{4}\right)^{x-1}$. Therefore, the only transformations necessary to transform $m(x) = -\left(\frac{3}{2}\right)^{2x-2}$ to $n(x) = -\left(\frac{9}{4}\right)^{-x+1} + 2$ area reflection in the y-axis and a vertical translation of 2 units up.

4.7 Applications Involving Exponential Functions, pp. 261–264

1. a) $A = 250(1.05)^{10}$
$= 407.22$

b) $P = 9000\left(\frac{1}{2}\right)^8$
$= 35.16$

c) $500 = N_0(1.25)^{1.25}$
$N_0 = \frac{500}{1.25^{1.25}}$
$= 378.30$

d) $625 = P(0.71)^9$
$P = \frac{625}{(0.71)^9}$
$= 13\,631.85$

2.

Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
$V = 20(1.02)^t$	Growth	20	2%
$P = (0.8)^n$	Decay	1	−20%
$A = 0.5(3)^x$	Growth	0.5	200%
$Q = 600\left(\frac{5}{8}\right)^w$	Decay	600	−37.5%

3. a) The initial population is 1250; it is the value for a in the general exponential function.
b) The growth rate is 3%; it is the base of the exponent minus 1.
c) $P(11) = 1250(1.03)^{11}$
$= 1730$
d) The population reaches 2000 persons in the first year n such that
$2000 = 1250(1.03)^n$
$1.03^n = \frac{2000}{1250}$
$= 1.6$
The first n satisfying this equation is $n = 16$, so the population will reach 2000 persons in 16 years. This is the year 2012.
4. a) The initial value $1500; it is the value for a in the general exponential function.
b) The growth rate is −5%; it is the base of the exponent minus 1.
c) $V(24) = 1500(0.95)^m$
$= \$437.98$
d) The computer's worth falls below $900 in the first month m such that
$900 = 1500(0.95)^m$
$0.95^m = \frac{900}{1500}$
$= 0.6$
The first m satisfying this equation is $m = 10$, so the computer's worth will fall below $900 in 10 months.
5. a) The growth rate is 6%.
b) The initial amount is $1000.
c) There are 15 growth periods.
d) An equation that models the growth of the investment is $V(n) = 1000(1.06)^n$.
The value of the investment after 15 years is $V(15) = 1000(1.06)^{15}$, or $2396.56.
6. a) The exponent $\frac{t}{10}$ reflects the doubling period of 10 hours.
b) The base 2 represents the population doubling in number (100% growth rate).
c) 500 is the initial population.
d) $P(12) = 500(2^{\frac{12}{10}})$
$= 1149$ bacteria
e) $P(24) = 500(2^{\frac{24}{10}})$
$= 2639$ bacteria
f) The time at which the population first exceeds 2000 will be the first hour t such that
$2000 = 500(2^{\frac{t}{10}})$
$2^{\frac{t}{10}} = \frac{2000}{500}$
$= 4$
Since 2^2 is 4, t is 20. So the population will exceed 2000 at 20 hours from the starting time, or at 8:00 A.M
7. c and d; both c and d have bases between 0 and 1, so they represent exponential decay.
8. a) $P(10) = 12(1.025^{10})$
$= 15\,361$ thousand people
b) The population will double when it reaches 24 000, and this will occur in the first year n such that
$24 = 12(1.025^n)$, or when
$1.025^n = 2$
The first n satisfying this equation is $n = 29$, so the population will double in 29 years.
c) The number of years ago n that the population was 8000 will be the first n such that
$8 = 12(1.025^n)$, or when
$1.025^n = \frac{8}{12}$
$= \frac{2}{3}$
The first n satisfying this equation is $n = -17$, so the population was 8000 17 years ago.
d) The domain is $\{n \in \mathrm{R}\}$; the range is $\{P \in \mathrm{R} \mid P \geq 0\}$.
9. a) The initial temperature of the sandwich was $T(0)$, which is equal to 63 + 19, or 82°C.
b) $T(20) = 63(0.5)^{\frac{20}{10}} + 19$
$= 63(0.5)^2 + 19$
$= 35$°C

c) The number of minutes t it took for the sandwich to reach an internal temperature of 30°C is the first t such that

$$30 = 63(0.5)^{\frac{t}{10}} + 19, \text{ or when}$$

$$(0.5)^{\frac{t}{10}} = \frac{30 - 19}{63} = \frac{11}{63}$$

The closest value of t satisfying this equation is $t = 25$, so it took about 25 minutes for the sandwich to reach an internal temperature of 30°C.

10. a) $C(w) = 100(0.99)^w$

100 refers to the percent of the colour at the beginning. 0.99 refers to the fact that 1% of the colour is lost during every wash, and w refers to the number of washes.

b) $P(t) = 2500(1.005)^t$

2500 refers to the initial population. 1.005 refers to the fact that the population grows 0.5% every year, and t refers to the number of years after 1990.

c) $P = P_0(2)^t$

P_0 refers to the initial population. 2 refers to the fact that the population doubles in one day, and t refers to the number of days.

11. a) The growth rate is 100% per hour.

b) The population in t hours can be determined by the equation $P(t) = 80(2^t)$ since the growth rate is 100% (making the exponent's base $1 + 1$, or 2).

c) $P(6) = 8(2^6)$
$= 5120$ cells

d) $P(1.5) = 8(2^{1.5})$
$= 226$ cells

e) $1\,000\,000 = 80(2^t)$

$$2^t = \frac{1\,000\,000}{80} = 12\,500$$

$t = 13.6$ hours

f) The domain is $\{t \in \mathrm{R} \mid t \geq 0\}$; the range is $\{P \in \mathrm{R} \mid P \geq 80\}$

12. a) $V(t) = 5(1.06^t)$, where t is the number of years since 1990.

b) The increase in value of the card in the 4th year after it was purchased is equal to the value of the card after the 4th year minus the value of the card after the 3rd year.

$$V(4) - V(3) = 5(1.06^4) - 5(1.06^3) = 6.31 - 5.95 = \$\,0.36$$

c) The increase in value of the card in the 20th year after it was purchased is equal to the value of the card after the 20th year minus the value of the card after the 19th year.

$$V(20) - V(19) = 5(1.06^{20}) - 5(1.06^{19}) = 16.04 - 15.13 = \$0.91$$

13. a) $I(d) = 100(0.91^d)$, where d is the depth per metre.

b) $I(7.5) = 100(0.91^{7.5})$
$= 49.3\%$

14. a) $P(n) = 100(0.01^n)$, where n is the number of applications.

b) The number n of applications required to eliminate all 1 billion germs will be the smallest n such that $10^{10}(0.01^n) < 1$.

$$10^{10}(0.01^n) = 10^{10}(10^{-2n}) = 10^{10-2n}$$

$10^{10-2n} < 1$ when n is greater than or equal to 6, so at least 6 applications will be required.

15. $8400(1 + n)^{15} = 12\,500$

$$(1 + n)^{15} = \frac{12\,500}{8400} = \frac{125}{84}$$

$$1 + n = \sqrt[15]{\frac{125}{8}} = 1.027$$

So the average annual growth n of this town's population is about 2.7%.

16. a) $P(t) = 200(1.75^{\frac{t}{3}})$, where t is the number of hours after 9 A.M.

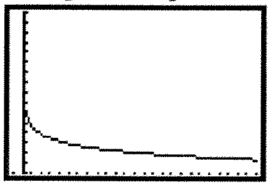
b) 200 refers to the initial count of yeast cells. 1.75 refers to the fact that the cells grow by 75% every 3 hours, and $\frac{t}{3}$ refers to the fact that the cells grow every 3 hours.

17. a) This may be an example of exponential growth.

b) A possible equation to model the growth is $f(x) = 4.25^x$, since $f(2) \doteq 18.06$ and $f(3) \doteq 76.77$, which approximates the data.

c) There are too few pieces of data to make a model, and the number of girls born may not be the same every year.

18. a) The graph is a quickly decreasing curve with a horizontal asymptote of $y = 0$. It is an example of exponential decay.



b) $R(24) = \dfrac{100}{1 + 1.08(24)^{0.21}}$

$= \dfrac{100}{3.1}$

$= 32.3\%$

Chapter Review, pp. 267–269

1. a) If $x > 1$, then $x^2 > x^{-2}$, since x^2 is greater than 1 and x^{-2}, or $\frac{1}{x^2}$, is less than 1.

b) yes; if $-1 < x < 1$ and $x \neq 0$, then x^{-2}, or $\frac{1}{x^2}$, is greater than 1 and x^2 is less than 1.

2. a) $(-7)^3(-7)^4 = (-7)^{3-4}$

$= (-7)^{-1}$

$= \dfrac{-1}{7}$

b) $\dfrac{(-2)^8}{(-2)^3} = (-2)^{8-3}$

$= (-2)^5$

$= -32$

c) $\dfrac{(5)^{-3}(5)^6}{5^3} = \dfrac{5^{-3+6}}{5^3}$

$= \dfrac{5^3}{5^3}$

$= 5^0$

$= 1$

d) $\dfrac{4^{-10}(4^{-3})^6}{(4^{-4})^8} = \dfrac{4^{-10}4^{-18}}{4^{-32}}$

$= \dfrac{4^{-28}}{4^{-32}}$

$= 4^{-28+32}$

$= 4^4$

$= 256$

e) $(11)^9\left(\dfrac{1}{11}\right)^7 = (11)^9(11^{-1})^7$

$= (11)^9(11)^{-7}$

$= 11^2$

$= 121$

f) $\left[\dfrac{(-3)^7(-3)^4}{(-3^4)^3}\right]^{-3} = \left[\dfrac{(-3)^{7+4}}{(-3)^{12}}\right]^{-3}$

$= \left[\dfrac{(-3)^{11}}{(-3)^{12}}\right]^{-3}$

$= ((-3)^{11-12})^{-3}$

$= ((-3)^{-1})^{-3}$

$= (-3)^3$

$= -27$

3. a) $\sqrt[3]{x^7} = x^{\frac{7}{3}}$

b) $y^{\frac{8}{5}} = \sqrt[5]{y^8}$

c) $\left(\sqrt{p}\right)^{11} = (p^{\frac{1}{2}})^{11}$

$= p^{\frac{11}{2}}$

d) $m^{1.25} = m^{\frac{5}{4}}$

$= \sqrt[4]{m^5}$

4. a) $\left(\dfrac{2}{5}\right)^{-3} = \dfrac{1}{\left(\frac{2}{5}\right)^3}$

$= \dfrac{1}{\left(\frac{8}{125}\right)}$

$= \dfrac{125}{8}$

b) $\left(\dfrac{16}{225}\right)^{-0.5} = \left(\dfrac{16}{225}\right)^{-\frac{1}{2}}$

$= \dfrac{1}{\sqrt{\frac{16}{225}}}$

$= \dfrac{1}{\left(\frac{4}{15}\right)}$

$= \dfrac{15}{4}$

c) $\dfrac{(81)^{-0.25}}{\sqrt[3]{-125}} = \dfrac{(81)^{-\frac{1}{4}}}{\sqrt[3]{-125}}$

$= \dfrac{1}{\sqrt[3]{-125}\sqrt[4]{81}}$

$= \dfrac{1}{(-5)(3)}$

$= \dfrac{-1}{15}$

d) $\left(\sqrt[3]{-27}\right)^4 = (-3)^4$

$= 81$

e) $\left(\sqrt[5]{-32}\right)\left(\sqrt[6]{64}\right)^5 = (-2)(2)^5$
$= (-2)(32)$
$= -64$

f) $\sqrt[6]{((-2)^3)^2} = \sqrt[6]{(-8)^2}$
$= \sqrt[6]{64}$
$= 2$

5. a) $a^{\frac{3}{2}}(a^{\frac{-3}{2}}) = a^{\frac{3}{2}-\frac{3}{2}}$
$= a^0$
$= 1$

b) $\frac{b^{0.8}}{b^{-0.2}} = b^{0.8-(-0.2)}$
$= b^{1.0}$
$= b$

c) $\frac{c(c^{\frac{5}{6}})}{c^2} = \frac{c^{1+\frac{5}{6}}}{c^2}$
$= c^{\frac{11}{6}-2}$
$= c^{\frac{11}{12}-\frac{12}{11}}$
$= c^{\frac{-1}{6}}$
$= \frac{1}{c^{\frac{1}{6}}}$

d) $\frac{d^{-5}d^{\frac{11}{2}}}{(d^{-3})^2} = \frac{d^{-5+\frac{11}{2}}}{d^{-6}}$
$= \frac{d^{\frac{-10}{2}+\frac{11}{2}}}{d^{-6}}$
$= \frac{d^{\frac{1}{2}}}{d^{-6}}$
$= d^{\frac{1}{2}+6}$
$= d^{\frac{1}{2}+\frac{12}{2}}$
$= d^{\frac{13}{2}}$

e) $((e^{-2})^{\frac{7}{2}})^{-2} = (e^{-7})^{-2}$
$= e^{14}$

f) $((f^{\frac{-1}{6}})^{\frac{6}{5}})^{-1} = (f^{\frac{-1}{5}})^{-1}$
$= f^{\frac{1}{5}}$

6. $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ for $a > 0$ and $b > 0$ since a counterexample can be given.
For example, if $a = 9$ and $b = 16$, then $\sqrt{a+b}$ is $\sqrt{9+16}$, which is equal to $\sqrt{25}$, or 5.
However, $\sqrt{a} + \sqrt{b}$ is $\sqrt{9} + \sqrt{16}$, which is equal to $3 + 4$, or 7, and $5 \neq 7$.

7. a) $(5x)^2(2x)^3 = (25x^2)(8x^3)$
$= 200x^5$

For $x = -2$, $200x^5$ is $200(-2)^5$, which is equal to $200(-32)$, or -6400.

b) $\frac{8m^{-5}}{(2m)^{-3}} = \frac{8m^{-5}}{2^{-3}m^{-3}}$
$= \frac{2^3m^{-5}}{2^{-3}m^{-5}}$
$= 2^{3-(-3)}m^{5-(-3)}$
$= 2^6m^{-2}$
$= 64m^{-2}$
$= \frac{64}{m^2}$

For $m = 4$, $\frac{64}{m^2}$ is $\frac{64}{16}$, or 4.

c) $\frac{2w(3w^{-2})}{(2w)^2} = \frac{6w^{1-2}}{4w^2}$
$= \frac{3w^{-1}}{2w^2}$
$= \frac{3}{2}w^{-1-2}$
$= \frac{3}{2w^3}$

For $w = -3$, $\frac{3}{2w^3}$ is $\frac{3}{2(-3)^3}$, which is equal to $\frac{3}{2(-27)}$, or $\frac{-1}{18}$.

d) $\frac{(9y)^2}{(3y^{-1})^3} = \frac{81y^2}{27y^{-3}}$
$= 3y^{2-(-3)}$
$= 3y^5$

For $y = -2$, $3y^5$ is $3(-2)^5$, which is equal to $3(-32)$, or -96.

e) $(6(x^{-4})^3)^{-1} = (6(x^{-12}))^{-1}$
$= (6x^{-12})^{-1}$
$= 6^{-1}x^{(-12)(-1)}$
$= \frac{x^{12}}{6}$

For $x = -2$, $\frac{x^{12}}{6}$ is $\frac{(-2)^{12}}{6}$, which is equal to $\frac{4096}{6}$ or $\frac{2048}{3}$.

f) $\frac{(-2x^{-2})^3(6x)^2}{2(-3x^{-1})^3} = \frac{(-8x^{-6})(36x^2)}{2(-27x^{-3})}$
$= \frac{-288x^{-4}}{-54x^{-3}}$
$= \frac{16x^{-4-(-3)}}{3}$

$$= \frac{16x^{-1}}{3}$$
$$= \frac{16}{3x}$$

For $x = \frac{1}{2}$, $\frac{16}{3x}$ is $\frac{16}{\left(\frac{3}{2}\right)}$, or $\frac{32}{3}$.

8. a) $\sqrt[3]{27x^3y^9} = \sqrt[3]{27}\,\sqrt[3]{x^3}\,\sqrt[3]{y^9}$
$$= 3x^{\frac{3}{3}}y^{\frac{9}{3}}$$
$$= 3xy^3$$

b) $\sqrt{\frac{a^6b^5}{a^8b^3}} = \left(\frac{a^6}{a^8}\right)^{\frac{1}{2}}\left(\frac{b^5}{b^3}\right)^{\frac{1}{2}}$
$$= (a^{-2})^{\frac{1}{2}}(b^2)^{\frac{1}{2}}$$
$$= a^{-1}b$$
$$= \frac{b}{a}$$

c) $\frac{m^{\frac{3}{2}}n^{-2}}{m^{\frac{7}{2}}n^{\frac{-3}{2}}} = (m^{\frac{3}{2}-\frac{7}{2}})(n^{-2-(\frac{-3}{2})})$
$$= m^{\frac{-4}{2}}n^{\frac{-1}{2}}$$
$$= m^{-2}n^{\frac{-1}{2}}$$
$$= \frac{1}{m^2n^{\frac{1}{2}}}$$

d) $\frac{\sqrt[4]{x^{-16}(x^6)^{-6}}}{(x^4)^{\frac{-11}{2}}} = \frac{(x^{-16}x^{-36})^{\frac{1}{4}}}{x^{\frac{-44}{2}}}$
$$= \frac{(x^{-52})^{\frac{1}{4}}}{x^{-22}}$$
$$= \frac{x^{-13}}{x^{-22}}$$
$$= x^{-13-(-22)}$$
$$= x^9$$

e) $((-x^{0.5})^3)^{-1.2} = (-x^{1.5})^{-1.2}$
$$= -x^{-1.8}$$
$$= \frac{-1}{x^{1.8}}$$

f) $\frac{\sqrt{x^6(y^3)^{-2}}}{(x^3y)^{-2}} = \frac{\sqrt{x^6y^{-6}}}{x^{-6}y^{-2}}$
$$= \frac{(x^6y^{-6})^{\frac{1}{2}}}{x^{-6}y^{-2}}$$
$$= \frac{x^3y^{-3}}{x^{-6}y^{-2}}$$
$$= x^{3-(-6)}y^{-3-(-2)}$$
$$= x^9y^{-1}$$
$$= \frac{x^9}{y}$$

9. a)

x	y	First differences	Second differences
−5	−38		
		35	
0	−3		10
		45	
5	42		10
		55	
10	97		10
		65	
15	162		10
		75	
20	237		

Quadratic; since the second differences of the function are constant, the function is quadratic.

b)

x	y	First differences
0	−45	
		30
2	−15	
		30
4	15	
		30
6	45	
		30
8	75	
		30
10	105	

Linear; since the first differences of the function are constant, the function is linear.

c)

x	y	First differences	Second differences
1	13		
		30	
2	43		90
		120	
3	163		360
		480	
4	643		1440
		1920	
5	2563		5760
		7680	
6	10 243		

Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

d)

x	y	First differences	Second differences
−2	40		
		20	
−1	20		10
		10	
0	10		5
		5	
1	5		2.5
		2.5	
2	2.5		1.25
		1.25	
3	1.25		

Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

e)

x	y	First differences	Second differences
−2	2000	1000	500
−1	1000	500	250
0	500	250	125
1	250	125	62.5
2	125	62.5	
3	62.5		

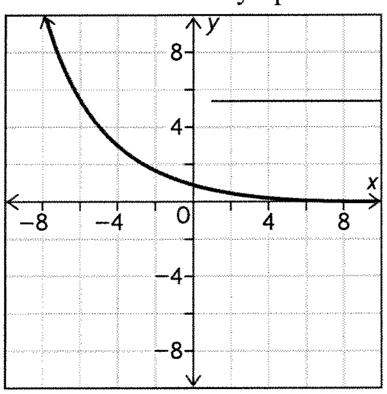
Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

f)

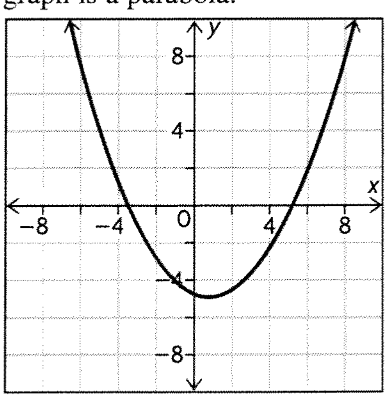
x	y	First differences	Second differences
0.2	−10.8	1.2	1.2
0.4	−9.6	2.4	2.4
0.6	−7.2	4.8	4.8
0.8	−2.4	9.6	9.6
1	7.2	19.2	
1.2	26.4		

Exponential; since neither the first differences nor the second differences of the function are constant, the function is exponential.

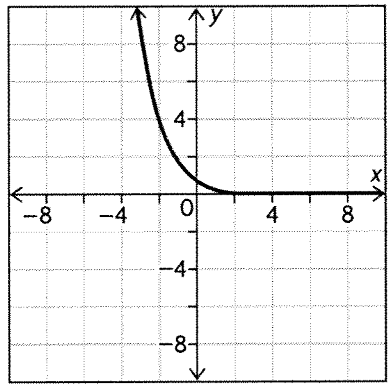
10. a) exponential; the function is exponential since its graph is a quickly decreasing curve with a horizontal asymptote of 0.



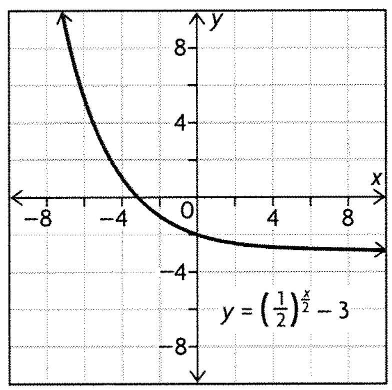
b) quadratic; the function is quadratic since its graph is a parabola.



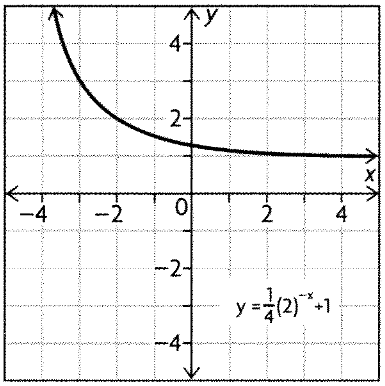
c) exponential; the function is exponential since its graph is a quickly decreasing curve with a horizontal asymptote of 0.



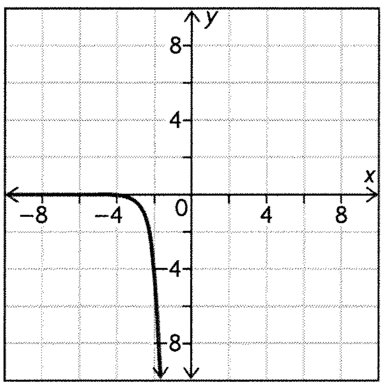
11. a) The base function is $y = \left(\frac{1}{2}\right)^x$. A horizontal stretch by a factor of $\frac{1}{\left(\frac{1}{2}\right)}$, or 2, along with a vertical translation of 3 down transforms $y = \left(\frac{1}{2}\right)^x$ into $y = \left(\frac{1}{2}\right)^{\frac{x}{2}} - 3$.



b) The base function is $y = 2^x$. A vertical compression by a factor of $\frac{1}{4}$, a reflection in the y-axis, and a vertical shift of one up transforms $y = 2^x$ into $y = \frac{1}{4}(2)^{-x} + 1$.



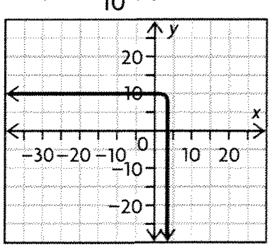
c) The base function is $y = 3^x$. A reflection in the x-axis, a vertical stretch of 2, a horizontal compression by a factor of 2, and a horizontal translation of 2 units to the left transforms $y = 3^x$ into $y = -2(3)^{2x+4}$.



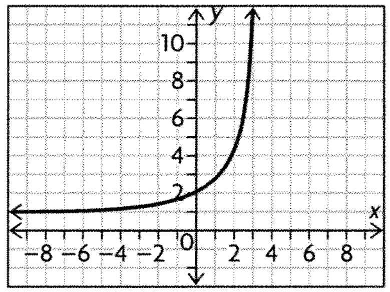
$y = -2(3)^{2x+4}$

d) The base function is $y = 5^x$. A reflection in the x-axis, a vertical compression of $\frac{1}{10}$, a horizontal compression by a factor of 3, and a horizontal translation of 3 units to the right, and a vertical translation 10 units up transforms $y = 5^x$ into $y = \frac{-1}{10}(5)^{3x-9} + 10$.

$y = -\frac{1}{10}(5)^{3x-9} + 10$



12. The function's y-intercept is 2, its asymptote is 1, and a possible equation for it is $y = 2^x + 1$.



13. a) $V(t) = 100(1.08)^t$ models exponential growth. Its initial value is 100, and its rate of growth is 8%.
b) $P(n) = 32(0.95)^n$ models exponential decay. Its initial value is 32, and its rate of decay is −5%.
c) $A(x) = 5(3)^x$ models exponential growth. Its initial value is 5, and its rate of growth is 200%.
d) $Q(n) = 600\left(\frac{5}{8}\right)^n$ represents exponential decay. Its initial value is 600, and its rate of decay is $\frac{3}{8}$, or −37.5%.

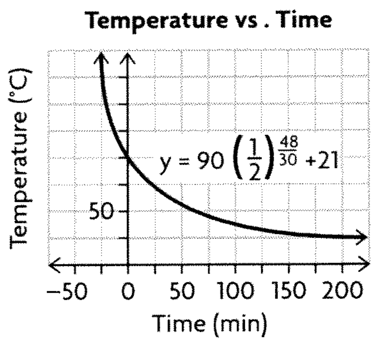
Function	Growth/ Decay	y-int	Growth/ Decay Rate
$V(t) = 100(1.08)^t$	growth	100	8%
$P(n) = 32(0.95)^n$	decay	32	−5%
$A(x) = 5(3)^x$	growth	5	200%
$Q(n) = 600\left(\frac{5}{8}\right)^n$	decay	600	−37.5%

14. a) The base of the exponent is $\frac{1}{2}$, which is less than 1, so this equation is an example of exponential decay.
b) The initial temperature of the coffee is given by the equation

$$\begin{aligned} T(48) &= 69\left(\frac{1}{2}\right)^0 + 21 \\ &= 69(1) + 21 \\ &= 90 \end{aligned}$$

So the initial temperature of the coffee was 90°C.

c)



d) After 48 minutes, the temperature of the coffee will be given by the equation

$$T(48) = 69\left(\frac{1}{2}\right)^{\frac{48}{30}} + 21$$
$$\doteq 69(0.33) + 21$$
$$\doteq 44°\text{C}$$

e) If the coffee cooled faster, the graph of the equation would have a greater horizontal compression, so the 30 in the exponent would be a lesser number.

f) If the coffee cooled faster, there would be a horizontal compression of the graph; that is, the graph would decrease more quickly.

15. a) The purchase price of the car is $28 000.

b) The annual rate of depreciation is 12.5%.

c) The car's value at the end of three years is given by the formula

$$V(3) = 28\,000(0.875)^3$$
$$= 18\,757.81$$

So the car is worth $18 757.81 at the end of 3 years.

d) The car's value at the end of 30 months, or 2.5 years, is given by the formula

$$V(2.5) = 28\,000(0.875)^{2.5}$$
$$= 20\,052.95$$

So the car is worth $20 052.95 at the end of 30 months.

e) The value the car loses in its first year is equal to the initial price of the car minus the price of the car after one year.
The price of the car after one year is given by the equation

$$V(1) = 28\,000(0.875)^1$$
$$= 24\,500$$

So value the car loses in its first year is 28 000 − 24 500, or $3500

f) The value the car loses in its fifth year is equal to the price of the car after 4 years minus the price of the car after 5 years.
The price of the car after 4 years is given by the equation

$$V(4) = 28\,000(0.875)^4$$
$$= 16\,413.09$$

The price of the car after 5 years is given by the equation

$$V(5) = 28\,000(0.875)^5$$
$$= 14\,361.45$$

So the value the car loses in its fifth year is 16 413.09 − 14 361.45, or $2051.64.

16. a) $P(n) = \frac{1}{3}(1.10)^n$

P is the percent of the pond covered by water lilies. $\frac{1}{3}$ refers to the fact that the pond is $\frac{1}{3}$ covered by lilies initially. 1.10 refers to the 10% increase in coverage each week, and n refers to the number of weeks. 10^9

b) $A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{4.5 \times 10^9}}$

A refers to the remaining amount of U_{238}. A_0 refers to the initial amount of U_{238}. $\frac{1}{2}$ refers to the half-life of the isotope, i.e. the rate of decay of 50%. t refers to the number of years. The fraction $\frac{1}{4.5 \times 10^9}$ in the exponent serves to stretch the graph horizontally so that the isotope decays 50% in the correct time period of 4.5×10^9 years.

c) $I(n) = 100(0.96)^n$

I refers to the intensity of light. 100 is the initial percent of intensity. 0.96 refers to the 4% decrease in intensity per gel, and n refers to the number of gels.

17. a) $P(n) = 45\,000(1.03)^n$

P refers to the population of the city. 45 000 is the initial population, i.e. the population in 1990. 1.03 refers to the growth rate of 3% per year, and n refers to the number of years.

b) The population of the city in 2007 is given by the equation

$$P(17) = 45\,000(1.03)^{17}$$
$$= 74\,378$$

So the population of the city in 2007 will be 74 378.

c) The year during which the population will have doubled is the smallest n such that $90\,000 \geq 45\,000(1.03)^n$, or $2 \geq (1.03)^n$. The smallest integer n that satisfies this inequality is $n = 24$, so the population will double during the year 1990 + 24, or 2014.
d) The growth rate r required for the population to take 10 years to double will be the r such that $90\,000 = 45\,000(1 + r)^{10}$, or

$$2 = (1 + r)^{10}$$
$$1 + r = \sqrt[10]{2}$$
$$= 1.072.$$

So r is 0.072, or 7.2%.

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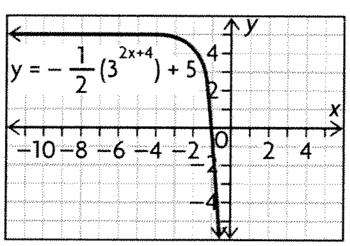
1. a) There is a variable in the exponent part of the equation, so it's an exponential function.
b)

x	$f(x)$	First differences	Second differences
−2	4.5		
−1	0.5	4	
0	−35.5	36	32
1	−359.5	324	288
2	−3276	2916.5	2592.5
3	−29 520	26 244	23 327.5

Since neither the first differences nor the second differences for $f(x)$ are constant, the function is exponential.
c) The transformations that map $g(x) = 3^x$ onto $f(x) = \frac{-1}{2}(3^{2x+4}) + 5$ are the following: a reflection in the x-axis, a vertical compression of $\frac{1}{2}$, a horizontal compression by a factor of 2, a horizontal translation of 4 units to the left, and a vertical translation of 5 units up. The equation of its asymptote is $y = 5$.



2. a) $(-5)^{-3} = \frac{1}{(-5)^3}$

$= \frac{-1}{125}$

b) $27^{\frac{2}{3}} = \sqrt[3]{27^2}$

$= \sqrt[3]{729}$

$= 9$

3. a) $(-3x^2y)^3(-3x^{-3}y)^2$

$= (-27x^6y^3)(9x^{-6}y^2)$

$= -243x^{6+(-6)}y^{3+2}$

$= -243x^0y^5$

$= -243y^5$

b) $\frac{(5a^{-1}b^2)^{-2}}{125a^5b^{-3}} = \frac{5^{-2}a^2b^{-4}}{5^3a^5b^{-3}}$

$= 5^{-2-3}a^{2-5}b^{-4-(-3)}$

$= 5^{-5}a^{-3}b^{-1}$

$= \frac{1}{5^5a^3b}$

$= \frac{1}{3125a^3b}$

c) $\sqrt[5]{\frac{1024(x^{-1})^{10}}{(2x^{-3})^5}} = \sqrt[5]{\frac{2^{10}x^{-10}}{2^5x^{-15}}}$

$= \sqrt[5]{2^{10-5}x^{-10-(-15)}}$

$= \sqrt[5]{2^5x^5}$

$= \sqrt[5]{2^5}\sqrt[5]{x^5}$

$= 2x$

d) $\frac{(8x^6y^{-3})^{\frac{1}{3}}}{(2xy)^3} = \frac{8^{\frac{1}{3}}x^2y^{-1}}{2^3x^3y^3}$

$= \frac{2x^2y^{-1}}{8x^3y^3}$

$= \frac{1}{4}x^{2-3}y^{-1-3}$

$= \frac{1}{4}x^{-1}y^{-4}$

$= \frac{1}{4xy^4}$

4. a) $I(n) = 100(0.964)^n$
b) The percentage of light left if three gels are used is given by the equation
$I(3) = 100(0.964)^3$
$= 89.6\%$
c) This is an example of exponential decay because as the number of gels increases the intensity of the light decreases exponentially.

5. a) $P(n) = 2(1.04)^n$, where P is the population in millions and n is the number of years since 1990.

b) The year during which the population will double is the smallest n such that

$4 \geq 2(1.04)^n$, or

$2 \geq (1.04)^n$.

The smallest integer n that satisfies this inequality is $n = 18$, so the population will double during the year 1990 + 18, or 2008.

6. D; the graph shows that there must be a reflection about the x-axis, leaving choices C and D. However, we can eliminate choice C because the y-intercept of the function is between -2 and -3, and the y-intercept of the function in will be greater than -1. This leaves us with choice D, which fits with the graph's y-intercept of about $-2\frac{2}{3}$.

7. The restriction on the value of n in $a^{\frac{1}{n}}$ if $a < 0$ is that $n \neq 0$ and n must be odd, since we cannot take the roots of negative numbers.