$$\doteq 77°$$

$$\frac{b}{\sin 77°} = \frac{2.1}{\sin 20°}$$

$$b \doteq (\sin 77°)\left(\frac{2.1}{\sin 20°}\right)$$

$$\doteq 6.0 \text{ cm}$$

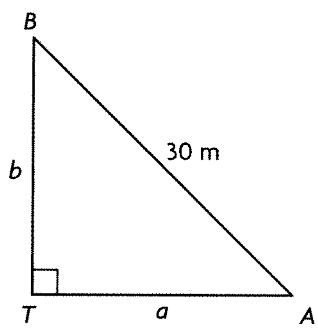
So for one triangle, $\angle C$ is about 83°, $\angle B$ is about 77°, and b is about 6.0 cm.
For the other triangle, $\angle C$ is about 180° − 83°, or 97°, $\angle B$ is about 180° − (97° + 20°), or 63°, and b is given by the formula

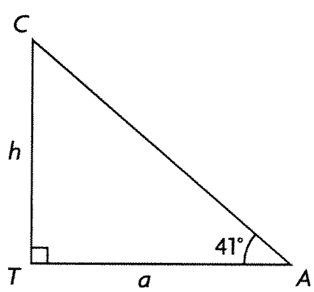
$$\frac{b}{\sin 63°} = \frac{2.1}{\sin 20°}$$

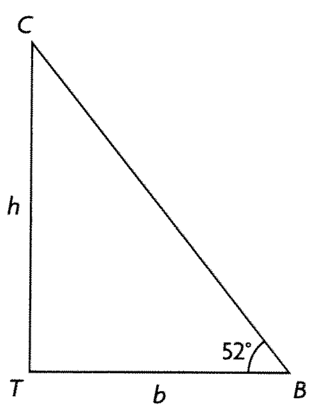
$$b \doteq (\sin 63°)\left(\frac{2.1}{\sin 20°}\right)$$

$$\doteq 5.5 \text{ cm}$$

8.







$$\tan 41° = \frac{h}{a}$$

$$a = \frac{h}{\tan 41°}$$

$$\tan 52° = \frac{h}{b}$$

$$b = \frac{h}{\tan 52°}$$

$$a^2 + b^2 = 30^2$$

$$a = \sqrt{30^2 - b^2}$$

$$\frac{h}{\tan 41°} = \sqrt{30^2 - b^2}$$

$$= \sqrt{30^2 - \left(\frac{h}{\tan 52°}\right)^2}$$

$$\frac{h^2}{(\tan 41°)^2} = 30^2 - \left(\frac{h}{\tan 52°}\right)^2$$

$$h^2 = (\tan 41°)^2\left(30^2 - \frac{h^2}{(\tan 52°)^2}\right)$$

$$= 30^2(\tan 41°)^2 - \frac{(\tan 41°)^2}{(\tan 52°)^2}h^2$$

$$\left(1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}\right)h^2 = 30^2(\tan 41°)^2$$

$$h^2 = \frac{30^2(\tan 41°)^2}{1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}}$$

$$h = \sqrt{\frac{30^2(\tan 41°)^2}{1 + \frac{(\tan 41°)^2}{(\tan 52°)^2}}}$$

$$\doteq 22 \text{ m}$$

CHAPTER 6: Sinusoidal Functions

NOTE: Answers are given to the same number of decimal points as the numbers in each question.

Getting Started, p. 344

1. a) x represents the number of times the price is reduced by \$2. The factor $(30 - 2x)$ represents the price of one T-shirt in terms of the number of times the price is reduced; the factor $(100 + 20x)$ represents the total number of T-shirts sold in terms of the number of times the price is reduced.

b) Since the revenue generated by T-shirt sales can be modelled by the function
$R(x) = (30 - 2x)(100 + 20x)$, for $R(x)$ to be equal to 0, one or both of the factors must be equal to 0. $100 + 20x$ can only be equal to 0 if x is negative, and this is not possible. Therefore, $30 - 2x = 0$.

$-2x = -30$

$x = 15$ times

c) Create a table with two columns. In the first column put the number of price reductions. In the second column, put the corresponding revenue. Continue until the revenue starts to decrease.

Number of Price Reductions	**Revenue (dollars)** $R(x) = (30 - 2x)(100 + 20x)$
0	3000
1	3360
2	3640
3	3840
4	3960
5	4000
6	3960

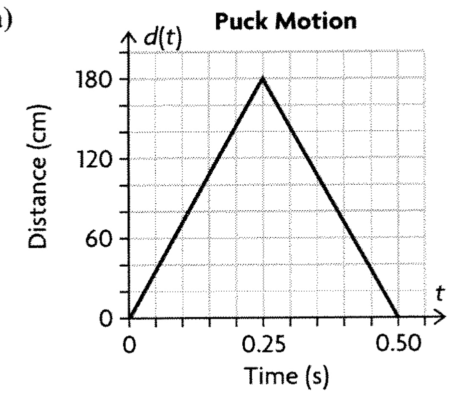
The price will have to be dropped 5 times to reach the maximum revenue.

d) From the table created in part c, it's apparent that the maximum revenue is \$4000.

e) The price of a T-shirt is equal to $30 - 2x$, so to obtain the maximum revenue, the T-shirts will sell for $30 - 2(5)$ or 20 dollars.

f) The number of T-shirts sold is equal to $100 + 20x$, so to obtain the maximum revenue, $100 + 20(5)$ or 200 T-shirts will be sold.

2. a)



The puck travelled 180 cm to the opposite end of the table in the first 0.25 s and 180 cm back in the next 0.25 s for a total distance of 360 cm.

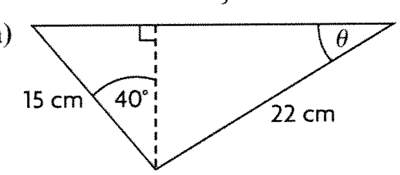
b) The puck was farthest away from where it was shot when it was 180 cm away, and this occurred after 0.25 s.

c) The puck travelled 180 cm in the first 0.25 s.

$$\frac{180 \text{ cm}}{0.25 \text{ s}} = \frac{720 \text{ cm}}{1 \text{ s}} = 720 \text{ cm/s}$$

d) The domain is all possible values of the time, or t, while the range is all possible values of the distance, or d. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 2.5\}$, and the range is $\{d \in \mathbf{R} \mid 0 \leq d \leq 180\}$.

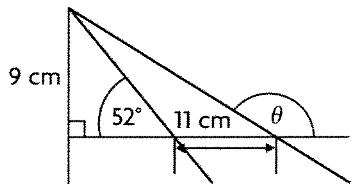
3. a)



Within the larger triangle there exist two smaller right triangles. The smaller right triangle with a hypotenuse of 15 cm has the measurements of two angles given: 40° and 90°. Therefore, the measurement of the unknown angle within that triangle is 50°. With that angle known, the Sine Law can be used to find the measurement of θ.

$$\frac{\sin 50°}{22} = \frac{\sin \theta}{15}$$
$$15 \times \sin 50° = 22 \times \sin \theta$$
$$15 \times 0.766 = 22 \times \sin \theta$$
$$11.491 = 22 \times \sin \theta$$
$$0.522 = \sin \theta$$
$$\theta = 32°$$

b)



Within the larger triangle there exist two smaller triangles, one of which is a right triangle. The right triangle has the measurements of two angles given: 52° and 90°. Therefore, the measurement of the unknown angle within that triangle is 38°. With that angle known, the Sine Law can be used to find the measurement of the unknown leg.

$$\frac{\sin 52°}{9} = \frac{\sin 38°}{x}$$
$$9 \times \sin 38° = x \times \sin 52°$$
$$9 \times 0.616 = x \times 0.788$$
$$5.541 = x \times 0.788$$
$$x = 7 \text{ cm}$$

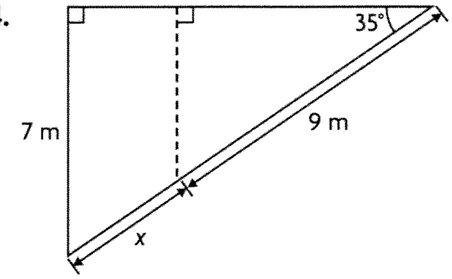
Now it can be determined that the measurements of the legs of the larger triangle are 9 cm and 18 cm, so by the Pythagorean Theorem, $9^2 + 18^2 = c^2$, and $c = 20$ cm. With the hypotenuse of the larger triangle known, the Sine Law can be used to find measurement of the angle supplementary to θ.

$$\frac{\sin 90°}{20} = \frac{\sin \beta}{9}$$
$$9 \times \sin 90° = 20 \times \sin \beta$$
$$9 \times 1 = 20 \times \sin \beta$$
$$9 = 20 \times \sin \beta$$
$$0.450 = \sin \beta$$
$$\beta = 26°$$

The angle β is supplementary to θ,.

$$\theta = 180° - \beta$$
$$= 180° - 26°$$
$$= 154°$$

4.

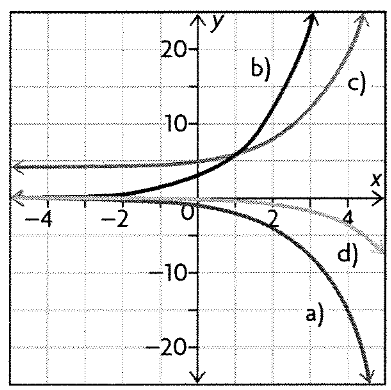


If the hypotenuse of the larger triangle is c, then the Sine Law can be used to find c.

$$\frac{\sin 35°}{7} = \frac{\sin 90°}{c}$$
$$7 \times \sin 90° = c \times \sin 35°$$
$$7 \times 1 = c \times 0.574$$
$$7 = c \times 0.574$$
$$c = 12.2 \text{ m}$$

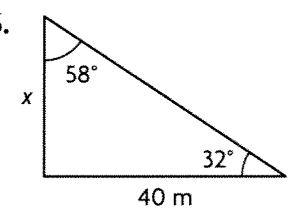
Since $c = 9 + x$, $12.2 = 9 + x$, and $x = 3.2$ m.

5.



a) The graph has been reflected in its axis.
b) The graph has been vertically stretched by a factor of 3 units.
c) The graph has been moved up 4 units.
d) The graph has been vertically stretched by a factor of 2 units, moved to the right 3 units, and reflected in its axis.

6.



Form a right triangle with the legs being the building and the shadow. Since the angle of elevation of the Sun was 32°, the measurement of the unknown angle within the triangle is 58°. The Sine Law can be used to determine the height of the building.

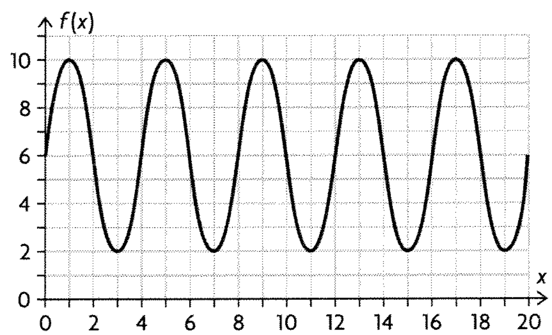
$$\frac{\sin 58°}{40} = \frac{\sin 32°}{x}$$
$$40 \times \sin 32° = x \times \sin 58°$$
$$40 \times 0.530 = x \times 0.848$$
$$21.197 = x \times 0.848$$
$$x = 25 \text{ m}$$

7. Answers will vary and may include the following: A vertical translation would move the graph up or down and result in the equation $y = x^2 + c$ or $y = x^2 - c$. A horizontal translation would move the graph left or right and result in the equation $y = (x + d)^2$ or $y = (x - d)^2$. A vertical stretch or compression would make the graph thinner or wider and result in the equation $y = ax^2$.

6.1 Periodic Functions and Their Properties, pp. 352–356

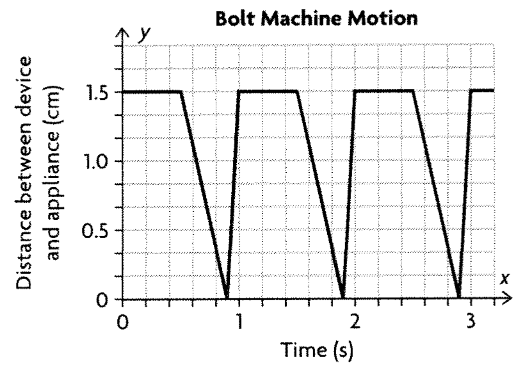
1. a) It is periodic because the cycle repeats.
b) It is not periodic because the cycle does not repeat.
c) It is periodic because the cycle repeats.
d) It is not periodic because the cycle does not repeat.

2.



The range is all possible values of $f(x)$. Since $f(x)$ oscillates between 2 and 10, the range is $\{f(x) \in \mathbf{R} \mid 2 \leq f(x) \leq 10\}$. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 3 to 7, for example, the period of the function is $7 - 3$, or 4. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 2 and the maximum is 10, the equation of the axis is $y = \frac{10 + 2}{2}$ or $y = 6$. The amplitude is the distance from the axis to the maximum. Since the axis is $y = 6$ and the maximum is 10, the amplitude is $10 - 6$ or 4. The amplitude is also half the distance between the maximum and minimum values. Since the minimum is 2 and the maximum is 10, the amplitude is $\frac{10 - 2}{2}$ or 4.

3.



a) The period is the time it takes the bolt machine to go through one complete cycle. Since the bolt machine goes through one complete cycle between about 0.95 s and 1.95 s, for example, the period is $1.95 - 0.95$ or 1 s.
b) The maximum distance d between the device and the appliance is the maximum value of d on the graph, which is 1.5 cm.
c) The range is all possible values of the distance d between the device and the appliance. Since d is always between 0 and 1.5 on the graph, the range is $\{d \in \mathbf{R} \mid 0 \leq d \leq 1.5\}$.
d) Since the bolt machine goes through one complete cycle every second, the time t required for it to go through five complete cycles is 5 s. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 5\}$.
e) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 0 and the maximum is 1.5, the equation of the axis is $y = \frac{1.5 + 0}{2}$ or $y = 0.75$.
f) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 0 and the maximum is 1.5, the amplitude is $\frac{1.5 - 0}{2}$ or 0.75 cm.
g) The horizontal component of graph: the device is not in motion; it remains fixed at 1.5 cm. The component of the graph with a negative slope: the device is approaching the

appliance and simultaneously attaching the bolt. The component of the graph with the positive slope: the device has finished attaching the bolt and is moving away from the appliance.

4. a) The function is periodic because it repeats at a regular interval. Since the function goes through one complete cycle between about $x = 11$ and $x = 17$, for example, the period is $17 - 11$, or 6.

b) The function is periodic because it repeats at a regular interval. Since the function goes through one complete cycle between about $x = 0$ and $x = 3$, for example, the period is $3 - 0$, or 3.

c) The function is not periodic because it does not repeat at a regular interval.

d) The function is not periodic because it does not repeat at a regular interval.

e) The function is periodic because it repeats at a regular interval. Since the function goes through one complete cycle between about $x = 5$ and $x = 10$, for example, the period is $10 - 5$, or 5.

f) The function is periodic because it repeats at a regular interval. Since the function goes through one complete cycle between about $x = 15$ and $x = 35$, for example, the period is $35 - 15$, or 20.

5. a) This would produce a periodic graph, since the height of the tooth of the chainsaw would rise and fall at a regular interval.

b) This would produce a periodic graph, since Alex's height above the ground would rise and fall at a regular interval.

c) This would not produce a periodic graph, since the cost of riding in the taxi would keep rising.

d) This would not produce a periodic graph, since the value of the Guaranteed Investment Certificate would keep rising.

e) This would not produce a periodic graph, since the ball would bounce a smaller height as time goes on.

f) This would produce a periodic graph, since the intensity of the antenna's signal would rise and fall at a regular interval as the antenna rotated.

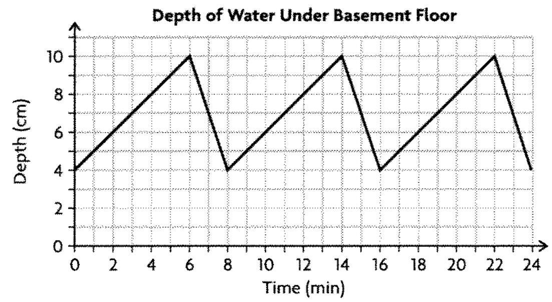
6. a) This table does not represent a periodic function, since there is no repeating cycle.

b) This table might represent a periodic function, since there is a repeating cycle.

c) This table does not represent a periodic function, since the values of y keep rising.

d) This table does not represent a periodic function, since although the values of y repeat in a cycle, the values of x are not evenly spaced.

7.



a) Yes, the function is periodic, since it repeats at a regular interval.

b) The pump turns on at 10 cm, since after the depth hits 10 cm, it begins to decrease.

c) The pump remains on for 2 min, since the depth decreases from 6 to 8 min and from 14 to 16 min, for example. $8 - 6 = 2$ min, and $16 - 14 = 2$ min.

d) The period of the function is 8 min, since the function goes through one complete cycle from 0 to 8 min and from 8 to 16 min, for example. $8 - 0 = 8$ min, and $16 - 8 = 8$ min.

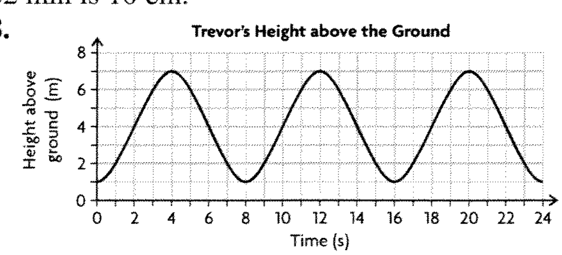
e) The range is all possible values of the depth, or d. Therefore, the range is $\{d \in \mathbf{R} \mid 4 \leq d \leq 10\}$

f) From the graph, at 3 min the depth is 7 cm.

g) The first time the depth of the water is 10 cm is at 6 min. Since the period of the function is 8 min, the depth of the water will be 10 cm every 8 min after that.

h) Since the period of the function is 8 min, and since the depth is 4 cm at the end of each 8 min interval, it can be determined that at 64 min, the depth will be 4 cm (64 is a multiple of 8). Moving back 2 min from the end of each 8 min interval, the depth is 10 cm, so the depth at 62 min is 10 cm.

8.



a) The function goes through one complete cycle from 8 to 16 s, for example, so the period of the function is 16 − 8 or 8 s. The period represents one revolution of the Ferris wheel.

b) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 7, the equation of the axis is $h = \frac{1 + 7}{2}$ or $h = 4$.

c) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 7, the amplitude is $\frac{7 - 1}{2}$ or 3 m.

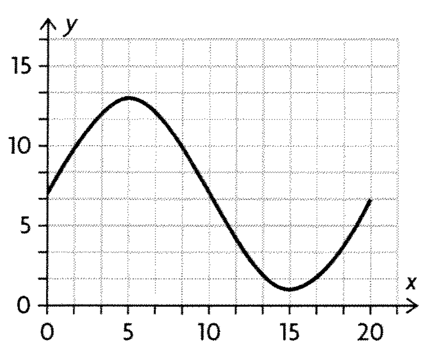
d) The range is all possible values of the height h above the ground. Since h is always between 1 and 7 on the graph, the range is $\{h \in \mathbf{R} \mid 1 \leq h \leq 7\}$.

e) Since the period of the function is 8 s, after 24 s, Trevor will be at the lowest height again at 24 + 8 or 32 s.

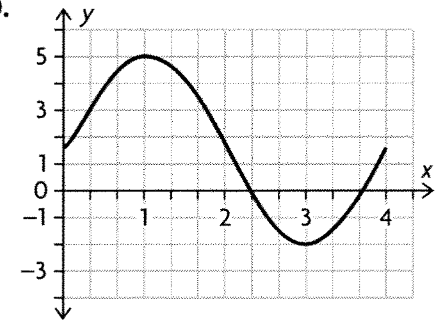
f) Trevor is at the top of the wheel when his height above the ground is at its maximum. From the graph, the first maximum occurs at 4 s, and since the period of the function is 8 s, a maximum occurs every 8 s after that. Therefore, Trevor is at the top of the wheel at 4 s, 12 s, 20 s, and 28 s, for example.

g) It's apparent from the graph that Trevor's height above the ground is 4 m at 2 s and 6 s after each minimum. Since a minimum occurs at 24 s, Trevor's height above the ground will be 4 m at 24 + 2 or 26 s, and at 24 + 6 or 30 s.

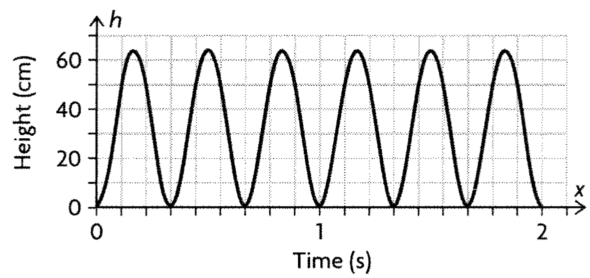
9.



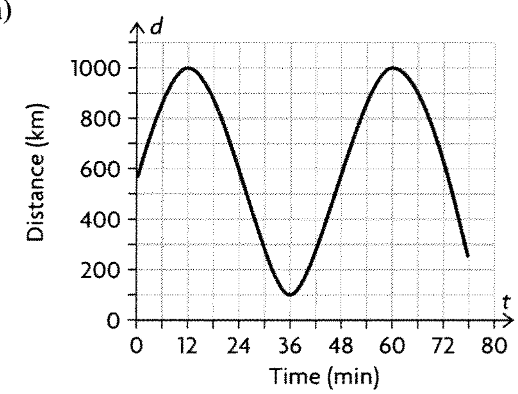
10.



11. Since Maria's riding at a speed of 21.6 km/h, she's riding at a speed of 21.6 × 100 000 or 2 160 000 cm/h. Therefore, she's riding at a speed of 2 160 000 ÷ 3600 or 600 cm/s, and in 15 s, she would travel 9000 cm. The circumference of her bicycle wheel is 64 × π or about 200 cm. Therefore, in 15 s, her bicycle wheel would make 9000 ÷ 200 or 45 revolutions. This means that her bicycle wheel would make 3 revolutions every second, so a graph that shows the stone's height above the ground would look like the following:



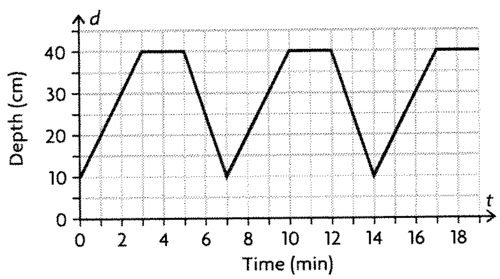
12. a)



b) Yes, the graph is periodic because it repeats at a regular interval.

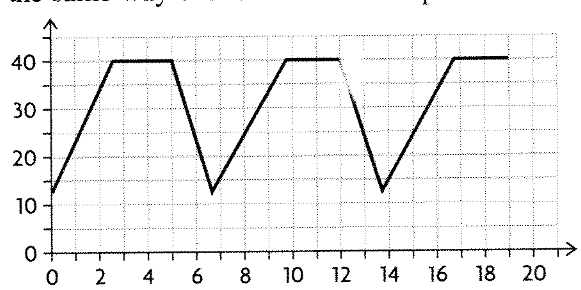
c) The period of the function is 48 min, since the function goes through one complete cycle from 0 to 48 min, for example. $48 - 0 = 48$ min. The period represents the time of one orbit of the spacecraft around Earth.
d) From the graph, the approximate distance between the spacecraft and Earth at 8 min is 900 km.
e) From the graph, the spacecraft is first farthest from Earth at $t = 12$ min, and since the period of the function is 48 min, the spacecraft is farthest from Earth every 48 min after that time.
f) The domain is all possible values of the time t. Since the period of the function is 48 min and this represents the time of one orbit of the spacecraft around Earth, if the spacecraft completes six orbits, the largest possible value of t is 48×6 or 288. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 288\}$.

13. a)



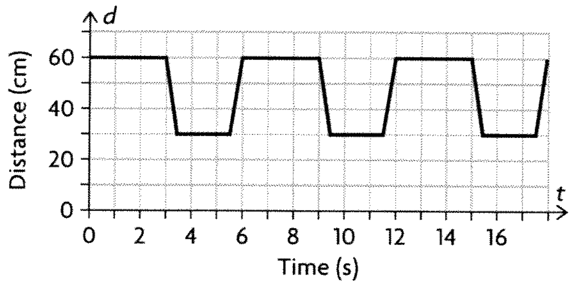
b) Yes, the graph is periodic because it repeats at a regular interval.
c) The period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between $t = 7$ and $t = 14$, for example, the period is $14 - 7$ or 7. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 10 and the maximum is 40, the equation of the axis is $d = \frac{40 + 10}{2}$ or $d = 25$.

The amplitude is half the distance between the maximum and minimum values. Since the minimum is 10 and the maximum is 40, the amplitude is $\frac{40 - 10}{2}$ or 15.
d) From the graph, the depth of the water increases from 10 cm to 40 cm as the time increases from 0 min to 3 min, for example. Therefore, the rate at which the depth of the water is increasing as the container is being filled is $\frac{40 - 10}{3 - 0}$ or 10 cm/min.
e) From the graph, the depth of the water decreases from 40 cm to 10 cm as the time increases from 5 min to 7 min, for example. Therefore, the rate at which the depth of the water is decreasing as the container is being drained is $\frac{40 - 10}{7 - 5}$ or 15 cm/min.
f) No, the container is never empty, since from the graph, it's apparent that the curve never intersects the t-axis, which represents a depth of 0 or an empty container.
14. A periodic function is a function that produces a graph that has a regular repeating pattern over a constant interval. It describes something that happens in a cycle, repeating in the same way over and over. Example:



The example is periodic because its pattern repeats every 7 units of the x-axis.

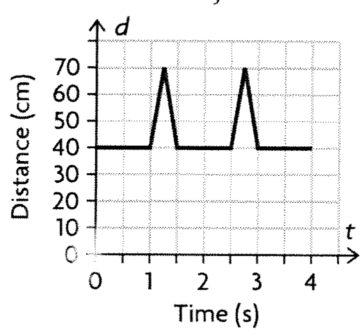
15. a)



b) The period of the function is 6 s, since the function goes through one complete cycle from 0 to 6 s, for example. $6 - 0 = 6$ s.

c) The range is all possible values of the distance d from the detector. Since d is always between 30 and 60 on the graph, the range is $\{d \in \mathbf{R} \mid 30 \leq d \leq 60\}$. The domain is all possible values of the time t. Since the period of the function is 6 s, if Denis repeats the process 3 times, the largest possible value of t is 6×3 or 18. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 18\}$.

16.



At time $t = 0$, the paddle is 40 cm in front of the CBR and doesn't move for 1 second. At 1 second, the paddle moves 30 cm further away from the CBR and then returns to its original position of 40 cm in front of the CBR at 1.5 seconds. For 1 second, the paddle doesn't move. At $t = 2.5$ seconds, the paddle moves 30 cm further away from the CBR and then returns to its original position of 40 cm in front of the CBR at $t = 3$ seconds where it remains for 1 second until 4 seconds.

6.2 Investigating the Properties of Sinusoidal Functions, pp. 363–364

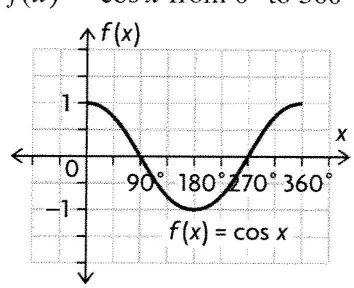
1. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -2 and the maximum is 4, the amplitude is $\frac{4 - (-2)}{2}$ or 3 m. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through eight complete cycles in 1440°, the period is $1440 \div 8$ or 180°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -2 and the maximum is 4, the equation of the axis is $y = \frac{4 + (-2)}{2}$ or $y = 1$.

b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -6 and the maximum is 2, the amplitude is $\frac{2 - (-6)}{2}$ or 4 m. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through two complete cycles in 1440°, the period is $1440 \div 2$ or 720°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -6 and the maximum is 2, the equation of the axis is $y = \frac{2 + (6)}{2}$ or $y = -2$.

2. a) If $h(x) = \sin(5x) - 1$, then $h(25°) = \sin(5 \times 25°) - 1$.

$$\begin{aligned}\sin(5 \times 25°) - 1 &= \sin(125°) - 1\\ &= 0.82 - 1\\ &= -0.18\end{aligned}$$

b) If $f(x) = \cos x$ and $f(x) = 0$, then $\cos x = 0$. Use a graphing calculator to graph the function $f(x) = \cos x$ from 0° to 360°.

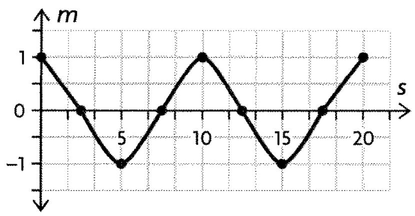


It's apparent from the graph that from 0° to 360°, $\cos x = 0$ at $x = 90°$ and $x = 270°$.

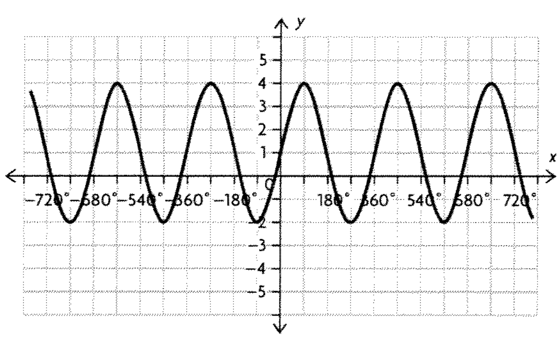
3. a) Create a table with two columns. In the first column put t from 0 s to 20 s in 2.5 s intervals. In the second column, put $h(t)$, which is equal to $\cos(36t)°$ m.

t (s)	$h(t) = \cos (36t)^\circ$ (m)
0	$\cos (36 \times 0)^\circ = \cos 0^\circ = 1$
2.5	$\cos (36 \times 2.5)^\circ = \cos 90^\circ = 0$
5	$\cos (36 \times 5)^\circ = \cos 180^\circ = -1$
7.5	$\cos (36 \times 7.5)^\circ = \cos 270^\circ = 0$
10	$\cos (36 \times 10)^\circ = \cos 360^\circ = 1$
12.5	$\cos (36 \times 12.5)^\circ = \cos 450^\circ = 0$
15	$\cos (36 \times 15)^\circ = \cos 540^\circ = -1$
17.5	$\cos (36 \times 17.5)^\circ = \cos 630^\circ = 0$
20	$\cos (36 \times 20)^\circ = \cos 720^\circ = 1$

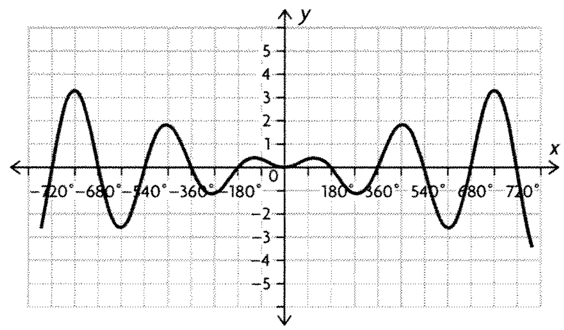
Plot the points obtained in the table and graph a sinusoidal function about the points.



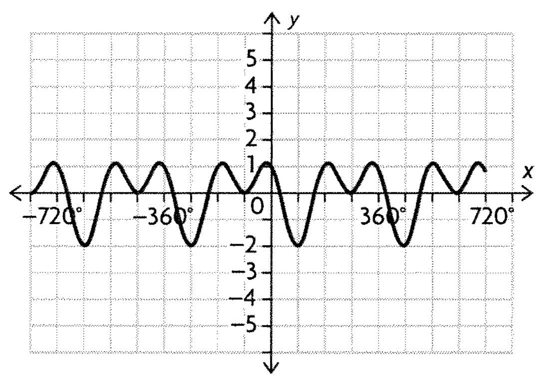
b) The period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 10$, for example, the period is $10 - 0$ or 10 s.
c) Since the displacement at 20 s is the same as the displacement at 0 s, the displacement at 20 s + 15 s or 35 s will be the same as the displacement at 0 s + 15 s or 15 s. The displacement at 15 s was −1 m, so the displacement at 35 s will also be −1 m.
d) From the graph, the displacement first reaches −0.8 m at about 4 s.
4. Consider the point (0, 0) the centre of a circle with radius 2, with the point (2, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 50° about (0, 0) from the point (2, 0) are (2 cos 50°, 2 sin 50°) or (1.29, 1.53).
5. a)



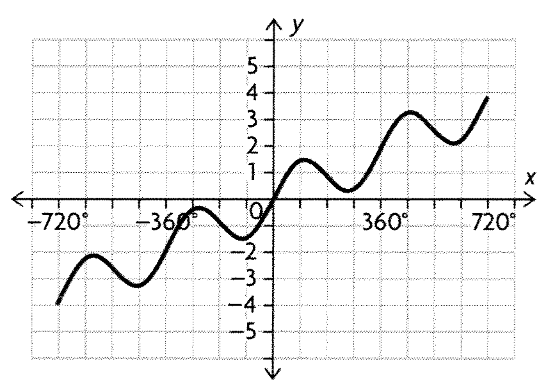
The function is periodic because its graph repeats at a regular interval. The function is sinusoidal because its graph looks like smooth symmetrical repeating waves.
b)



The function is not periodic because its graph does not repeat at a regular interval. The function is not sinusoidal because its graph does not look like smooth symmetrical repeating waves.
c)

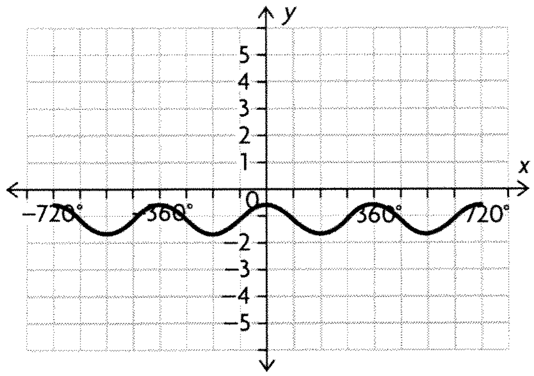


The function is periodic because its graph repeats at a regular interval. The function is not sinusoidal because its graph does not look like smooth symmetrical repeating waves.
d)



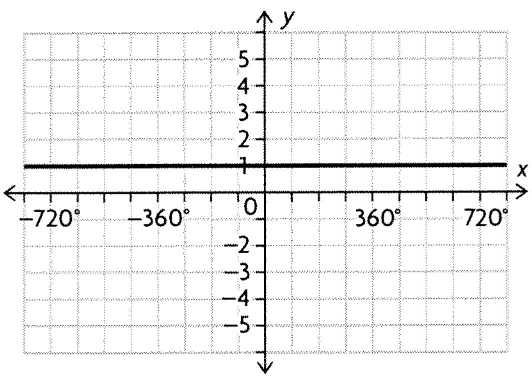
The function is not periodic because its graph does not repeat at a regular interval. The function is not sinusoidal because its graph does not look like smooth symmetrical repeating waves.

e)



The function is periodic because its graph repeats at a regular interval. The function is sinusoidal because its graph looks like smooth symmetrical repeating waves.

f)



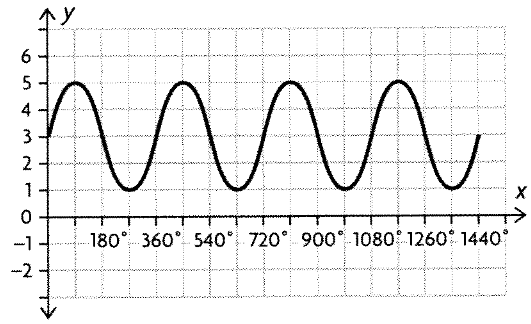
The function is not periodic because its graph does not repeat at a regular interval. The function is not sinusoidal because its graph does not look like smooth symmetrical repeating waves.

6. It cannot necessarily be concluded that a function that possesses sine or cosine in its equation is periodic or sinusoidal. In other words, nothing can be concluded.

7. a) If $g(x) = \sin x$, then $g(90°) = \sin 90° = 1$. This means that when $x = 90°$, $y = 1$, or that the sine (y-coordinate of a point on the unit circle) of 90° is 1.

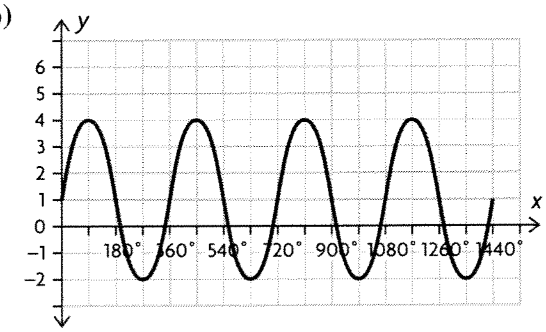
b) If $h(x) = \cos x$, then $h(90°) = \cos 90° = 0$. This means that when $x = 90°$, $y = 0$, or that the cosine (x-coordinate of a point on the unit circle) of 90° is 0.

8. a)



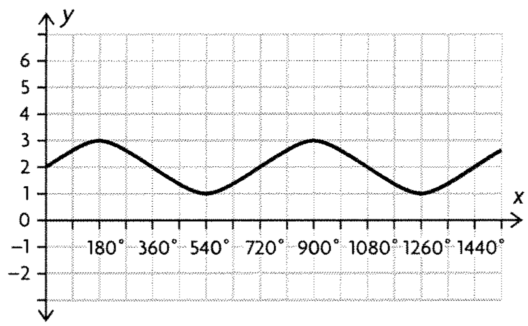
The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 5, the amplitude is $\frac{5-1}{2}$ or 2. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 360°, for example, the period of the function is $360 - 0$, or 360°. It's apparent from the graph that the increasing intervals are 0° to 90°, 270° to 450°, 630° to 810°, 990° to 1170°, and 1350° to 1440°. It's apparent from the graph that the decreasing intervals are 90° to 270°, 450° to 630°, 810° to 990°, and 1170° to 1350°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 5, the equation of the axis is $y = \frac{5+1}{2}$ or $y = 3$.

b)



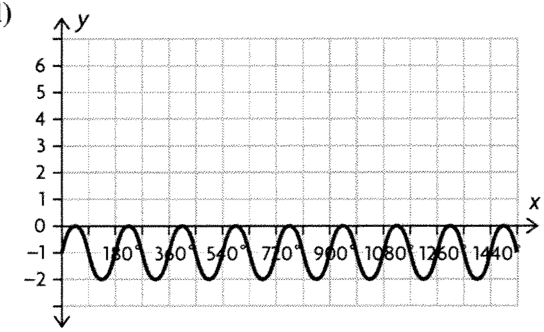
The amplitude is half the distance between the maximum and minimum values. Since the minimum is -2 and the maximum is 4, the amplitude is $\frac{4-(-2)}{2}$ or 3. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 360°, for example, the period of the function is 360 − 0, or 360°. It's apparent from the graph that the increasing intervals are 0° to 90°, 270° to 450°, 630° to 810°, 990° to 1170°, and 1350° to 1440°. It's apparent from the graph that the decreasing intervals are 90° to 270°, 450° to 630°, 810° to 990°, and 1170° to 1350°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −2 and the maximum is 4, the equation of the axis is $y = \frac{4 + (2)}{2}$ or $y = 1$.

c)



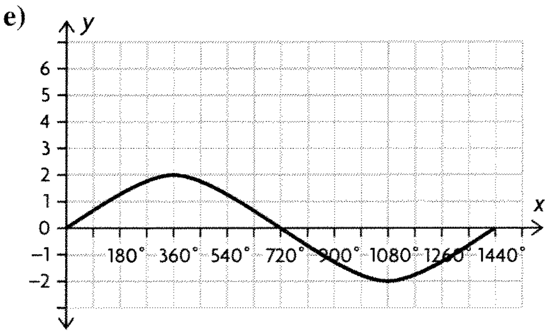
The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 3, the amplitude is $\frac{3 - 1}{2}$ or 1. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 720°, for example, the period of the function is 720 − 0, or 720°. It's apparent from the graph that the increasing intervals are 0° to 180°, 540° to 900°, and 1260° to 1440°. It's apparent from the graph that the decreasing intervals are 180° to 540° and 900° to 1260°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 3, the equation of the axis is $y = \frac{3 + 1}{2}$ or $y = 2$.

d)



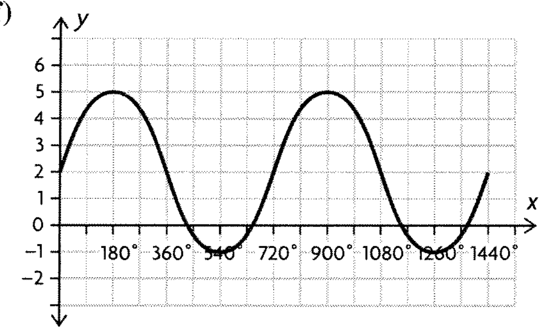
The amplitude is half the distance between the maximum and minimum values. Since the minimum is −2 and the maximum is 0, the amplitude is $\frac{0 - (-2)}{2}$ or 1. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 180°, for example, the period of the function is 180 − 0, or 180°. It's apparent from the graph that the increasing intervals are 0° to 45°, 135° to 225°, 315° to 405°, 495° to 585°, 675° to 765°, 855° to 945°, 1035° to 1125°, 1215° to 1305°, and 1395° to 1440°. It's apparent from the graph that the decreasing intervals are 45° to 135°, 225° to 315°, 405° to 495°, 585° to 675°, 765° to 855°, 945° to 1035°, 1125° to 1215°, and 1305° to 1395°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −2 and the maximum is 0, the equation of the axis is $y = \frac{0 + (-2)}{2}$ or $y = -1$.

e)



The amplitude is half the distance between the maximum and minimum values. Since the minimum is −2 and the maximum is 2, the amplitude is $\frac{2 - (-2)}{2}$ or 2. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 1440°, for example, the period of the function is 1440 − 0, or 1440°. It's apparent from the graph that the increasing intervals are 0° to 360° and 1080° to 1440°. It's apparent from the graph that the decreasing interval is 360° to 1080°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −2 and the maximum is 2, the equation of the axis is $y = \frac{2 + (-2)}{2}$ or $y = 0$.

f)



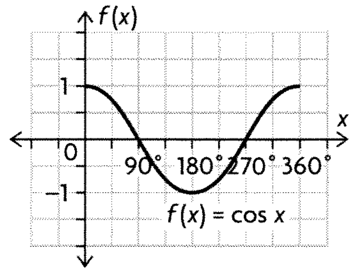
The amplitude is half the distance between the maximum and minimum values. Since the minimum is −1 and the maximum is 5, the amplitude is $\frac{5 - (-1)}{2}$ or 3. The period is the change in x that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle as x changes from 0° to 720°, for example, the period of the function is 720 − 0, or 720°. It's apparent from the graph that the increasing intervals are 0° to 180°, 540° to 900°, and 1260° to 1440°. It's apparent from the graph that the decreasing intervals are 180° to 540° and 900° to 1260°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −1 and the maximum is 5, the equation of the axis is $y = \frac{5 + (-1)}{2}$ or $y = 2$.

9. a) If $f(x) = \cos x$, then $f(35°) = \cos 35°$
$= 0.82$

b) If $g(x) = \sin(2x)$, then $g(10°)$
$= \sin(2 \times 10°) = \sin 20° = 0.34$.

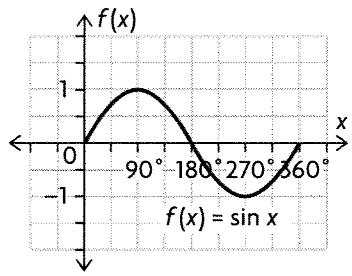
c) If $h(x) = \cos(3x) + 1$, then $h(20°)$
$= \cos(3 \times 20°) + 1$
$= \cos 60° + 1 = 0.5 + 1$
$= 1.5$.

d) If $f(x) = \cos x$ and $f(x) = -1$ then $\cos x = -1$. Use a graphing calculator to graph the function $f(x) = \cos x$ from 0° to 360°.



It's apparent from the graph that from 0° to 360°, $\cos x = -1$ at $x = 180°$.

e) If $f(x) = \sin x$ and $f(x) = -1$, then $\sin x = -1$. Use a graphing calculator to graph the function $f(x) = \sin x$ from 0° to 360°.

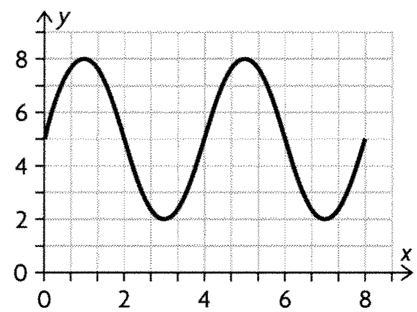


It's apparent from the graph that from 0° to 360°, $\sin x = -1$ at $x = 270°$.

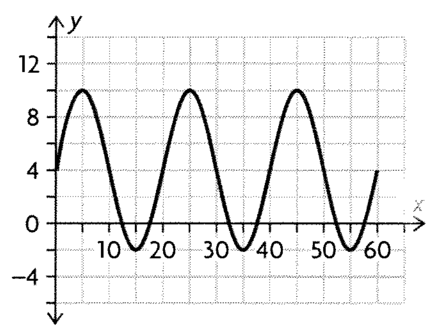
10. In a right triangle, $\sin x = \frac{\text{opp}}{\text{hyp}}$, and $\cos x = \frac{\text{adj}}{\text{hyp}}$. Therefore, for $\sin x$ to equal $\cos x$, $\frac{\text{opp}}{\text{hyp}} = \frac{\text{adj}}{\text{hyp}}$ or opp = adj. If the two legs of a right triangle are equal to each other, then the angles adjacent to them must also be equal, and since the sum of the non-right angles must equal 90°, each non-right angle must measure 45°. It can therefore be determined that $\sin x = \cos x$ when $x = 45°$. By viewing the graphs of $\sin x$ and $\cos x$, it's apparent that every 180°, both $\sin x$ and $\cos x$ are equal to their opposites, so $\sin x = \cos x$ at 180° increments from 45°. Therefore, for $-360° \le x \le 360°$, $\sin x = \cos x$ at $x = -315°$, −135°, 45°, and 225°.

11. a) Consider the point (0, 0) the centre of a circle with radius 1, with the point (1, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 25° about (0, 0) from the point (1, 0) are (1 cos 25°, 1 sin 25°) or (0.91, 0.42).
b) Consider the point (0, 0) the centre of a circle with radius 5, with the point (5, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 80° about (0, 0) from the point (5, 0) are 5 cos 25°, 5 sin 25°) or (0.87, 4.92).
c) Consider the point (0, 0) the centre of a circle with radius 4, with the point (4, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 180° about (0, 0) from the point (4, 0) are (4 cos 120°, 4 sin 120°) or (−2, 3.46).
d) Consider the point (0, 0) the centre of a circle with radius 3, with the point (3, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 230° about (0, 0) from the point (3, 0) are (3 cos 230°, 3 sin 230°) or (−1.93, −2.30).

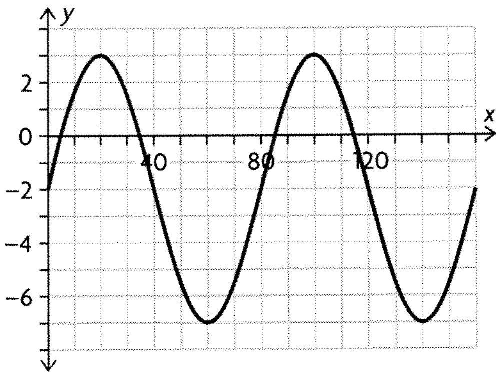
12. a) The period is the change in x that occurs as the function goes through one complete cycle. Since the period is 4 and the number of cycles is 2, the function should go through two complete cycles over 4 × 2 or 8 units. The amplitude is half the distance between the maximum and minimum values, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the amplitude is 3 and the equation of the axis is $y = 5$, the maximum should be 5 + 3 or 8, and the minimum should be 5 − 3 or 2. Therefore, the graph should look as follows:



b) The period is the change in x that occurs as the function goes through one complete cycle. Since the period is 20 and the number of cycles is 3, the function should go through three complete cycles over 20 × 3 or 60 units. The amplitude is half the distance between the maximum and minimum values, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the amplitude is 6 and the equation of the axis is $y = 4$, the maximum should be 4 + 6 or 10, and the minimum should be 4 − 6 or −2. Therefore, the graph should look as follows:



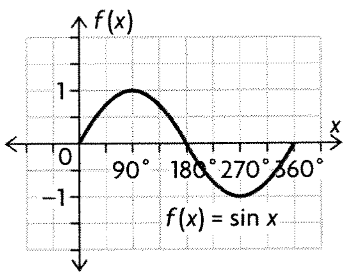
c) The period is the change in x that occurs as the function goes through one complete cycle. Since the period is 80 and the number of cycles is 2, the function should go through two complete cycles over 80 × 2 or 160 units. The amplitude is half the distance between the maximum and minimum values, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the amplitude is 5 and the equation of the axis is $y = -2$, the maximum should be $-2 + 5$ or 3, and the minimum should be $-2 - 5$ or -7. Therefore, the graph should look as follows:



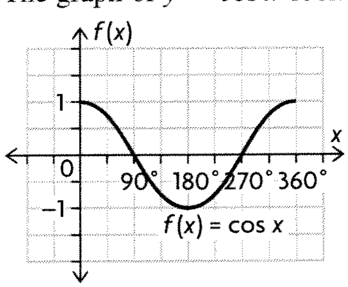
13. a) $h(10)$ represents where Jim is on the Ferris wheel at 10 sec. $h(10) = 5\cos(18 \times 10)°$ $= 5\cos(180°) = 5 \times -1 = -5$. Since the cosine of any angle can never be less than -1, $h(10)$ also represents the Ferris wheel at its lowest point.

b) $h(10)$ represents where Jim is on the Ferris wheel at 10 sec. $h(10) = 5\sin(18 \times 10)°$ $= 5\sin(180°) = 5 \times 0 = 0$. Since 0 is halfway between the maximum and minimum values of the sine function, $h(10)$ also represents the Ferris wheel at its midpoint.

14. The graph of $y = \sin x$ looks like this:



The graph of $y = \cos x$ looks like this:



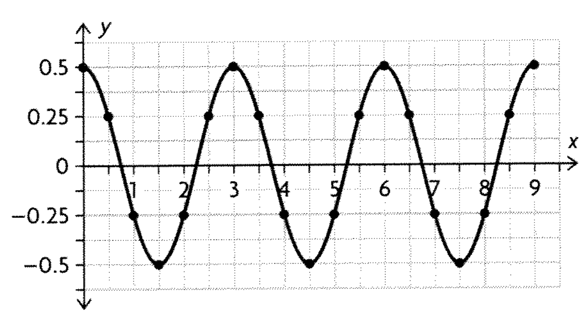
Both graphs have the same period, amplitude, and equation of the axis. However, there is a different starting point for each cycle.

15. When the water level was such that half the wheel was exposed, the equation of the sinusoidal function that described the height of the nail in m above the surface of the water in terms of the rotation was $y = \sin x°$. Since three-quarters of the wheel is now exposed, and since the radius of the wheel is 1 m, at the start the nail will be 0.5 m above the surface of the water. Therefore, the equation of the sinusoidal function that describes the height of the nail in m above the surface of the water in terms of the rotation is $y = \sin x° + 0.5$.

16. a)

$t(s)$	$d(t)$ cm
0	0.5
0.5	0.25
1	−0.25
1.5	−0.5
2	−0.25
2.5	0.25
3	0.5
3.5	0.25
4	−0.25
4.5	−0.5
5	−0.25
5.5	0.25
6	0.5
6.5	0.25
7	−0.25
7.5	−0.5
8	−0.25
8.5	0.25
9	0.5

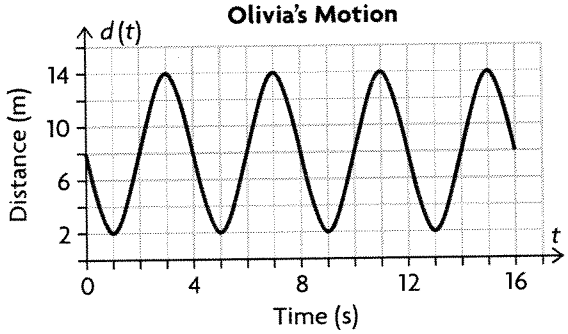
b) Here is a scatter plot with a curve of good fit.



c) A periodic function repeats at a regular interval. Since the function repeats itself every 3 s, the function models periodic behaviour.
d) The amplitude is half the distance between the maximum and minimum values. Therefore, the amplitude of the function is equal to the maximum displacement of the spring from its rest position (or the absolute value of the minimum displacement of the spring from its rest position).

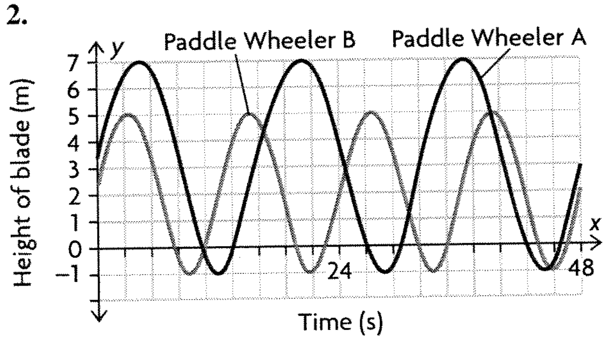
6.3 Interpreting Sinusoidal Functions, pp. 370–373

1.



a) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 2 and the maximum is 14, the equation of the axis is $y = \frac{14 + 2}{2}$ or $y = 8$. In this case it represents the resting position of the swing.
b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 2 and the maximum is 14, the amplitude is $\frac{14 - 2}{2}$ or 6 m.
c) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 s and 4 s, for example, the period is $4 - 0$ or 4 s. In this case it represents the time to complete one full swing.
d) From the graph, the minimum distance between Olivia and the motion detector was 2 m, so the closest Olivia got to the motion detector was 2 m.
e) At $t = 7$ s, the distance between Olivia and the motion detector is 14 m. However, over the course of the next 2 s, that distance will decrease to 2 m. Therefore, whether it would be safe to run between Olivia and the motion detector would depend on how long it would take and how close to the motion detector you ran. If you ran less than 2 m from the motion detector, it would definitely be safe.
f) If the motion detector was activated as soon as Olivia started to swing from at rest, the amplitude of each cycle of the graph would increase up until the amplitude became 6 m. Therefore, the graph would not be sinusoidal because the amplitude would be changing.

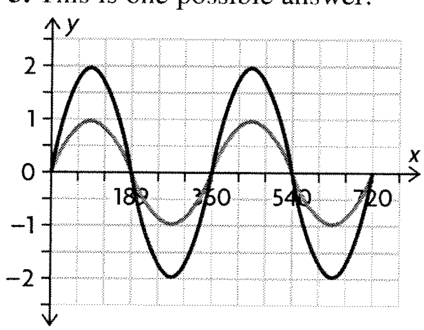
2.



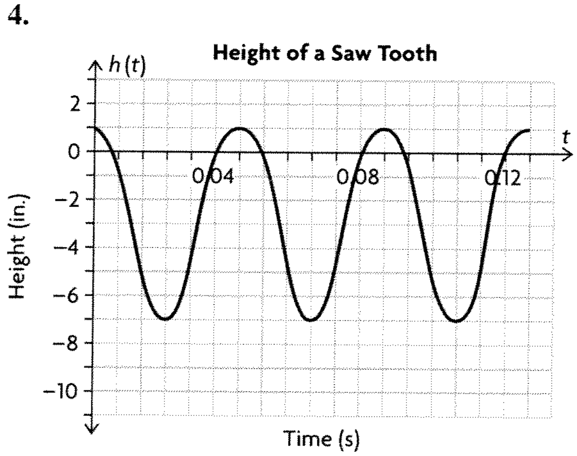
The radius of each wheel is equal to the amplitude of its respective graph, and the amplitude is half the distance between the maximum and minimum values. Since the minimum is -1 and the maximum is 5 for paddle wheel A, its radius is $\frac{5 - (-1)}{2}$ or 3 m. Since the minimum is -1 and the maximum is 7 for paddle wheel B, its radius is $\frac{7 - (-1)}{2}$ or 4 m. The height of the axel of each wheel relative to the water is equal to the equation of the axis of its respective graph, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -1 and the maximum is 5 for paddle wheel A, the height of its axis relative to the water is $y = \frac{5 + (-1)}{2}$ or $y = 2$ m. Since the minimum is -1 and the maximum is 7 for paddle wheel B, the height of its axis relative to the water is $y = \frac{7 + (-1)}{2}$ or $y = 3$ m.

The time taken for each wheel to complete one revolution is equal to the period of its respective graph, and the period is the time it takes the graph to go through one complete cycle. Since the graph for paddle wheel A goes through one complete cycle between 0 s and 12 s, for example, the time it takes for it to complete one revolution is 12 − 0 or 12 s. Since the graph for paddle wheel B goes through one complete cycle between 0 s and 16 s, for example, the time it takes for it to complete one revolution is 16 − 0 or 16 s. The speed of each wheel is equal to its circumference divided by the time it takes for it to complete one revolution. Since the circumference of paddle wheel A is $\pi \times 2 \times 3$ m or 6π m, and since the time it takes for it to complete one revolution is 12 s, its speed is $\frac{6\pi \text{ m}}{12 \text{ s}}$ or 1.57 m/s. Since the circumference of paddle wheel B is $\pi \times 2 \times 4$ m or 8π m, and since the time it takes for it to complete one revolution is 16 s, its speed is $\frac{8\pi \text{ m}}{16 \text{ s}}$ or 1.57 m/s.

3. This is one possible answer:

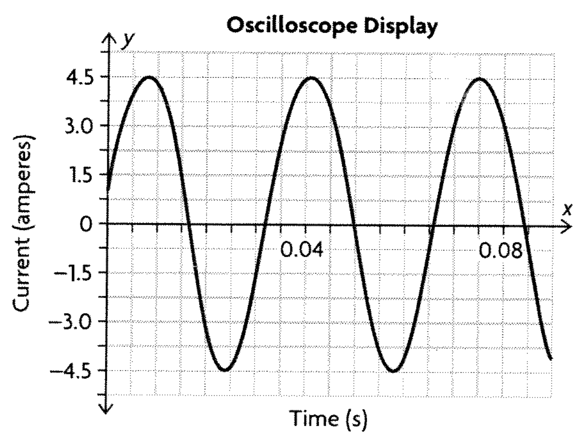


4.



a) The height above the cutting surface that the blade is set is equal to the maximum of the graph. Since the maximum is 1 in, the blade is set 1 in above the cutting surface.
b) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 s and 0.04 s, for example, the period is 0.04 − 0 or 0.04 s. In this case it represents how long it takes the blade to make a full rotation.
c) The amplitude is half the distance between the maximum and minimum values. Since the minimum is −7 and the maximum is 1, the amplitude is $\frac{1 - (-7)}{2}$ or 4 in. In this case it represents the radius of the saw blade.
d) The speed of a tooth on the saw blade is equal to the saw blade's circumference divided by the time it takes for it to make a full rotation. Since the circumference of the saw blade is $\pi \times 2 \times 4$ in or 8π in, and since the time it takes for it to complete one revolution is 0.04 s, its speed is $\frac{8\pi \text{ in}}{0.04 \text{ s}}$ or 628 in/s.

5.

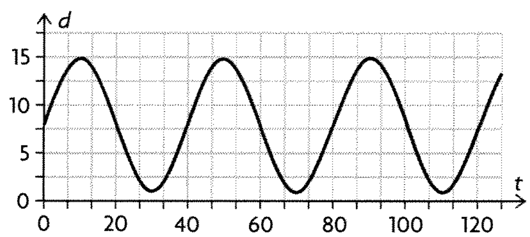


a) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 s and about 0.0325 s, for example, the period is about 0.0325 − 0 or 0.0325 s.
b) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the

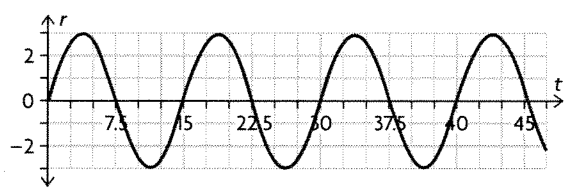
minimum is -4.5 amperes and the maximum is 4.5 amperes, the equation of the axis is $y = \frac{4.5 + (-4.5)}{2}$ or $y = 0$ amperes.

c) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -4.5 amperes and the maximum is 4.5 amperes, the amplitude is $\frac{4.5 - (-4.5)}{2}$ or 4.5 amperes.

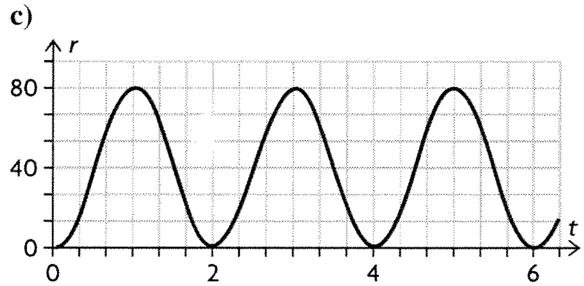
6. a)



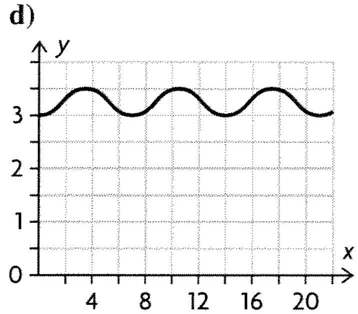
b)



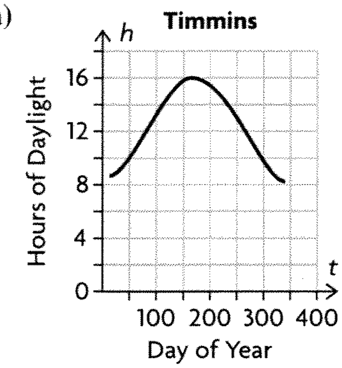
c)

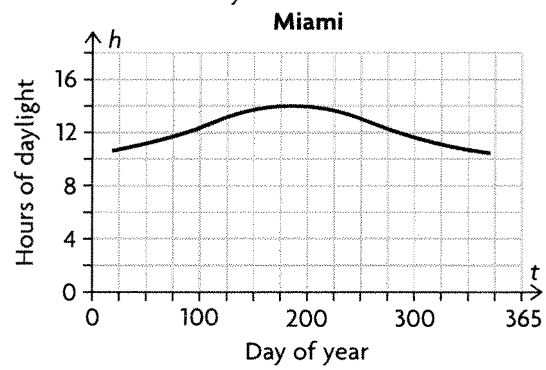


d)



7. a)

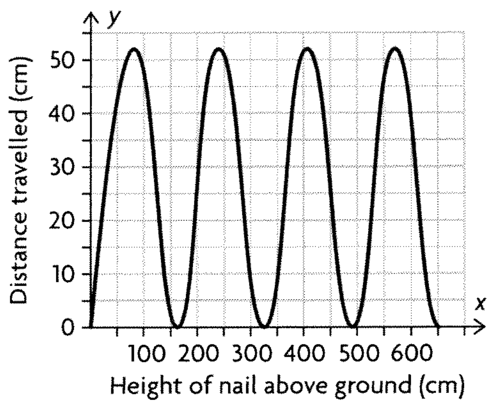




b) The period is the time it takes the graph to go through one complete cycle. Since the graph for Timmins goes through one complete cycle in about 365 days, its period is 365 days. Since the graph for Miami also goes through one complete cycle in about 365 days, its period is also 365 days. The amplitude is half the distance between the maximum and minimum values. Since the minimum for Timmins is 8.3 hours and the maximum is 16.1 hours, the amplitude is $\frac{16.1 - 8.3}{2}$ or 3.9 hours. Since the minimum for Miami is 10.5 hours and the maximum is 13.8 hours, the amplitude is $\frac{13.8 - 10.5}{2}$ or 1.7 hours. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for Timmins is 8.3 hours and the maximum is 16.1 hours, the equation of the axis is $h = \frac{16.1 + 8.3}{2}$ or $h = 12.2$ hours. Since the minimum for Miami is 10.5 hours and the maximum is 13.8 hours, the equation of the axis is $h = \frac{13.8 + 10.5}{2}$ or $h = 12.2$ hours.

c) The farther north one goes, more extreme differences occur in the hours of daylight throughout the year.

8. a)

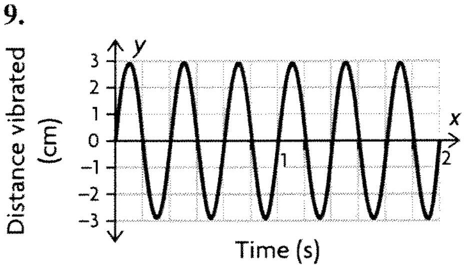


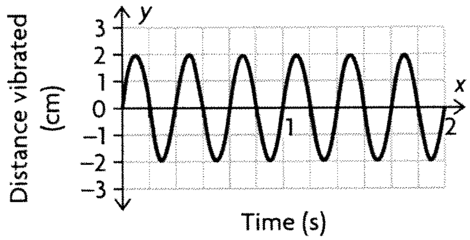
b) 0.1 km = 100 m = 10 000 cm. Since the car travels 163.36 cm for each revolution of the tire, after it has travelled 10 000 cm, the tire has made approximately 10 000 ÷ 163.36 or 61.21 revolutions. Since the height of the nail above the ground is 0 cm after 1 revolution, it is also 0 cm above the ground after 61 revolutions. Therefore, the height of the nail above the ground after 61.21 revolutions is the same as it is after 61.21 − 61 or 0.21 revolutions. During 1 revolution, the distance travelled is 163.36 cm, so during 0.21 revolutions, the distance traveled is 163.36 × 0.21 or 34.31 cm. From the graph in part a, the height of the nail above the ground after the car has travelled 34.31 cm is approximately 29 cm.

c) From the graph, when the nail reaches a height of 20 cm above the ground for the fifth time, the car will have travelled approximately 360 cm.

d) For the function to give an accurate height, you must assume that the driver doesn't spin the wheels.

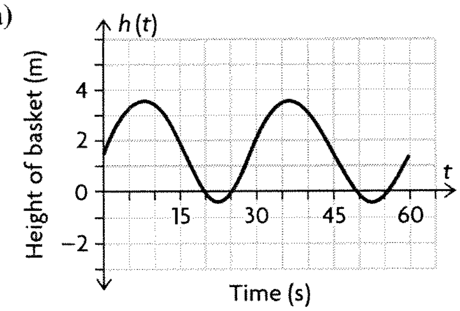
9.





The reduced wind speed does not affect the period or the equation of the axis. However, it does change the amplitude from 3 cm to 2 cm. Therefore, it can be determined that if the wind speed is lowered, the distance the post shakes back and forth decreases.

10. a)



b) The time it takes for the wheel to complete one revolution is the period of the graph, and the period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between 0 s and 30 s, for example, the period is 30 − 0 or 30 s. Therefore, the wheel completes one revolution in 30 s.

c) The radius of the wheel is the amplitude of the graph, and the amplitude is half the distance between the maximum and minimum values. Since the minimum is −0.5 and the maximum is 3.5, the amplitude is $\frac{3.5 - (-0.5)}{2}$ or 2 m. Therefore, the radius of the wheel is 2 m.

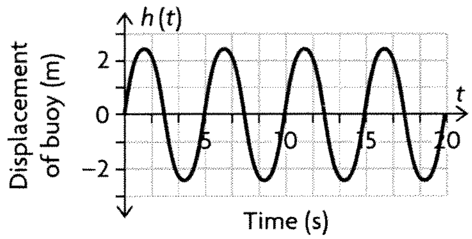
d) Where the centre of the wheel is located in terms of the water level is the equation of the axis of the graph, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −0.5 and the maximum is 3.5, the equation of the axis is $y = \frac{3.5 + (-0.5)}{2}$ or $y = 1.5$ m. Therefore, the centre of the wheel is located 1.5 m above the water.

e) Since $h(t) = 2\sin(12t)° + 1.5$,

$$\begin{aligned} h(10) &= 2\sin(12 \times 10)° + 1.5 \\ &= 2\sin(120)° + 1.5 \\ &= 2 \times 0.866 + 1.5 \\ &= 3.232 \end{aligned}$$

$h(10)$ represents the height of the basket at $t = 10$ s, so the height at this time is 3.232 m.

11. a)

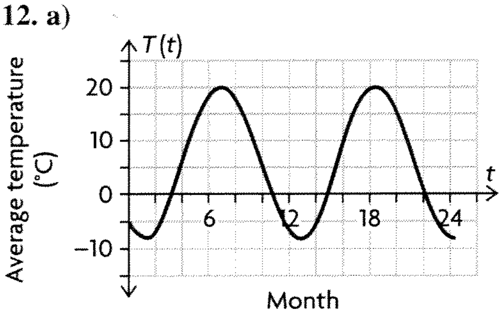


b) The time it takes for the buoy to travel from the peak of a wave to the next peak is the period of the graph, and the period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between 0 s and 5 s, for example, the period is 5 − 0 or 5 s. Therefore, the buoy travels from the peak of a wave to the next peak in 5 s.

c) The number of waves that will cause the buoy to rise and fall in 1 min is the number of complete cycles that the graph goes through in 1 min. Since the period of the graph is 5 s and since there are 60 s in 1 min, the graph goes through 60 ÷ 5 or 12 complete cycles in 1 min. Therefore, 12 waves will cause the buoy to rise and fall in 1 min.

d) The buoy at its highest point is the maximum of the graph, and the buoy at its lowest point is the minimum of the graph. Since the minimum is −2.5 and the maximum is 2.5, the buoy drops 2.5 − (−2.5) or 5 m from its highest to its lowest point.

12. a)



b) The period represents 12 months or 1 year.

c) The highest temperature in Kingston is the maximum of the graph, and the lowest temperature in Kingston is the minimum of the graph. Since the minimum is −8.3 and the maximum is 20.1, the average temperature range in Kingston is 20.1 − (−8.3) or 28.4°C. In other words, the average temperature range is between an average high of 20.1°C and an average low of −8.3°C.

d) The mean temperature in Kingston is the equation of the axis of the graph, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −8.3 and the maximum is 20.1, the equation of the axis is $y = \frac{20.1 + (-8.3)}{2}$ or $y = 5.9$°C. Therefore, the mean temperature in Kingston is 5.9°C.

e) Since $T(t) = 14.2\sin(30(t - 4.2))° + 5.9$,

$$\begin{aligned} T(30) &= 14.2\sin(30(30 - 4.2))° + 5.9 \\ &= 14.2\sin(30(25.8))° + 5.9 \\ &= 14.2\sin(774)° + 5.9 \\ &= 11.5 + 5.9 \\ &= 17.4 \end{aligned}$$

$T(30)$ represents the average monthly temperature of the 30th month, or the sixth month of the third year, so the average temperature of this month is 17.4°C.

13. a) The period is the time it takes the graph to go through one complete cycle. Since the graph for ball A goes through one complete cycle between $t = 0$ and $t = 8$, for example, the period is 8 − 0 or 8 s. Since the graph for ball B goes through one complete cycle between $t = 0$ and $t = 6$, for example, the period is 6 − 0 or 6 s. In this situation, the periods for the graphs represent the time it takes for each wrecking ball to complete a swing back and forth.

b) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for ball A is −4 m and the maximum is 4 m, the equation of the axis is $d = \frac{4 + -(4)}{2}$ or $d = 0$ m. Since the minimum for ball B is −3 m and the maximum is 3 m, the equation of the axis is $d = \frac{3 + (-3)}{2}$ or $d = 0$ m. In this situation, the equations of the axes represent the resting position of each wrecking ball.

c) The amplitude is half the distance between the maximum and minimum values. Since the minimum for ball A is -4 m and the maximum is 4 m, the amplitude is $\frac{4 - (-4)}{2}$ or 4 m. Since the minimum for ball B is -3 m and the maximum is 3 m, the amplitude is $\frac{3 - (-3)}{2}$ or 3 m. In this situation, the amplitudes represent the maximum distance the balls swing back and forth from their resting positions.

d) The range is all possible values of y. Since y oscillates between -4 and 4 for ball A, the range is $\{y \in \mathbf{R} \mid -4 \leq y \leq 4\}$. Since y oscillates between -3 and 3 for ball B, the range is $\{y \in \mathbf{R} \mid -3 \leq y \leq 3\}$.

e) Ball A swings farther left and right than ball B, and because of this, it takes ball A longer to complete a swing back and forth than it takes for ball B.

14. You need to know 4 pieces of information. They would be the amplitude, where the sinusoidal function starts on the graph, the equation of the axis, and the period.

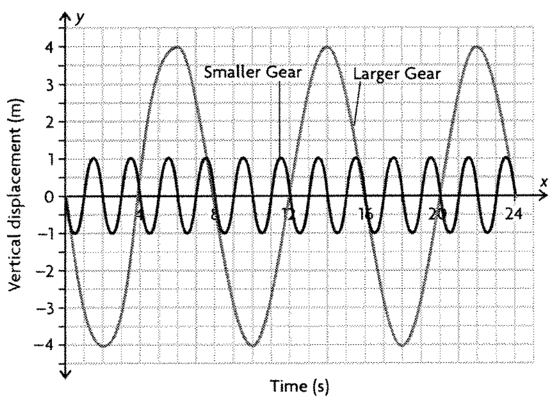
15. a) The larger gear is turning clockwise.

b) The smaller gear has a radius of 1 m and a circumference of $2 \times \pi \times 1$ or 2π m. The larger gear has a radius of 4 m and a circumference of $2 \times \pi \times 4$ or 8π m. Since the period of the smaller gear is 2 s, a tooth on the smaller gear travels 2π m in 2 s, and since a tooth on the larger gear is traveling at the same speed, it also travels 2π m in 2 s. With the circumference of the larger gear being 8π m and with a tooth on it traveling at $\frac{2\pi}{2}$ or π m/s, it would take 8 s for the tooth to make a complete revolution. Therefore, the period of the larger gear is 8 s.

c)

$t(s)$	$d(t)$ for smaller gear (m)
0	0
0.5	-1
1	0
1.5	1
2	0
2.5	-1
3	0
3.5	1
4	0
4.5	-1
5	0
5.5	1
6	0
6.5	-1
7	0
7.5	1
8	0
8.5	-1
9	0
9.5	1
10	0
10.5	-1
11	0
11.5	1
12	0
12.5	-1
13	0
13.5	1
14	0
14.5	-1
15	0
15.5	1
16	0
16.5	-1
17	0
17.5	1
18	0
18.5	-1
19	0
19.5	1
20	0
20.5	-1
21	0
21.5	1
22	0
22.5	-1
23	0
23.5	1
24	0

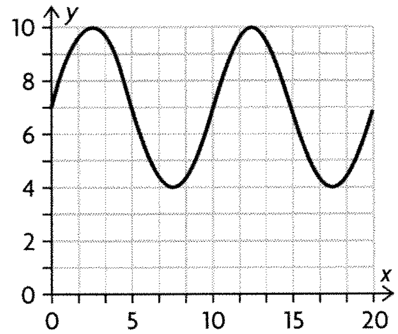
$t(s)$	$d(t)$ for larger gear (m)
0	0
2	−4
4	0
6	4
8	0
10	−4
12	0
14	4
16	0
18	−4
20	0
22	4
24	0



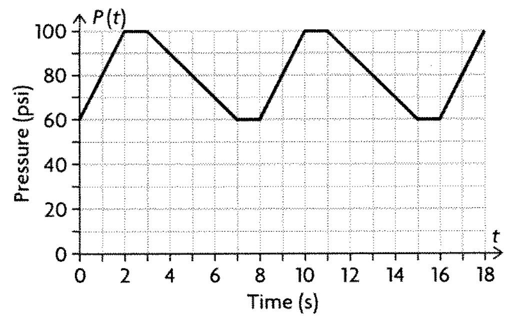
d) From the graph in part c, it's apparent that when the smaller gear first has a vertical displacement of −0.5 m, the larger gear also has a vertical displacement of about −0.5 m.
e) From the graph in part c, it's apparent that when the larger gear first has a vertical displacement of 2 m, the smaller gear has a vertical displacement of about 0 m.
f) 5 min = 5 × 60 or 300 s. Since the period of the larger gear is 8 s, in 300 s it goes through 300 ÷ 8 or 37.5 revolutions. After 37 revolutions (or any number of complete revolutions), the vertical displacement is 0 m, so after 37.5 revolutions, the vertical displacement is the same as after 0.5 revolutions. 0.5 revolutions occur in 8 × 0.5 or 4 s, and from part c, the vertical displacement after 4 s is 0 m. Therefore, the vertical displacement after 5 min is 0 m.

Mid-Chapter Review, p. 376

1. One possible answer:



2. a)

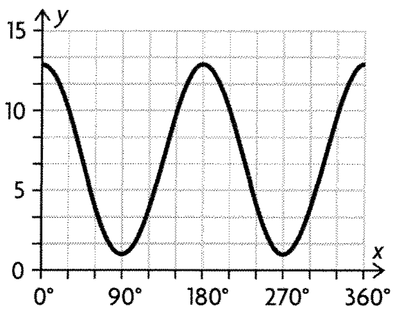


b) It is periodic because the cycle repeats.
c) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 s and 8 s, for example, the period is 8 − 0 or 8 s.
d) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 60 psi and the maximum is 100 psi, the equation of the axis is $P = \frac{100 + 60}{2}$ or $P = 80$ psi.
e) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 60 psi and the maximum is 100 psi, the amplitude is $\frac{100 - 60}{2}$ or 20 psi.
f) The compressor is on from $t = 1$ s to $t = 3$ s, for example. From $t = 1$ s to $t = 3$ s, the air pressure increases from 60 psi to 100 psi. Therefore, the rate at which the air pressure is increasing when the compressor is on is $\frac{100 - 60}{3 - 1}$ or 20 psi/s.

g) The equipment is in operation from $t = 4$ s to $t = 8$ s, for example. From $t = 4$ s to $t = 8$ s, the air pressure decreases from 100 psi to 60 psi. Therefore, the rate at which the air pressure is decreasing when the equipment is in operation is $\frac{100 - 60}{8 - 4}$ or 10 psi/s.

h) No, the container is never empty because its lowest pressure value is 60 psi.

3. a)



The period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between 0° and 180°, for example, the period is 180° − 0° or 180°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 2 and the maximum is 12 m, the equation of the axis is $g = \frac{12 + 2}{2}$ or $g = 7$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 2 and the maximum is 12, the amplitude is $\frac{12 - 2}{2}$ or 5. The range is all possible values of g. Since g oscillates between 2 and 12, the range is $\{g \in \mathbf{R} \mid 2 \le g \le 12\}$.

b) The function is sinusoidal because its graph looks like smooth symmetrical repeating waves.

c) Since $g(x) = 5 \cos (2x)° + 7$,

$$\begin{aligned} g(125) &= 5 \cos (2 \times 125)° + 7 \\ &= 5 \cos (250)° + 7 \\ &= 5 \times (-0.342) + 7 \\ &= -1.710 + 7 \\ &= 5.3 \end{aligned}$$

d) From the graph in part a, $g(x) = 12$ at $x = 0°$, 180°, and 360°.

4. Consider the point (0, 0) the centre of a circle with radius 7, with the point (7, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos \theta, r \sin \theta)$. Therefore, the coordinates of the new point after a rotation of 64° about (0, 0) from the point (7, 0) are (7 cos 64°, 7 sin 64°) or (3.1, 6.3).

5. a) The period is the time it takes the function to go through one complete cycle. Since the function for mark 1 goes through one complete cycle between 0 s and 0.25 s, for example, its period is 0.25 − 0 or 0.25 s. Since the function for mark 2 also goes through one complete cycle between 0 s and 0.25 s, for example, its period is also 0.25 − 0 or 0.25 s. In this situation the period represents the time it takes for the tire to complete one revolution.

b) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for mark 1 is 0 cm and the maximum is 60 cm, the equation of the axis is $h = \frac{60 + 0}{2}$ or $h = 30$ cm. Since the minimum for mark 2 is 10 cm and the maximum is 50 cm, the equation of the axis is $h = \frac{50 + 10}{2}$ or $h = 30$ cm. In this situation the equation of the axis represents the height of the axle.

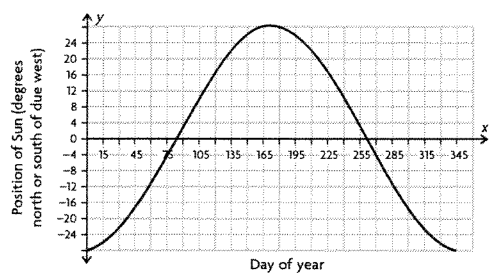
c) The amplitude is half the distance between the maximum and minimum values. Since the minimum for mark 1 is 0 cm and the maximum is 60 cm, the amplitude is $\frac{60 - 0}{2}$ or 30 cm. Since the minimum for mark 2 is 10 cm and the maximum is 50 cm, the amplitude is $\frac{50 - 10}{2}$ or 20 cm. In this situation the amplitude represents the distance from the white mark to the centre of the wheel.

d) The range is all possible values of h. Since h oscillates between 0 and 60 for mark 1, the range is $\{h \in \mathbf{R} \mid 0 \le h \le 60\}$. Since h oscillates between 10 and 50 for mark 2, the range is $\{h \in \mathbf{R} \mid 10 \le h \le 50\}$.

e) The speed of each mark is equal to the circumference of the circle it traces divided by the time it takes for it to complete one revolution. Since the circumference that mark 1 traces is $\pi \times 2 \times 30$ cm or 60π cm,, and since the time it takes for it to complete one revolution is 0.25 s, its speed is $\frac{60\pi \text{ cm}}{0.25 \text{ s}}$ or 754 cm/s. Since the circumference that mark 2 traces is $\pi \times 2 \times 20$ cm cm or 40π cm, and since the time it takes for it to complete one revolution is 0.25 s, its speed is $\frac{40\pi \text{ cm}}{0.25 \text{ s}}$ or 502 cm/s.

f) This graph would be periodic in nature and have a smaller amplitude than the graph of mark 2 (the red graph). However, it would have the same period and the same equation of the axis as the graphs for marks 1 and 2.

6. a)



b) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 and 365, its period is $365 - 0$ or 365 days. In this situation the period represents the amount of time it takes for the Sun to return to its original position at sunset with respect to due west. This amount of time happens to be 365 days or 1 year.

c) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -28 and the maximum is 28, the equation of the axis is $P = \frac{28 + (-28)}{2}$ or $P = 0$. In this situation the equation of the axis represents the position of 0° with respect to due west.

d) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -28 and the maximum is 28, the amplitude is $\frac{28 - (-28)}{2}$ or 28. In this situation the amplitude represents the maximum number of degrees north or south of due west the Sun can be at sunset for this particular latitude.

e) The range is all possible values of P. Since P oscillates between –28 and 28, the range is $\{P \in \mathbf{R} \mid -28 \leq P \leq 28\}$.

f) February 15 is the 46th day of the year.

Since $P(d) = 28 \sin\left(\frac{360}{360}d - 81\right)^\circ$

$$\begin{aligned} P(46) &= 28 \sin\left(\frac{360}{360} \times 46 - 81\right)^\circ \\ &= 28 \sin (45.37 - 81)^\circ \\ &= 28 \sin (-35.63)^\circ \\ &= 28 \times -0.58 \\ &= -16.3 \end{aligned}$$

Therefore, the angle of sunset on February 15 is $-16.3°$.

6.4 Exploring Transformations of Sinusoidal Functions, p. 379

1. a) Changing the value of a from 1 in the function $y = a \cos x$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|3| > 1$, $y = 3 \cos x$ results in a vertical stretch by a factor of 3.

b) Changing the value of d from 0 in the function $y = \sin(x - d)$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since 50 is positive, $y = \sin (x - 50°)$ results in a horizontal translation of 50° to the right.

c) Changing the sign of a in the function $y = a \cos x$ results in a reflection of the function in the x-axis. Therefore, $y = -\cos x$ results in a reflection in the x-axis.

d) Changing the value of k from 1 in the function $y = \sin(kx)$ results in a horizontal stretch or compression of the function. If $|K| > 1$, a horizontal compression occurs, and if $|K| < 1$, a horizontal stretch occurs. Since $|5| > 1$, $y = \sin(5x)$ results in a horizontal compression by a factor of $\frac{1}{5}$.

e) Changing the value of c from 0 in the function $y = \cos x + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since -6 is negative, $y = \cos x - 6$ results in a vertical translation of 6 units down or a vertical translation of -6.

f) Changing the value of d from 0 in the function $y = \cos(x - d)$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since -20 is negative, $y = \cos(x + 20°)$ results in a horizontal translation of 20° to the left or a horizontal translation of $-20°$.

2. a) Changing the value of c from 0 in the function $y = \sin x + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \sin x$ is $y = 0$, and since the function $y = \sin x + 2$ slides up 2 units, the equation of the axis of $y = \sin x + 2$ is $y = 2$.

b) Changing the value of a from 1 in the function $y = a \sin x$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \sin x$ is 1, and since the function $y = 4 \sin x$ results in a vertical stretch by a factor of 4, the amplitude of $y = 4 \sin x$ is 4.

c) Changing the value of k from 1 in the function $y = \cos(kx)$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \cos x$ is 360°, and since the function $y = \cos(8x)$ results in a horizontal compression by a factor of $\frac{1}{8}$, the period of

$y = \cos(8x)$ is $\frac{360}{8}$ or 45°.

d) Changing the values of k from 1 and d from 0 in the function $y = \sin(k(x - d))$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \sin x$ is 360°, and since the function $y = \sin(2x + 30°)$ results in a horizontal compression by a factor of $\frac{1}{2}$, the period of

$y = \sin(2x + 30°)$ is $\frac{360}{2}$ or 180°. Also, $y = \sin(2x + 30°)$ results in a horizontal translation of 15° to the left or $-15°$.

e) Changing the value of a from 1 in the function $y = a \cos x$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \cos x$ is 1, and since the function $y = 0.25 \cos x$ results in a vertical compression by a factor of 0.25, the amplitude of $y = 0.25 \cos x$ is 0.25.

f) Changing the value of k from 1 in the function $y = \sin(kx)$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \sin x$ is 360°, and since the function $y = \sin(0.5x)$ results in a horizontal stretch by a factor of 2,

the period of $y = \sin(0.5x)$ is $\frac{360}{0.5}$ or 720°.

3. A vertical stretch/vertical compression affects the amplitude of a sinusoidal function, but not the period or the equation of the axis. A vertical translation affects the equation of the axis of a sinusoidal function, but not the amplitude or the period. A horizontal stretch/horizontal compression affects the period of a sinusoidal function, but not the amplitude or equation of the axis. Therefore, the only two transformations that do not affect the period, amplitude, or equation of the axis of a sinusoidal function are a) a reflection in the x-axis; and e) a horizontal translation.

6.5 Using Transformations to Sketch the Graphs of Sinusoidal Functions, pp. 383–385

1. a) Changing the value of k from 1 in the function $f(x) = \sin(kx) + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|4| > 1$, $f(x) = \sin(4x) + 2$ results in a horizontal compression by a factor of $\frac{1}{4}$. Changing the value of c from 0 in the function $f(x) = \sin(kx) + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since 2 is positive, $y = \sin(4x) + 2$ results in a vertical translation of 2 units up or a vertical translation of 2. These transformations can be applied in either order.

b) Changing the value of a from 1 in the function $y = a\cos(x - d)$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|0.25| > 1$, $y = 0.25\cos(x - 20°)$ results in a vertical compression by a factor of $\frac{1}{4}$. Changing the value of d from 0 in the function $y = a\cos(x - d)$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since 20 is positive, $y = 0.25\cos(x - 20°)$ results in a horizontal translation of 20° to the right. These transformations can be applied in either order.

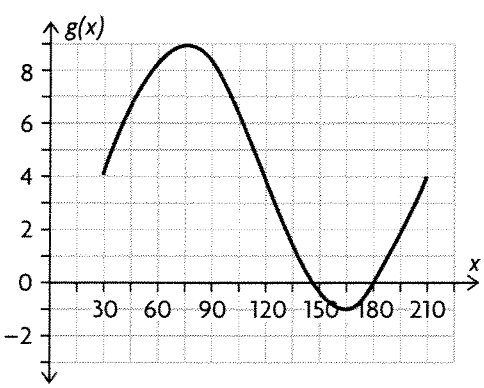
c) Changing the sign of a in the function $g(x) = a\sin(kx)$ results in a reflection of the function in the x-axis. Therefore, $g(x) = -\sin(0.5x)$ results in a reflection in the x-axis. Changing the value of k from 1 in the function $g(x) = a\sin(kx)$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|0.5| > 1$, $g(x) = -\sin(0.5x)$ results in a horizontal stretch by a factor of 2. These transformations can be applied in either order.

d) Changing the value of a from 1 in the function $y = a\cos(kx) + c$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|12| > 1$, $y = 12\cos(18x) + 3$ results in a vertical stretch by a factor of 12. Changing the value of c from 0 in the function $y = a\cos(kx) + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since 3 is positive, $y = 12\cos(18x) + 3$ results in a vertical translation of 3 units up or a vertical translation of 3. Changing the value of k from 1 in the function $y = a\cos(kx) + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|18| < 1$, $y = 12\cos(18x) + 3$ results in a horizontal compression by a factor of $\frac{1}{18}$. The vertical stretch should be applied before the vertical translation.

e) Changing the sign of a in the function $f(x) = a\sin[k(x - d)]$ results in a reflection of the function in the x-axis. Therefore, $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$ results in a reflection in the x-axis. Changing the value of a from 1 in the function $f(x) = a\sin[k(x - d)]$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|-20| > 1$, $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$ results in a vertical stretch by a factor of 20. Changing the value of k from 1 in the function $f(x) = a\sin[k(x - d)]$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $\frac{1}{3} < 1$, $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$ results in a horizontal stretch by a factor of 3. Changing the value of d from 0 in the function $f(x) = a\sin[k(x - d)]$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since 40 is positive, $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$ results in a horizontal translation of 40° to the right. The horizontal stretch should be applied before the horizontal translation.

2. Changing the value of a from 1 in the function $f(x) = a\cos(kx) + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $f(x) = \cos x$ is 1, and since the function $f(x) = 4\cos(3x) + 6$ results in a vertical stretch by a factor of 4, the amplitude of $f(x) = 4\cos(3x) + 6$ is 4. Changing the value of k from 1 in the function $f(x) = a\cos(kx) + c$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \cos x$ is 360°, and since the function $f(x) = 4\cos(3x) + 6$ results in a horizontal compression by a factor of $\frac{1}{3}$, the period of $f(x) = 4\cos(3x) + 6$ is $\frac{360}{3}$ or 120°. Changing the value of c from 0 in the function $f(x) = a\cos(kx) + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \cos x$ is $y = 0$, and since the function $f(x) = 4\cos(3x) + 6$ slides up 6 units, the equation of the axis of $f(x) = 4\cos(3x) + 6$ is $y = 6$. The domain is all possible values of x. Since the period is the change in x that occurs as the function completes one full cycle, and since the period is 120°, the function completes two complete cycles in 120° × 2 or 240°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0 \le x \le 240\}$. The range is all possible values of y. Since the equation of the axis is $y = 6$ and since the amplitude is 4, y oscillates between $6 - 4$ or 2 and $6 + 4$ or 10. Therefore, the range is $\{y \in \mathbf{R} \mid 2 \le y \le 10\}$.

3. If the graph of $g(x) = 5\sin(2(x - 30°)) + 4$ is transformed from the graph of $g(x) = \sin x$, there is a vertical stretch by a factor of 5, a vertical translation of 4 units up, a horizontal compression by a factor of $\frac{1}{2}$, and a horizontal translation of 30° to the right. Therefore, the graph of $g(x) = 5\sin(2(x - 30°)) + 4$ will look as follows:



4. a) Changing the sign of a in the function $y = a\sin(x - d)$ results in a reflection of the function in the x-axis. Therefore, $y = -2\sin(x + 10°)$ results in a reflection in the x-axis. Changing the value of a from 1 in the function $y = a\sin(x - d)$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|-2| > 1$, $y = -2\sin(x + 10°)$ results in a vertical stretch by a factor of 2. Changing the value of d from 0 in the function $y = a\sin(x - d)$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since -10 is negative, $y = -2\sin(x + 10°)$ results in a horizontal translation of 10° to the left or a horizontal translation of $-10°$. The reflection and the vertical stretch should occur one after the other in either order.

b) Changing the value of k from 1 in the function $y = \cos(kx) + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|5| > 1$, $y = \cos(5x) + 7$ results in a horizontal compression by a factor of $\frac{1}{5}$. Changing the value of c from 0 in the function $y = \cos(kx) + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since 7 is positive, $y = \cos(5x) + 7$ results in a vertical translation of 7 units up or a vertical translation of 7. These transformations can be applied in either order.

c) Changing the value of a from 1 in the function $y = a\cos[k(x - d)] + c$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|9| > 1$, $y = 9\cos(2\,(x + 6°)) - 5$ results in a vertical stretch by a factor of 9. Changing the value of c from 0 in the function $y = a\cos[k(x - d)] + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since -5 is negative, $y = 9\cos(2\,(x + 6°)) - 5$ results in a vertical translation of 5 units down or a vertical translation of -5. Changing the value of k from 1 in the function $y = a \cos[k(x - d)] + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|2| > 1$, $y = 9\cos(2(x + 6°)) - 5$ results in a horizontal compression by a factor of $\frac{1}{2}$.

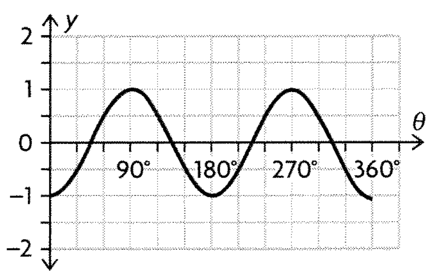
Changing the value of d from 0 in the function $y = a \cos[k(x - d)] + c$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since -6 is negative, $y = 9\cos(2(x + 6°)) - 5$ results in a horizontal translation of 6° to the left or a horizontal translation of $-6°$. The vertical stretch should be applied before the vertical translation, and the horizontal stretch should be applied before the horizontal translation.

d) Changing the value of a from 1 in the function $g(x) = a \sin(x - d) + c$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $\left|\frac{1}{5}\right| < 1$, $g(x) = \frac{1}{5}\sin(x - 15°) + 1$ results in a vertical compression by a factor of $\frac{1}{5}$. Changing the value of c from 0 in the function $g(x) = a \sin(x - d) + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since 1 is positive $g(x) = \frac{1}{5}\sin(x - 15°) + 1$ results in a vertical translation of 1 unit up or a vertical translation of 1. Changing the value of d from 0 in the function $g(x) = a \sin(x - d) + c$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since 15 is positive, $g(x) = \frac{1}{5}\sin(x - 15°) + 1$ results in a horizontal translation of 15° to the right or a horizontal translation of 15°. The vertical compression should be applied before the vertical translation.

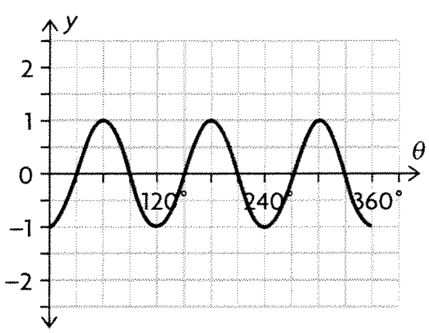
e) Changing the sign of a in the function $h(x) = a \sin[k(x - d)] + c$ results in a reflection of the function in the x-axis. Therefore, $h(x) = -\sin\left[\frac{1}{4}(x + 37°)\right] - 2$ results in a reflection in the x-axis. Changing the value of c from 0 in the function $h(x) = a \sin[k(x - d)] + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since -2 is negative, $h(x) = -\sin\left[\frac{1}{4}(x + 37°)\right] - 2$ results in a vertical translation of 2 units down or a vertical translation of -2. Changing the value of k from 1 in the function $h(x) = a \sin[k(x - d)] + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $\left|\frac{1}{4}\right| < 1$, $h(x) = -\sin\left[\frac{1}{4}(x + 37°)\right] - 2$ results in a horizontal stretch by a factor of 4. Changing the value of d from 0 in the function $h(x) = a \sin[k(x - d)] + c$ results in a horizontal translation and slides the graph to the left or right. If d is positive, the graph slides to the right, and if d is negative, the graph slides to the left. Since -37 is negative $h(x) = -\sin\left[\frac{1}{4}(x + 37°)\right] - 2$ results in a horizontal translation of 37° to the left or a horizontal translation of $-37°$. The reflection should be applied before the vertical translation, and the horizontal stretch should be applied before the horizontal translation.

f) Changing the sign of a in the function $d = a \cos(kx) + c$ results in a reflection of the function in the x-axis. Therefore, $d = -6\cos(3t) + 22$ results in a reflection in the x-axis. Changing the value of a from 1 in the function $d = a \cos(kx) + c$ results in a vertical stretch or compression of the function. If $|a| > 1$, a vertical stretch occurs, and if $|a| < 1$, a vertical compression occurs. Since $|-6| > 1$, $d = -6\cos(3t) + 22$ results in a vertical stretch by a factor of 6. Changing the value of c from 0 in the function $d = a \cos(kx) + c$ results in a vertical translation and slides the graph up or down. If c is positive, the graph slides up, and if c is negative, the graph slides down. Since 22 is positive $d = -6\cos(3t) + 22$ results in a vertical translation of 22 units up or a vertical translation of 22. Changing the value of k from 1 in the function $d = a\cos(kx) + c$ results in a horizontal stretch or compression of the function. If $|k| > 1$, a horizontal compression occurs, and if $|k| < 1$, a horizontal stretch occurs. Since $|3| > 1$, $d = -6\cos(3t) + 22$ results in a horizontal compression by a factor of $\frac{1}{3}$. The reflection and vertical stretch should be applied before the vertical translation.

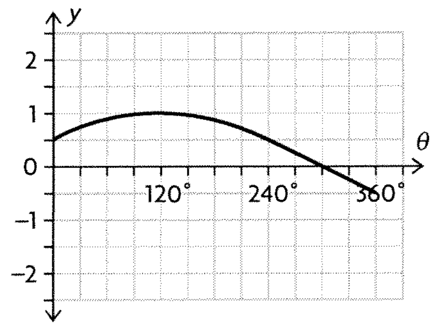
5. a) If the graph of $y = \sin(2\theta - 90°)$ is transformed from the graph of $y = \sin\theta$, there is a horizontal compression by a factor of $\frac{1}{2}$ and a horizontal translation of 45° to the right. Therefore, the graph of $y = \sin(2\theta - 90°)$ will look as follows, and the answer is ii:



b) If the graph of $y = \sin(3\theta - 90°)$ is transformed from the graph of $y = \sin\theta$, there is a horizontal compression by a factor of $\frac{1}{3}$ and a horizontal translation of 30° to the right. Therefore, the graph of $y = \sin(3\theta - 90°)$ will look as follows, and the answer is iii:



c) If the graph of $y = \sin\left(\frac{\theta}{2} + 30°\right)$ is transformed from the graph of $y = \sin\theta$, there is a horizontal stretch by a factor of 2 and a horizontal translation of 60° to the left. Therefore, the graph of $y = \sin\left(\frac{\theta}{2} + 30°\right)$ will look as follows, and the answer is i:



6. a) The period of the function $y = 3\sin x + 2$ is 360°, since the period of the function $y = \sin x$ is 360°. Changing the value of a from 1 in the function $y = a\sin x + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \sin x$ is 1, and since the function $y = 3\sin x + 2$ results in a vertical stretch by a factor of 3, the amplitude of $y = 3\sin x + 2$ is 3. Changing the value of c from 0 in the function $y = a\sin x + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \sin x$ is $y = 0$, and since the function $y = 3\sin x + 2$ slides up 2 units, the equation of the axis of $y = 3\sin x + 2$ is $y = 2$. The domain is all possible values of x. Since the period is the change in x that occurs as the function completes one full cycle, and since the period is 360°, the function completes three complete cycles in $360° \times 3$ or 1080°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0° \le x \le 1080°\}$. The range is all possible values of y. Since the equation of the axis is $y = 2$ and since the amplitude is 3, y oscillates between $2 - 3$ or -1 and $2 + 3$ or 5. Therefore, the range is $\{y \in \mathbf{R} \mid -1 \le y \le 5\}$.

b) Changing the value of k from 1 in the function $g(x) = a\cos(kx) + c$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \cos x$ is 360°, and since the function $g(x) = -4\cos(2x) + 7$ results in a horizontal compression by a factor of $\frac{1}{2}$, the period of $g(x) = -4\cos(2x) + 7$ is $\frac{360}{2}$ or 180°.

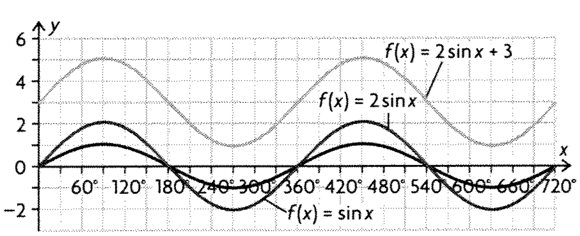
Changing the value of a from 1 in the function $g(x) = a\cos(kx) + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \cos x$ is 1, and since the function $g(x) = -4\cos(2x) + 7$ results in a vertical stretch by a factor of 4, the amplitude of $g(x) = -4\cos(2x) + 7$ is 4. Changing the value of c from 0 in the function $g(x) = a\cos(kx) + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \cos x$ is $y = 0$, and since the function $g(x) = -4\cos(2x) + 7$ slides up 7 units, the equation of the axis of $g(x) = -4\cos(2x) + 7$ is $g = 7$. The domain is all possible values of x. Since the period is the change in x that occurs as the function completes one full cycle, and since the period is 180°, the function completes three complete cycles in 180° × 3 or 540°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0° \le x \le 540°\}$. The range is all possible values of y. Since the equation of the axis is $y = 7$ and since the amplitude is 4, y oscillates between $7 - 4$ or 3 and $7 + 4$ or 11. Therefore, the range is $\{g \in \mathbf{R} \mid 3 \le g \le 11\}$.

c) The period of the function $h = \frac{-1}{2}\sin t - 5$ is 360°, since the period of the function $h = \sin t$ is 360°. Changing the value of a from 1 in the function $h = a\sin t + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $h = \sin t$ is 1, and since the function $h = \frac{-1}{2}\sin t - 5$ results in a vertical compression by a factor of $\frac{1}{2}$, the amplitude of $h = \frac{-1}{2}\sin t - 5$ is $\frac{1}{2}$. Changing the value of c from 0 in the function $h = a\sin t + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $h = \sin t$ is $h = 0$, and since the function $h = \frac{-1}{2}\sin t - 5$ slides down 5 units, the equation of the axis of $h = \frac{-1}{2}\sin t - 5$ is $h = -5$. The domain is all possible values of t. Since the period is the change in t that occurs as the function completes one full cycle, and since the period is 360°, the function completes three complete cycles in 360° × 3 or 1080°. Therefore, the domain is $\{t \in \mathbf{R} \mid 0° \le t \le 1080°\}$. The range is all possible values of h. Since the equation of the axis is $h = -5$ and since the amplitude is $\frac{1}{2}$, y oscillates between $-5 - \frac{1}{2}$ or -5.5 and $-5 + \frac{1}{2}$ or -4.5. Therefore, the range is $\{h \in \mathbf{R} \mid -5.5 \le h \le -4.5\}$.

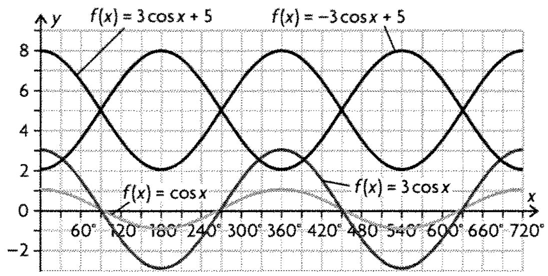
d) Changing the value of k from 1 in the function $h(x) = \cos(k(x - d)) + c$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \cos x$ is 360°, and since the function $h(x) = \cos(4(x - 12°)) - 9$ results in a horizontal compression by a factor of $\frac{1}{4}$, the period of $h(x) = \cos(4(x - 12°)) - 9$ is $\frac{360}{4}$ or 90°. The amplitude of the function $h(x) = \cos(4(x - 12°)) - 9$ is 1, since the amplitude of the function $y = \cos x$ is 1. Changing the value of c from 0 in the function $h(x) = \cos(4(x - 12°)) - 9$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \cos x$ is $y = 0$, and since the function $h(x) = \cos(4(x - 12°)) - 9$ slides down 9 units, the equation of the axis of

$h(x) = \cos(4(x - 12°)) - 9$ is $h = -9$.
The domain is all possible values of x. Since the period is the change in x that occurs as the function completes one full cycle, and since the period is 90°, the function completes three complete cycles in $90° \times 3$ or 270°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0° \le x \le 270°\}$. The range is all possible values of h. Since the equation of the axis is $h = -9$ and since the amplitude is 1, y oscillates between $-9 - 1$ or -10 and $-9 + 1$ or -8. Therefore, the range is $\{h \in \mathbf{R} \mid -10 \le h \le -8\}$.
e) Changing the value of k from 1 in the function $d = a \sin(k(t - d)) + c$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $d = \sin t$ is 360°, and since the function $d = 10 \sin(180(t - 17°)) - 30$ results in a horizontal compression by a factor of $\frac{1}{180}$, the period of $d = 10 \sin(180(t - 17°)) - 30$ is $\frac{360}{180}$ or 2°. Changing the value of a from 1 in the function $d = a \sin(k(t - d)) + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $d = \cos t$ is 1, and since the function $d = 10 \sin(180(t - 17°)) - 30$ results in a vertical stretch by a factor of 10, the amplitude of function $d = 10 \sin(180(t - 17°)) - 30$ is 10. Changing the value of c from 0 in the function $d = a \sin(k(t - d)) + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $d = \cos t$ is $d = 0$, and since the function $d = 10 \sin(180(t - 17°)) - 30$ slides down 30 units, the equation of the axis of $d = 10 \sin(180(t - 17°)) - 30$ is $d = -30$.
The domain is all possible values of t. Since the period is the change in t that occurs as the function completes one full cycle, and since the period is 2°, the function completes three complete cycles in $2° \times 3$ or 6°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0° \le t \le 6°\}$. The range is all possible values of d. Since the equation of the axis is $d = -30$ and since the amplitude is 10, y oscillates between $-30 - 10$ or -40 and $-30 + 10$ or -20. Therefore, the range is $\{d \in \mathbf{R} \mid -40 \le d \le -20\}$.

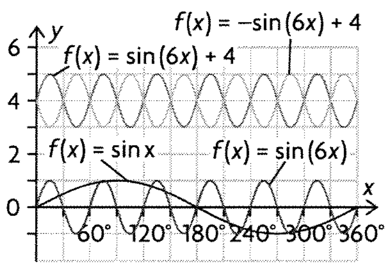
f) Changing the value of k from 1 in the function $j(x) = a \sin(kx - d)$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \sin x$ is 360°, and since the function $j(x) = 0.5 \sin(2x - 30°)$ results in a horizontal compression by a factor of $\frac{1}{2}$, the period of $j(x) = 0.5 \sin(2x - 30°)$ is $\frac{360}{2}$ or 180°. Changing the value of a from 1 in the function $j(x) = a \sin(kx - d)$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \cos x$ is 1, and since the function of $j(x) = 0.5 \sin(2x - 30°)$ results in a vertical compression by a factor of 0.5, the amplitude of function of $j(x) = 0.5 \sin(2x - 30°)$ is 0.5.
Since the equation of the axis of $y = \sin x$ is $y = 0$, the equation of the axis $j(x) = 0.5 \sin(2x - 30°)$ is $j = 0$. The domain is all possible values of x. Since the period is the change in x that occurs as the function completes one full cycle, and since the period is 180°, the function completes three complete cycles in $180° \times 3$ or 540°. Therefore, the domain is $\{x \in \mathbf{R} \mid 0° \le x \le 540°\}$. The range is all possible values of j. Since the equation of the axis is $j = 0$, and since the amplitude is 0.5, j oscillates between $0 - 0.5$ or -0.5 and $0 + 0.5$ or 0.5. Therefore, the range is $\{j \in \mathbf{R} \mid -0.5 \le j \le 0.5\}$.
7. a) If the graph of $y = 2 \sin x + 3$ is transformed from the graph of $y = \sin x$, there is a vertical stretch by a factor of 2 and a vertical translation of 3 units up. Therefore, the graph of $y = 2 \sin x + 3$ will look as follows:



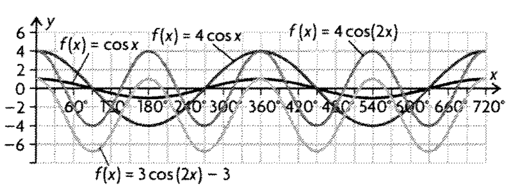
b) If the graph of $y = -3 \cos x + 5$ is transformed from the graph of $y = \cos x$, there is a vertical stretch by a factor of 3, a vertical translation of 5 units up, and a reflection in the x-axis. Therefore, the graph of $y = -3 \cos x + 5$ will look as follows:



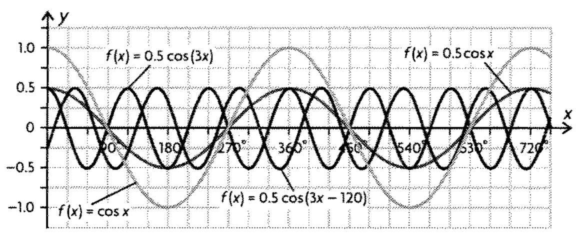
c) If the graph of $y = -\sin(6x) + 4$ is transformed from the graph of $y = \sin x$, there is a horizontal compression by a factor of $\frac{1}{6}$, a vertical translation of 4 units up, and a reflection in the x-axis. Therefore, the graph of $y = -\sin(6x) + 4$ will look as follows:



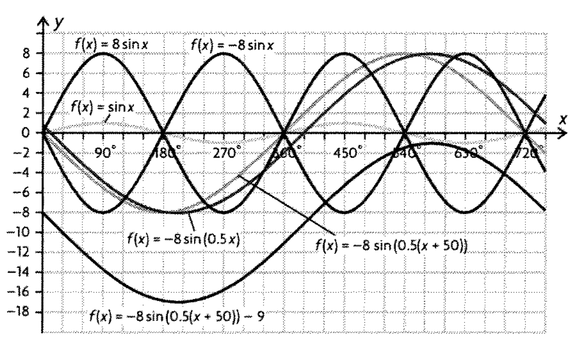
d) If the graph of $y = 4 \cos(2x) - 3$ is transformed from the graph of $y = \cos x$, there is a vertical stretch by a factor of 4, a horizontal compression by a factor of $\frac{1}{2}$, and a vertical translation of 3 units down. Therefore, the graph of $y = 4 \cos(2x) - 3$ will look as follows:



e) If the graph of $y = \frac{1}{2} \cos (3x - 120°)$ is transformed from the graph of $y = \cos x$, there is a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{3}$, and a horizontal translation of 40° to the right. Therefore, the graph of $y = \frac{1}{2} \cos (3x - 120°)$ will look as follows:



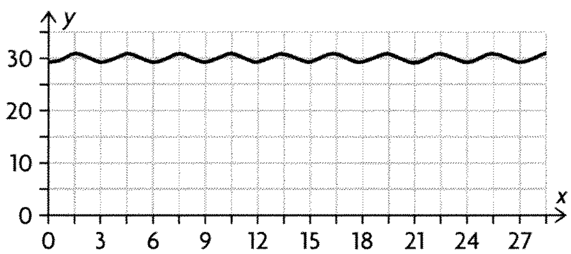
f) If the graph of $y = -8 \sin \left[\frac{1}{2} (x + 50°)\right] - 9$ is transformed from the graph of $y = \sin x$, there is a vertical stretch by a factor of 8, a reflection in the x-axis, a horizontal stretch by a factor of 2, a horizontal translation of 50° to the left or –50°, and a vertical translation of 9 units down. Therefore, the graph $y = -8 \sin \left[\frac{1}{2} (x + 50°)\right] - 9$ will look as follows:



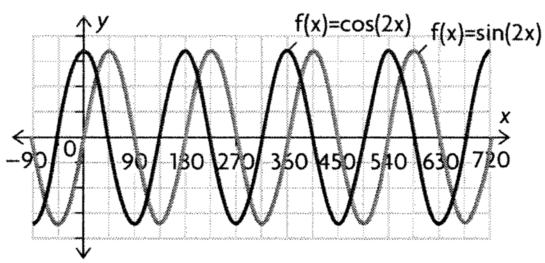
8. a) The period is 180°, so the difference between X max and X min must be at least 180°. The equation of the axis is $y = 6$ and the amplitude is 1, so Y min cannot be greater than 5,

and Y max cannot be less than 7. Possible settings are as follows:
X min: 0°
X max: 180°
Y min: 5
Y max: 7
b) The period is 720°, so the difference between X max and X min must be at least 720°. The equation of the axis is $y = 20$ and the amplitude is 5, so Y min cannot be greater than 15, and Y max cannot be less than 25. Possible settings are as follows:
X min: 0°
X max: 720°
Y min: 15
Y max: 25
c) The period is 4°, so the difference between X max and X min must be at least 4°. The equation of the axis is $y = 82$ and the amplitude is 7, so Y min cannot be greater than 75, and Y max cannot be less than 89. Possible settings are as follows:
X min: 0°
X max: 4°
Y min: 75
Y max: 89
d) The period is 1°, so the difference between X max and X min must be at least 1°. The equation of the axis is $y = -27$ and the amplitude is 0.5, so Y min cannot be greater than -27.5, and Y max cannot be less than -26.5. Possible settings are as follows:
X min: 0°
X max: 1°
Y min: -27.5
Y max: -26.5
9. a) Changing the value of k from 1 in the function $0P(t) = a\cos(kt) + c$ affects the period of the function, since there is a horizontal stretch or compression. Since the period of $y = \cos x$ is 360°, and since the function $P(t) = -20\cos(300t) + 100$ results in a horizontal compression of $\frac{1}{300}$, the period of $P(t) = -20\cos(300t) + 100$ is $\frac{360}{300}$ or 1.2 s. In this case, the period represents one heart beat of an individual.
b) Changing the value of c from 0 in the function $P(t) = a\cos(kt) + c$ affects the equation of the axis of the function, since the graph slides up or down. Since the equation of the axis of $y = \sin x$ is $y = 0$, and since the function $P(t) = -20\cos(300t) + 100$ slides up 100 units, the equation of the axis of $P(t) = -20\cos(300t) + 100$ is $P = 100$. Changing the value of a from 1 in the function $P(t) = a\cos(kt) + c$ affects the amplitude of the function, since there is a vertical stretch or compression. Since the amplitude of $y = \sin x$ is 1, and since the function $P(t) = -20\cos(300t) + 100$ results in a vertical stretch by a factor of 20, the amplitude of $P(t) = -20\cos(300t) + 100$ is 20. Since the equation of the axis is $P = 100$ and the amplitude is 20, P must be greater than or equal to $100 - 20$ or 80 and less than or equal to $100 + 20$ or 120. Therefore, the range is $\{P \in \mathbf{R} \mid 80 \leq P \leq 120\}$. This means that an individual has a maximum blood pressure of 120 and a minimum blood pressure of 80.
10. a) Since the range is $\{y \in \mathbf{R} \mid -1 \leq y \leq 7\}$, the equation of the axis must be $y = \frac{7 + (-1)}{2}$ or $y = 3$. If the amplitude is a, then $3 + a$ must equal the maximum, or 7, and $3 - a$ must equal the minimum, or -1. Therefore, the amplitude, or a, must equal 4. Since the period is 720°, there must be a horizontal stretch by a factor of 2. Therefore, the function must be $y = 4\sin(0.5x) + 3$.
b) The sine function is the result of a horizontal translation of the cosine function of 90° to the right. Therefore, the cosine function that results in the same graph of the sine function from part a is $4\cos(0.5x - 90) + 3$.
11. If the graph of $f(x) = \frac{-1}{2}\cos(120x) + 30$ is transformed from the graph of $f(x) = \cos x$, there is a reflection in the x-axis, a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{120}$, and a vertical translation of 30 units up. Therefore, the graph

of $f(x) = \frac{-1}{2}\cos(120x) + 30$ will look as follows:



12. If the functions $y = \sin x$ and $y = \cos x$ are subjected to a horizontal compression by a factor of 0.5, the functions become $y = \sin(2x)$ and $y = \cos(2x)$. Graph the functions $y = \sin(2x)$ and $y = \cos(2x)$ on the same graph.



It's apparent from the graph that in order to map the sine curve onto the cosine curve, the sine curve would have to undergo a horizontal translation of 45° to the left or −45°.

13. a) The number of hours of daylight increases to a maximum and decreases to a minimum in a regular cycle as Earth revolves around the Sun.

b) March 21 is the 80th day of the year, so if

$$D(t) = 4\sin\left[\frac{360}{365}(t - 80°)\right] + 12\text{, then}$$

$$\begin{aligned} D(80) &= 4\sin\left[\frac{360}{365}(80 - 80)\right] + 12, \\ &= 4\sin(0°) + 12 \\ &= 0 + 12 \\ &= 12\,h \end{aligned}$$

September 21 is the 264th day of the year, so if

$$D(t) = 4\sin\left[\frac{360}{365}(t - 80°)\right] + 12\text{, then}$$

$$\begin{aligned} D(264) &= 4\sin\left[\frac{360}{365}(264 - 80)\right] + 12, \\ &= 4\sin(181°) + 12 \\ &= 0 + 12 \\ &= 12\,h \end{aligned}$$

The significance of these days is that they are the spring and fall equinoxes.

c) June 21 is the 172nd day of the year, so if

$$D(t) = 4\sin\left[\frac{360}{365}(t - 80°)\right] + 12\text{, then}$$

$$\begin{aligned} D(172) &= 4\sin\left[\frac{360}{365}(172 - 80)\right] + 12 \\ &= 4\sin(91°) + 12, \\ &= 4 + 12 \\ &= 16\,h \end{aligned}$$

December 21 is the 355th day of the year, so if

$$D(t) = 4\sin\left[\frac{360}{365}(t - 80°)\right] + 12\text{, then}$$

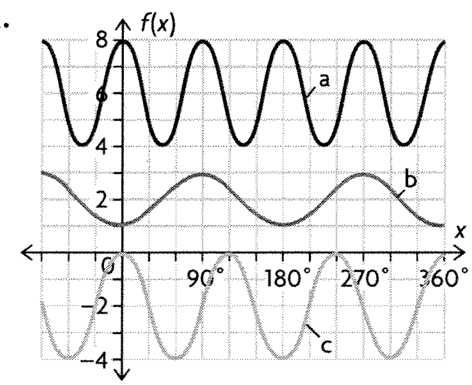
$$\begin{aligned} D(355) &= 4\sin\left[\frac{360}{365}(355 - 80)\right] + 12, \\ &= 4\sin(271°) + 12 \\ &= -4 + 12 \\ &= 8\,h \end{aligned}$$

The significance of these days is that they are the longest and shortest days of the year, or the summer and winter solstices.

d) 12 is the axis of the curve and is half the distance between the maximum and minimum hours of daylight.

6.6 Investigating Models of Sinusoidal Functions, pp. 391–393

1.



a) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0°$ and $x = 90°$, for example, the period is $90 - 0$ or $90°$. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 4 and the maximum is 8, the equation of the axis is $y = \frac{8 + 4}{2}$ or $y = 6$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 4 and the maximum is 8, the amplitude is $\frac{8 - 4}{2}$ or 2. Also, the function crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 2\cos\left(\frac{360}{90}x\right)^\circ + 6$ or $y = 2\cos(4x)^\circ + 6$. This can also be expressed as $y = 2\sin(4x + 90)^\circ + 6$.

b) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0^\circ$ and $x = 180^\circ$, for example, the period is $180 - 0$ or 180°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 3, the equation of the axis is $y = \frac{3 + 1}{2}$ or $y = 2$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 3, the amplitude is $\frac{3 - 1}{2}$ or 1. Also, if the function had not been shifted 90° to the right, then it would have crossed the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = \cos\frac{360}{180}(x - 90)^\circ + 2$ or $y = \cos(2x - 180)^\circ + 2$. This can also be expressed as $y = \sin(2x - 90)^\circ + 2$.

c) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0^\circ$ and $x = 120^\circ$, for example, the period is $120 - 0$ or 120°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -4 and the maximum is 0, the equation of the axis is $y = \frac{0 + (-4)}{2}$ or $y = -2$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -4 and the maximum is 0, the amplitude is $\frac{0 - (-4)}{2}$ or 2. Also, the function crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 2\cos\left(\frac{360}{120}x\right)^\circ - 2$ or $y = 2\cos(3x)^\circ - 2$. This can also be expressed as $y = 2\sin(3x + 90)^\circ - 2$

2. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0^\circ$ and $x = 180^\circ$, for example, the period is $180 - 0$ or 180°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 5 and the maximum is 9, the equation of the axis is $y = \frac{9 + 5}{2}$ or $y = 7$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 5 and the maximum is 9, the amplitude is $\frac{9 - 5}{2}$ or 2. Also, the function crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 2\cos\left(\frac{360}{180}x\right)^\circ + 7$ or $y = 2\cos(2x)^\circ + 7$. This can also be expressed as $y = 2\sin(2x + 90)^\circ + 7$.

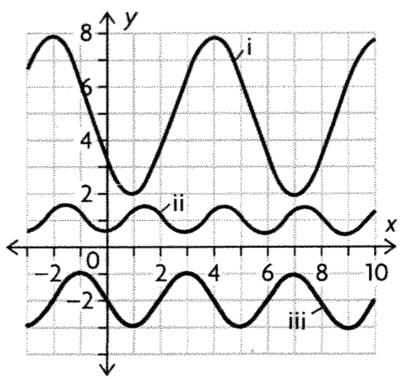
3. Since the sinusoidal function has an amplitude of 4 units and a maximum at (0, 9), the equation of the axis must be $y = 9 - 4$ or $y = 5$. Also, since the maximum is at (0, 9), the graph crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, since the period of 120° is already known, the equation for the sinusoidal function is

$y = 4\cos\left(\frac{360}{120}x\right)^\circ + 5$ or $y = 4\cos(3x)^\circ + 5$.

This can also be expressed as

$y = 4\sin(3x + 90)^\circ + 5$.

4. a)



i) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for i goes through one complete cycle between $x = 4$ and $x = 10$, for example, the period is $10 - 4$ or 6. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 2 and the maximum is 8, the equation of the axis is $y = \frac{8 + 2}{2}$ or $y = 5$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 2 and the maximum is 8, the amplitude is $\frac{8 - 2}{2}$ or 3. Also, if the function had not been shifted 4 units to the right, it would have crossed the y-axis at its maximum, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is

$y = 3\cos\left[\frac{360}{6}(x - 4)\right]^\circ + 5$ or

$y = 3\cos[60(x - 4)]^\circ + 5$. This can also be expressed as $y = 3\sin[60(x - 2.5)]^\circ + 5$.

ii) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for ii goes through one complete cycle between $x = 0$ and $x = 3$, for example, the period is $3 - 0$ or 3. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 0.5 and the maximum is 1.5, the equation of the axis is $y = \frac{1.5 + 0.5}{2}$ or $y = 1$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 0.5 and the maximum is 1.5, the amplitude is $\frac{1.5 - 0.5}{2}$ or 0.5. Also, if the function had not been shifted 1.5 units to the right, then it would have crossed the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is

$y = 0.5\cos\left[\frac{360}{3}(x - 1.5)\right]^\circ + 1$ or

$y = 0.5\cos(120x - 180)^\circ + 1$. This equation can also be expressed as

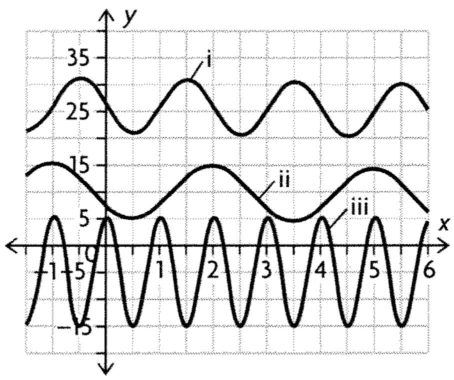
$y = [-0.5\cos(120x)]^\circ + 1$ or

$y = [-0.5\sin(120x + 90)]^\circ + 1$.

iii) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for iii goes through one complete cycle between $x = 0$ and $x = 4$, for example, the period is $4 - 0$ or 4. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -3 and the maximum is -1, the equation of the axis is $y = \frac{-1 + (-3)}{2}$ or $y = -2$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -3 and the maximum is -1, the amplitude is $\frac{-1 - (-3)}{2}$ or 1. Also, if the function had not been shifted 3 units to the right, then it would have crossed the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = \cos\left[\frac{360}{4}(x - 3)\right]^\circ - 2$ or

$y = \cos(90x - 270)^\circ - 2$. This equation can also be expressed as $y = \cos[90(x - 3)]^\circ - 2$ or as $y = \sin[90(x - 2)]^\circ - 2$.

b)



i) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for i goes through one complete cycle between $x = 0$ and $x = 2$, for example, the period is $2 - 0$ or 2. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 20 and the maximum is 30, the equation of the axis is $y = \frac{30 + 20}{2}$ or $y = 25$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 20 and the maximum is 30, the amplitude is $\frac{30 - 20}{2}$ or 5. Also, the function crosses the y-axis at its midpoint and is reflected in the y-axis, so the equation for it should use the sine function and the value of k should be negative. Therefore, the equation for the sinusoidal function is $y = 5 \sin\left(\frac{-360}{2}x\right)^\circ + 25$ or $y = 5 \sin(-180x)^\circ + 25$. This equation can also be expressed as $y = 5 \cos[180(x - 1.5)]^\circ + 25$.

ii) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for ii goes through one complete cycle between $x = 0.5$ and $x = 3.5$, for example, the period is $3.5 - 0.5$ or 3. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 5 and the maximum is 15, the equation of the axis is $y = \frac{15 + 5}{2}$ or $y = 10$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 5 and the maximum is 15, the amplitude is $\frac{15 - 5}{2}$ or 5. Also, if the function had not been shifted 2 units to the right, then it would have crossed the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 5 \cos\left[\frac{360}{3}(x - 2)\right]^\circ + 10$ or $y = 5 \cos(120x - 240)^\circ + 10$. This equation can also be expressed as
$y = 5 \cos[120(x - 2)]^\circ + 10$ or
$y = 5 \sin[120(x - 1.25)]^\circ + 10$.

iii) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph for iii goes through one complete cycle between $x = 0$ and $x = 1$, for example, the period is $1 - 0$ or 1. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -15 and the maximum is 5, the equation of the axis is $y = \frac{5 + (-15)}{2}$ or $y = -5$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -15 and the maximum is 5, the amplitude is $\frac{5 - (-15)}{2}$ or 10. Also, the function crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 10 \cos\left(\frac{360}{1}x\right)^\circ - 5$ or $y = 10 \cos(360x)^\circ - 5$. This can also be expressed as $y = 10 \sin(360x + 90)^\circ - 5$.

5. a) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0^\circ$ and $x = 120^\circ$, for example, the period is $120 - 0$ or 120°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 3, the equation of the axis is $y = \frac{3 + 1}{2}$ or $y = 2$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 3, the amplitude is $\frac{3 - 1}{2}$ or 1. Also, the function crosses the y-axis at its maximum value, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = \cos\left(\frac{360}{120}x\right)^{\circ} + 2$ or $y = \cos(3x)^{\circ} + 2$. This can also be expressed as $y = \sin(3x + 90)^{\circ} + 2$.

b) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0^{\circ}$ and $x = 720^{\circ}$, for example, the period is $720 - 0$ or 720°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 13 and the maximum is 21, the equation of the axis is $y = \frac{13 + 21}{2}$ or $y = 17$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 13 and the maximum is 21, the amplitude is $\frac{21 - 13}{2}$ or 4. Also, the function crosses the y-axis at its minimum value, so the equation for it should use the cosine function and be shifted 180° to the right. Therefore, the equation for the sinusoidal function is $y = 4\cos\left[\frac{360}{720}(x - 180)^{\circ}\right] + 17$ or $y = 4\cos\left[\frac{1}{2}(x - 180)^{\circ}\right] + 17$. This can also be expressed as or $y = 4\sin\left(\frac{1}{2}x\right)^{\circ} + 17$.

c) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = -120^{\circ}$ and $x = 120^{\circ}$, for example, the period is $120 - (-120)$ or 240°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -7 and the maximum is -1, the equation of the axis is $y = \frac{-1 + (-7)}{2}$ or $y = -4$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -7 and the maximum is -1, the amplitude is $\frac{(-1) - (-7)}{2}$ or 3. Also, the function crosses the y-axis at its midpoint, so the equation for it should use the sine function. Therefore, the equation for the sinusoidal function is $y = \sin\left(\frac{360}{240}x\right)^{\circ} - 4$ or $y = 3\sin\left(\frac{3}{2}x\right)^{\circ} - 4$. This function can also be expressed as $y = 3\cos\left[\frac{3}{2}(x - 60)\right]^{\circ} - 4$.

d) The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = -20^{\circ}$ and $x = 100^{\circ}$, for example, the period is $100 - (-20)$ or 120°. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -1 and the maximum is 5, the equation of the axis is $y = \frac{5 + (-1)}{2}$ or $y = 2$. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -1 and the maximum is 5, the amplitude is $\frac{5 - (-1)}{2}$ or 3. Also, the function would cross the y-axis at its maximum value, except that the function is shifted 10° to the right, so the equation for it should use the cosine function. Therefore, the equation for the sinusoidal function is $y = 3\cos\left[\frac{360}{120}(x - 10)\right]^{\circ} + 2$ or $y = 3\cos[3(x - 10)]^{\circ} + 2$. This can also be expressed as $y = 3\sin[3(x + 20)]^{\circ} + 2$.

6. a) Since the amplitude is 3, a in the function $y = a\cos(k(x - d)) + c$ is 3. Since the period is 360°, k in the function

$y = a\cos(k(x - d)) + c$ is $\frac{360}{360}$ or 1. Since the equation of the axis is $y = 11$, c in the equation $y = a\cos(k(x - d)) + c$ is 11. Since the horizontal translation is 0°, d in the equation $y = a\cos(k(x - d))$ is 0. Therefore, the equation is $y = 3\cos x° + 11$. This can also be expressed as is $y = 3\sin(x + 90)° + 11$.

b) Since the amplitude is 4, a in the function $y = a\cos(k(x - d)) + c$ is 4. Since the period is 180°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{180}$ or 2. Since the equation of the axis is $y = 15$, c in the equation $y = a\cos(k(x - d)) + c$ is 15. Since the horizontal translation is 30°, d in the equation $y = a\cos(k(x - d))$ is 30. Therefore, the equation is $y = 4\cos[2(x - 30)]° + 15$. This can also be expressed as $y = 4\sin[2(x + 15)]° + 15$.

c) Since the amplitude is 2, a in the function $y = a\cos(k(x - d)) + c$ is 2. Since the period is 40°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{40}$ or 9. Since the equation of the axis is $y = 0$, c in the equation $y = a\cos(k(x - d)) + c$ is 0. Since the horizontal translation is 7°, d in the equation $y = a\cos(k(x - d))$ is 7. Therefore, the equation is $y = 2\cos 9(x - 7)°$. This can also be expressed as is $y = 2\sin 9(x + 3)°$.

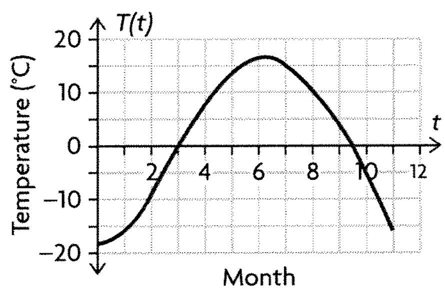
d) Since the amplitude is 0.5, a in the function $y = a\cos(k(x - d)) + c$ is 0.5. Since the period is 720°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{720}$ or $\frac{1}{2}$. Since the equation of the axis is $y = -3$, c in the equation $y = a\cos(k(x - d)) + c$ is -3. Since the horizontal translation is $-56°$, d in the equation $y = a\cos(k(x - d))$ is -56. Therefore, the equation is $y = 0.5\cos\left[\frac{1}{2}(x + 56)\right]° - 3$.

This can also be expressed as

$y = 0.5\sin\left[\frac{1}{2}(x + 236)\right]° - 3.$

7. Since the amplitude is 6, a in the function $y = a\cos(k(x - d)) + c$ is 6. Since the period is 45°, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{45}$ or 8. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since a minimum is at (0, 1), the maximum is at $1 + 2 \times 6$ or 13, and the equation of the axis is $y = \frac{13 + 1}{2}$ or $y = 7$. Since the equation of the axis is $y = 7$, c in the equation $y = a\cos(k(x - d)) + c$ is 7. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the equation $y = a\cos(k(x - d))$ should be negative. Since there is no horizontal translation, d in the equation $y = a\cos(k(x - d))$ is 0. Therefore, the equation is $y = -6\cos(8x)° + 7$. This can also be expressed as $y = -6\sin(8x + 90)° + 7$.

8. a)



b) A sinusoidal model fits the graph because it changes with a cyclical pattern over time.

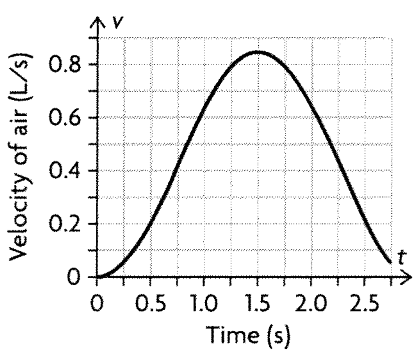
c) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -18.6 and the maximum is 17.0, the amplitude is $\frac{17.0 - (-18.6)}{2}$ or 17.8. Since the amplitude is 17.8, a in the function $T(t) = a\cos(k(t - d)) + c$ is 17.8. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 12$, the period is $12 - 0$ or 12. Since the period is 12, k in the function

$T(t) = a\cos(k(x - d)) + c$ is $\frac{360}{12}$ or 30. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -18.6 and the maximum is 17.0, the equation of the axis is $y = \frac{17.0 + (-18.6)}{2}$ or $y = -0.8$. Since the equation of the axis is $y = -0.8$, c in the function $T(t) = a\cos(k(t - d)) + c$ is -0.8. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the function $T(t) = a\cos(k(t - d))$ should be negative. Since there is no horizontal translation, d in the function $T(t) = a\cos(k(t - d))$ is 0. Therefore, the equation for the function is $T(t) = -17.8\cos(30t)° - 0.8$. This can also be expressed as is $T(t) = -17.8\sin(30t + 90)° - 0.8$.

d) The first cycle goes from month 0 to month 11. Therefore, the second cycle goes from month 12 to month 23. Since each cycle is the same, the average monthly temperature for month 20 is the same as the average monthly temperature for month 8. Therefore, the average monthly temperature for month 20 is 10.3 °C.

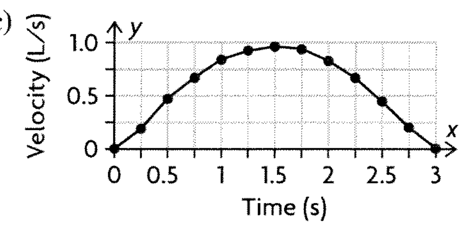
9. a) The respiratory cycle is an example of a periodic function because we inhale, rest, exhale, rest, inhale, and so on in a cyclical pattern.

b)



The amplitude is half the distance between the maximum and minimum values. Since the minimum is 0 and the maximum is 0.85, the amplitude is $\frac{0.85 - 0}{2}$ or 0.425. Since the amplitude is 0.425, a in the equation $v = a\cos(k(t - d)) + c$ is 0.425. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 3$, the period is $3 - 0$ or 3. Since the period is 3, k in the equation $v = a\cos(k(x - d)) + c$ is $\frac{360}{3}$ or 120. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 0 and the maximum is 0.85, the equation of the axis is $y = \frac{0.85 + 0}{2}$ or $y = 0.425$. Since the equation of the axis is $y = 0.425$, c in the equation $v = a\cos(k(t - d)) + c$ is 0.425. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the equation $v = a\cos(k(t - d))$ should be negative. Since there is no horizontal translation, d in the equation $v = a\cos(k(t - d))$ is 0. Therefore, the equation is $v = -0.425\cos(120t)° + 0.425$. This can also be expressed as $v = -0.425\sin(120t + 90)° + 0.425$.

c)



The equation is almost an exact fit on the scatter plot.

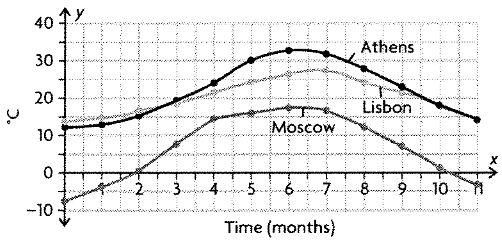
d) The first cycle goes from 0 s to 3 s. Therefore, the second cycle goes from 3 s to 6 s. Since each cycle is the same, the velocity of Nicole's breathing at 3 s is the same as the velocity of Nicole's breathing at 6 s. Therefore, the velocity of Nicole's breathing at 6 s is 0 L/s.

e) Since the equation for this situation is

$$v = -0.425 \cos (120t)^\circ + 0.425,$$
$$0.5 = -0.425 \cos (120t)^\circ + 0.425$$
$$0.075 = -0.425 \cos (120t)^\circ$$
$$-0.176 = \cos (120t)^\circ$$
$$100.164 = 120t$$
$$t = 0.8\text{ s}$$

Since 0.8 s have passed when the velocity is 0.5 L/s, and since the maximum occurs at 1.5 s, the difference between the time at the maximum and the time at velocity 0.5 L/s is 1.5 s − 0.8 s or 0.7 s. Since the curve is symmetrical, a velocity of 0.5 L/s also occurs at 1.5 s + 0.7 s or 2.2 s.

10. a)



b) i) The amplitude is half the distance between the maximum and minimum values. Since the minimum for Athens is 12 and the maximum is 33, the amplitude is $\frac{33 - 12}{2}$ or 10.5. Since the amplitude is 10.5, a in the function $T(t) = a\cos(k(t - d)) + c$ is 10.5. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 12$, the period is $12 - 0$ or 12. Since the period is 12, k in the function $T(t) = a\cos(k(t - d)) + c$ is $\frac{360}{12}$ or 30. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for Athens is 12 and the maximum is 33, the equation of the axis is $y = \frac{33 + 12}{2}$ or $y = 22.5$. Since the equation of the axis is $y = 22.5$, c in the function $T(t) = a\cos(k(t - d)) + c$ is 22.5. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the function $T(t) = a\cos(k(t - d))$ should be negative. Since there is no horizontal translation, d in the function $T(t) = a\cos(k(t - d))$ is 0. Therefore, the equation for the function for Athens is $T(t) = -10.5 \cos 30t + 22.5$. This can also be expressed as $T(t) = -10.5 \sin (30t + 90) + 22.5$.

ii) The amplitude is half the distance between the maximum and minimum values. Since the minimum for Lisbon is 13 and the maximum is 27, the amplitude is $\frac{27 - 13}{2}$ or 7. Since the amplitude is 7, a in the function $T(t) = a\cos(k(t - d)) + c$ is 7. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 12$, the period is $12 - 0$ or 12. Since the period is 12, k in the function $T(t) = a\cos(k(x - d)) + c$ is $\frac{360}{12}$ or 30. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for Lisbon is 13 and the maximum is 27, the equation of the axis is $y = \frac{27 + 13}{2}$ or $y = 20$. Since the equation of the axis is $y = 20$, c in the function $T(t) = a\cos(k(t - d)) + c$ is 20. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the function $T(t) = a\cos(k(t - d))$ should be negative. Since there is no horizontal translation, d in the function $T(t) = a\cos(k(t - d))$ is 0. Therefore, the equation for the function for Lisbon is $T(t) = -7 \cos 30t + 20$. This can also be expressed as $T(t) = -7 \sin (30t + 90) + 20$.

iii) The amplitude is half the distance between the maximum and minimum values. Since the minimum for Moscow is -9 and the maximum is 23, the amplitude is $\frac{23-(-9)}{2}$ or 16. Since the amplitude is 16, a in the function $T(t) = a\cos(k(t-d)) + c$ is 16. The period is the change in t that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $t = 0$ and $t = 12$, the period is $12 - 0$ or 12. Since the period is 12, k in the function $T(t) = a\cos(k(x-d)) + c$ is $\frac{360}{12}$ or 30. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum for Moscow is -9 and the maximum is 23, the equation of the axis is $y = \frac{23+(9)}{2}$ or $y = 7$. Since the equation of the axis is $y = 7$, c in the function $T(t) = a\cos(k(t-d)) + c$ is 7. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Since there is a reflection in the x-axis, the sign of a in the function $T(t) = a\cos(k(t-d))$ should be negative. Since there is no horizontal translation, d in the function $T(t) = a\cos(k(t-d))$ is 0. Therefore, the equation for the function for Moscow is $T(t) = -16\cos 30t + 7$. This can also be expressed as $T(t) = -16\sin(30t + 90) + 7$.

c) The latitude of a city affects its amplitude and vertical translation.

d) Athens and Lisbon are close to the same latitude; Moscow is farther north.

11. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 5 and the maximum is 11, the amplitude is $\frac{11-5}{2}$ or 3. Since the amplitude is 3, a in the equation $y = a\cos(k(x-d)) + c$ is 3. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0$ s and $x = 0.04$ s, for example, the period is $0.04 - 0$ or 0.04 s. Since the period is 0.04 s, k in the equation $y = a\cos(k(x-d)) + c$ is $\frac{360}{0.04}$ or 9000. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 5 and the maximum is 11, the equation of the axis is $y = \frac{11+5}{2}$ or $y = 8$. Since the equation of the axis is $y = 8$, c in the equation $y = a\cos(k(x-d)) + c$ is 8. Since the function would have crossed the y-axis at its maximum value had it not been shifted 0.01 units to the right, the equation for it should use the cosine function and be shifted 0.01 units to the right. Therefore, d in the equation $y = a\cos(k(x-d))$ is 0.01. Therefore, the equation for the function is $y = 3\cos 9000(x - 0.01)^\circ + 8$. This can also be expressed as $y = 3\sin(9000x)^\circ + 8$.

b) The peaks of the function represent the maximum equivalent stress.

c) Since the equation for this situation is $y = 3\cos 9000(x - 0.01)^\circ + 8$, at 0.143 s the amount of stress the motor was undergoing was

$$\begin{aligned} y &= 3\cos 9000(0.143 - 0.01)^\circ + 8 \\ &= 3\cos 9000(0.133)^\circ + 8 \\ &= 3\cos 1197^\circ + 8 \\ &= -1.36 + 8 \\ &= 6.64 \text{ MPa} \end{aligned}$$

12. Find the amplitude. Whatever the amplitude is, a in the equation $y = a\cos(k(x-d)) + c$ will be equal to it. Find the period. Whatever the period is, k in the equation $y = a\cos(k(x-d)) + c$ will be equal to 360 divided by it. Find the equation of the axis. Whatever the equation of the axis is, c in the equation $y = a\cos(k(x-d)) + c$ will be equal to it. Find the phase shift. Whatever the phase shift is, d in the equation $y = a\cos(k(x-d)) + c$ will be equal to it. Determine if the function is reflected in its axis. If it is, the sign of a will be negative; otherwise, it will be positive. Determine if the function is reflected in the y-axis. If it is, the sign of k will be negative; otherwise, it will be positive.

13. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 0 and the maximum is 60, the amplitude is $\frac{60 - 0}{2}$ or 30. Since the amplitude is 30, a in the equation $y = a\cos(k(x - d)) + c$ is 30. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0$ cm and $x = \pi \times 60$ or 188.5 cm, the period is $188.5 - 0$ or 188.5 cm. Since the period is 188.5 cm, k in the equation

$y = a\cos(k(x - d)) + c$ is $\frac{360}{188.5}$ or 1.909859.

The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 0 and the maximum is 60, the equation of the axis is $y = \frac{60 + 0}{2}$ or $y = 30$. Since the equation of the axis is $y = 30$, c in the equation $y = a\cos(k(x - d)) + c$ is 30. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $y = -30\cos(1.909\,859x)° + 30$. This can also be expressed as

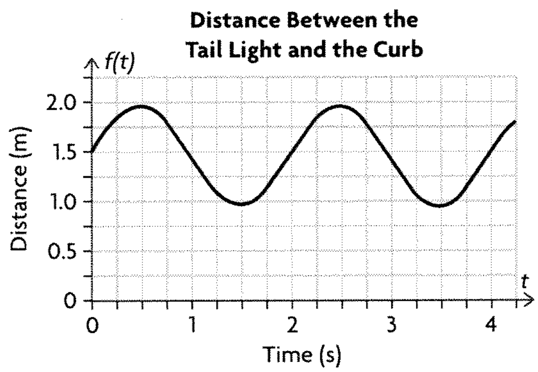
$y = -30\sin(1.909\,859x + 90)° + 30$. The height of the nail above the ground after 1 km is

$$
\begin{aligned}
y &= -30\cos(1.909\,859 \times 100\,000)° + 30\\
&= -30 \times -1 + 30\\
&= 60 \text{ cm}
\end{aligned}
$$

14. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1 and the maximum is 15, the amplitude is $\frac{15 - 1}{2}$ or 7. Since the amplitude is 7, a in the equation $h = a\cos(k(t - d)) + c$ is 7. The period is the change in t that occurs as the function goes through one complete cycle. The circumference of the Ferris wheel in m is $2 \times \pi \times 7$ or 44.0 m. The speed of the Ferris wheel in m/s is $10 \times 1000 \times \frac{1}{3600}$ or 2.8 m/s. Therefore, since the graph goes through one complete cycle between $t = 0$ s and $t = 44.0 \div 2.8$ or 15.7 s, the period is $15.7 - 0$ or 15.7 s. Since the period is 15.7 s, k in the equation $h = a\cos(k(t - d)) + c$ is $\frac{360}{15.7}$ or 22.9. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1 and the maximum is 15, the equation of the axis is $y = \frac{15 + 1}{2}$ or $y = 8$. Since the equation of the axis is $y = 8$, c in the equation $h = a\cos(k(t - d)) + c$ is 8. Since the function crosses the y-axis at its maximum value, the equation for it should use the cosine function. Therefore, the equation for the function is $h = 7\cos(22.9t)° + 8$. This can also be expressed as $h = 7\sin(22.9t + 90)° + 8$.

6.7 Solving Problems Using Sinusoidal Models, pp. 398–401

1.



a) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1.0 and the maximum is 2.0, the equation of the axis is $d = \frac{2.0 + 1.0}{2}$ or $d = 1.5$ m. In this situation it represents the distance between the tail lights and the curb if the trailer isn't swinging back and forth.

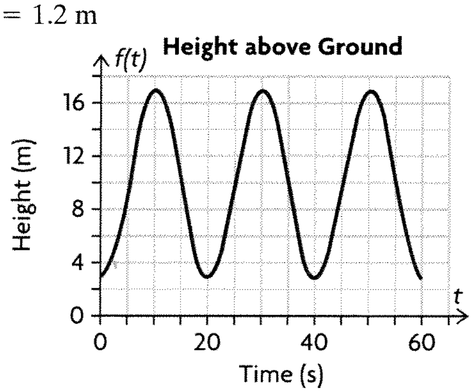
b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1.0 and the maximum is 2.0, the amplitude is $\frac{2.0 - 1.0}{2}$ or 0.5 m. In this situation it represents the distance the trailer swings to the left and right.

c) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between

0 s and 2 s, for example, the period is 2 – 0 or 2 s. In this situation it represents the time it takes the trailer to swing back and forth.
d) Since the amplitude is 0.5 m, a in the function $y = a\cos(k(x - d)) + c$ is 0.5. Since the period is 2 s, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{2}$ or 180. Since the equation of the axis is $y = 1.5$ m, c in the function $y = a\cos(k(x - d)) + c$ is 1.5. Since the function would have crossed the y-axis at its maximum had it not been shifted 0.5 units to the right, d in the equation $y = a\cos(k(x - d))$ is 0.5. Therefore, the equation is $d = 0.5\cos[180(t - 0.5)]° + 1.5$. The range is all possible values of y. Since y oscillates between 1 and 2, the range is $\{d \in \mathbf{R} \mid 1 \le d \le 2\}$.
e) The range is the distance the trailer swings back and forth. The domain is the time.
f) Since the equation that represents the situation is $d = 0.5\cos[180(t - 0.5)]° + 1.5$, the distance from the tail light to the curve at $t = 3.2$ s is

$$\begin{aligned} d &= 0.5\cos[180(3.2 - 0.5)]° + 1.5 \\ &= 0.5\cos 486° + 1.5 \\ &= 0.5 \times -0.59 + 1.5 \\ &= 1.2 \text{ m} \end{aligned}$$

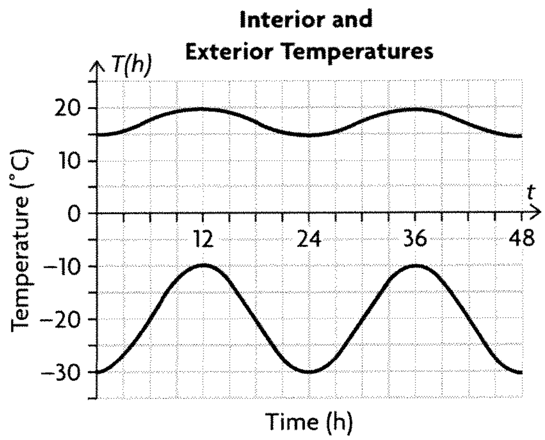
2.



a) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 3 and the maximum is 17, the equation of the axis is $h = \frac{17 + 3}{2}$ or $h = 10$ m. In this situation it represents the axle height.
b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 3 and the maximum is 17, the amplitude is $\frac{17 - 3}{2}$ or 7 m. In this situation it represents the length of the blade.
c) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between 0 s and 20 s, for example, the period is 20 – 0 or 20 s. In this situation it represents the time to complete one revolution.
d) The domain is all possible values of t. Since Don Quixote remains snagged for 7 complete cycles and each cycle takes 20 s, the domain is $\{t \in \mathbf{R} \mid 0 \le t \le 140\}$. The range is all possible values of h. Since h oscillates between 3 and 17, the range is $\{h \in \mathbf{R} \mid 3 \le h \le 17\}$.
e) Since the amplitude is 7 m, a in the function $y = a\cos(k(x - d)) + c$ is 7. Since the period is 20 s, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{20}$ or 18. Since the equation of the axis is $h = 10$ m, c in the function $y = a\cos(k(x - d)) + c$ is 10. Since the function crosses the y-axis at its minimum and is reflected in its axis, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation is $h = -7\cos(18x)° + 10$.
f) If the wind speed decreased, the period would be larger.
3. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 4 and the maximum is 12, the amplitude is $\frac{12 - 4}{2}$ or 4. Since the amplitude is 4, a in the equation $y = a\cos(k(x - d)) + c$ is 4. The period is the change in x that occurs as the function goes through one complete cycle. Since Chantelle goes through one complete cycle between $x = 1$ s and $x = 5$ s, for example, the period is $5 - 1$ or 4 s. Since the period is 4 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{4}$ or 90. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 4 and the maximum is 12, the equation of the axis is $y = \frac{12 + 4}{2}$ or $y = 8$. Since the equation of the axis is $y = 8$, c in the equation $y = a\cos(k(x - d)) + c$ is 8. Since the function would have crossed the y-axis at its maximum value had it not been shifted 1 unit to the right, the equation for it should use the

cosine function and be shifted 1 unit to the right. Therefore, d in the equation $y = a\cos(k(x - d))$ is 1. Therefore, the equation for the function is $d = 4\cos(90(t - 1))° + 8$.

4.



a) They have the same period (24) and the same horizontal translation (12), but different amplitudes (2.5 and 10) and different equations of the axis ($T = 17.5$ and $T = -20$). The top curve is probably the interior temperature (higher, with less fluctuation), while the bottom curve is probably the exterior temperature.

b) The domain is all possible values of t. Since both curves cover a 48 h period, the domain for both curves is $\{t \in \mathbf{R} \mid 0 \le t \le 48\}$. The range is all possible values of T. Since T oscillates between 15 and 20 for the top curve, the range for the top curve is $\{T \in \mathbf{R} \mid 15 \le T \le 20\}$. Since T oscillates between -30 and -10 for the bottom curve, the range for the bottom curve is $\{T \in \mathbf{R} \mid -30 \le T \le -10\}$.

c) i) Since the amplitude for the top curve is 2.5, a in the function $y = a\cos(k(x - d)) + c$ is 2.5. Since the period is 24 h, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{24}$ or 15. Since the equation of the axis is $y = 17.5$, c in the function $y = a\cos(k(x - d)) + c$ is 17.5. Since the function would have crossed the y-axis at its maximum had it not been shifted 12 units to the right, d in the equation $y = a\cos(k(x - d))$ is 12. Therefore, the equation for the top curve is $T = 2.5\cos[15(h - 12)]° + 17.5$.

ii) Since the amplitude for the bottom curve is 10, a in the function $y = a\cos(k(x - d)) + c$ is 10. Since the period is 24 h, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{24}$ or 15. Since the equation of the axis is $y = -20$, c in the function $y = a\cos(k(x - d)) + c$ is -20. Since the function would have crossed the y-axis at its maximum had it not been shifted 12 units to the right, d in the equation $y = a\cos(k(x - d))$ is 12. Therefore, the equation for the top curve is $T = 10\cos[15(h - 12)]° - 20$.

5. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -30 and the maximum is 30, the amplitude is $\frac{30 - (-30)}{2}$ or 30. Since the amplitude is 30, a in the equation $y = a\cos(k(x - d)) + c$ is 30. The period is the change in x that occurs as the function goes through one complete cycle. Since the building goes through one complete cycle between $t = 2$ s and $t = 22$ s, for example, the period is $22 - 2$ or 20 s. Since the period is 20 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{20}$ or 18. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -30 and the maximum is 30, the equation of the axis is $y = \frac{30 + (-30)}{2}$ or $y = 0$. Since the equation of the axis is $y = 0$, c in the equation $y = a\cos(k(x - d)) + c$ is 0. Since the function would have crossed the y-axis at its maximum value had it not been shifted 12 units to the right, the equation for it should use the cosine function and be shifted 12 units to the right. Therefore, d in the equation $y = a\cos(k(x - d))$ is 12. Therefore, the equation for the function is $d = 30\cos(18(t - 12))°$.

b) If the sway is reduced by 70 percent, the amplitude of the function is reduced by 70 percent. Everything else about the function remains the same. Therefore, the function becomes $d = 30 \times 0.3\cos(18(t - 12))°$ or $d = 9\cos(18(t - 12))°$.

6. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1.5 m and the maximum is 2.1 m, the amplitude is $\frac{2.1 - 1.5}{2}$ or 0.3 m. Since the amplitude is 0.3 m, a in the equation $y = a\cos(k(x - d)) + c$ is 0.3. The period is the change in t that occurs as the function goes through one complete cycle. Since Milton goes through one complete cycle between $t = 0$ s and $t = 2.5$ s, for example, the period is $2.5 - 0$ or 2.5 s. Since the period is 2.5 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{2.5}$ or 144. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1.5 m and the maximum is 2.1 m, the equation of the axis is $y = \frac{2.1 + 1.5}{2}$ or $y = 1.8$. Since the equation of the axis is $y = 1.8$, c in the equation $y = a\cos(k(x - d)) + c$ is 1.8. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $d = -0.3\cos(144t)° + 1.8$.

b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1.5 m and the maximum is 2.1 m, the amplitude is $\frac{2.1 - 1.5}{2}$ or 0.3 m. In this situation it represents the height of the crest relative to the normal water level.

c) Since the equation for this situation is $d = -0.3\cos(144t)° + 1.8$, at $t = 4$ s, Milton's height above the bottom of the pool is

$$\begin{aligned} d &= -0.3\cos(144 \times 4)° + 1.8 \\ &= -0.3\cos(576)° + 1.8 \\ &= -0.3 \times -0.8 + 1.8 \\ &= 0.2 + 1.8 \\ &= 2 \text{ m} \end{aligned}$$

d) If data are collected for only 40 s, there will be $40 \div 2.5$ or 16 complete cycles.

e) If the period of the function changes to 3 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{3}$ or 120. Therefore, the equation becomes $d = -0.3\cos(120t)° + 1.8$.

7. The amplitude is half the distance between the maximum and minimum values. Since the minimum is -4.5 A and the maximum is 4.5 A, the amplitude is $\frac{4.5 - (-4.5)}{2}$ or 4.5 A. Since the amplitude is 4.5 A, a in the equation $y = a\cos(k(x - d)) + c$ is 4.5. The period is the change in t that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle between $t = 0$ s and $t = \frac{1}{60}$ s, for example, the period is $\frac{1}{60} - 0$ or $\frac{1}{60}$ s. Since the period is $\frac{1}{60}$ s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{\frac{1}{60}}$ or 21 600. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -4.5 A and the maximum is 4.5 A, the equation of the axis is $y = \frac{4.5 + (-4.5)}{2}$ or $y = 0$. Since the equation of the axis is $y = 0$, c in the equation $y = a\cos(k(x - d)) + c$ is 0. Since the function crosses the y-axis at its maximum value, the equation for it should use the cosine function. Therefore, the equation for the function is $C = 4.5\cos(21\,600t)°$.

8. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 4 cm and the maximum is 20 cm, the amplitude is $\frac{20 - 4}{2}$ or 8 cm. Since the amplitude is 8 cm, a in the equation $y = a\cos(k(x - d)) + c$ is 8. The period is the change in t that occurs as the function goes through one complete cycle. Since the function goes through one complete cycle between $t = 0.2$ s and $t = 1$ s, for example, the period is $1 - 0.2$ or 0.8 s. Since the period is 0.8 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{0.8}$ or 450. The equation of the axis is the equation of the horizontal line that is halfway between

the maximum and the minimum. Since the minimum is 4 cm and the maximum is 20 cm, the equation of the axis is $y = \frac{20 + 4}{2}$ or $y = 12$.
Since the equation of the axis is $y = 12$, c in the equation $y = a\cos(k(x - d)) + c$ is 12. Since the function would have crossed the y-axis at its maximum value had it not been shifted 0.2 units to the right, the equation for it should use the cosine function and be shifted 0.2 units to the right. Therefore, d in the equation $y = a\cos(k(x - d))$ is 0.2. Therefore, the equation for the function is
$h = 8\cos(450(t - 0.2))° + 12$.
b) The domain is all possible values of t. Since there is no limit to the amount of time that Candice can hold onto the spring with the lead ball, the domain is $\{t \in \mathbf{R}\}$. The range is all possible values of h. Since the height of the ball oscillates between 4 cm and 20 cm, the range is $\{h \in \mathbf{R} \mid 4 \leq h \leq 20\}$.
c) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 4 cm and the maximum is 20 cm, the equation of the axis is $h = \frac{20 + 4}{2}$ or $h = 12$ cm.
In this situation it represents the resting position of the spring.
d) Since the equation for the situation is $h = 8\cos(450(t - 0.2))° + 12$, the height of the lead ball at $t = 1.3$ s is

$$\begin{aligned} h &= 8\cos(450(1.3 - 0.2))° + 12 \\ &= 8\cos(450 \times 1.1)° + 12 \\ &= 8\cos(495)° + 12 \\ &= 8 \times -0.71 + 12 \\ &= -5.7 + 12 \\ &= 6.3 \text{ cm} \end{aligned}$$

9. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 10 and the maximum is 70, the amplitude is $\frac{70 - 10}{2}$ or 30. Since the amplitude is 30, a in the equation $y = a\cos(k(x - d)) + c$ is 30. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0$ cm and $x = 2 \times \pi \times 40$ or 251.3 cm, the period is $251.3 - 0$ or 251.3 cm. Since the period is 251.3 cm, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{251.3}$ or 1.43. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 10 and the maximum is 70, the equation of the axis is $y = \frac{70 + 10}{2}$ or $y = 40$. Since the equation of the axis is $y = 40$, c in the equation $y = a\cos(k(x - d)) + c$ is 40. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $h = -30\cos(1.43x)° + 40$.
b) The domain is all possible values of d. Since the wheel completes 5 revolutions before it stops, it travels $2 \times \pi \times 40 \times 5$ or 400π cm before it stops. Therefore, the domain is $\{d \in \mathbf{R} \mid 0 \leq d \leq 400\pi\}$. The range is all possible values of h. Since the height of the circular mark oscillates between 10 cm and 70 cm, the range is $\{h \in \mathbf{R} \mid 10 \leq h \leq 70\}$.
c) Since the equation for the situation is $h = -30\cos(1.43x)° + 40$, the height of the mark when the wheel has traveled 120 cm is

$$\begin{aligned} h &= -30\cos(1.43 \times 120)° + 40 \\ &= -30\cos(171.6)° + 40 \\ &= -30 \times -0.99 + 40 \\ &= 69.7 \text{ cm} \end{aligned}$$

10. The periods of the two functions are the same, so the population of rabbits and the population of foxes both go through one complete cycle in the same amount of time. That's the only way the population of rabbits and the population of foxes appear to be related. The amplitudes, equations of the axis, and horizontal translations of the two functions all differ.
11. The period and amplitude, as well as where it starts on the x-axis and the position on the y-axis when it started.
12. The amplitude is half the distance between the maximum and minimum values. Since the minimum is 7 cm and the maximum is 19 cm, the amplitude is $\frac{19 - 7}{2}$ or 6 cm. Since the

amplitude is 6 cm, a in the equation $y = a\cos(k(x - d)) + c$ is 6. The period is the change in x that occurs as the function goes through one complete cycle. Since pulley A has half the circumference as pulley B, pulley A makes 2 rotations for every 1 rotation of pulley B. Therefore, the change in x in terms of pulley A that occurs as the function goes through one complete cycle is $2 \times 360°$ or $720°$. Since the graph goes through one complete cycle between $x = 0$ and $x = 720$, the period is $720 - 0$ or 720. Since the period is 720, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{720}$ or 0.5. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 7 cm and the maximum is 19 cm, the equation of the axis is $y = \frac{19 + 7}{2}$ or $y = 13$. Since the equation of the axis is $y = 13$, c in the equation $y = a\cos(k(x - d)) + c$ is 13. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $h = -6\cos(0.5x)° + 13$.

13. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is -4 and the maximum is 2, the amplitude is $\frac{2 - (-4)}{2}$ or 3. Since the amplitude is 3, a in the equation $y = a\cos(k(x - d)) + c$ is 3. The period is the change in x that occurs as the function goes through one complete cycle. Since the graph goes through one complete cycle between $x = 0$ and $x = 360$, the period is $360 - 0$ or 360. Since the period is 360, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{360}$ or 1. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -4 and the maximum is 2, the equation of the axis is $y = \frac{2 + (-4)}{2}$ or $y = -1$. Since the equation of the axis is $y = -1$, c in the equation $y = a\cos(k(x - d)) + c$ is -1. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $f(x) = -3\cos x - 1$.

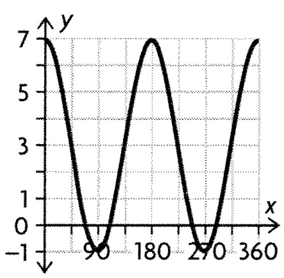
b) If $f(x) = -3\cos x - 1$, then

$$\begin{aligned} f(20) &= -3\cos 20° - 1 \\ &= -3 \times 0.94 - 1 \\ &= -2.8 - 1 \\ &= -3.8 \end{aligned}$$

c) $f(x)$ first equals 2 when x equals 180°, and the period of the function is 360°. Therefore, $f(x) = 2$ when $x = 180° + 360°k$, $k \in I$, and the answer is i.

d) $f(x)$ first equals -1 when x equals 90°, and it equals -1 every 180° after that. Therefore, $f(x) = -1$ when $x = 90° + 180°k$, $k \in I$, and the answer is iv.

14.



From the graph, when $f(x) = 7$, $x = 0°$, 180°, and 360°.

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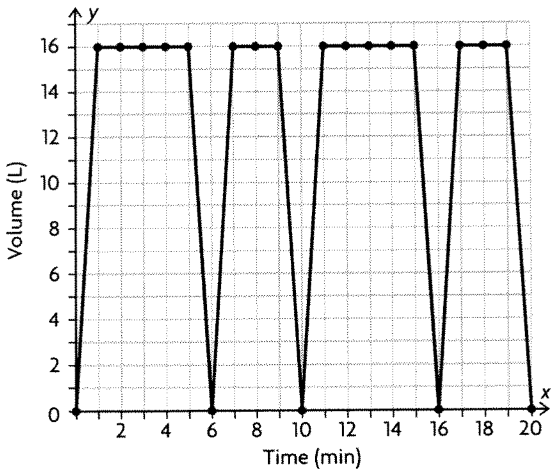
1. a)

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0	16	16	16	16	16	0	16

Time (min)	8	9	10	11	12	13	14	15
Volume (L)	16	16	0	16	16	16	16	16

Time (min)	16	17	18	19	20
Volume (L)	0	16	16	16	0



b) Yes, the graph repeats itself, and so is periodic.
c) The part of the graph from $x = 0$ to $x = 10$ is an exact copy of the part from $x = 10$ to $x = 20$. So the period is of length $10 - 0 = 10$ min.
In this case, the period represents the time to complete one washing cycle.
d) The equation of the axis is

$$y = \frac{\text{minimum} + \text{maximum}}{2}$$

$$= \frac{0 + 16}{2}$$

$$= 8 \text{ L}$$

e) The amplitude is

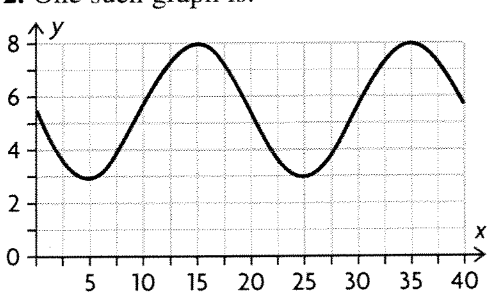
$$a = \text{maximum} - \text{axis}$$

$$= 16 - 8$$

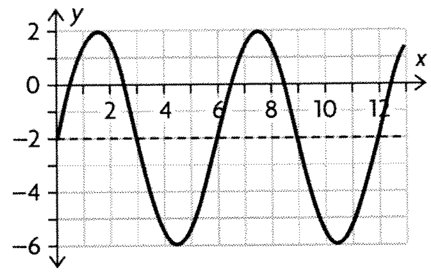
$$= 8 \text{ L}$$

f) The range of y values is from $y = 0$ to $y = 16$. That is, $\{y \in \mathbf{R} \mid 0 \leq y \leq 16\}$.

2. One such graph is:



3. One such graph is:

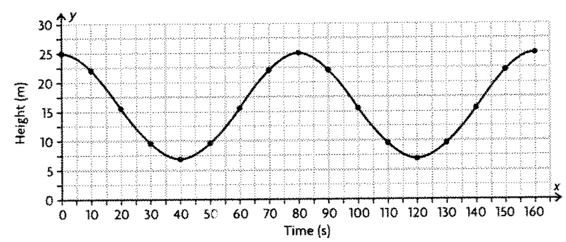


4. Here is the data representing Colin's height (in meters) at a given time (in seconds), and the corresponding graph:

Time (s)	0	10	20	30	40	50
Height (m)	25	22.4	16	9.7	7	9.7

Time (s)	60	70	80	90	100	110
Height (m)	16	22.4	25	22.4	16	9.7

Time (s)	120	130	140	150	160
Height (m)	7	9.7	16	22.4	25



a) The graph repeats itself every 80 s, and so has period 80. In this case, the period represents the time to complete one revolution of the Ferris wheel.
b) The equation of the axis is

$$y = \frac{\text{minimum} + \text{maximum}}{2}$$

$$= \frac{25 + 7}{2}$$

$$= 16 \text{ m}$$

In this case, the axis represents those times when Colin is halfway between the peak and bottom of the Ferris wheel (either moving in an upward or downward direction).
c) The amplitude is

$$a = \text{maximum} - \text{axis}$$

$$= 25 - 16$$

$$= 9 \text{ m}$$

In this case, the amplitude represents the amount of height gained or lost as Colin moves from the halfway point of the Ferris wheel to the peak or to the bottom. In other words, the amplitude is the radius of the Ferris wheel.

d) Yes, because Colin was already at the peak height of the Ferris wheel when data was first recorded.

e) By part c), the radius of the Ferris wheel is 9 m, and therefore the circumference of the Ferris wheel is $2\pi(9) = 18\pi$ m

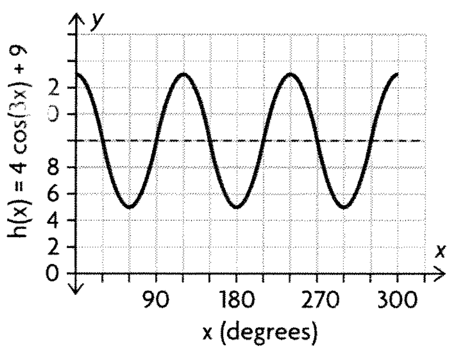
By part a), Colin completes one revolution around the Ferris wheel in 80 s, and therefore the Ferris wheel is travelling at a rate of

$\frac{18\pi}{80} \doteq .71$ metres per second

f) By observing the graph and accompanying data, we see that the range of the function is $\{y \in \mathbf{R} \mid 7 \le y \le 25\}$.

g) The Ferris wheel is 7 m off the ground at its lowest point. So if the building is 6 m tall, Colin boarded the Ferris wheel exactly 1 m off the top of the building. That is, the Ferris wheel is situated exactly 1 m off the top of the building.

5. a) Here's the graph:



Since $h(x) = 4\cos(3x) + 9$, we have horizontal compression factor $k = 3$, so

$$\text{period} = \frac{360°}{k} = \frac{360°}{3} = 120°$$

This makes sense from looking at the graph. Also, a quick look at the graph and the equation shows that we have amplitude $a = 4$, axis $y = 9$ and range $\{y \in \mathbf{R} \mid 5 \le y \le 13\}$.

b) Yes. This is clear since the function $h(x)$ is of cosine-type.

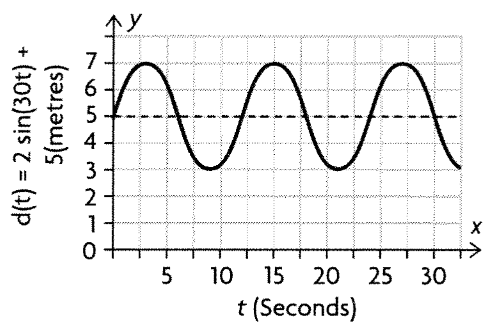
c)
$$\begin{aligned} h(45) &= 4\cos(3(45)) + 9 \\ &= 4\cos(135) + 9 \\ &= -4\frac{\sqrt{2}}{2} + 9 \\ &\doteq 6.2 \end{aligned}$$

d) These are the degree measures x where the graph has a minimum. This happens when the argument of cosine is 180° + some integer multiple of 360° (since at these angles cosine will equal -1). So when

$3x = 180$
$x = 60°$
$3x = 540$
$x = 180°$
$3x = 900$
$x = 300°$

In summary, $h(x) = 5$ at $x = 60°, 180°, 300°$.

6. Here's the graph:



a) The graph completes exactly one period from $30t = 0°$ to $30t = 360°$, that is from $t = 0$ to $t = \frac{360}{30} = 12$ seconds. So the period is 12 seconds, which makes sense from looking at the graph. In our situation, this means that the boat rises, falls and returns to its original depth to restart the cycle every 12 seconds. In other words, 12 s is the time between waves.

b) If there were no waves, the propeller would just sit at the depth representing the axis, which is at

$$\begin{aligned} y &= \frac{\text{maximum} + \text{minimum}}{2} \\ &= \frac{3 + 7}{2} \\ &= 5 \text{ m} \end{aligned}$$

c) Just plug $t = 5.5$ into the equation:

$$\begin{aligned} d(5.5) &= 2\sin(30(5.5)) + 5 \\ &= 2\sin(165°) + 5 \\ &\doteq 2(0.26) + 5 \\ &\doteq 5.5 \text{ m} \end{aligned}$$

d) By looking at the graph, we see that the range is $\{y \in \mathbf{R} \mid 3 \leq y \leq 7\}$.

e) A depth of 3 m occurs when the graph is at a minimum. Restricting our attention to the first 10 seconds, this only happens once, when the argument of sine is 270°, that is, when

$$\begin{aligned} 30t &= 270 \\ t &= 9 \text{ s} \end{aligned}$$

7. Consider the point (0,0) the centre of a circle with radius 4, with the point (4,0) lying on the circle. Any point on a circle centred at (0,0) with radius r and rotated through an angle θ can be expressed as an ordered pair ($r \cos \theta$, $r \sin \theta$). Therefore, the coordinates of the new point after a rotation of 25° about (0,0) from the point (4,0) are (4 cos 25°, 4 sin 25°) or (3.63, 1.69).

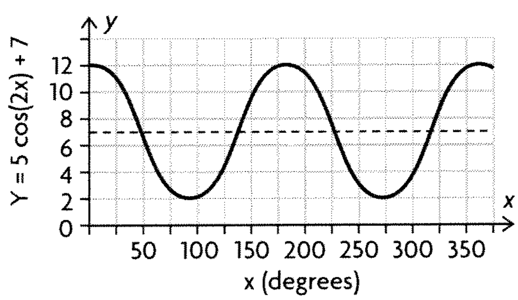
8. a) The axis has been shifted from $y = 0$ to $y = -3$.

b) The period has been changed from 360° to $\frac{360}{4} = 90°$.

c) The amplitude has changed from 1 to 7.

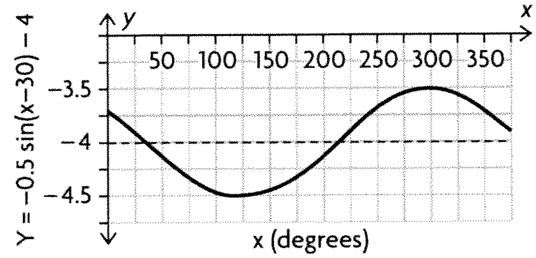
d) The graph of $y = \cos x$ has been shifted horizontally to the right by 70°.

9. a)



The graph of $y = \cos x$ has its amplitude changed from 1 to 5, its axis changed from $y = 0$ to $y = 7$, and its period changed from 360° to $\frac{360}{2} = 180°$.

b)



The graph of $y = \sin x$ has its amplitude changed from 1 to 0.5, its axis changed from $y = 0$ to $y = -4$, and is shifted horizontally to the right by 30°.

10. a) $y = -3\sin(4x) + 2$. The minimum value the sine function achieves is -1, and this happens when the argument of sine is 270° + an integer multiple of 360°. Whenever this happens, we will have

$$\begin{aligned} y &= -3(-1) + 2 \\ &= 5 \end{aligned}$$

So this is the maximum value of this function. On the other hand, the maximum value the sine function achieves is $+1$, and this happens when the argument of sine is 90° + an integer multiple of 360°. Whenever this happens, we will have

$$\begin{aligned} y &= -3(+1) + 2 \\ &= -1 \end{aligned}$$

So this is the minimum value of this function. In summary, the range of $y = -3\sin(4x) + 2$ is $\{y \in \mathbf{R} \mid -1 \leq y \leq 5\}$.

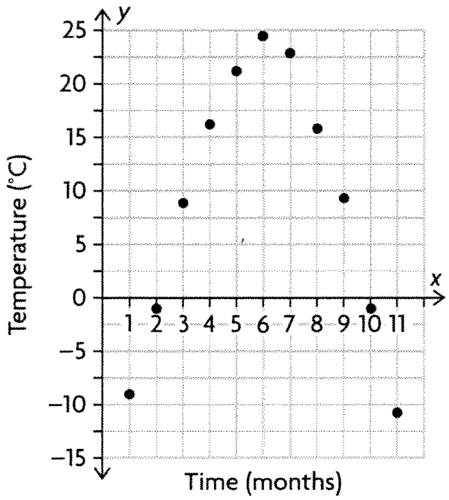
b) By reasoning similar to part a), we find the range of the function $y = 0.5\cos(3(x - 40°))$ to be $\{y \in \mathbf{R} \mid -0.5 \leq y \leq 0.5\}$.

11. a) Here's a scatter plot, and the corresponding data:

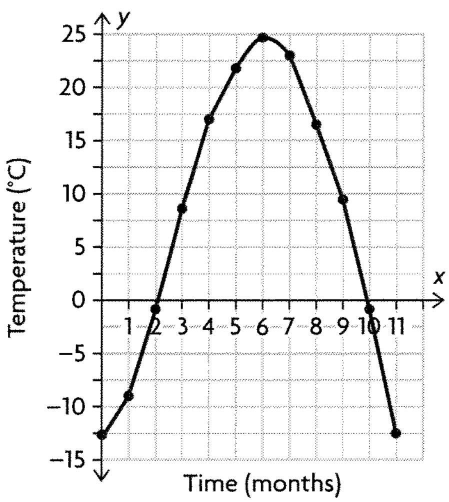
Time (months)	J	F	M	A
Temperature (°C)	−13.1	−9.0	−1.1	8.5

Time (months)	M	J	J	A
Temperature (°C)	16.8	21.6	24.7	22.9

Time (months)	S	O	N	D
Temperature (°C)	16.3	9.3	−1.2	−10.2



b) Here's a "curve of good fit" to the scatter plot in a):



The curve appears to be periodic, of period 12 months. This makes sense, because temperature cycles will roughly repeat themselves every 12 months, as the seasons run their course.
c) The maximum is 24.7°C, and occurs at month 6, or July. The minimum is -13.1°C, and occurs at month 0, or January.
d) The period is 12 months, and we discussed why this makes sense in part b).
e) The axis is

$$y = \frac{\text{maximum} - \text{minimum}}{2}$$
$$= \frac{24.7 - 13.1}{2}$$
$$= 5.8°\text{C}$$

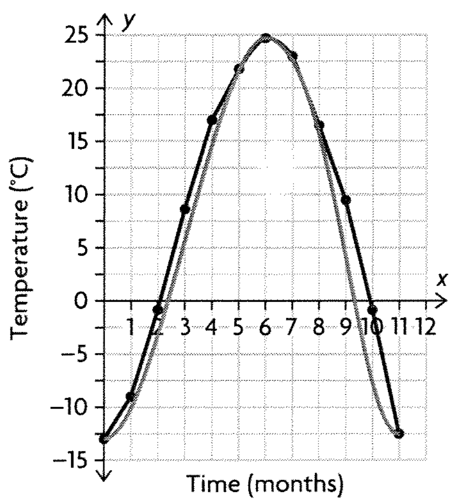
f) Usually, the cosine function has its maximum at $x = 0$. Now the maximum occurs at $x = 6$. So if the cosine function acts as the base curve, then its phase has been shifted by 6 units to the right.
g) Using the previous parts,

$$\text{amplitude} = \text{maximum} - \text{axis}$$
$$= 24.7 - 5.8$$
$$= 18.9$$

For the compression factor, k, we have period 12 so

$$\text{period} = \frac{360}{k}$$
$$12 = \frac{360}{k}$$
$$k = 30$$

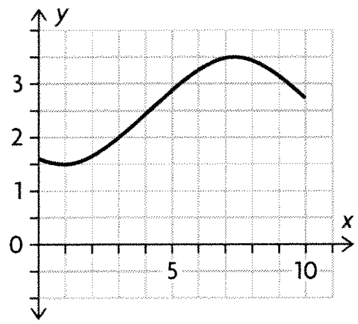
So the previous parts suggest that the sinusoidal function
$y = 18.9\cos(30(x - 6)) + 5.8$
might be a good fit for the data. Of course, there are other equivalent possibilities here. Here's a graph of this function, together with a "curve of good fit" from part b):



So this sinusoidal curve appears to model the data quite well.
h) Plug $x = 38$ into the equation found in part g).
$18.9\cos(30(38 - 6)) + 5.8$
$= 18.9\cos(960°) + 5.8$
$\doteq -3.7°\text{C}$
According to the table, the 12th month would be January, as would the 24th and 36th months. So the 38th month would be March, which had an average daily maximum temperature of -1.1°C

in the table. This is pretty close to what the curve predicts, $-3.7°C$.

12. a)

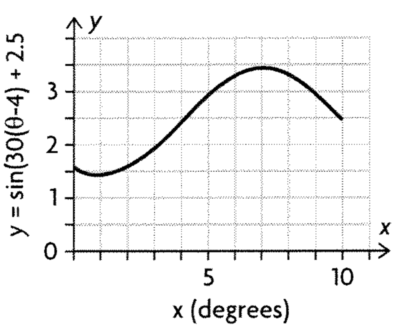


$y = a\sin(k(\theta - d)) + c$. First of all, the amplitude is still 1, so $a = 1$. The sine graph has been shifted upward by 2.5 units, so $c = 2.5$. The sine graph has been shifted to the right by 4 units, so $d = 4$. Finally, the period has now changed from 360° to 12°, which means that

$12 = \frac{360}{k}$

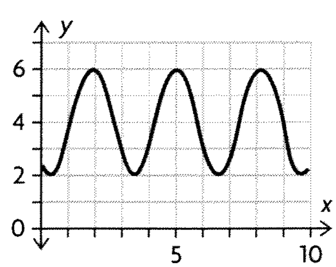
$k = 30$

So we guess that $y = \sin(30(\theta - 4)) + 2.5$ might be a good fit. Here's a graph of this function:



It appears to be a pretty good match to the given graph. There are other equivalent possibilities here.

b)

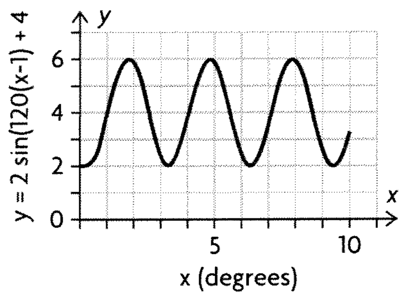


$y = a\sin(k(\theta - d)) + c$. First of all, the amplitude has changed from 1 to 2, so $a = 2$. The sine graph has been shifted upward by 4 units, so $c = 4$. The sine graph has been shifted to the right by 1 unit, so $d = 1$. Finally, the period has now changed from 360° to 3°, which means that

$3 = \frac{360}{k}$

$k = 120$

So we guess that $y = 2\sin(120(\theta - 1)) + 4$ might be a good fit. Here's a graph of this function:



It appears to be a pretty good match to the given graph. There are other equivalent possibilities here.

13. a) At $t = 1$ s, Meagan is closest to the wall, and at $t = 1.75$ s, she is furthest away. So after another 0.75 s, or at 2.5 s, she will be back to her original position, closest to the wall, and the cycle will repeat. So, we see that the period is $2.5 - 1 = 1.5$ s long. In this case, one period represents one "back-and-forth" in the rocking chair.

b) The closest she is to the wall is 18 cm, and the furthest away is 34 cm. So when the chair is at rest, it is halfway between these distances away from the wall, or $\frac{34 + 18}{2} = 26$ cm away from the wall. This is the axis of the sinusoidal function.

c) By part a), each "back-and-forth" accounts for 1.5 s added to $t = 0$ s. So going "back-and forth" 40 times will take us to

$t = 40 \times 1.5$

$= 60\ s$

So, in this case, the domain of the function would be $\{t \in \mathbf{R} \mid 0 \le t \le 60\}$.

d) Meagan will go "back-and-forth" 40 times in the case of part c), so will achieve both the minimum (18 cm) and maximum (34 cm) distances away from the wall. So the range of the function in part c) is $\{y \in \mathbf{R} \mid 18 \leq y \leq 34\}$.

e) By part b), the axis is $y = 26$. So the amplitude is

$$\begin{aligned} a &= \text{maximum} - \text{axis} \\ &= 34 - 26 \\ &= 8 \text{ cm} \end{aligned}$$

This measures the distance travelled from the rocking chair's resting position, to either its closest or furthest positions from the wall.

f) Let's use sine as our base curve, for instance. We wish to find all of the missing variables in the equation $d(t) = a \sin(k(t - d)) + c$ (other than t, of course). The chair is closest to the wall at $t = 1$ s, and is furthest away at $t = 1.75$ s. This means that the chair is at rest at

$$\begin{aligned} t &= \frac{1 + 1.75}{2} \\ &= 1.375 \text{ s} \end{aligned}$$

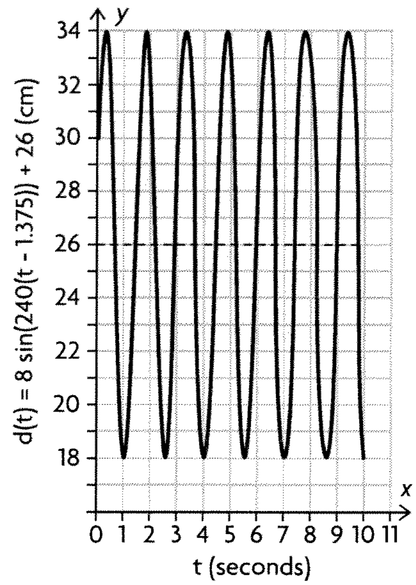
So the sine curve has been shifted to the right by 1.375 units, that is, $d = 1.375$. We already know that $a = 8$ and $c = 26$ from the previous parts, so it only remains to find k. For this, we know that the period is now 1.5 s, so

$$\begin{aligned} 1.5 &= \frac{360}{k} \\ k &= 240 \end{aligned}$$

So the sinusoidal curve we want is

$d(t) = 8 \sin(240(t - 1.375)) + 26$

Of course, there are many other equivalent solutions here. Here's a graph of this particular one:



g) We need only plug $t = 8$ into the sinusoidal function we found in part g):

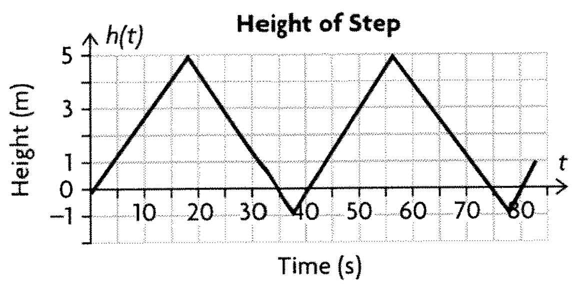
$$\begin{aligned} d(8) &= 8 \sin(240(8 - 1.375)) + 26 \\ &= 8 \sin(1590°) + 26 \\ &= 8 \times \frac{1}{2} + 26 \\ &= 30 \text{ cm} \end{aligned}$$

14. We have seen how to do this time and again above, but we will repeat the process here for completeness. Let's use the sine function as our base curve. Then the general form of the sinusoidal curve is $y = a \sin(k(x - d)) + c$
To find the axis, $y = c$, we only need to know the minimum and maximum y-values of our process. Then

$$\begin{aligned} y &= c \\ &= \frac{\text{maximum} + \text{minimum}}{2} \end{aligned}$$

This represents a sort of "midpoint" for our process. The amplitude is just
$a = \text{maximum} - \text{axis}$, and is a measure of how far our extreme values are from the "midpoint," or axis. The period is the length of one cycle of our process (in x-values), and we find k in the following way:
$k = 360° \div (\text{period length})$
Finally, we think of being on the axis $y = c$ as being at "rest," in some sense. The measurement d is how far to the right this resting point has moved away from the y-axis, or $x = 0$, which often serves as the "beginning" of our process.

Chapter Self-Test, p. 406



1. a) The period is the time it takes the function to go through one complete cycle. Since the function goes through one complete cycle between about 35 s and 75 s, for example, the period is 75 − 35 or 40 s. In this situation it represents the time for a stair to return to its initial position.

b) The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is −1 m and the maximum is 5 m, the equation of the axis is $h = \frac{5 + (-1)}{2}$ or $h = 2$ m.

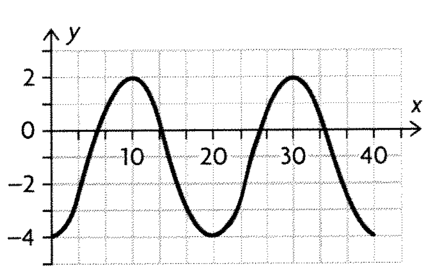
c) In this situation the peaks represent the height at the top of the escalator.

d) The range is all possible values of h. Since h varies between −1 and 5, the range is $\{h \in \mathbf{R} \mid -1 \le h \le 5\}$.

e) The domain is all possible values of t. If the escalator completes only 10 cycles before being shut down, it is operational for 40 × 10 or 400 s. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \le t \le 400\}$.

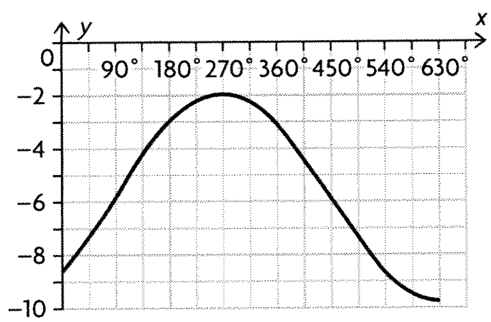
f) No, the stair will not be at ground level at $t = 300$ s, since 300 is not a multiple of 40. Since the stair started at the ground, it would need to be for this to be true.

2.



3. Consider the point (0, 0) the centre of a circle with radius 7, with the point (7, 0) lying on the circle. Any point on a circle centred at (0, 0) with radius r and rotated through an angle θ can be expressed as an ordered pair $(r \cos\theta, r \sin\theta)$. Therefore, the coordinates of the new point after a rotation of 65° about (0, 0) from the point (7, 0) are (7 cos 65°, 7 sin 65°) or (2.96, 6.34).

4. a) If the graph of $f(x) = -4\cos(0.5(x + 90°)) - 6$ is transformed from the graph of $f(x) = \cos x$, there is a reflection in the x-axis, a vertical stretch by a factor of 4, a horizontal stretch by a factor of 2, a horizontal translation of 90° to the left or −90°, and a vertical translation of 6 units down. Therefore, the graph of $f(x) = -4\cos(0.5(x + 90°)) - 6$ will look as follows:



b) The amplitude is the value of a in the function $f(x) = a\cos(k(x-d)) + c$. Since the value of a in the function $f(x) = -4\cos(0.5(x + 90°)) - 6$ is 4, the amplitude of the function is 4.

The period is the value of $\frac{360}{k}$ in the function $f(x) = a\cos(k(x-d)) + c$. Since the value of k in the function $f(x) = -4\cos(0.5(x + 90°)) - 6$ is 0.5, the period of the function is $\frac{360}{0.5}$ or 720°.

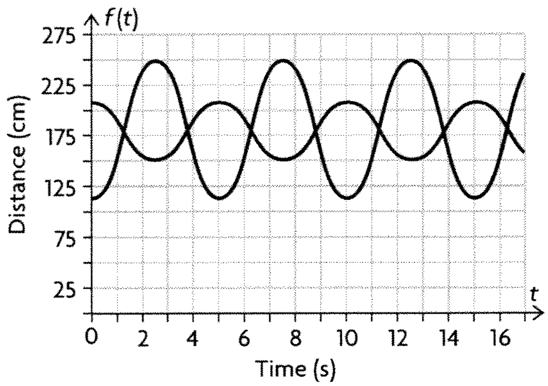
The equation of the axis is the value of c in the function $f(x) = a\cos(k(x - d)) + c$. Since the value of c in the function $f(x) = -4\cos(0.5(x + 90°)) - 6$ is −6, the equation of the axis of the function is $y = -6$.

c) If $f(x) = -4\cos(0.5(x + 90°)) - 6$, then

$$\begin{aligned} f(135°) &= -4\cos(0.5(135° + 90°)) - 6 \\ &= -4\cos(0.5(225°)) - 6 \\ &= -4\cos(112.5°) - 6 \\ &= -4 \times -0.38 - 6 \\ &= 1.53 - 6 \\ &= -4.47 \end{aligned}$$

d) The range is all possible values of $f(x)$. Since $f(x)$ varies between −6 − 4 or −10 and −6 + 4 or −2, the range is $\{y \in \mathbf{R} \mid -10 \le y \le -2\}$.

5.



a) In this situation the troughs represent the minimum distance between the tip of the metre stick and the edge of the plywood.
b) The periods are the same. Even though you are tracking different ends of the metre stick, the ends do belong to the same metre stick.
c) The distance of the nail from the top of the plywood is equivalent to the equation of the axis of either of the functions. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum of the blue function is 110 cm and the maximum is 250 cm, the equation of the axis is $y = \frac{250 + 110}{2}$ or $y = 180$ cm. Therefore, the distance of the nail from the top of the plywood is 180 cm.
d) The amplitude is half the distance between the maximum and minimum values. Since the minimum of the blue function is 110 cm and the maximum is 250 cm, the amplitude of the blue function is $\frac{250 - 110}{2}$ or 70 cm. Since the minimum of the red function is 150 cm and the maximum is 210 cm, the amplitude of the red function is $\frac{250 - 150}{2}$ or 30 cm. In this case the amplitudes represent the distance from the nail to the ends of the metre stick.
e) The range is all possible values of d. Since the blue function oscillates between 110 cm and 250 cm, the range for the blue function is $\{d \in \mathbf{R} \mid 110 \leq d \leq 250\}$. Since the red function oscillates between 150 cm and 210 cm, the range for the red function is $\{d \in \mathbf{R} \mid 150 \leq d \leq 210\}$.
f) The domain is all possible values of t, and the period is the time it takes for one of the functions to go through one complete cycle. Since both functions have the same period, and since the blue function goes through one complete cycle between $t = 0$ s and $t = 5$ s, for example, the period of both functions is 5 s. If Kerrie rotates the metre stick 5 complete revolutions, it is being rotated for 5 s $\times$ 5 or 25 s. Therefore, the domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 25\}$.
g) i) Since the amplitude for the blue function is 70 cm, a in the function $y = a\cos(k(x - d)) + c$ is 70. Since the period is 5 s, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{5}$ or 72. Since the equation of the axis is $y = 180$ cm, c in the function $y = a\cos(k(x - d)) + c$ is 180. Since the function crosses the y-axis at its minimum value, the sign of a in the function $y = a\cos(k(x - d)) + c$ is negative. Since the function has no horizontal translation, d in the function $y = a\cos(k(x - d)) + c$ is 0. Therefore, the equation for the blue function is $d = -70\cos(72t)° + 180$.
ii) Since the amplitude for the red function is 30 cm, a in the function $y = a\cos(k(x - d)) + c$ is 30. Since the period is 5 s, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{5}$ or 72. Since the equation of the axis is $y = 180$ cm, c in the function $y = a\cos(k(x - d)) + c$ is 180. Since the function crosses the y-axis at its maximum value, the sign of a in the function $y = a\cos(k(x - d)) + c$ is positive. Since the function has no horizontal translation, d in the function $y = a\cos(k(x - d)) + c$ is 0. Therefore, the equation for the red function is $d = 30\cos(72t)° + 180$.
h) Since the equation for the situation is $d = 30\cos(72t)° + 180$, at 19 s

$$\begin{aligned} d &= 30\cos(72 \times 19)° + 180 \\ &= 30\cos(1368)° + 180 \\ &= 30 \times 0.31 + 180 \\ &= 9.3 + 180 \\ &= 189.3 \text{ cm} \end{aligned}$$

Therefore, the distance between the short end of the metre stick and the top of the plywood at $t = 19$ s is 189.3 cm.

Chapters 4–6 Cumulative Review, pp. 408–411

1. B;
$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = (2)^2 = 4$$
$$81^{\frac{3}{4}} = (\sqrt[4]{81})^3 = 3^3 = 27$$
$$4^2 = 16$$
$$4 - 27 + 16 = -7$$

2. A, D;
$$3^{(2(2)-1)} = 3^3 = 27$$
$$2^{2(2)} \times 2^{(2-1)} = 2 \times 2^4 = 32$$

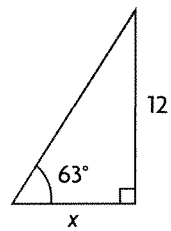
3. C; $[(c)^{(2n-3m)}]\dfrac{(c^3)^m}{(c^2)^n}$
$$= [(c)^{(2n-3m)}]\frac{(c^{3m})}{(c^{2n})}$$
$$= [(c)^{(2n-3m)}][(c)^{(3m-2n)}]$$
$$= c^{((2n-3m)+(3m-2n))}$$
$$= c^0$$
$$= 1$$

4. A; the estimate of the town's population is given by (15 000) $(1.05)^n$, where n is the number of years.
$$(15\,000)(1.05)^{20} \doteq 39\,799 \doteq 40\,000$$

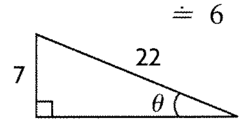
5. A; by the Pythagorean Theorem, the length of the line from the origin to the point $(-7, 24)$ is $\sqrt{(-7)^2 - 24^2}$, or 25. By the Sine Law, $\dfrac{25}{\sin 90°} = \dfrac{24}{\sin \theta°}$, where θ is the related acute angle.
$$\theta = \sin^{-1}\left(\frac{24}{25}\right) \doteq 74°$$
The principle angle is $180° - 74°$, or $106°$.

6. C; $\csc 300° = \dfrac{1}{\sin 300°}$
$$\sin 300° = \sin(-60°) = -\frac{\sqrt{3}}{2}$$
$$\csc 300° = \frac{1}{\frac{-\sqrt{3}}{2}}$$
$$= \frac{-2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$= \frac{-2\sqrt{3}}{3}$$

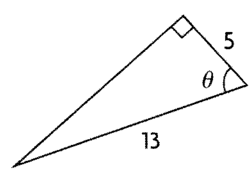
7. B; $\dfrac{\tan x \sin x}{\tan x + \sin x}$ does not equal $\dfrac{\tan x \sin x}{\tan x \sin x}$ since $\dfrac{\tan x \sin x}{\tan x \sin x}$ is 1, which would mean that $\dfrac{\tan x \sin x}{\tan x + \sin x}$ is also 1; that is, that $\tan x \sin x$ is equal to $\tan x + \sin x$, which is false.



8. C; $\tan 63° = \dfrac{12}{x}$
$$= 1.96$$
$$1.96x = 12$$
$$x = \frac{12}{1.96} \doteq 6.12 \doteq 6$$

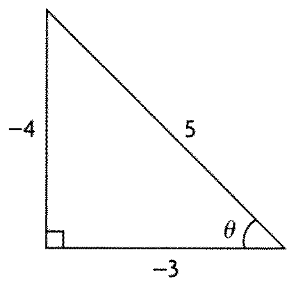


9. A; $\sin \theta = \dfrac{7}{22}$
$$\doteq 18.6 \doteq 19$$



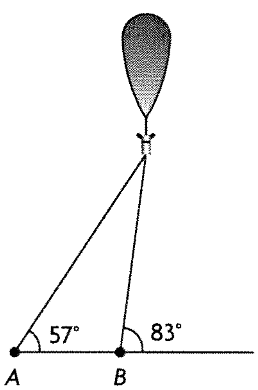
10. C; by the Pythagorean Theorem, the unlabeled leg of the triangle is $\sqrt{13^2 - 5^2}$, or 12.
$$\csc \theta = \frac{1}{\sin \theta}$$
$$\sin \theta = \frac{12}{13}$$
$$\csc \theta = \frac{1}{\frac{12}{13}} = \frac{13}{12}$$

11.



B; if $\tan\theta = \frac{4}{3}$, then by the Pythagorean Theorem, the hypotenuse of the right triangle in question is $\sqrt{3^2 - 4^2}$, or 5. Since θ lies in the third quadrant, the lengths of each leg of the triangle will be negative, i.e. the side of the triangle adjacent to θ will measure -3, and the side of the triangle opposite of θ will measure -4.

This means that $\cos\theta$ is $-\frac{3}{5}$.



12. A; since supplementary angles add up to 180°, the supplementary angle to 83° must be 97°. Call the location of the weather balloon (the top vertex of the triangle) C. Since the angles of a triangle add to 180°, the angle C must be 26°.

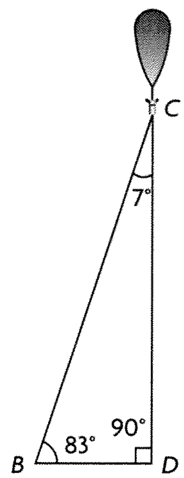
$AB = 15$ km, so by the Sine Law,

$$\frac{15}{\sin 26^\circ} = \frac{a}{\sin 57^\circ}$$

$$a = \frac{15\sin 57^\circ}{\sin 26^\circ}$$

$$\doteq 28.7$$

Form a right triangle with the straight line from the balloon to the ground, the line BC, and the line from B to the point at which the line from the balloon touches the ground. Call that point D.



By the Sine Law, $\frac{28.7}{\sin 90^\circ} = \frac{b}{\sin 83^\circ}$, where b is the altitude of the balloon.

$$B = 28.7(\sin 83^\circ)$$

$$= 28.5 \text{ km}$$

13. A; the complimentary angle to 35°, the angle of depression, is 55°.

$$\cos 55^\circ = \frac{12}{x}$$

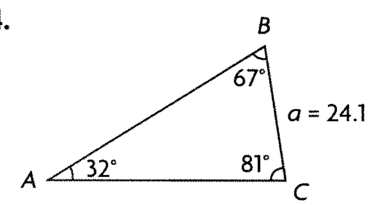
$$\cos 55^\circ \doteq 0.57$$

$$0.57 = \frac{12}{x}$$

$$x = \frac{12}{0.57}$$

$$x \doteq 20.9\text{m}$$

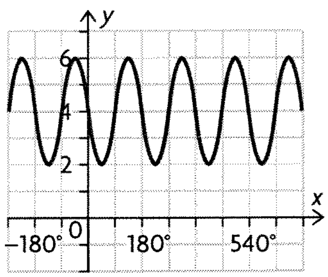
14.



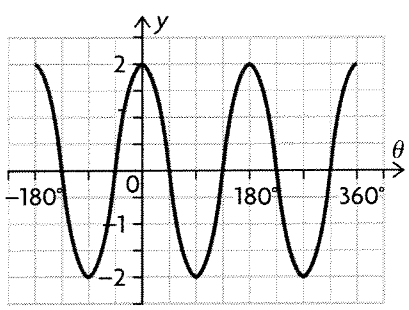
D; since all angles of a triangle add up to 180°, the missing angle B must be $180^\circ - 32^\circ - 81^\circ = 67^\circ$.

By the Sine Law, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

$$\frac{a}{\sin A} = \frac{24.1}{\sin 32°}$$
$$\doteq 45.5$$
$$\frac{b}{\sin 67°} = 45.5$$
$$b = 45.5(\sin 67°)$$
$$\doteq 41.9$$
$$= AC$$
$$\frac{c}{\sin 81°} = 45.5$$
$$c = 45.5(\sin 81°)$$
$$\doteq 44.9$$
$$= AB$$



15. D; the graph of $y = 2\cos 2(\theta + 45°) + 4$ is the graph of $y = \cos\theta$ with a vertical translation of +4, a horizontal translation of 45° to the left, a vertical stretch by a factor of 2 times the original graph, and a shortening of the period to $\frac{360}{[illegible]}$, or



16. A; the graph of $y = 2\cos 2\theta$ is the graph of $y = \cos\theta$ with a vertical stretch by a factor of 2 times the original graph, and a shortening of the period to $\frac{360°}{2}$, or 180°.

17. C; given the equation $y = a\sin(k(\theta - d)) + c$, for a change in amplitude to 5, there must be a vertical stretch by a factor of 5 times the original graph, meaning a is 5; for a change in the period to 720°, there must be a change of k to 0.5, since $\frac{360°}{0.5}$ is 720°; and for a change in the range to $\{y \in \mathbf{R} \mid 2 \le y \le 12\}$, there must be a vertical translation of + 7, meaning $c = 7$. So the new equation will be $y = 5\sin 0.5\theta + 7$.

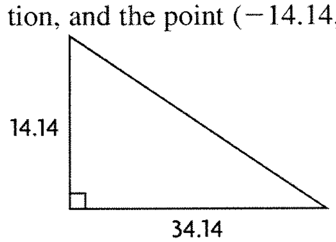
18. A; changing the value of a results in a vertical stretch or compression, which affects the maximum and minimum values, the amplitude, and the range.

19. B; at $t = 18$, the coordinates of the woman's position are

$$(x, y) = (20\cos 135°, 20\sin 135°).$$
$$= (-14.14, 14.14)$$

The woman's starting coordinates were $(x, y) = (20, 0)$.

For her to walk the shortest distance to retrieve her purse, the woman should walk along the hypotenuse of the right triangle formed by the points of her starting location, her current location, and the point (−14.14, 0).



The hypotenuse of this triangle will be $\sqrt{34.14^2 + 14.14^2} \doteq 372$

20. D; the range of $y = \sin x$ is $\{y \in \mathbf{R} \mid -1 \le y \le 1\}$.

21. 85.7; The octagon can be divided into 8 isosceles triangles, each with two sides equal to the radius of the circle, or 14. The angles of the triangle are 67.5°, 67.5°, and 45°, since 360° divided 8 times is 45°. The length of the unknown side of the triangle can be calculated by splitting the isosceles triangle into two right triangles and using the Sine Law:

$$\frac{14}{\sin 90°} = \frac{x}{\sin 22.5°}$$
$$x = 5.35$$

$2x$ is 10.7, which is the length of one side of the octagon.

The perimeter of the octagon will be 8(10.7), or 85.7 cm.

22. A; by the Cosine Law,

$a^2 = 10^2 + 15^2 - 2(10)(15)\cos 85°$
$= 298.85$
$a = \sqrt{298.85}$
$\doteq 17.3$ cm

23. A; $\cos(-420°) = \cos(-60°)$
$= \cos(60°)$
$= \frac{1}{2}$

24. B; $\sin^2\theta + \cos^2\theta$ is 1, so numerator of the expression simplifies to 1.

$\frac{\cos\theta}{\sin\theta} = \frac{xr}{ry}$
$= \frac{x}{y}$

So the expression simplifies to $\frac{1}{\frac{x}{y}} = \frac{y}{x}$.

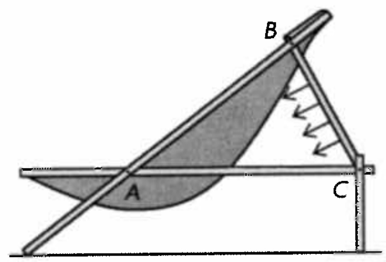
25. C; $\frac{\sin x \sin x}{(1 - \sin x)(1 + \sin x)}$
$= \frac{\sin^2 x}{1 + \sin x - \sin x - \sin^2 x}$
$= \frac{\sin^2 x}{1 - \sin^2 x}$
$= \frac{\sin^2 x}{\cos^2 x}$
$= \tan^2 x$

26. C; the period of the function $y = \sin 4\theta$ is $\frac{360°}{4}$, or 90°.

27. A; $\left(\left(\frac{1}{a}\right)\left(\frac{1}{b^{-1}}\right)\right)^{-1} = \frac{1}{\left(\frac{1}{a}\right)\left(\frac{1}{b^{-1}}\right)}$
$= \frac{1}{\left(\frac{1}{a}\right)(b)}$
$= \frac{1}{\left(\frac{b}{a}\right)}$
$= \frac{a}{b}$

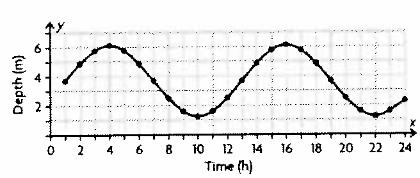
28. C; $3x^{\frac{1}{2}} = 12$
$x^{\frac{1}{2}} = \frac{12}{3}$
$= 4$
$x = 4^2$
$= 16$

29. The thickness of the paper is given by $0.1(2)^n$, where n is the number of folds.
Since 553 m is 553 000 mm, the number of folds required will be the first n such that
$0.1(2)^n > 553\ 000$
$n = 23$



30. The angle of the lawn chair is the angle supplementary to angle A, so $A = 180° - \theta$, where θ is the angle of the lawn chair.
For each θ, by the Cosine Law,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(180° - \theta)$.
So, for $\theta = 105°$,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(75°)$
$= 80.7$ cm
For $\theta = 125°$,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(55°)$
$= 62.6$ cm
For $\theta = 145°$,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(35°)$
$= 43.5$ cm
For $\theta = 165°$,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(15°)$
$= 26.1$ cm
For $\theta = 175°$,
$BC = 75^2 + 55^2 - (2)(75)(55)\cos(5°)$
$= 20.8$ cm

31. a)



Given the original equation $y = \sin h$, first, the period must be changed from 360 to 12.

$$\frac{360}{x} = 12$$

$$x = 30$$

So, in the sinusoidal function $y = a\sin(k(h - d)) + c$ modeling this situation, k is 30.

The amplitude should be changed from 1 to 2.5, so a is 2.5.

The graph should be translated 1 to the right, so that it starts at 1 instead of 0, so d is 1.

Finally, there should be a vertical translation of 3.75, so $c = 3.75$. Now the range will be $\{y \in \mathbf{R} \mid 1.25 \leq y \leq 6.25\}$.

Or, given the original equation $y = \cos h$, first, the period must be changed from 360 to 12.

$$\frac{360}{x} = 12$$

$$x = 30$$

So, in the sinusoidal function $y = a\sin(k(h - d)) + c$ modeling this situation, k is 30.

The amplitude should be changed from 1 to 2.5, so a is 2.5.

The graph should be translated 4 to the right, so that it reaches its peak at 4 instead of 0, so d is 4.

Finally, there should be a vertical translation of 3.75, so $c=3.75$. Now the range will be $\{y \in \mathbf{R} \mid 1.25 \leq y \leq 6.25\}$.

So the function modeling this situation is either $y = 2.5\sin(30(h - 1)) + 3.75$ or $y = 2.5\cos(30(h - 4)) + 3.75$.

b) The maximum of the function is 6.25, so the maximum depth of the water will be 6.25 m.

c) The minimum depth of the water at this location is 1.25 m. Therefore, since the hull of the boat must have a clearance of at least 1 m at all times, if the bottom of the hull is more than 0.25 m below the surface of the water, then this location is not suitable for the dock. However, if the bottom of the hull is less than or equal to 0.25 m below the surface of the water, then this location is suitable for the dock.