4.1 The Vertex Form of a Quadratic Function

In this Chapter, we will be exploring the relationship between the Standard Form and the Vertex Form of quadratic functions. The Standard Form is $f(x)=ax^2+bx+c$ and the ~~Factored~~ Vertex Form is $f(x)=a(x-h)^2+k$. These two forms are equivalent, meaning that they generate the exact same information or graph. In the Vertex Form, the *h* and *k* form together to give the coordinate of the vertex (h,k). The value *k* is also known as the maximum or minimum value, which is handy when writing out the range of the function.

Expand!

Question 1: Convert $f(x)=3\boxed{(x+4)^2}-18$ to the Standard Form.

$f(x)=3(x+4)(x+4)-18$

$f(x)=3(x^2+8x+16)-18$

$f(x)=3x^2+24x+48-18$

$f(x)=3x^2+24x+30$

Question 2: Given $g(x)=\overset{a}{-4}(x+5)^2+3$, state the vertex (1), axis of symmetry (2), direction of opening (3) and range (4).

$(x-\boxed{-5})$ (h,k) $x=h$

1. Vertex $(-5,3)$

2. $x=-5$

3. Down because $a<0$

$a=-4$

thank Joel.

4. $\{g(x)\in\mathbb{R}\mid g(x)\leq +3\}$

k

comes from a.

Question 3: Given the vertex (-3, -8) and the coordinate (-6, 37), determine the equation of the parabola.

h K x $y=f(x)$

$f(x) = a(x-h)^2 + K$

$37 = a(-6+3)^2 - 8$

$37 = a(-3)^2 - 8$

$37 = 9a - 8$ ($+8$, $+8$)

$\frac{45}{9} = \frac{9a}{9}$

$5 = a$

$\therefore f(x) = 5(x+3)^2 - 8$

Question 4: The height above the ground of a bungee jumper is modelled by the quadratic function $h(t) = -5(t-0.3)^2 + 110$, where height, *h(t)*, is in metres and time, *t*, is in seconds.

a) When does the bungee jumper reach a maximum height? Why is it a maximum? → because $a = -5 < 0$

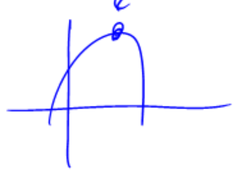
time → at 0.3 seconds

b) What is the maximum height reached by the jumper?

110 metres

c) Determine the height of the platform from which the bungee jumper jumps.

→ Vertex is $(0.3, 110)$ (t, $h(t)$)



At $t=0$, the jumper is still on the platform.

$h(0) = -5(0-0.3)^2 + 110$

$h(0) = -5(0.09) + 110$

$h(0) = -0.45 + 110$

$h(0) = 109.55$ metres.

Class/Homework for Section 4.1

Pg. 203 #1, 3, 4, 6, 8, 9, 10.

4.2 Completing the Square

As we learned in 4.1, the Vertex Form gives the vertex, which in turn tells us the maximum or minimum value of the parabola. Given the Standard Form, it would be handy to have a way to convert it to the Vertex Form so we could determine the maximum or minimum. Thankfully, we have a method to do just that!

First, we need to explore perfect squares, and given missing pieces, how it can be completed.

$f(x) = a(x-h)^2 + k$

Expand the following: (notice the pattern!)

$(x+4)^2 = (x+4)(x+4) = x^2 + 8x + 16$

$(x-1)^2 = x^2 - 2x + 1$ Perfect Square Trinomial

$(x \underline{-\boxed{7}})^2 = x^2 - 14x + \underline{49}$

$\frac{14}{2} = 7$

$(x \underline{+5})^2 = x^2 + 10x + \underline{25}$

$\frac{10}{2} = 5$ then $5^2 = 25$

$(x \underline{-174})^2 = x^2 - 348x + \underline{30276}$

$\frac{-348}{2} = -174$

$(x \underline{-16.5})^2 = x^2 - 33x + \underline{272.25}$

$\frac{-33}{2} = -16.5$ then $(-16.5)^2 =$

$(x \underline{-1.42})^2 = x^2 - 2.84x + \underline{2.0164}$

$\frac{-2.84}{2} = -1.42$, then $(-1.42)^2 = 2.0164$

Important: Vertex Form has $(x-h)^2$ which is a perfect square. We will need to create this when completing the square.

Example 1: Complete the Square to convert to Vertex Form, then state the Vertex.

$f(x)=2x^2-12x+13$

$\frac{6}{2}=3,\ 3^2=9$

① $f(x)=2(x^2-6x+0)+13$

② $f(x)=2(x^2-6x+9-9)+13$

③ $f(x)=2(x^2-6x+9)+13-18$

④ $f(x)=2(x-3)^2-5$

Vertex is $(3,-5)$

Algorithm:

① Factor (divide) the "a" from only the first two terms. Add zero in the bracket.

② Look inside brackets. Divide the x-term by 2, then square the result. Then replace the zero with that # − #

③ Multiply the "a" with the −#, and place it at the end of the equation.

④ Simplify the perfect square trinomial to $(x-h)^2$ and simplify the end.

Example 2:

$g(x)=-4x^2-40x-7$

$\frac{10}{2}=5,\ 5^2=\boxed{25}$

① $g(x)=-4(x^2+10x+0)-7$

② $g(x)=-4(x^2+10x+25-25)-7$

multiply

③ $g(x)=-4(x^2+10x+25)-7+100$

④ $g(x)=-4(x+5)^2+93$

Vertex is $(-5, 93)$

Example 3:

$f(x)=\frac{1}{2}x^2+7x+6$

$\frac{14}{2}=7,\ 7^2=49$

$f(x)=\frac{1}{2}(x^2+14x+0)+6$

$f(x)=\boxed{\frac{1}{2}}(x^2+14x+49\boxed{-49})+6$

$f(x)=\frac{1}{2}(x^2+14x+49)+6-24.5$

$f(x)=\frac{1}{2}(x+7)^2-18.5$ Vertex: $(-7,-18.5)$

Example 4:

$g(x)=4.5x^2-102.1x+23.4$

$\frac{22.69}{2}=11.35,\ 11.35^2=128.82$

$g(x)=4.5(x^2-22.69x+0)+23.4$

multiply

$g(x)=\boxed{4.5}(x^2-22.69x+128.82\boxed{-128.82})+23.4$

$g(x)=4.5(x^2-22.69x+128.82)+23.4-579.69$

$g(x)=4.5(x-11.35)^2-556.29$

Vertex is $(11.35,-556.29)$

Class/Homework for Section 4.2

Pg. 214 #6, 7 (do not graph) 8-11

4.3 Quadratic Formula

The quadratic formula is the one formula to solve them all. With great power, comes great responsibility. It is a tricky formula, but once you learn how to properly use it, you will be that much happier. The key is to write and communicate your math carefully.

Given $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$-\boxed{5^2}^{=25} = -25$

$(-5)^2 = 25$

Two notes:

1. Inside the square root, you start with b^2. No matter what you plug in, you get a positive number. If $b=9,\ \ b^2=81$. If $b=-5,\ \ b^2=25$. Why do I make note of this? I have seen many people do it wrong, so don't be one of those people. Don't type – in your calculator.

2. Since we are calculating a square root, we have three options. If the number inside the square root is positive, there are two solutions. If the number is zero, there is only one solution. If the number is negative, there are no solutions since you cannot square root a negative. This is a fact. Don't try to square root a negative as it absolutely 100% cannot be done. Please don't try to do it. It doesn't work. No solution is an answer, so do not fret. If you don't like the negative, double check your work.

Example 1:

$\overset{a}{3x^2}\overset{b}{\boxed{-24}}x\overset{c}{\boxed{+45}}=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{24\pm\sqrt{24^2-4(3)(45)}}{2(3)}$$

$$x=\frac{24\pm\sqrt{576-540}}{6}$$

$$x=\frac{24\pm\sqrt{36}}{6}$$

$$x=\frac{24\pm 6}{6}$$

$\oplus\ x=\dfrac{24+6}{6}=\dfrac{30}{6}=5$

$\ominus\ x=\dfrac{24-6}{6}=\dfrac{18}{6}=3$

Example 2:

a b c

$3x^2 + 2x + 15 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(15)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - 180}}{6}$$

$$x = \frac{-2 \pm \sqrt{-176}}{6}$$

∴ no solutions!

or no ~~zeros~~

x-int.

Example 3:

$4x^2 - 8x + 10 = 2x + 7$

$-2x \quad -7$

$4x^2 - 10x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{10 \pm \sqrt{10^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{10 \pm \sqrt{100 - 48}}{8}$$

$$x = \frac{10 \pm \sqrt{52}}{8}$$

$$x = \frac{10 \pm 7.2}{8}$$

$(+)\ x = \frac{10 + 7.2}{8} = \frac{17.2}{8} = 2.2$

$(-)\ x = \frac{10 - 7.2}{8} = \frac{2.8}{8} = 0.4$

Example 4:

The profit on a school drama production is modelled by the quadratic equation $P(x) = -60x^2 + 790x - 1000$, where $P(x)$ is the profit in dollars and x is the price of the ticket, also in dollars.

a) Use the quadratic formula to determine the break-even price for the tickets. $P(x) = 0$

b) At what price should the drama department set the tickets to maximize their profit?

a)
$$x = \frac{-790 \pm \sqrt{790^2 - 4(-60)(-1000)}}{2(-60)}$$

$$x = \frac{-790 \pm \sqrt{624100 - 240000}}{-120}$$

$$x = \frac{-790 \pm \sqrt{384100}}{-120}$$

$$x = \frac{-790 \pm 619.76}{-120}$$

(+) $x = \frac{-790 + 619.76}{-120}$

$x = \frac{-170.24}{-120} = \$1.42$

(−) $x = \frac{-790 - 619.76}{-120}$

$x = \frac{-1409.76}{-120} = \$11.75$

b) AoS ⇒ $x = \frac{1.42 + 11.75}{2}$

$x = \frac{13.17}{2}$

$x = \$6.59$ → put this into the equation to get the actual profit.

Class/Homework for Section 4.3

Pg. 222 #3, 6, 8, 9