

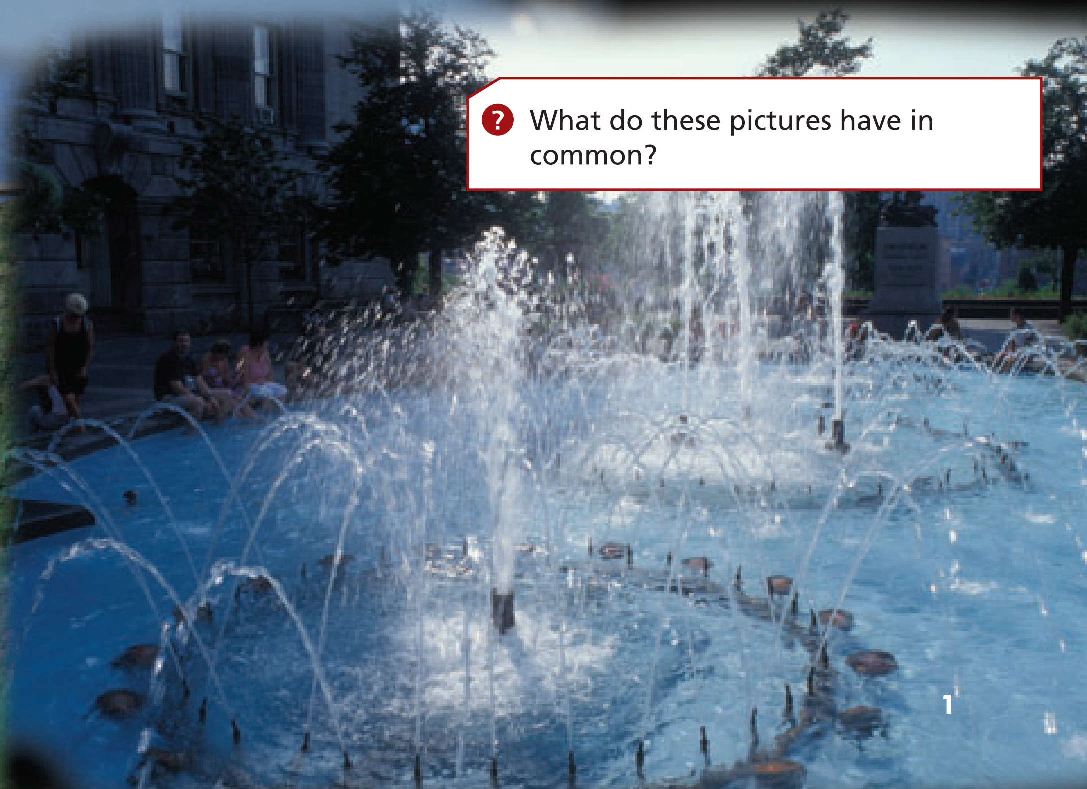

Chapter 1

Introduction to the Quadratic Function

GOALS

You will be able to

- Use appropriate notation to describe relationships as functions
- Graph linear and quadratic functions
- Model linear and quadratic relationships
- Use transformations to relate different quadratic functions

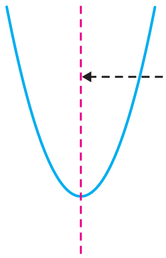
What do these pictures have in common?

1 Getting Started

WORDS You Need to Know

1. Match the term with the picture or example that best illustrates its definition.

a) linear relation
b) quadratic relation
c) vertex of a parabola
d) axis of symmetry of a parabola
e) line of best fit
f) intercepts

i)


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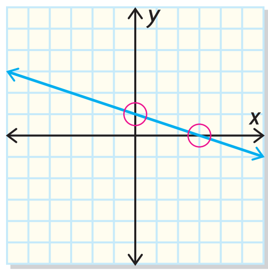

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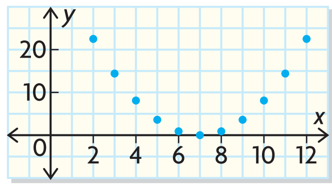

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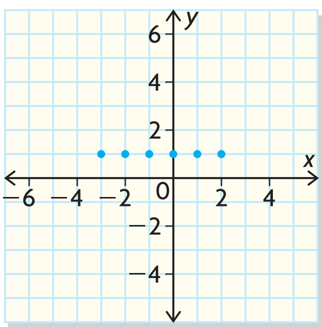

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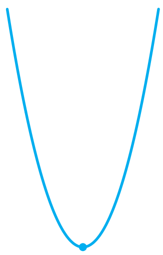

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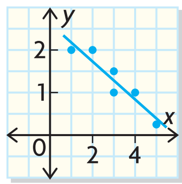


SKILLS AND CONCEPTS You Need

Study *Aid*

For help, see Essential Skills Appendix, A-5.

Evaluating Algebraic Expressions

An expression may be evaluated for different values of the variables. Substitute the given numerical value of the letter in brackets and evaluate.

EXAMPLE

Evaluate $3x^2 - 2x + 1$, when $x = -3$.
Substitute $x = -3$ (in brackets) into the above algebraic expression.

$3(-3)^2 - 2(-3) + 1$
$= 3(9) + 6 + 1$
$= 27 + 7$
$= 34$

2. Evaluate each algebraic expression if $a = 0$, $b = 1$, $c = -1$, and $d = 2$.

a) $b + 3c$
b) $3b + 2c - d$
c) $2a^2 + b^2 - d^2$
d) $(a + 3b)(2c - d)$

Creating a Table of Values and Sketching Graphs of Quadratic Relations

When creating a table of values, select various x-values and substitute them into the quadratic relation and solve for y. Plot the ordered pairs to determine the graph.

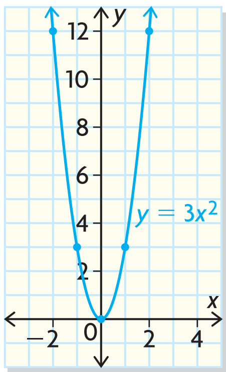
Study Aid

For help, see Essential Skills Appendix, A-8.

EXAMPLE

Create a table of values and sketch the graph of $y = 3x^2$.
Select x-values ranging from -2 to 2 and solve for each value of y.

x	$y = 3x^2$	Ordered Pair
-2	$y = 3(-2)^2 = 12$	$(-2, 12)$
-1	$y = 3(-1)^2 = 3$	$(-1, 3)$
0	$y = 3(0)^2 = 0$	$(0, 0)$
1	$y = 3(1)^2 = 3$	$(1, 3)$
2	$y = 3(2)^2 = 12$	$(2, 12)$



3. Create a table of values and sketch the graph of each quadratic relation.

a) $y = 2x^2$
b) $y = -4x^2 - 3$
c) $y = 0.5x^2 + 5$
d) $y = -5x^2 + 5$

PRACTICE

Study Aid

- For help, see Essential Skills Appendix.

Question	Appendix
4, 5, 6	A-11
7	A-6
10	A-7
11	A-13

4. Solve each equation for y. Then evaluate y for the given value of x.

a) $y - 5 = -3x; x = 2$
b) $3x + y = 3; x = 2$

5. a) Does the point $(2, -1)$ lie on the line $y = -3x + 5$?
b) Does the point $(-4, 10)$ lie on the parabola $y = -2x^2 - 5x + 22$?

6. a) Is $(2, -1)$ a solution of $2x - y = 5$? Explain.
b) Is $(-1, 29)$ a solution of $y = -2x^2 - 5x + 22$? Explain.

7. For each linear relation, determine the x-intercept, the y-intercept, and the slope.

a) $2x + 3y = 12$
b) $-x + 4y = 8$

8. For each quadratic relation, determine the y-intercept and the axis of symmetry.

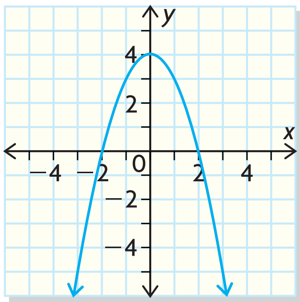
a) $y = x^2 + 2$ **b)** $y = -x^2 - 4$

9. Match the equation with its graph.

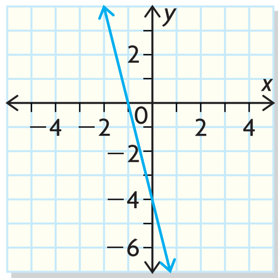
a) $y = 4x - 8$ **c)** $y = x^2 - 3$

b) $y = -4x - 4$ **d)** $y = -x^2 + 4$

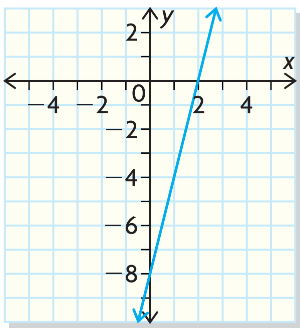
A.



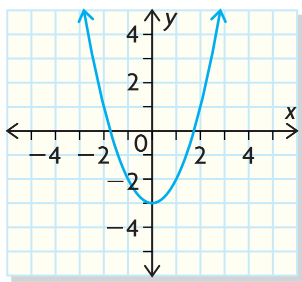
C.



B.



D.



10. Use the suggested method to sketch the graph of each linear relationship.

a) $y = -4x + 3$ (slope, y-intercept method)

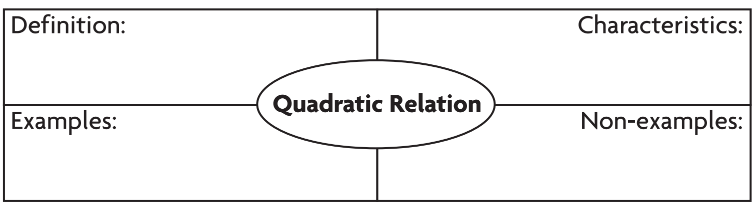
b) $2x - y = 4$ (x-, y-intercept method)

c) $3x = 6y - 3$ (table of values)

11. Create a scatter plot, and then use it to find the equation that defines the relation shown in the table.

x	−3	−2	−1	0	1	2	3
y	11	9	7	5	3	1	−1

12. Complete the chart by writing down what you know about the term Quadratic Relation.



APPLYING What You Know

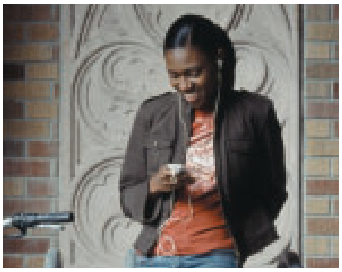
Selling Tickets

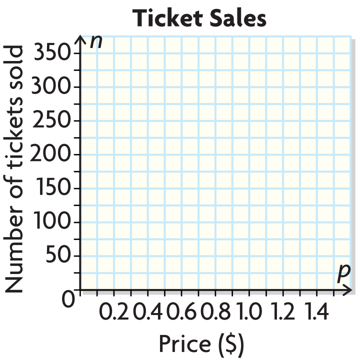
YOU WILL NEED

- graph paper

The student council is raising money by holding a raffle for an MP3 player. To determine the price of a raffle ticket, the council surveys students and teachers to find out how many tickets would be bought at different prices. The council found that

- if they charge \$0.50, they will be able to sell 200 tickets; and
- if they raise the price to \$1.00, they will sell only 50 tickets.





? What ticket price will raise the most money?

A. Create a scatter plot that shows the relationship between the price per ticket (p) and the number of tickets sold (n) by graphing the two pieces of information surveyed.

B. Assume that the relationship is linear. Draw a line joining the two given points. Calculate the slope of the line. Use the slope and one of the points to write an equation for the relationship.

C. Use the equation from part B to complete the table of values for other ticket prices.

Price (\$)	0.1	0.2	0.3	0.4	0.50	0.60	0.70	0.80	0.90	1.00
Number of Tickets Sold					200					50

D. Determine the total amount of money raised for each ticket price. (amount raised = price × number sold)

Amount Raised (\$)										

E. Use the table to estimate the price per raffle ticket that will raise the most money.

F. Sketch a new graph that shows the relationship between the price per ticket (p) and the amount of money raised (A).

G. How could you use the graph to determine the best price to charge per ticket?

1.1 The Characteristics of a Function

YOU WILL NEED

- graph paper
- graphing calculator (*optional*)

GOAL

Identify the difference between a relation and a function.

INVESTIGATE the Math

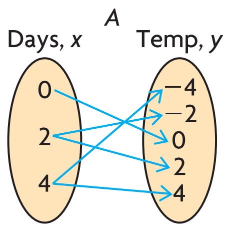
Nathan examines two temperature **relations** for the month of March. In relation *A*, the **dependent variable** *y* takes on the same or opposite value of the **independent variable** *x*. In relation *B*, the dependent variable *y* takes on the same value of the independent variable *x*. Each relation is represented by a table of values and a mapping diagram.

mapping diagram
a drawing with arrows to show the relationship between each value of *x* and the corresponding values of *y*

Tech Support
For help using the graphing calculator to create a scatter plot, see Technical Appendix, B-10.

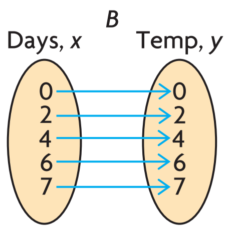
Relation *A*

Number of Days in the Month, *x*	0	2	2	4	4
Minimum Daily Temperature (°C), *y*	0	2	−2	−4	4



Relation *B*

Number of Days in the Month, *x*	0	2	4	6	7
Maximum Daily Temperature (°C), *y*	0	2	4	6	7



? Which relation is also a function?

A. The table of values of relation A represents the set of ordered pairs $(x, y) =$ (days, minimum temp.). Create a scatter plot of the ordered pairs of A on a coordinate grid.

B. The table of values of relation B represents the set of ordered pairs $(x, y) =$ (days, maximum temp.) Create a scatter plot of the ordered pairs of B on a different coordinate grid.

C. The **domain** of relation A can be expressed using **set notation**, $D = \{0, 2, 4\}$. What do these numbers represent? Express the domain of relation B in set notation.

D. The **range** of relation A is $R = \{-4, -2, 0, 2, 4\}$. What do these numbers represent? Express the range of relation B in set notation.

E. Examine the mapping diagram for relation B. Each x-value in the domain (number of days) is mapped to exactly one y-value (maximum temp.) in the range by an arrow. Examine the mapping diagram for relation A. How are the mappings in A different from the mappings in B?

F. Explain how following the arrows of a mapping diagram helps you see which relation is a **function**.

G. Draw the vertical line $x = 2$ on your scatter plots of relation A and relation B. What do you notice? Explain how drawing a vertical line helps you see which relation is a function.

domain

the set of all values for which the independent variable is defined

set notation

a way of writing a set of items or numbers within curly brackets, { }

range

the set of all values of the dependent variable. All such values are determined from the values in the domain

function

a relation in which there is only one value of the dependent variable for each value of the independent variable (i.e., for every x-value, there is only one y-value)

Reflecting

H. Relations can be expressed by equations. Write the equations for relations A and B that define the relationship between the number of days in the month and the maximum or minimum temperature.

I. A relation can be represented as a table of values, a mapping diagram, a graph, and an equation. Which methods make it easier to help you decide whether a relation is a function? Explain.

J. Write a definition for the term *function* in terms of the domain and range.

APPLY the Math

EXAMPLE 1 Using reasoning to decide whether a relation is a function

For each of the following relations, determine

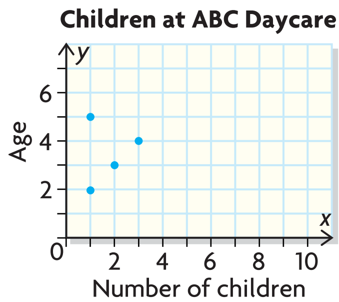
- the domain and the range
- whether or not it is a function

a) $G: (x, y) =$ (number of golfers, score below or above par)
$= \{(0, -2), (0, -1), (0, 0)\ (0, 5)\}$

b)

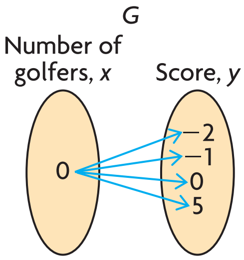
x	*y*
−1	−3
0	1
1	5
2	9

c)



Anisha's Solution

a)



I drew a mapping diagram to represent the ordered pairs. The *x*-coordinate, 0, maps onto four different *y*-coordinates.

Domain $= \{0\}$
Range $= \{-2, -1, 0, 5\}$

This relation is not a function.

b)

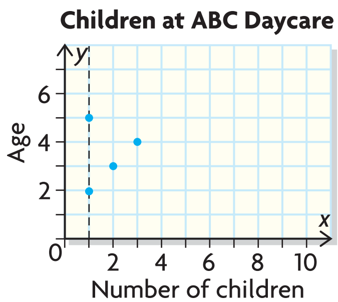
x	*y*
−1	−3
0	1
1	5
2	9

Each *x*-value in the table corresponds to one unique *y*-value.

Domain $= \{-1, 0, 1, 2\}$
Range $= \{-3, 1, 5, 9\}$

This relation is a function.

c)



The vertical line crosses two points, (1, 2) and (1, 5), when *x* equals 1. The *x*-coordinate, 1, corresponds to two different *y*-coordinates.

Domain $= \{1, 2, 3\}$
Range $= \{2, 3, 4, 5\}$

This relation is not a function.

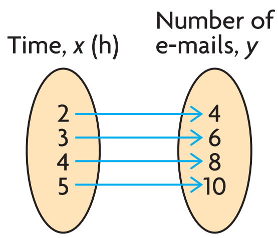
For any relation that is a function, if you know the value of the independent variable, you can predict the value of the dependent variable.

EXAMPLE 2 Connecting mapping diagrams with functions

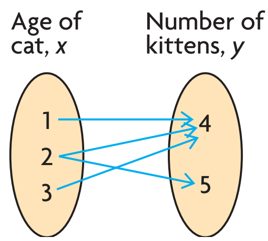
For each relation, determine

- the domain and the range
- whether or not it is a function

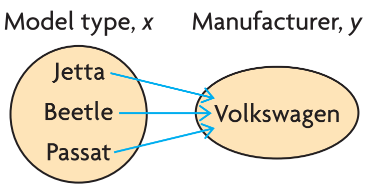
a)



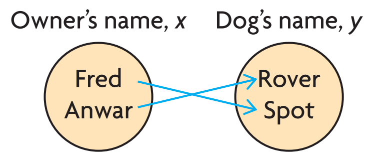
c)



b)



d)



Kinson's Solution

a) The domain is $\{2, 3, 4, 5\}$.

The range is $\{4, 6, 8, 10\}$.

This relation is a function.

Each *x*-value corresponds to only one *y*-value. If I know the quantity of time, I can predict the number of e-mails received.

b) The domain is {Jetta, Beetle, Passat}.

The range is {Volkswagen}.

This relation is a function.

Each model of car in the domain corresponds with only one manufacturer in the range. If I know the model of car, I can predict the manufacturer.

c) The domain is $\{1, 2, 3\}$.

The range is $\{4, 5\}$.

This relation is not a function.

The 2 in the domain corresponds to two different values in the range. I can't predict the number of kittens born from knowing the age of the mother.

d) The domain is {Fred, Anwar}.

The range is {Rover, Spot}.

This relation is a function.

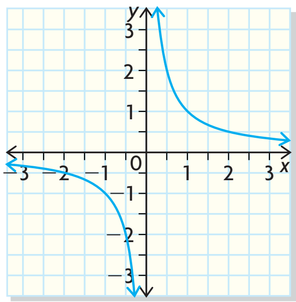
Each name in the domain corresponds to one name in the range. If I know the owner's name, I can predict the dog's name.

If you are given the graph of a relation, you can also identify if it is a function by comparing the values of the independent variable with the values of the dependent variable.

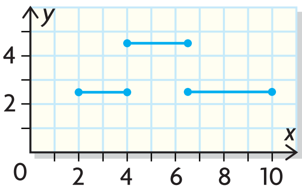
EXAMPLE 3 Connecting functions with graphs

For each of the following relations, determine the domain and the range, using **real numbers.** State whether or not the relation is a function.

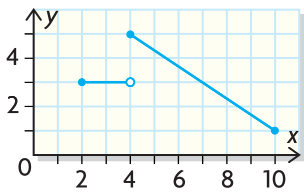
a)



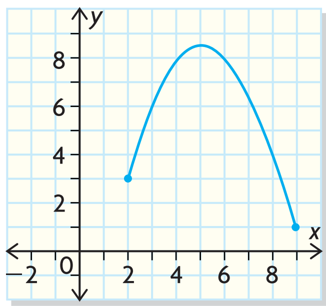
c)



b)



d)



real numbers

the set of real numbers is the set of all decimals—positive, negative, and 0, terminating and nonterminating. This statement is expressed mathematically with the set notation $\{x \in \mathbf{R}\}$

Communication *Tip*

$\{x \in \mathbf{R} \mid x \neq 0\}$ is the mathematical notation for saying "the set of x such that x can be any real number except 0."

Dimitri's Solution

a) The domain is $D = \{x \in \mathbf{R} \mid x \neq 0\}$.

The range is $R = \{y \in \mathbf{R} \mid y \neq 0\}$.

This relation is a function.

This relation passes the **vertical-line test**. This means that each x-coordinate corresponds to exactly one y-coordinate.

b) The domain is $D = \{x \in \mathbf{R} \mid 2 \leq x \leq 10\}$.

The range is $R = \{y \in \mathbf{R} \mid 1 \leq y \leq 5\}$.

This relation is a function.

This relation passes the vertical-line test. The open dot at the point (4, 3) means that $y \neq 3$ when $x = 4$, and the closed dot at the point (4, 5) means that $y = 5$ when $x = 4$. So when $x = 4$, there is only one y-value.

c) The domain is $D = \{x \in \mathbf{R} \mid 2 \leq x \leq 10\}$.

The range is $R = \{y \in \mathbf{R} \mid y = 2.5, 5.5\}$.

This relation is a not function.

This relation fails the vertical-line test. At $x = 4$, $y = 2.5$ and 5.5; and at $x = 6.5$, $y = 2.5$ and 5.5.

vertical-line test

a test to determine whether the graph of a relation is a function. The relation is not a function if at least one vertical line drawn through the graph of the relation passes through two or more points

d) The domain is
$D = \{x \in \mathbf{R} | 2 \leq x \leq 9\}$.

The range is
$R = \{y \in \mathbf{R} | 1 \leq y \leq 8.5\}$.

This relation is a function.

The relation passes the vertical-line test; for each x-value, there is exactly one y-value.

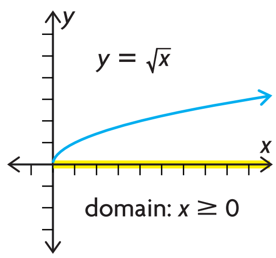
In Summary

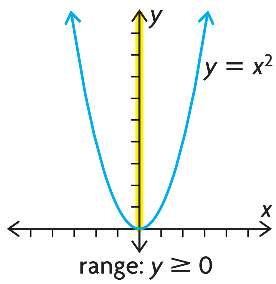
Key Ideas

- A function is a relation where each value of the independent variable corresponds with only one value of the dependent variable. The dependent variable is then said to be a function of the independent variable.
- For any function, knowing the value of the independent variable enables you to predict the value of the dependent variable.
- Functions can be represented in words, by a table of values, by a set of ordered pairs, in set notation, by a mapping diagram, by a graph, or by an equation.

Need to Know

- The set of all values of the independent variable is called the domain. On a Cartesian graph, the domain is the set of all possible values of the independent variable x. *Example*: The relation $y = \sqrt{x}$ exists only for positive values of x. The domain of this relation is $x = 0$ and all positive values of x.
- The set of all values of the dependent variable is called the range. On a Cartesian graph, the range is the set of all possible values of the dependent variable y. *Example*: $y = x^2$. The range is $y = 0$ and all positive values of y.
- A function can also be defined as a relation in which each element of the domain corresponds to only one element of the range.
- The vertical-line test can be used to check whether the graph of a relation represents a function. If two or more points lie on the same vertical line, then the relation is not a function.
- It is often necessary to define the domain and range using set notation. For example, the set of numbers ..., −2, −1, 0, 1, 2, ... is the set of integers and can be written in set notation as $\{x | x \in \mathbf{I}\}$, where the symbol "|" means "such that" and the symbol "∈" means "belongs to" or "is a member of." So set notation for the integers would be read as "The set of all x-values such that x belongs to the integers."



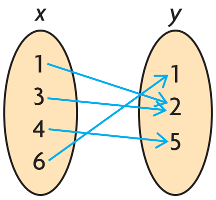


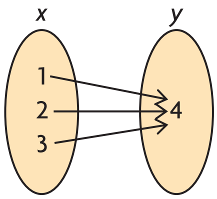
CHECK Your Understanding

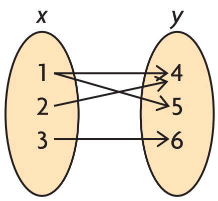
1. For each relation, state
   i) the domain and the range
   ii) whether or not it is a function, and justify your answer

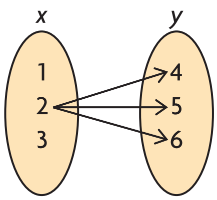
a) $g = \{(1, 2), (3, 1), (4, 2), (7, 2)\}$
b) $h = \{(1, 2), (1, 3), (4, 5), (6, 1)\}$
c) $f = \{(1, 0), (0, 1), (2, 3), (3, 2)\}$
d) $m = \{(1, 2), (1, 5), (1, 9), (1, 10)\}$

2. For each relation, state
   i) the domain and the range
   ii) whether or not it is a function, and justify your answer

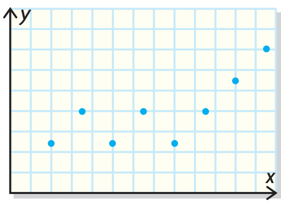
a)


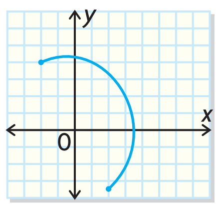
c)


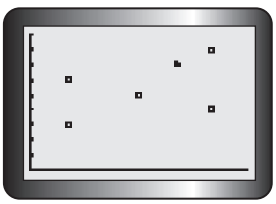
b)


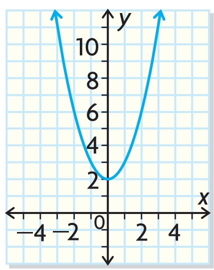
d)


3. For each relation, state
   i) the domain and the range
   ii) whether or not it is a function, and justify your answer

a)


c)


b)


d)


PRACTISING

4. Is the relation a function? Why or why not?

a) $h: (x, y) = (\text{age of child, number of siblings})$
$= \{(1, 4), (2, 4), (3, 3), (4, 2)\}$

b) $k: (x, y) = (\text{number of households, number of TV's})$
$= \{(1, 5), (1, 4), (2, 3), (3, 2)\}$

c)

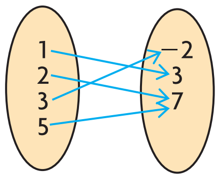
x	*y*
0	−2
2	0
5	−2

d)

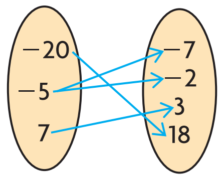
Number of Drivers, *x*	Number of Speeding Tickets, *y*
1	6
1	4
2	3
7	8

5. Is each relation a function? If not, which ordered pair should be removed to make the relation a function?

a)

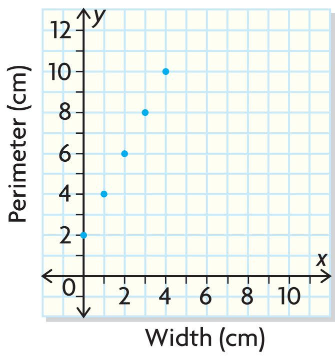


b)

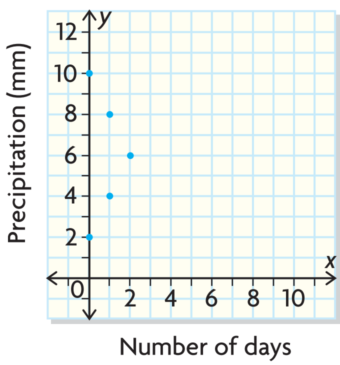


6. Which scatter plot represents a function? Explain. For each graph, state the domain and range.

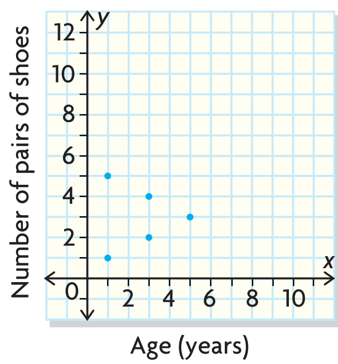
a)



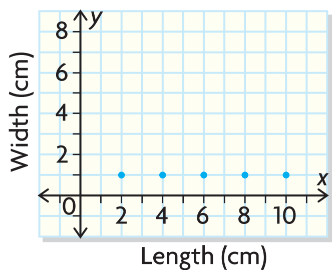
b)



c)

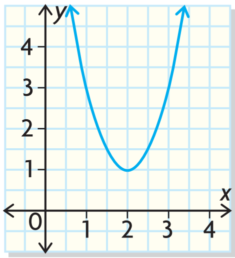


d)

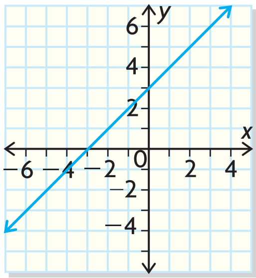


7. **K** Which relations are functions? Explain. For each graph, state the domain and range.

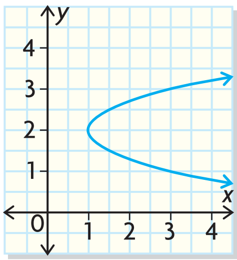
a)



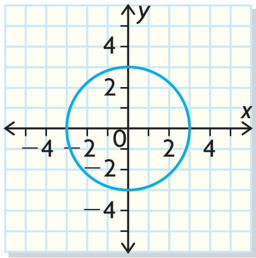
c)



b)

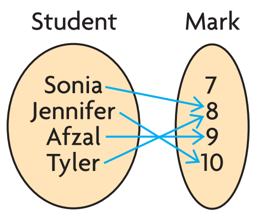


d)



8. **a)** Draw a line that is both a relation and a function.
b) Draw a line that is a relation but is not a function.
c) Which type of line cannot be the graph of a linear function? Explain.

9. The mapping diagram at the right shows the relation between students and their marks on a math quiz.
a) Write the relation as a set of ordered pairs.
b) State the domain and range.
c) Is the relation a function? Explain.

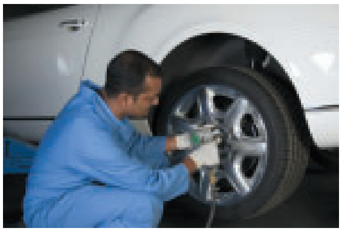


10. **A** The table gives typical resting pulse rates for six different mammals.

Mammal, *x*	Cat	Elephant	Human	Mouse	Rabbit	Rat
Pulse Rate, *y* (beats/minute)	176	27	67	667	260	315

a) Is the resting pulse rate a function of the type of mammal? Explain.
b) If more mammals and their resting pulse rates were included, would the extended table of values be a function? Explain.

11. Which variable would be associated with the domain for the following pairs of related quantities? Which variable would be associated with the range? Explain.
a) heating bill, outdoor temperature
b) report card mark, time spent doing homework
c) person, date of birth
d) number of slices of pizza, number of cuts



12. Bill called a garage to ask for a price quote on the size and type of tire he needed.

a) Explain why this scenario represents a function.

b) If Bill had given the clerk the tire price, would the clerk be able to tell Bill the tire size and type? Would this scenario represent a function?

13. T Dates and outdoor temperatures are related. The hottest temperature recorded in Canada was 45 °C at Midvale and Yellow Grass, Saskatchewan, on July 5, 1937. The coldest temperature recorded in Canada was −63 °C, in Snag, in the Yukon Territories, on February 3, 1947.

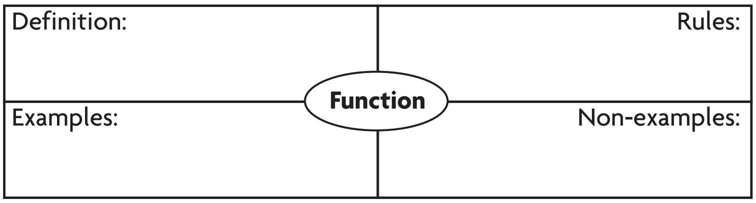
a) What is the independent variable in this relation? What is the dependent variable?

b) What is the domain?

c) What is the range?

d) Is one variable a function of the other? Explain.

14. C Summarize your understanding of functions in a chart similar to the one shown. Use at least three different types of representation for the examples and the non-examples.



Extending

15. A rock rolls off a cliff 66 m high. Which set of values best represents the range in the relationship between the time elapsed, in seconds, and the resultant height, in metres? Explain.

a) 100 to 200

b) −100 to 0

c) −100 to 100

d) 0 to 66

16. If a company's profit or loss depends on the number of items sold, which is the most reasonable set of values for the domain in this relation? Explain.

a) negative integers

b) positive integers

c) integers between −1500 and 1500

1.2 Comparing Rates of Change in Linear and Quadratic Functions

YOU WILL NEED

- graph paper

GOAL

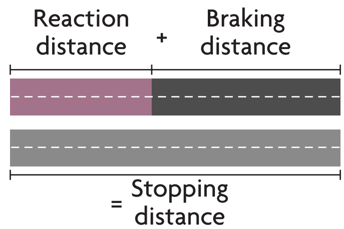
Identify and compare the characteristics of linear and quadratic functions.

LEARN ABOUT the Math

Stopping distance is the distance a car travels from the time the driver decides to apply the brakes to the time the car stops. Stopping distance is split into two types of distances.

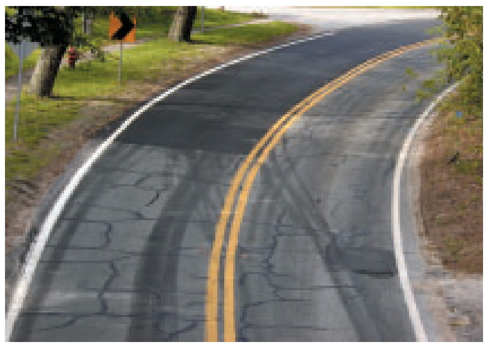
1. **Reaction Distance**: the distance the car travels from the time you first think "stop" to the time your foot hits the brake
2. **Braking Distance**: the distance the car travels from the time you first apply the brakes to the time the car stops

Stopping Distance = Reaction Distance + Braking Distance



The table shows some average values of these three distances, in metres, for different speeds.

Speed (km/h)	Speed (m/s)	Reaction Distance (m)	Braking Distance (m)	Stopping Distance (m)
0	0.00	0.00	0.00	0.00
20	5.56	8.33	1.77	10.11
40	11.11	16.67	7.10	23.76
60	16.67	25.00	15.96	40.96
80	22.22	33.33	28.38	61.71
100	27.78	41.67	44.35	86.01



? How does the relationship between speed and reaction distance compare with the relationship between speed and braking distance?

EXAMPLE 1 Representing and comparing linear and quadratic functions

Compare the relationship between speed and reaction distance with the relationship between speed and braking distance.

Kayla's Solution

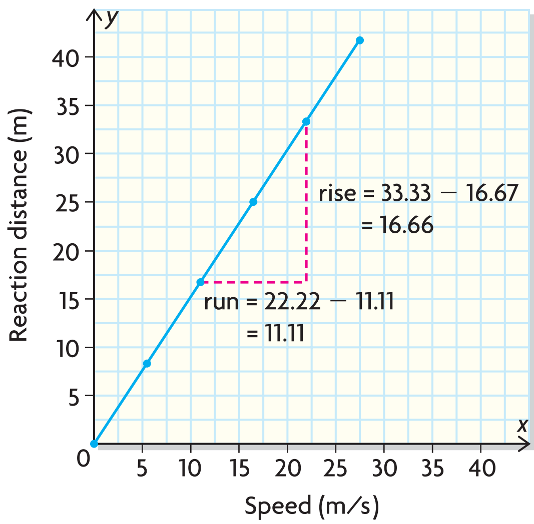
Part 1: Comparing speed and reaction distance

Speed (m/s)	Reaction Distance (m)	First Differences
0.00	0.00	
		8.33 − 0 = 8.33
5.56	8.33	
		16.67 − 8.33 = 8.34
11.11	16.67	
		25.00 − 16.67 = 8.33
16.67	25.00	
		33.33 − 25.00 = 8.33
22.22	33.33	
		41.67 − 33.33 = 8.34
27.78	41.67	

I calculated the first differences of the dependent variable, reaction distance.

This relationship is linear.

The first differences appear to be close to a constant, about 8.33.



I drew a scatter plot and a line of best fit to help me determine an equation for this relationship.

$$\text{slope} = \frac{16.66}{11.11} \doteq 1.5 \text{ and } y\text{-intercept} = 0.$$

I used the slope of the line and the y-intercept to determine that the equation of the line of best fit is $y = 1.5x$.

So, $y = 1.5x$ is the equation that relates speed to the reaction distance.

The degree of this relation is 1.

degree
the degree of a polynomial with a single variable, say, x, is the value of the highest exponent of the variable. For example, for the polynomial $5x^3 - 4x^2 + 7x - 8$, the highest power or exponent is 3; the degree of the polynomial is 3

$y = f(x) = 1.5x$

Since there is a unique reaction distance for each value of speed, the reaction distance is a function of speed. I can express this relationship in function notation.

function notation

$f(x)$ is called function notation and represents the value of the dependent variable for a given value of the independent variable, x

Part 2: Comparing speed and braking distance

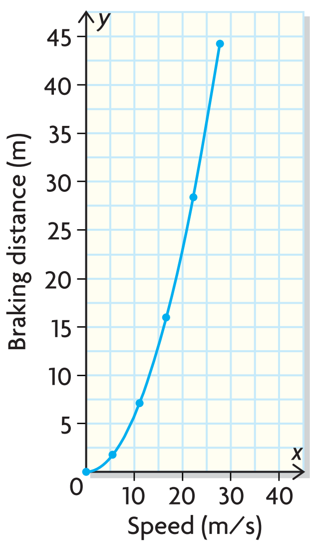
Speed (m/s)	Braking Distance (m)	First Differences	Second Differences
0.00	0.00		
		1.77	
5.56	1.77		3.56
		5.33	
11.11	7.10		3.53
		8.86	
16.67	15.96		3.56
		12.42	
22.22	28.38		3.55
		15.97	
27.78	44.35		

I calculated the first differences. They are not constant, so the relationship is nonlinear.

I calculated the second differences. They are almost the same value, so the relationship is quadratic.

Communication *Tip*

$f(x)$ is read as "f of x" or "f at x." The symbols $f(x)$, $g(x)$, and $h(x)$ are often used to name functions, but other letters may be used. When working on a problem, it is sometimes easier to choose letters that match the quantities in the problem. For example, use $v(t)$ to write velocity as a function of time.



I drew a scatter plot and a smooth curve to help me find the equation of this quadratic relationship.

$y = ax^2$

$a = \frac{y}{x^2}$

$a = \frac{7.10}{11.11^2}$

I used $y = ax^2$ since it looks like the parabola's vertex is (0, 0). I chose the point (11.11, 7.10) and substituted $y = 7.10$ and $x = 11.11$ into the equation to solve for a.

$a \doteq 0.0575$

$y = 0.0575x^2$

is the equation that relates speed to braking distance.

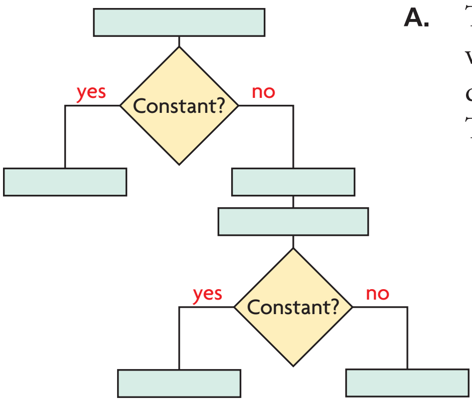
The degree of this relation is 2.

The highest power of the variable, x, is 2.

$y = g(x) = 0.055x^2$

Since there is a unique breaking distance for each value of speed, braking distance is a function of speed. I wrote the equation using function notation using the letter g as the name of the function since I already named the stopping distance function f.

Reflecting



A. The chart at the left summarizes the sequence of steps used to decide whether data for a function are linear, quadratic, or neither. The diamonds in the chart represent decisions that have yes-or-no answers. The rectangles represent steps in the sequence.

i) Copy the chart. Then complete it by matching each empty rectangle with one of the six phrases.

1. calculate second differences

2. linear

3. neither quadratic nor linear

4. nonlinear

5. calculate first differences

6. quadratic

ii) Use the steps in the chart to determine whether stopping distance is a linear or quadratic relation. The data for stopping distance are provided in the table on page 17.

B. What is the relationship between the differences in a table of values and the degree of a function?

C. How can you tell whether a function is linear or quadratic if you are given
- a table of values?
- a graph?
- an equation?

APPLY the Math

EXAMPLE 2 Representing volume as a function of time

Water is poured into a tank at a constant rate. The volume of water in the tank is measured every minute until the tank is full. The measurements are recorded in the table.

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0.0	1.3	2.6	3.9	5.2	6.5	7.8	9.1

a) Use difference tables to determine whether the volume of water poured into the tank, $V(t)$, is a linear or quadratic function of time. Explain.
b) State the domain and range using set notation.

Brian's Solution

a)

Time (min)	0	1	2	3	4	5	6	7
Volume (L)	0.0	1.3	2.6	3.9	5.2	6.5	7.8	9.1
First Differences		1.3	1.3	1.3	1.3	1.3	1.3	1.3

Every minute, an additional 1.3 L of water is added to the tank. Since the first differences are constant, $V(t)$ is a linear function.

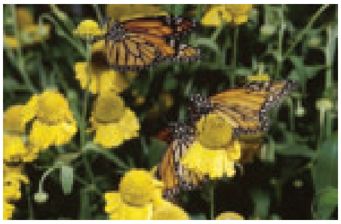
b) Domain $= \{t \in \mathbf{R} \mid 0 \le t \le 7\}$

It takes 7 min to fill the tank, and time can't be negative.

Range $= \{V(t) \in \mathbf{R} \mid 0 \le V(t) \le 9.1\}$

The tank holds a minimum of 0 L when empty and a maximum of 9.1 L of water when full.

EXAMPLE 3 Representing distance travelled as a function of time



A migrating butterfly travels about 128 km each day. The distance it travels, $g(t)$, in kilometres, is a function of time, t, in days.

a) Write an equation using function notation to represent the distance a butterfly travels in t days.

b) State the degree of this function and whether it is linear or quadratic.

c) Use the function to calculate the distance the butterfly travels in 20 days.

d) What are the domain and range of this function, assuming that a butterfly lives 40 days, on average? Express your answers in set notation.

Rashmika's Solution

a) $g(t) = 128t$

In this equation, t is the number of days and $g(t)$ is the total distance travelled for the given value of t.

b) The degree is 1.

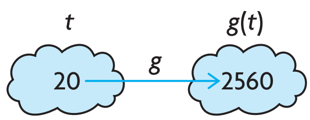
The function is linear.

The highest exponent of t is 1. This means the function $g(t)$ is linear.

c)

$$g(t) = 128t$$
$$g(20) = 128(20)$$
$$= 2560$$

Substitute $t = 20$ into the function $g(t)$.



The butterfly travels 2560 km in 20 days.

d) Domain $= \{t \in \mathbf{R} \mid 0 \le t \le 40\}$

The maximum number of days is 40.

Range $= \{g(t) \in \mathbf{R} \mid 0 \le g(t) \le 5120\}$

The maximum distance is $g(40) = 5120$ km.

In this case, neither distance nor time can be negative.

If you know the equation of a function, its degree indicates whether it is linear or quadratic.

EXAMPLE 4 Connecting degree to type of function

State the degree of each function, and identify which functions are linear and which are quadratic.

a) $k(x) = 3x(x + 1)$ **b)** $m(x) = (x + 2)^2 - x^2$

Akiko's Solution

a) $k(x) = 3x(x + 1)$ — I used the distributive property to expand.

$= 3x^2 + 3x.$

The degree is 2. — The highest exponent of x is 2.

The function is quadratic.

b) $m(x) = (x + 2)^2 - x^2$

$= (x + 2)(x + 2) - x^2$ — I used the distributive property to expand. I simplified by collecting like terms.

$= x(x + 2) + 2(x + 2) - x^2$

$= x^2 + 2x + 2x + 4 - x^2$

$= x^2 + 4x + 4 - x^2$

$= 4x + 4$

The degree is 1. — The highest exponent of x is 1.

The function is linear.

Study Aid

- For help, see Essential Skills Appendix, A-9.

In Summary

Key Ideas

- Linear functions have constant first differences, a degree of 1, and graphs that are lines.
- Quadratic functions have constant second differences, a degree of 2, and graphs that are parabolas.

Need to Know

- $f(x)$ is called function notation and is used to represent the value of the dependent variable for a given value of the independent variable, x.
- The degree of a function is the highest power of the independent variable.

CHECK Your Understanding

1. A ball is dropped from a distance 10 m above the ground. The height of the ball from the ground is measured every tenth of a second, resulting in the following data:

Time (s)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Height (m)	10.00	9.84	9.36	8.56	7.44	6.00	4.24	2.16	0.00

a) Use difference tables to determine whether distance, $d(t)$, is a linear or quadratic function of time. Explain.

b) What are the domain and range of this function? Express your answer in set notation.

2. A math textbook costs $60.00. The number of students who need the book is represented by x. The total cost of purchasing books for a group of students can be represented by the function $f(x)$.

a) Write an equation in function notation to represent the cost of purchasing textbooks for x students.

b) State the degree of this function and whether it is linear or quadratic.

c) Use your equation to calculate the cost of purchasing books for a class of 30 students.

d) What are the domain and range of this function, assuming that books can be purchased for two classes of students? Assume that the maximum number of students in a class is 30. Express your answers in set notation.

3. State the degree of each function, and identify which are linear and which are quadratic.

a) $f(x) = -7 + 2x$

b) $g(x) = 3x^2 + 5$

c) $g(x) = (x - 4)(x - 3)$

d) $3x - 4y = 12$

PRACTISING

4. Use difference tables to determine whether the data represent a linear or quadratic relationship.

Time (s)	0	1	2	3	4	5
Height (m)	0	15	20	20	15	0

Time (h)	**Bacteria Count**
0	12
1	23
2	50
3	100

5. **K** The population of a bacteria colony is measured every hour and results in the data shown in the table at the left. Use difference tables to determine whether the number of bacteria, $n(t)$, is a linear or quadratic function of time. Explain.

6. State the degree of each function and whether it is linear or quadratic.

a) $f(x) = -4x(x - 1) - x$

b) $m(x) = -x^2 + (x + 3)^2$

c) $g(x) = 3x^2 + 35$

d) $g(x) = 3(x - 5)$

7. **T** A function has the following domain and range:

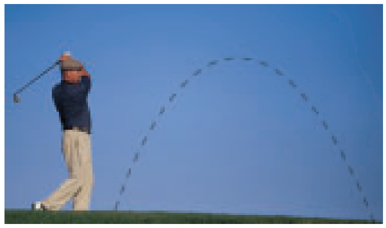
$$\text{Domain} = \{t \in \mathbf{R} \mid -3 \leq t \leq 5\}$$

$$\text{Range} = \{g(t) \in \mathbf{R} \mid 0 \leq g(t) \leq 10\}$$

a) Draw a sketch of this function if it is linear.

b) Draw a sketch of this function if it is quadratic.

8. **A** A golf ball is hit and its height is given by the equation $h = 29.4t - 4.9t^2$, where t is the time elapsed, in seconds, and h is the height, in metres.



a) Write an equation in function notation to represent the height of the ball as a function of time.

b) State the degree of this function and whether the function is linear or quadratic.

c) Use difference tables to confirm your answer in part (b).

d) Graph the function where $\{t \in \mathbf{R} \mid t \geq 0\}$.

e) At what time(s) is the ball at its greatest height? Express the height of the ball at this time in function notation.

f) At what time(s) is the ball on the ground? Express the height of the ball at this time in function notation.

9. **C** List the methods for determining whether a function is linear or quadratic. Use examples to explain the advantages and disadvantages of each method.

Extending

10. A baseball club pays a vendor \$50 per game for selling bags of peanuts for \$2.50 each. The club also pays the vendor a commission of \$0.05 per bag.

a) Determine a function that describes the income the vendor makes for each baseball game. Define the variables in your function.

b) Determine a function that describes the revenue the vendor generates each game for the baseball club. Define the variables in your function.

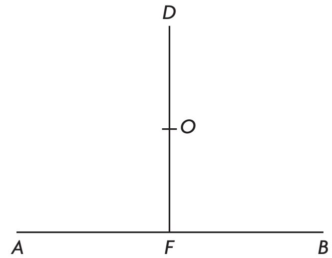
c) Determine the number of bags of peanuts the vendor must sell before the baseball club makes a profit from his efforts.

Curious | Math

Using Straight Lines to Draw a Parabola

How can you use a series of lines to create a parabola?

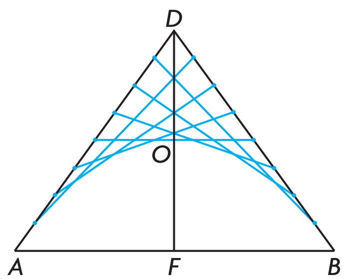
1. Draw lines *AB* and *FD* so that line *AB* is perpendicular to *FD* and $AF = FB$. Mark *O* on line *DF* so that $DO = OF$.



2. Draw lines *DA* and *DB*.

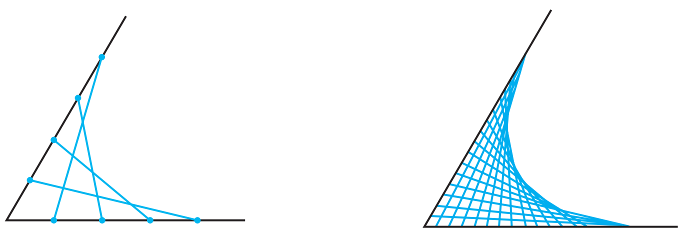
3. Divide *DA* into 8 equal parts. Divide *DB* into 8 equal parts.

4. Connect the new points as shown in the diagram to create additional lines. What do you notice?



5. Divide lines *DA* and *DB* into smaller equal parts. Connect the new points as in step 4. What do you notice?

Here are two similar sketches. The one on the left uses few lines, and the one on the right uses many. As the number of lines used increases, the shape of the parabola becomes more noticeable.



Why do you think this series of lines creates a parabola?

1.3 Working with Function Notation

YOU WILL NEED

- graph paper

GOAL

Interpret and evaluate relationships expressed in function notation.

LEARN ABOUT the Math

A rocket is shot into the air from the top of a building. Its height, in metres, after t seconds is modelled by the function $h(t) = -4.9t^2 + 19.6t + 34.3$.

? What is the meaning of $h(t)$ and how does it change for different values of t?

EXAMPLE 1 Connecting the rocket's height to function notation

Use function notation and a table of values to show the height of the rocket above the ground during its flight.

Leila's Solution: Using a Table of Values

$$h(t) = -4.9t^2 + 19.6t + 34.3$$

$$h(0) = -4.9(0)^2 + 19.6(0) + 34.3$$

I substituted 0 for t in the function. I started at $t = 0$ because time can never be negative.

$$= 0 + 0 + 34.3$$
$$= 34.3$$
$$h(0) = 34.3 \text{ m}$$

$h(0) = 34.3$ means that when the rocket was first shot, at 0 s, it was 34.3 m above the ground. This is the height of the building.

$$h(3) = -4.9(3)^2 + 19.6(3) + 34.3$$

I evaluated the height of the rocket at $t = 3$.

$$= -4.9(9) + 58.8 + 34.3$$
$$= -44.1 + 58.8 + 34.3$$
$$= 49$$
$$h(3) = 49 \text{ m}$$

$h(3) = 49$ means that after 3 s, the rocket is 49 m above the ground.

t	0	1	2	3	4	5	6
$h(t)$	34.3	49.0	53.9	49.0	34.3	9.8	−24.5

I found the height of the rocket at different times and created a table of values. $h(6)$ gave a negative result. Since the height is always positive, the rocket hit the ground between 5 s and 6 s.

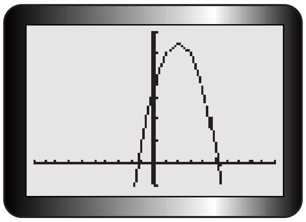
t	5.1	5.2	5.3	5.4
$h(t)$	6.81	3.72	0.54	−2.744

I evaluated the function for different times between 5 s and 6 s. The rocket hit the ground between 5.3 s and 5.4 s after it was shot into the air.

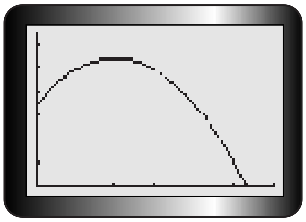
Tina's Solution: Using a Graph of the Height Function

Tech Support

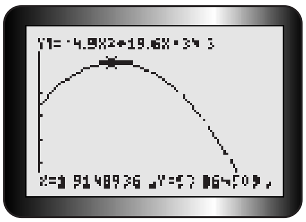
For help graphing and tracing along functions, see Technical Appendix, B-2.

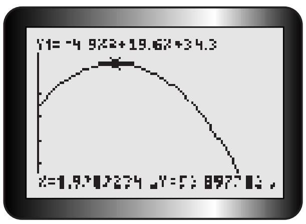


I used the equivalent equation $y = -4.9x^2 + 19.6x + 34.3$ and entered this for Y1 in the equation editor to plot the graph on a graphing calculator.



I adjusted the calculator window to show the part of the graph where the rocket is in the air. The height of the rocket is negative after about 5.3 s, so the domain is from 0 to 5.3 s.





By tracing along the graph, I can see that the maximum height, 53.89 m, occurs sometime between 1.91 s and 1.97 s.

Reflecting

A. How are the two methods similar? How are they different?

B. How did each student determine the domain? What is the domain of the function $h(t)$?

C. How did each student determine the valid values for the range? What is the range of the function $h(t)$?

D. Tina determined that the maximum height of the rocket would occur between 1.91 s and 1.97 s. Explain how Tina could find a more exact answer.

APPLY the Math

EXAMPLE 2 Using a substitution strategy to evaluate a function

Given $f(x) = 2x^2 + 3x - 1$, evaluate

a) $f(3)$ **b)** $f\left(\frac{1}{2}\right)$ **c)** $f(5 - 3)$ **d)** $f(5) - f(4)$

Richard's Solution

a)
$$\begin{aligned} f(x) &= 2x^2 + 3x - 1 \\ f(3) &= 2(3)^2 + 3(3) - 1 \\ &= 2(9) + 9 - 1 \\ f(3) &= 26 \end{aligned}$$

$f(3)$ means "find the value of the dependent variable or output when the independent variable or input is $x = 3$." I substituted 3 for x and then evaluated the expression.

b)
$$\begin{aligned} f(x) &= 2x^2 + 3x - 1 \\ f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) - 1 \\ &= 2\left(\frac{1}{4}\right) + \frac{3}{2} - 1 \\ &= \frac{2}{4} + \frac{6}{4} - \frac{4}{4} \\ f\left(\frac{1}{2}\right) &= 1 \end{aligned}$$

I substituted $\frac{1}{2}$ for x and then evaluated the expression using a common denominator of 4.

c)
$$\begin{aligned} f(5 - 3) &= 2(5 - 3)^2 + 3(5 - 3) - 1 \\ &= 2(2)^2 + 3(2) - 1 \\ &= 2(4) + 6 - 1 \\ f(5 - 3) &= 13 \end{aligned}$$

$f(5 - 3)$ is the same as $f(2)$, so I evaluated $f(5 - 3)$ by evaluating $f(2)$.

d) $f(5) - f(4)$

$= [2(5)^2 + 3(5) - 1] - [2(4)^2 + 3(4) - 1]$

$= [2(25) + 15 - 1] - [2(16) + 12 - 1]$

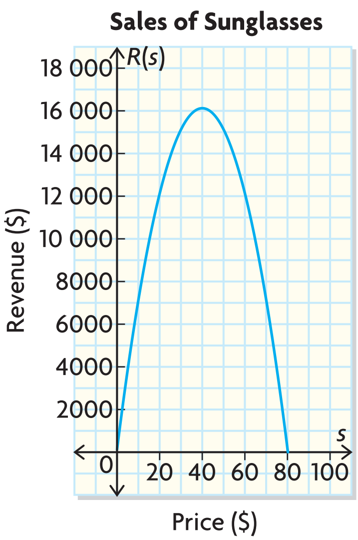
$= 64 - 43$

$f(5) - f(4) = 21$

$f(5) - f(4)$ is the difference in the value of the function evaluated at $x = 5$ and at $x = 4$.

So, I subtracted the value for $f(4)$ from the value for $f(5)$.

EXAMPLE 3 Representing and comparing the value of a function



The relationship between the selling price of a new brand of sunglasses and revenue, $R(s)$, is represented by the function $R(s) = -10s^2 + 800s + 120$ and its graph at the left.

a) Determine the revenue when the selling price is $5.

b) Explain what $R(20) = 12\ 120$ means.

c) If $R(s) = 16\ 120$, determine the selling price, s.

Kedar's Solution

a) $R(s) = -10s^2 + 800s + 120$

$R(5) = -10(5)^2 + 800(5) + 120$

$= -10(25) + 4000 + 120$

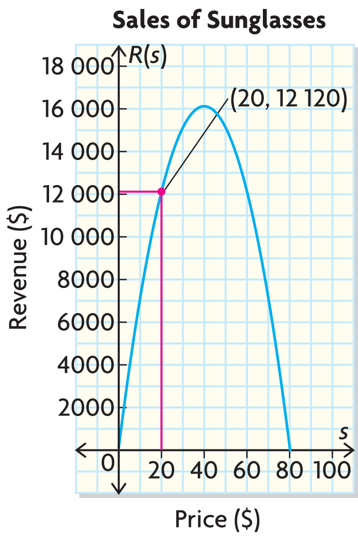
$= 3870$

$R(5)$ is the revenue generated at a selling price of $5. I substituted 5 for s into the equation and evaluated the expression.

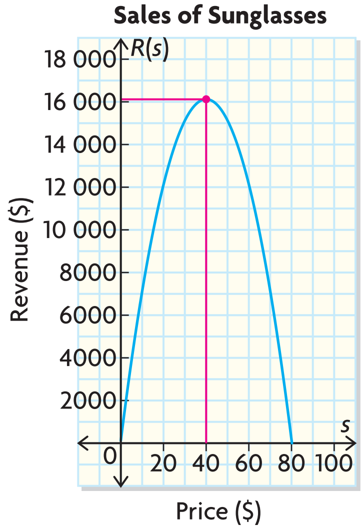
When the selling price is $5, the revenue is $3870.

b) $R(20)$ represents the revenue generated from a selling price of $20. In this case $R(20)$ is $12 120. This corresponds to the point (20, 12 120) on the graph.

When the value of the independent variable, s, is 20, the dependent variable, $R(s)$, has a value of 12 120.



c)



I used the graph to estimate the selling price that corresponds to a revenue of $16 120. I drew a horizontal line from 16 120 on the revenue axis until it touched the curve. From this point I drew a vertical line down to the selling price axis.

For a revenue of $16 120 to occur, the selling price must be $40.

EXAMPLE 4 Representing new functions

If $g(x) = 2x^2 - 3x + 5$, determine

a) $g(m)$ **b)** $g(3x)$

Sinead's Solution

a) $g(x) = 2x^2 - 3x + 5$

$g(m) = 2m^2 - 3m + 5$

In this case, x is replaced by the variable m, not by a number.

b) $g(3x) = 2(3x)^2 - 3(3x) + 5$

$= 2(9x^2) - 9x + 5$

$g(3x) = 18x^2 - 9x + 5$

I substituted $3x$ for x in the equation for $g(x)$ and then simplified.

In Summary

Key Idea

- $f(x)$ is called function notation and is used to represent the value of the dependent variable for a given value of the independent variable, x. For this reason, y and $f(x)$ are interchangeable in the equation or graph of a function, so $y = f(x)$.

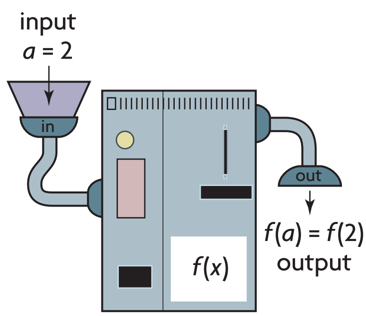
Need to Know

- When a function is defined by an equation, it is convenient to name the function to distinguish it from other equations of other functions.

  For example, the set of ordered pairs (x, y) that satisfies the equation $y = 3x + 2$ forms a function. By naming the function f, we can use function notation.

(x, y) Notation	Function Notation
(x, y) is a solution of $y = 3x + 2$.	$(x, f(x))$ is a solution of $f(x) = 3x + 2$.

- $f(a)$ represents the value or output of the function when the input is $x = a$. The output depends on the equation of the function. To evaluate $f(a)$, substitute a for x in the equation for $f(x)$.
- $f(a)$ is the y-coordinate of the point on the graph of f with x-coordinate a. For example, if $f(x)$ takes the value 3 at $x = 2$, then $f(2) = 3$ and the point (2, 3) lies on the graph of f.



CHECK Your Understanding

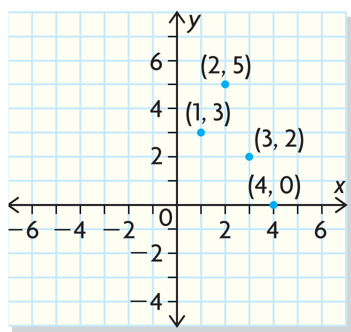
1. Explain the meaning of $f(3) = \frac{1}{2}$.
2. Evaluate $f(3)$ for each of the following.

   a) $\{(1, 2), (2, 0), (3, 1), (4, 2)\}$

   b)

x	1	2	3	4
y	2	3	4	5

   c)



3. Determine $f(-2)$, $f(0)$, $f(2)$, and $f(2x)$ for each function.

a) $f(x) = -3x^2 + 5$ **b)** $f(x) = 4x^2 - 2x + 1$

4. A stone is thrown from a bridge into a river. The height of the stone above the river at any time after it is released is modelled by the function $h(t) = 72 - 4.9t^2$. The height of the stone, $h(t)$, is measured in centimetres and time, t, is measured in seconds.

a) Evaluate $h(0)$. What does it represent?
b) Evaluate $h(2.5)$. What does it represent?
c) If $h(3) = 27.9$, explain what you know about the stone's position.

PRACTISING

5. Evaluate $f(4)$ for each of the following.

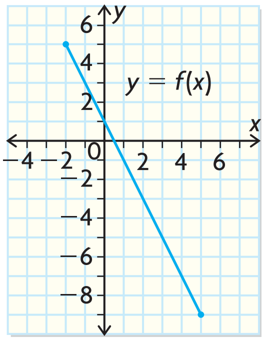
a) $f = \{(1, 5), (3, 2), (4, 1), (6, 2)\}$

b)

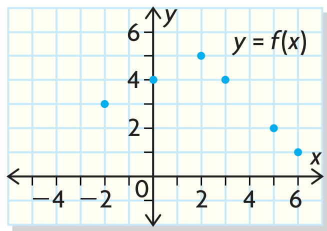
x	2	4	6	8
f(x)	4	8	12	16

c) $f(x) = 3x^2 - 2x + 1$

d)



6. The graph of $y = f(x)$ is shown at the right.



a) State the domain and range of f.
b) Evaluate.
i) $f(3)$ **ii)** $f(5)$ **iii)** $f(5 - 3)$ **iv)** $f(5) - f(3)$
c) In part (b), why is the value in (iv) not the same as that in (iii)?
d) $f(2) = 5$. What is the corresponding ordered pair? What does 2 represent? What does $f(2)$ represent?

7. If the point (2, 6) is on the graph of $y = f(x)$, what is the value of $f(2)$? Explain.

8. If $f(-2) = 6$, what point must be on the graph of f? Explain.

9. Evaluate each function for the given x-values.

a) $f(x) = 9x + 1$; $x = 0$, $x = 2$
b) $f(x) = -2x - 3$; $x = -1$, $x = 3$
c) $f(x) = 2x^2 + 5$; $x = 2$, $x = 3$
d) $f(x) = 3x^2 - 4$; $x = 0$, $x = 4$

10. Consider the function $g(t) = 3t + 5$.

a) Determine

i) $g(0)$ **iii)** $g(2)$ **v)** $g(1) - g(0)$

ii) $g(1)$ **iv)** $g(3)$ **vi)** $g(2) - g(1)$

b) In part (a), what are the answers to (v) and (vi) commonly called?

11. Consider the function $f(x) = s^2 - 6s + 9$.

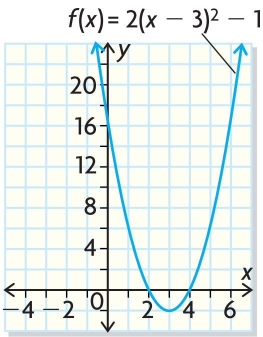
a) Determine each value.

i) $f(0)$ **iv)** $f(3)$

ii) $f(1)$ **v)** $[f(2) - f(1)] - [f(1) - f(0)]$

iii) $f(2)$ **vi)** $[f(3) - f(2)] - [f(2) - f(1)]$

b) In part (a), what are the answers to (v) and (vi) commonly called?



12. The graph shows $f(x) = 2(x - 3)^2 - 1$.

a) Evaluate $f(0)$.

b) What does $f(0)$ represent on the graph of f?

c) If $f(x) = 6$, determine possible values of x.

d) Does $f(3) = 4$ for this function? Explain.

13. **T** The sum of two whole numbers is 10. Their product can be modelled by the function $P(x) = x(10 - x)$.

a) What does x represent? What does $(10 - x)$ represent? What does $P(x)$ represent?

b) What is the domain of this function?

c) Evaluate the function for all valid values of the domain. Show the results in a table of values.

x	0									
***P*(*x*)**	0									

d) What two numbers do you estimate give the largest product? What is the largest product? Explain.

14. **A** A farming cooperative has recorded information about the relationship between tonnes of carrots produced and the amount of fertilizer used. The function $f(x) = -0.53x^2 + 1.38x + 0.14$ models the effect of different amounts of fertilizer, x, in hundreds of kilograms per hectare (kg/ha), on the yield of carrots, in tonnes.

a) Evaluate $f(x)$ for the given values of x and complete the table.

Fertilizer, *x* (kg/ha)	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Yield, *y*(*x*) (tonnes)									

b) According to the table, how much fertilizer should the farmers use to produce the most tonnes of carrots?

c) Check your answer with a graphing calculator or by evaluating $f(x)$ for values between 1.25 and 1.50. Why does the answer change? Explain.

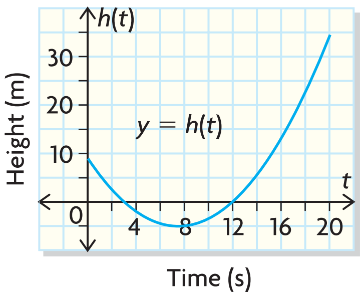
15. Sam read the following paragraph in his textbook. He does not understand the explanation. **C**

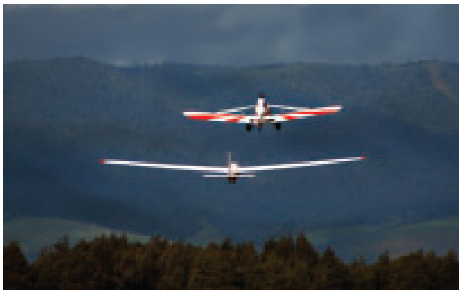
> "$f(x)$ represents an expression for determining the value of the function f for any value of x. $f(3)$ represents the value of the function (the output) when x (the input) is 3. If $f(x)$ is graphed, then $f(3)$ is the y-coordinate of a point on the graph of f, and the x-coordinate of that point is 3."

Create an example (equation and graph) using a quadratic function to help him understand what he has read.

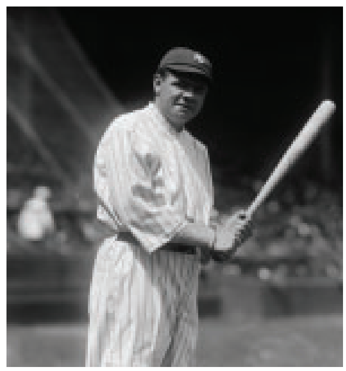
Extending

16. A glider is launched from a tower on a hilltop. The height, in metres, is negative whenever the glider is below the height of the hilltop. The equation representing the flight is $h(t) = \frac{1}{4}(t - 3)(t - 12)$, where time, t, is measured in seconds.





a) What does $h(0)$ represent?
b) What does $h(3)$ represent?
c) When is the glider at its lowest point? What is the vertical distance between the top of the tower and the glider at this time?



17. Babe Ruth, a baseball player, hits a "major league pop-up" so that the height of the ball, in metres, is modelled by the function $h(t) = 1 + 30t - 5t^2$, where t is time in seconds.
a) Evaluate the function for each of the given times to complete the table of values below.
b) Graph the function.
c) When does the ball reach its maximum height?
d) What is the ball's maximum height?
e) How long does it take for the ball to hit the ground?

Time (s)	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0
Height (m)																	

1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Study *Aid*

- See Lesson 1.1, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 1, 2, and 3.

Q: What is the difference between a relation and a function?

A: A function is a special kind of relation in which there is only one value of the dependent variable for each value of the independent variable. This means that for each input number, the equation generates only one output value. As a result, every number in the domain of the function corresponds to only one number in the range.

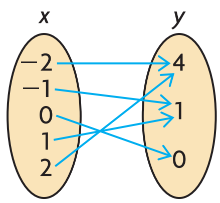
Q: What are some ways to represent a function?

A: You may describe a function as

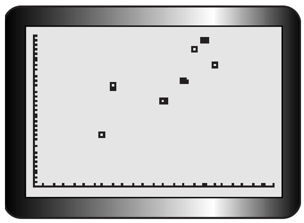
- a set of ordered pairs: $\{(0, 1), (3, 4), (2, -5)\}$
- a table of values:

x	1	2	3
y	5	7	9

- a description in words: the height of a ball is a function of time
- an equation: $y = 2x + 1$
- function notation: $f(x) = 2x + 1$
- a mapping diagram:



- a graph or a scatter plot:



Study *Aid*

- See Lesson 1.2, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 4 and 5.

Q: What do first and second differences indicate about a function?

A: For linear functions, the first differences are constant, indicating a function of degree 1.

For quadratic functions, the first differences are not constant but the second differences are, indicating a function of degree 2.

Study *Aid*

- See Lesson 1.3, Examples 1, 2, 3, and 4.
- Try Mid-Chapter Review Questions 6 and 7.

Q: What is function notation and what does it mean?

A:
- $f(x)$ is called function notation and is read "f at x" or "f of x."
- $f(x)$ represents the value of the dependent variable for a given value of the independent variable, x. This means y and $f(x)$ are interchangeable, so $y = f(x)$.
- $f(3)$ represents the value of the function when x is 3. So $(3, f(3))$ is the ordered pair for the point on the graph of f.

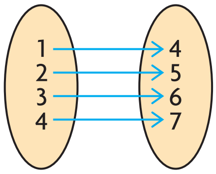
PRACTICE Questions

Lesson 1.1

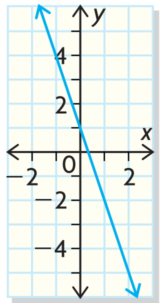
1. Determine whether the following relations are functions. Explain.

a) (1, 3), (2, 3), (3, 2), (1, 4), (4, 1)

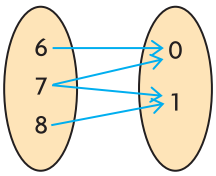
b)



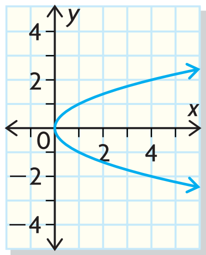
d)



c)



e)



2. Relations f and g are defined by

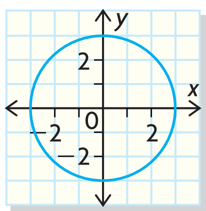
$f = \{(1, 2), (2, 3), (3, 4)\}$ and
$g = \{(1, 2), (2, 1), (2, 3), (3, 0), (3, 4)\}$.

a) State the domain and range of each relation.

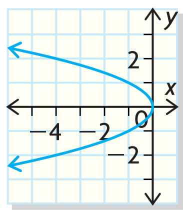
b) Is f a function? Is g? Explain.

3. Given the following, state the domain and range and whether the relation is a function.

a)



b)



Lesson 1.2

4. The time it takes for a pendulum to make one complete swing and return to the original position is called the period. The period changes according to the length of the pendulum.

Length of Pendulum (cm)	6.2	24.8	55.8	99.2	155.0
Period (s)	0.5	1.0	1.5	2.0	2.5

a) Identify whether the relationship between the length of the pendulum and the period is linear, quadratic, or neither.

b) If a pendulum is 40 cm long, estimate its period.

c) Predict the length of a pendulum if its period is 2.2 s.

5. The distance a car skids depends on the speed of the car just before the brakes are applied. The chart shows the car's speed and the length of the skid.

Speed (km/h)	1	10	20	30	40	50
Length of Skid (m)	0.0	0.7	2.8	6.4	11.4	17.8

Speed (km/h)	60	70	80	90	100
Length of Skid (m)	25.7	35.0	45.7	57.8	71.4

a) Create a scatter plot for the data. Draw a curve of good fit for the data.

b) Estimate the initial speed of the car if the skid mark is 104 m long.

c) Determine whether a linear or a quadratic relation can model the data.

Lesson 1.3

6. A function h is defined by $h(x) = 2x - 5$. Evaluate.

a) $h(-2)$ **b)** $h(2m)$ **c)** $h(3) + h(n)$

7. A function g is given by $g(x) = 2x^2 - 3x + 1$. Evaluate.

a) $g(-1)$ **b)** $g(3m)$ **c)** $g(0)$

1.4 Exploring Transformations of Quadratic Functions

YOU WILL NEED

- graphing calculator or graphing software
- graph paper

GOAL

Understand how the parameters a, h, and k in $f(x) = a(x - h)^2 + k$ affect and change the graph of $f(x) = x^2$.

EXPLORE the Math

transformations
transformations are operations performed on functions to change the position or shape of the associated curves or lines

Transformations of curves, lines, and shapes form the basis for many patterns, like those you see on fabrics, wallpapers, and gift-wrapping paper. They are also used in designing tessellations such as those used in the graphic art of Maurits Cornelis Escher (1898–1972).

? How do the parameters a, h, and k in a quadratic function of the form $f(x) = a(x - h)^2 + k$ change the graph of $f(x) = x^2$?

Tech *Support*
For help on graphing functions, see Technical Appendix, B-2.

A. Use a graphing calculator or graphing software to graph the quadratic functions shown in the table on the same set of axes. Sketch and label the graph of each function on the same set of axes. Then copy and complete the table.

	Function	Value of k in $f(x) = x^2 + k$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = x^2 + 2$	2				
c)	$f(x) = x^2 + 4$					
d)	$f(x) = x^2 - 1$					
e)	$f(x) = x^2 - 3$					

B. Use the graphs and the table from part A to answer the following:

i) What information about the graph does the value of k provide in functions of the form $f(x) = x^2 + k$?

ii) What happens to the graph of the function $f(x) = x^2 + k$ when the value of k is changed? Consider both positive and negative values of k.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = x^2 + k$? What happens to the y-coordinates?

C. Clear all previous equations from your calculator or graphing program. Repeat part A for the quadratic functions shown in the table.

	Function	Value of h in $f(x) = (x - h)^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	0	up	(0, 0)	$x = 0$	yes
b)	$f(x) = (x - 2)^2$	2				
c)	$f(x) = (x - 4)^2$					
d)	$f(x) = (x + 2)^2$					
e)	$f(x) = (x + 4)^2$					

D. Use the graphs and the table from part C to answer the following:

i) What information about the graph does the value of h provide in functions of the form $f(x) = (x - h)^2$?

ii) What happens to the graph of the function $f(x) = (x - h)^2$ when the value of h is changed? Consider both positive and negative values of h.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = (x - h)^2$? What happens to the y-coordinates?

E. Clear all previous equations from your calculator or graphing program. Repeat part A for the quadratic functions shown in the table.

	Function	Value of a in $f(x) = ax^2$	Direction of Opening	Vertex	Axis of Symmetry	Congruent to $f(x) = x^2$?
a)	$f(x) = x^2$	1	up	(0, 0)	$x = 0$	yes
b)	$f(x) = 2x^2$					
c)	$f(x) = 0.5x^2$					
d)	$f(x) = -2x^2$					
e)	$f(x) = -0.5x^2$					

F. Use the graphs and the table from part E to answer the following:

i) What information about the graph does the value of a provide in functions of the form $f(x) = ax^2$? Consider values of a that are greater than 1 and values of a between 0 and 1.

ii) What happens to the graph of the function $f(x) = ax^2$ when the value of a is changed? Consider both positive and negative values of a.

iii) What happens to the x-coordinates of all points on $f(x) = x^2$ when the function is changed to $f(x) = ax^2$? What happens to the y-coordinates?

Reflecting

G. Use examples and labelled graphs that compare the transformed functions with the function $f(x) = x^2$. Summarize the effect that a, h, and k have on the parabola's

a) direction of opening

b) vertex

c) axis of symmetry

d) congruency to $f(x) = x^2$

e) x-coordinates and y-coordinates

In Summary

Key Ideas

- Quadratic functions can be written in the form $g(x) = a(x - h)^2 + k$.
- Each of the constants a, h, and k changes the position and/or shape of the graph of $f(x) = x^2$.

Need to Know

- Changing the values of h and k changes the position of the parabola and, as a result, the locations of the vertex and the axis of symmetry. The new parabola is congruent to the parabola $f(x) = x^2$.
- Changing the value of a can change the shape of the parabola, as well as the direction in which the parabola opens. The new parabola is not congruent to the parabola $f(x) = x^2$ when $a \neq 1$ or -1.

FURTHER Your Understanding

1. The parabola $f(x) = x^2$ is transformed as described. The equation of the transformed graph has the form $f(x) = a(x - h)^2 + k$. Determine the values of a, h, and k for each of the following transformations.

a) The parabola moves 4 units to the right.

b) The parabola moves 5 units up.

c) The parabola moves 2 units to the left

d) The parabola moves 3 unit down.

e) The parabola is congruent to $f(x) = x^2$ and opens down.

f) The parabola is narrower and opens upward. The y-coordinates have been multiplied by a factor of 2.

g) The parabola is wider and opens downward. The y-coordinates have been multiplied by a factor of $\frac{1}{2}$.

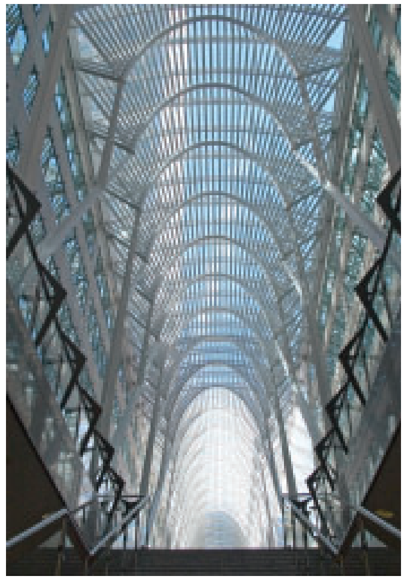
1.5 Graphing Quadratic Functions by Using Transformations

GOAL

Use transformations to sketch the graphs of quadratic functions.

LEARN ABOUT the Math

This photograph shows the interior of BCE Place in Toronto. Architects design structures that involve the quadratic model because it combines strength with elegance.



You have seen how changing the values of a, h, and k changes the shape and position of the graph of $f(x)$ in functions of the form $f(x) = x^2 + k$, $f(x) = (x - h)^2$, and $f(x) = ax^2$. This information can be used together with the properties of the quadratic function $f(x) = x^2$ to sketch the graph of the **transformed function**.

transformed function
the resulting function when the shape and/or position of the original graph of $f(x)$ are changed

? How do you use transformations and the properties of the quadratic function $f(x) = x^2$ to graph the function $g(x) = (x + 2)^2 - 4$?

EXAMPLE 1 Graphing quadratic functions by using a transformation strategy: translating

Use transformations to sketch the graph of $g(x) = (x + 2)^2 - 4$.

Dave's Solution

$g(x) = (x + 2)^2 - 4$

$g(x) = (x - (-2))^2 - 4$

$a = 1, h = -2, k = -4$

I expressed the relation in the form $g(x) = a(x - h)^2 + k$.

The values of h and k show that I must apply a horizontal and a vertical **translation** to the graph of $f(x) = x^2$ by moving each point on this graph 2 units to the left and 4 units down.

translation
two types of translations can be applied to the graph of a function:

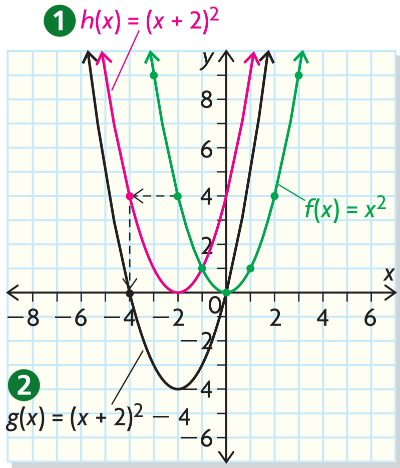
- Horizontal translations—all points on the graph move to the right when $h > 0$ and to the left when $h < 0$
- Vertical translations—all points on the graph move up when $k > 0$ and down when $k < 0$

key points

points of any function that define its general shape

Key Points of $f(x) = x^2$

x	$f(x) = x^2$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



1. I subtracted 2 from the x-coordinates of the **key points**. The base curve $f(x) = x^2$ (in green) moved 2 units to the left. The resulting graph is $h(x) = (x + 2)^2$ (in red).
2. I subtracted 4 from the y-coordinates of the key points of $h(x) = (x + 2)^2$. Each point of the curve $h(x) = (x + 2)^2$ moves down 4 units. The resulting graph is $g(x) = (x + 2)^2 - 4$ (in black).

The vertex changed from $(0, 0)$ to $(-2, -4)$.

The axis of symmetry changed from $x = 0$ to $x = -2$.

The shape of the graph did not change.

Reflecting

A. Consider Dave's solution. Given three points $O(0, 0)$, $A(-2, 4)$, and $B(1, 1)$ on the graph of $f(x) = x^2$, what would the coordinates of the corresponding images of the points on $f(x) = (x + 2)^2 - 4$ be if they were labelled O', A', and B'?

B. Dave translated the graph of $f(x) = x^2$ to the left 2 units and then 4 units down. Had he translated the graph 4 units down and then 2 units to the left, would the resulting graph be the same? Explain.

C. If the graph of $f(x) = x^2$ is only translated up/down and left/right, will the resulting graph always be congruent to the original graph? Explain.

APPLY the Math

EXAMPLE 2 Graphing quadratic functions by using a transformation strategy: stretching vertically

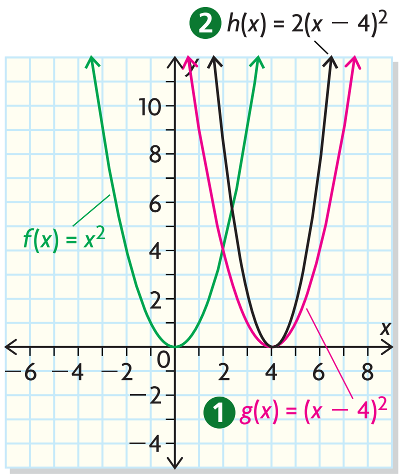
Use transformations to sketch the graph of $h(x) = 2(x - 4)^2$.

Amanda's Solution

$h(x) = 2(x - 4)^2$

$a = 2$, $h = 4$, and $k = 0$.

$h(x)$ is in the form $h(x) = a(x - h)^2 + k$. No vertical translation is required, because $k = 0$.



1. Since $h > 0$, I added 4 to the x-coordinates of the key points of the graph $f(x) = x^2$ (in green). This moved the graph 4 units to the right. The resulting graph is $g(x) = (x - 4)^2$ (in red).
2. I multiplied the y-coordinates of the key points of $g(x) = (x - 4)^2$ by 2 because $a = 2$. This resulted in a vertical stretch and gave me the graph of $h(x) = 2(x - 4)^2$ (in black).

vertical stretch

when $a > 1$, the graph of the function $f(x)$ is stretched vertically

The vertex changed from (0, 0) to (4, 0).

The axis of symmetry changed from $x = 0$ to $x = 4$.

The shape of the parabola also changed—it's narrower than $f(x) = x^2$.

EXAMPLE 3 Graphing quadratic functions by using transformation strategies: reflecting and compressing vertically

Use transformations to sketch the graph of $g(x) = -0.5(x + 2)^2$.

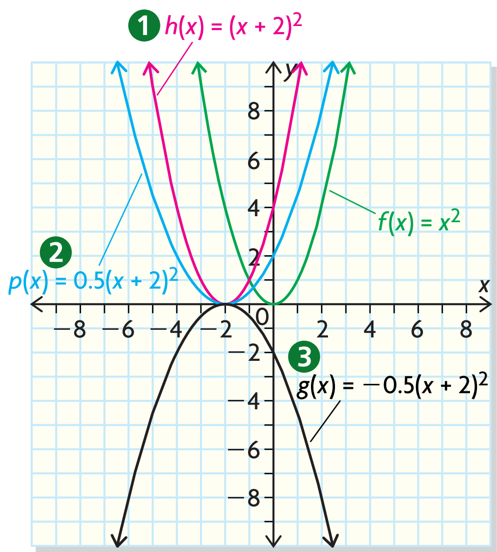
Chantelle's Solution

$g(x) = -0.5(x + 2)^2$

$g(x) = -0.5(x - (-2))^2 + 0$

$a = -0.5$, $h = -2$, and $k = 0$.

I wrote $g(x)$ in the form $g(x) = a(x - h)^2 + k$. No vertical translation is required because $k = 0$.



1 I subtracted 2 from each of the x-coordinates of the key points. The graph of $f(x) = x^2$ (in green) moved 2 units to the left, since $h < 0$. This resulted in the graph of $h(x) = (x + 2)^2$ (in red).

2 I multiplied the y-coordinates of the key points of $h(x) = (x + 2)^2$ by 0.5 to get the graph of $p(x) = 0.5(x + 2)^2$. This resulted in a **vertical compression** and gave me the graph of $p(x) = 0.5(x + 2)^2$ (in blue).

3 Since a is also negative, I reflected the graph of $p(x) = 0.5(x + 2)^2$ in the x-axis to get the graph $g(x) = -0.5(x + 2)^2$ (in black). This resulted in a **vertical reflection**.

The vertex changed from (0, 0) to $(-2, 0)$.

The axis of symmetry changed from $x = 0$ to $x = -2$.

The shape of the parabola also changed—it's wider than $f(x) = x^2$.

The parabola opens downward.

vertical compression

when $0 < a < 1$, the graph is compressed vertically

vertical reflection

when $a < 0$, the graph is reflected in the x-axis

EXAMPLE 4 Graphing quadratic functions by using transformation strategies

Use transformations to sketch the graph of $m(x) = \frac{1}{3}x^2 + 2$.

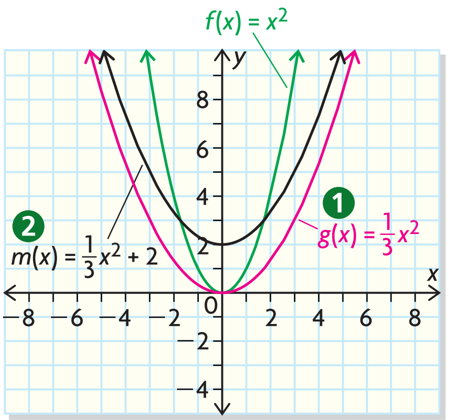
Craig's Solution

$$m(x) = \frac{1}{3}x^2 + 2$$

$$m(x) = \frac{1}{3}(x - 0)^2 + 2$$

I wrote $m(x)$ in the form $m(x) = a(x - h)^2 + k$. No horizontal translation is required because $h = 0$.

$a = \frac{1}{3}$, $h = 0$, and $k = 2$.



1. I multiplied the y-coordinates of $f(x) = x^2$ (in green) by $\frac{1}{3}$. The graph was vertically compressed by a factor of 3 and resulted in the graph of $g(x) = \frac{1}{3}x^2$ (in red).
2. Then I moved each point on $g(x)$ up 2 units. This gave me the graph of $m(x) = \frac{1}{3}x^2 + 2$ (in black).

The vertex changed from (0, 0) to (0, 2).

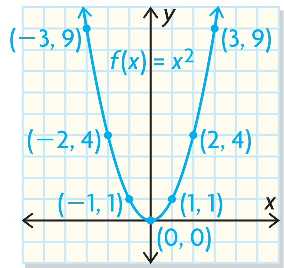
The axis of symmetry remained at $x = 0$.

The shape of the graph also changed—it's wider than $f(x) = x^2$.

In Summary

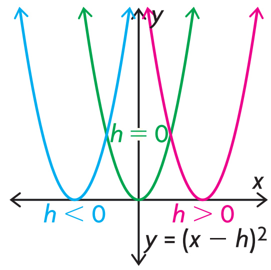
Key Ideas

- Functions of the form $g(x) = a(x - h)^2 + k$ can be graphed by applying transformations, one at a time, to the key points on the graph of $f(x) = x^2$.
- In graphing $g(x)$, the transformations apply to every point on the graph of $f(x)$. However, to sketch the new graph, you only need to apply the transformations to the key points of $f(x) = x^2$.

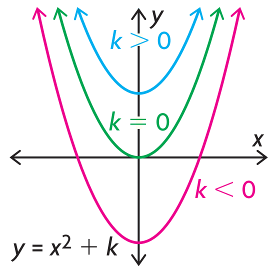


Need to Know

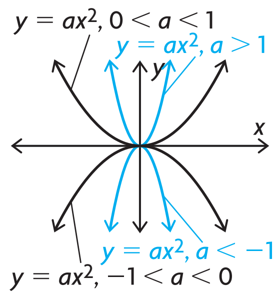
- For a quadratic function in the form $g(x) = a(x - h)^2 + k$,
  - horizontal translations: The graph moves to the right when $h > 0$ and to the left when $h < 0$.



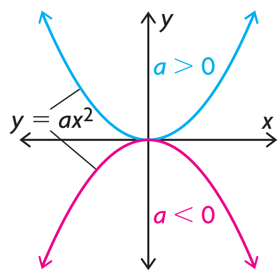
  - vertical translations: The graph moves up when $k > 0$ and down when $k < 0$.



  - vertical stretches: The graph is stretched vertically when $a > 1$.
  - vertical compressions: The graph is compressed vertically when $0 < a < 1$.



  - vertical reflections: The graph is reflected in the x-axis when $a < 0$.
  - the axis of symmetry is the line $x = h$.
  - the vertex is the point (h, k).



CHECK Your Understanding

1. Match each equation with its corresponding graph. Explain how you made your decision.

a) $y = -(x - 2)^2 - 3$

b) $y = -0.5x^2 - 4$

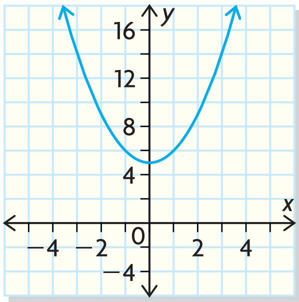
c) $y = x^2 + 5$

d) $y = 2(x + 2)^2$

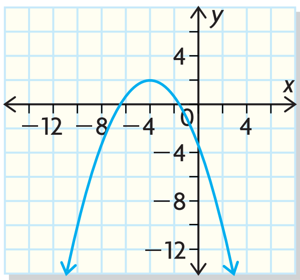
e) $y = (x - 2)^2$

f) $y = -\frac{1}{3}(x + 4)^2 + 2$

i)



ii)



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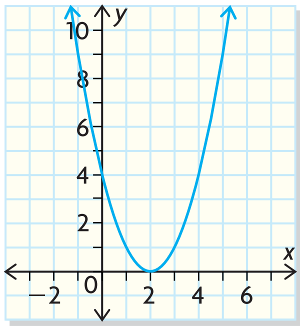

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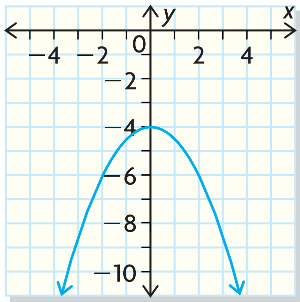

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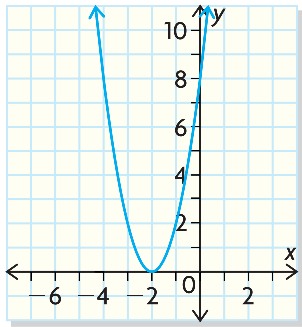

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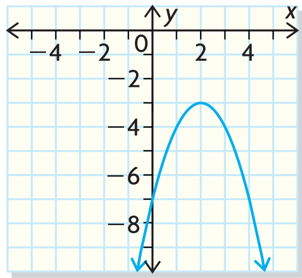


2. For each function,

i) identify the values of the parameters a, h, and k

ii) identify the transformations

iii) use transformations to graph the function and check that it is correct with a table of values or a graphing calculator

a) $f(x) = -3x^2$

b) $f(x) = (x + 3)^2 - 2$

c) $f(x) = (x - 1)^2 + 1$

d) $f(x) = -x^2 - 2$

e) $f(x) = -(x - 2)^2$

f) $f(x) = \frac{1}{2}(x + 3)^2$

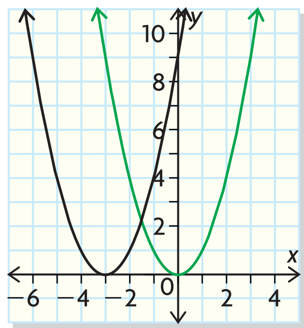
PRACTISING

3. Match each graph with the correct equation. The graph of $y = x^2$ is shown in green in each diagram.

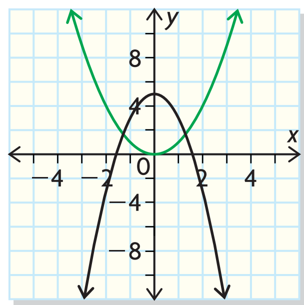
a) $y = x^2 + 5$ **c)** $y = -2x^2 + 5$

b) $y = (x + 5)^2$ **d)** $y = 2(x + 5)^2$

i)



iii)



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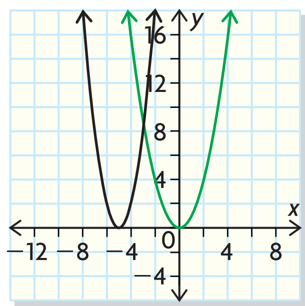

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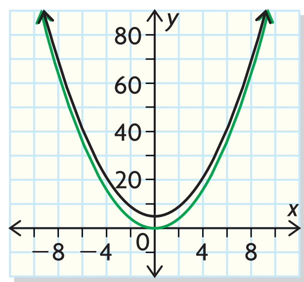


4. K Describe the translations applied to the graph of $y = x^2$ to obtain a graph of each quadratic function. Sketch the graph.

a) $y = (x - 5)^2 + 3$ **c)** $y = 3x^2 - 4$

b) $y = (x + 1)^2 - 2$ **d)** $y = -\frac{1}{3}(x + 4)^2$

5. Consider a parabola P that is congruent to $y = x^2$ and with vertex at $(0, 0)$. Find the equation of a new parabola that results if P is

a) stretched vertically by a factor of 5

b) compressed vertically by a factor of 2

c) translated 2 units to the right and reflected in the x-axis

d) compressed vertically by 3, reflected in the x-axis, and translated 2 units up

6. Consider a parabola P that is congruent to $y = x^2$ and with vertex $(2, -4)$. Find the equation of a new parabola that results if P is

a) translated 2 units down

b) translated 4 units to the left

c) translated 2 units to the left and translated 3 units up

d) translated 3 units to the right and translate 1 unit down

7. Write an equation of a parabola that satisfies each set of conditions.

a) opens upward, congruent to $y = x^2$, and vertex $(0, 4)$
b) opens upward, congruent to $y = x^2$, and vertex $(5, 0)$
c) opens downward, congruent to $y = x^2$, and vertex $(5, 0)$
d) opens upward, narrower than $y = x^2$, and vertex $(2, 0)$
e) opens downward, wider than $y = x^2$, and vertex $(-2, 0)$
f) opens upward, wider than $y = x^2$, and vertex $(1, 0)$

8. Determine the answers to the following questions for each of the given transformed quadratic functions.

i) How does the shape of the graph compare with the graph of $f(x) = x^2$?
ii) What are the coordinates of the vertex and the equation of the axis of symmetry?
iii) Graph the transformed function and $f(x) = x^2$ on the same set of axes.
iv) Label the points $O(0, 0)$, $A(-2, 4)$, and $B(1, 1)$ on the graph of $f(x) = x^2$. Determine the images of these points on the transformed function. Label the images O', A', and B'.

a) $f(x) = -(x - 2)^2$
b) $f(x) = \frac{1}{2}x^2 + 2$
c) $f(x) = (x + 2)^2 - 2$

9. For each of the following, state the equation of a parabola congruent to $y = x^2$ with the given property.

a) The graph is 2 units to the right of the graph of $y = x^2$.
b) The graph is 4 units to the left of the graph of $y = x^2$.
c) The graph is 4 units to the left and 5 units down from the graph of $y = x^2$.
d) The graph is vertically compressed by a factor of 4.
e) The graph is vertically stretched by a factor of 2 and is 4 units to the left of the graph of $y = x^2$.
f) The graph is vertically stretched by a factor of 3 and is 2 units to the right and 1 unit down from the graph of $y = x^2$.

10. T For each of the following, state the condition on a and k such that the parabola $y = a(x - h)^2 + k$ has the given property.

a) The parabola intersects the x-axis at two distinct points.
b) The parabola intersects the x-axis at one point.
c) The parabola does not intersect the x-axis.

11. The acceleration due to gravity, g, is 9.8 m/s^2 on Earth, 3.7 m/s^2 on Mars, 10.5 m/s^2 on Saturn, and 11.2 m/s^2 on Neptune. The height, $h(t)$, of an object, in metres, dropped from above each surface is given by $h(t) = -0.5gt^2 + k$.

A

a) Describe how the graphs will differ for an object dropped from a height of 100 m on each of the four planets.

b) On which planet will the object be moving fastest when it hits the surface?

c) On which planet will it be moving slowest?

12. Describe how the x- and y-coordinates of the given quadratic functions differ from the x- and y-coordinates of corresponding points of $y = x^2$.

C

a) $y = (x + 7)^2$

b) $y = x^2 + 7$

c) $y = -2(x - 4)^2$

d) $y = -\frac{1}{2}x^2 - 4$

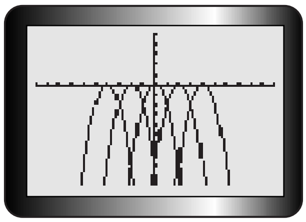
Extending

13. Predict what the graphs of each group of equations would look like. Check your predictions by using graphing technology.

a) $f(x) = 10x^2$
$f(x) = 100x^2$
$f(x) = 1000x^2$
$f(x) = 10000x^2$

b) $f(x) = 0.1x^2$
$f(x) = 0.01x^2$
$f(x) = 0.001x^2$
$f(x) = 0.0001x^2$

14. a) If $y = x^2$ is the base curve, write the equations of the parabolas that produce the following pattern shown on the calculator screen below. The scale on both axes is 1 unit per tick mark.



b) Create your own pattern using parabolas, and write the associated equations. Use $y = x^2$ as the base parabola.

1.6 Using Multiple Transformations to Graph Quadratic Functions

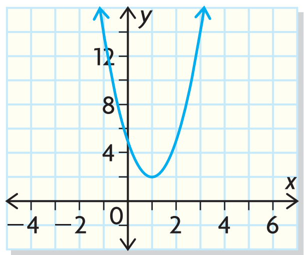
GOAL

Apply multiple transformations to $f(x) = x^2$ to graph quadratic functions defined by $f(x) = a(x - h)^2 + k$.

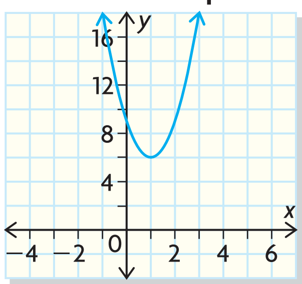
LEARN ABOUT the Math

Dave and Nathan have each graphed the function $f(x) = 3(x - 1)^2 + 2$. Dave's graph is different from Nathan's graph. They both applied the transformations in different orders.

Dave's Graph



Nathan's Graph



? Does the order in which transformations are performed matter?

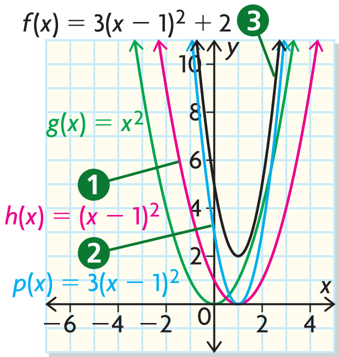
EXAMPLE 1 Reasoning about order: following the order of operations

Dave's Solution: Stretching and then Translating

$f(x) = 3(x - 1)^2 + 2$

$a = 3, h = 1, k = 2$

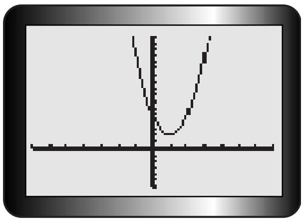
I began with $g(x) = x^2$. I applied the transformations by following the order of operations.



1. I performed the operation inside the bracket first. I translated the graph 1 unit to the right to get the graph $h(x) = (x - 1)^2$ (in red).

2. Next, I performed the multiplication. The translated graph stretches vertically by a factor of 3 to get the graph $p(x) = 3(x - 1)^2$ (in blue).

3. I translated the stretched graph 2 units up to get the graph $f(x) = 3(x - 1)^2 + 2$ (in black).

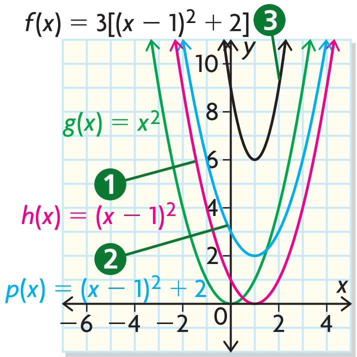


I checked my graph on the graphing calculator. My graph looks the same. The order I used must be correct.

Nathan's Solution: Translating and then Stretching

$f(x) = 3(x - 1)^2 + 2$
$a = 3, h = 1, k = 2$

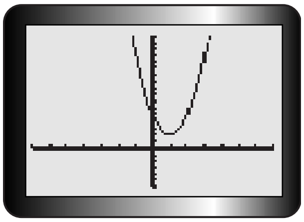
I began with $g(x) = x^2$. I applied the translations first.



1 I shifted the graph 1 unit to the right, since $h = 1$, to get the graph $h(x) = (x - 1)^2$ (in red).

2 I translated the graph 2 units up, since $k = 2$, to get the graph $p(x) = (x - 1)^2 + 2$ (in blue).

3 Next, I applied the vertical stretch. The graph stretches vertically by a factor of 3, since $a = 3$, to get the graph $f(x) = 3[(x - 1)^2 + 2]$ (in black).



I checked my graph on the graphing calculator. My graph looks different. The order I used can't be correct.

Reflecting

A. List the y-coordinates for $x = -2, -1, 0, 1,$ and 2 for the final graph of each student's solution.

B. List the y-coordinates for $x = -2, -1, 0, 1,$ and 2 for the graph of the function $f(x) = 3(x - 1)^2 + 2$.

C. How do the coordinates from part B compare with those from each student's solution?

D. Explain why one of the solutions is incorrect.

E. List the order of transformations of the correct solution. Does the order of transformations of the correct solution apply to all functions of the form $f(x) = a(x - h)^2 + k$?

APPLY the Math

EXAMPLE 2 Applying multiple transformations to sketch the graph of a quadratic function

Sketch $h(x) = 2(x + 3)^2 - 1$ by applying the appropriate transformations to the graph of $f(x) = x^2$.

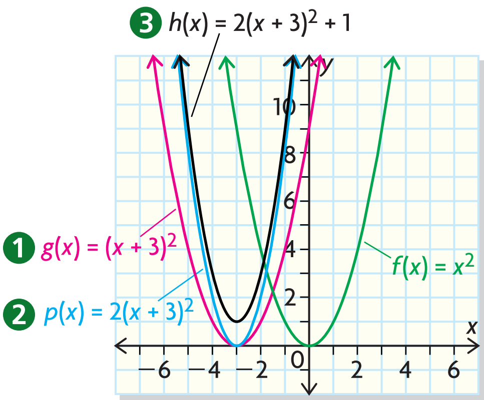
Mei's Solution

$h(x) = 2(x + 3)^2 + 1$

$h(x) = 2(x - (-3))^2 + 1$

$a = 2, h = -3, k = 1$

I applied the transformations one at a time, following the order of operations.



1. I moved the graph of $f(x) = x^2$ (in green) 3 units to the left to get the graph of $g(x) = (x + 3)^2$ (in red).
2. I multiplied the y-coordinates of $g(x) = (x + 3)^2$ by 2 to get the graph of $p(x) = 2(x + 3)^2$ (in blue).
3. I moved the resulting graph 1 unit up to get the graph of $h(x) = 2(x + 3)^2 + 1$ (in black).

The vertex changed from (0, 0) to (−3, 1).
The axis of symmetry changed from $x = 0$ to $x = -3$.
The final graph is narrower than $f(x) = x^2$.

EXAMPLE 3 Identifying transformations from the quadratic function

Describe the transformations you would use to graph the function $f(x) = -(x - 2.5)^2 - 5$.

Deirdre's Solution

$f(x) = -(x - 2.5)^2 - 5$

$a = -1, h = 2.5, k = -5$

Step 1.	Horizontal translation 2.5 units to the right	$h = 2.5$
Step 2.	No vertical stretch or compression	$a = -1$
Step 3.	Reflection in the x-axis	$a < 0$
Step 4.	Vertical translation 5 units down	$k = -5$

EXAMPLE 4 Applying multiple transformations to sketch the graph of a quadratic function

Graph $g(x) = -7 - (x + 3)^2$ by using transformations.

Jared's Solution

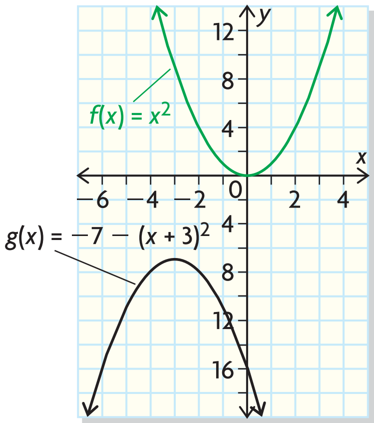
$g(x) = -7 - (x + 3)^2$

$g(x) = -(x + 3)^2 - 7$

$g(x) = -(x - (-3))^2 - 7$

I wrote the function in the form $g(x) = a(x - h)^2 + k$ to help me identify the values of a, h, and k.

$a = -1, h = -3, k = -7$



Since $h < 0$, I moved the graph of $f(x) = x^2$ (in green) 3 units to the left. Next, I reflected it in the x-axis because $a = -1$. Then I moved it 7 units down because $k < 0$. This gave me the graph of $g(x)$ (in black).

The vertex changed from (0, 0) to $(-3, -7)$.
The axis of symmetry changed from $x = 0$ to $x = -3$.
The shape did not change, since no stretching or compressing occurred.

In Summary

Key Idea

- Functions of the form $g(x) = a(x - h)^2 + k$ can be graphed by hand by applying the appropriate transformations, one at a time, to the graph of $f(x) = x^2$.

Need to Know

- Transformations can be applied following the order of operations:
  - horizontal translations
  - vertical stretches or compressions
  - reflections, if necessary
  - vertical translations
- You can use fewer steps if you combine the stretch/compression with the reflection and follow this with the necessary translations. This works because you are multiplying before adding/subtracting like the order of operations

CHECK Your Understanding

1. List the sequence of steps required to graph each function.

a) $f(x) = 3(x + 2)^2$

b) $f(x) = -2(x - 3)^2 + 1$

c) $f(x) = \frac{1}{3}x^2 - 3$

d) $f(x) = -\frac{1}{2}(x + 2)^2 + 4$

2. Sketch the final graph for each of the functions in question 1. Verify at least one of the key points, other than the vertex, by substituting its x-value into the equation and solving for y.

PRACTISING

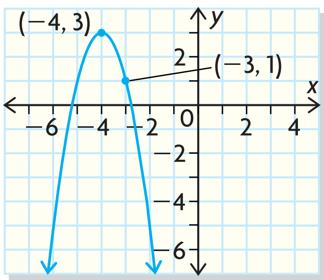
3. Match each function to its graph.

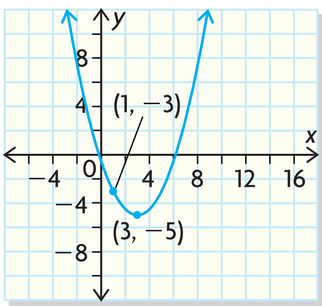
a) $f(x) = (x + 3)^2 + 1$

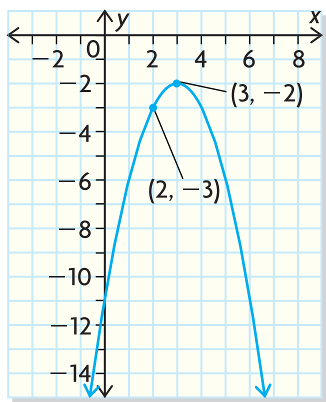
b) $f(x) = -2(x + 4)^2 + 3$

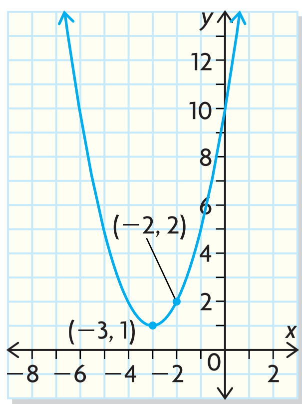
c) $f(x) = -(x - 3)^2 - 2$

d) $f(x) = \frac{1}{2}(x - 3)^2 - 5$

i) 

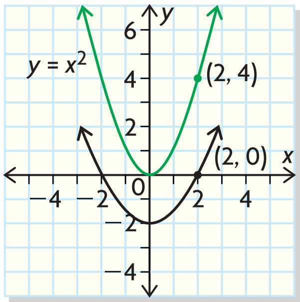
ii) 

iii) 

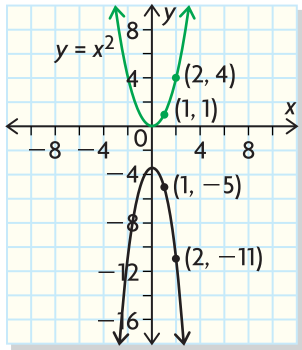
iv) 

4. Transformations are applied to the graphs of $y = x^2$ to obtain the black parabolas. Describe the transformations that were applied. Write an equation for each black parabola.

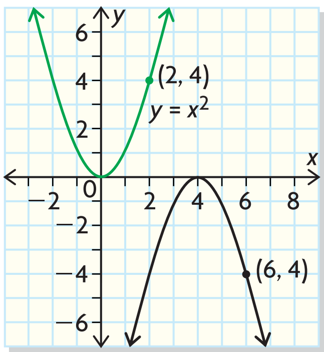
a)



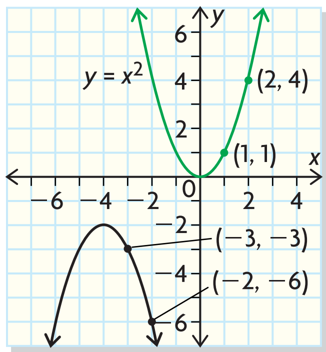
c)



b)



d)



5. Write an equation of a parabola that satisfies each set of conditions.

a) opens upward, congruent with $y = 2x^2$, vertex $(4, -1)$

b) opens downward, congruent with $y = \frac{1}{3}x^2$, vertex $(-2, 3)$

c) opens downward, congruent with $y = \frac{1}{2}x^2$, vertex $(-3, 2)$

d) vertex $(2, -2)$, x-intercepts of 0 and 4

e) vertex $(-1, 4)$, y-intercept of 2

f) vertex $(4, 5)$, passes through $(2, 9)$

6. Consider a parabola P that is congruent to $y = x^2$, opens upward, and has vertex $(2, -4)$. Now find the equation of a new parabola that results if P is

a) stretched vertically by a factor of 5

b) compressed by a factor of $\frac{1}{2}$

c) translated 2 units to the left

d) translated 3 units up

e) reflected in the x-axis and translated 2 units to the right and 4 units down

7. Describe the transformations applied to the graph of $y = x^2$ to obtain a graph of each quadratic relation. Sketch the graph by hand. Start with the graph of $y = x^2$ and use the appropriate transformations.

a) $y = -4(x - 5)^2 + 3$

b) $y = 2(x + 1)^2 - 8$

c) $y = \frac{2}{3}(x + 2)^2 + 1$

d) $y = -\frac{1}{2}(x - 1)^2 - 5$

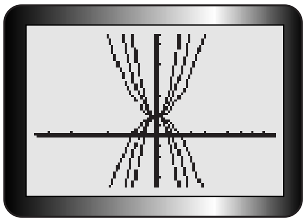
e) $y = -(x - 3)^2 + 2$

f) $y = 2(x + 1)^2 + 4$

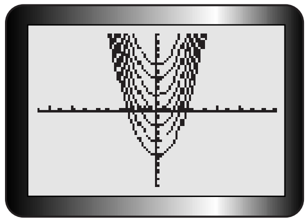
8. K Sketch the graph of the transformed function $g(x) = -3(x - 2)^2 + 5$. Start with the graph of $f(x) = x^2$ and use the appropriate transformations.

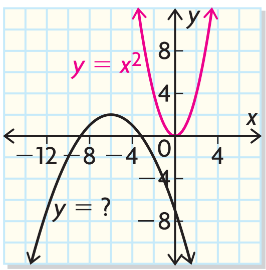
9. T If $y = x^2$ is the base curve for the graphs shown, what equations could be used to produce these screens on a graphing calculator? The scale on both axes is 1 unit per tick mark.

a)



b)





10. A The graphs of $y = x^2$ (in red) and another parabola (in black) are shown at the left.

a) Determine a combination of transformations that would produce the second parabola from the first.

b) Determine a possible equation for the second parabola.

11. C Describe the transformations applied to the graph of $y = x^2$ to obtain the graph of each quadratic relation.

a) $y = 2(x + 7)^2 - 3$

b) $y = 2x^2 + 7$

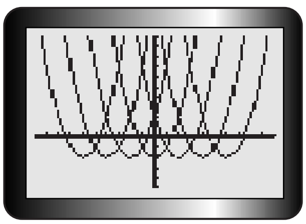
c) $y = -3(x - 4)^2 + 2$

d) $y = -3x^2 - 4$

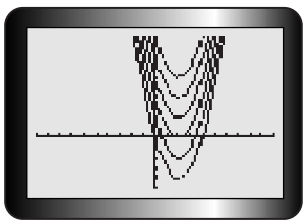
Extending

12. A graphing calculator was used together with the vertex form $y = a(x - h)^2 + k$ to graph the screens shown. For the set of graphs on each screen, tell which of the variables a, h, and k remained constant and which changed. Give possible values for the variables that remained constant.

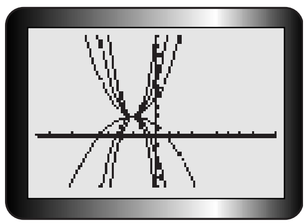
a)



b)



c)



1.7 The Domain and Range of a Quadratic Function

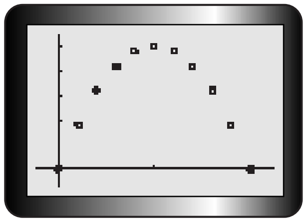
GOAL

Determine the domain and range of quadratic functions that model situations.

LEARN ABOUT the Math

A flare is shot vertically upward. A motion sensor records its height above ground every 0.2 s. The results are shown in the table.

Time (s)	0.0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Height (m)	0.0	1.8	3.2	4.2	4.8	5.0	4.8	4.2	3.2	1.8	0.0

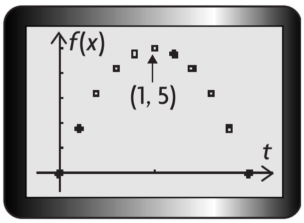


The data are plotted. The function $h(t) = -5t^2 + 10t$ models the height of the flare, in metres, as a function of time from the time the flare is first shot into the air to the time that it returns to the ground.

? How does the motion of the flare affect the domain and range of this quadratic function?

EXAMPLE 1 Reasoning about restricting the domain and range of a function

Wanda's Solution



The flare is initially shot at $t = 0$ s. The height of the flare when it is first shot is 0 m.

The flare reaches a maximum height of 5 m at $t = 1.0$ s.

After reaching 5 m, the flare starts to fall back to the ground.

At $t = 2$ s, the flare is on the ground.

The domain of this function is $\{t \in \mathbf{R} \mid 0 \leq t \leq 2\}$.

In many situations, the domain of a function is the set of all **real numbers**. However, in this situation, where time is the independent variable, we are interested only in the time from when the flare was first shot into the air until the time it hits the ground—the first 2 s of its flight.

The range is $\{h(t) \in \mathbf{R} \mid 0 \leq h(t) \leq 5\}$.

Since the height depends on the time the flare is in the air, the range of the function $h(t)$ is the set of all possible heights for the function. The maximum height is 5 m; the minimum is 0 m.

Reflecting

A. Use your graphing calculator to graph the quadratic function defined by $f(x) = -5x^2 + 10x$. Then identify the domain and range. Explain why these differ from the domain and range of the function used to model the height of the flare.

B. For each of the following descriptions, identify the independent variable and the dependent variable. Then describe and justify reasonable values for the domain.

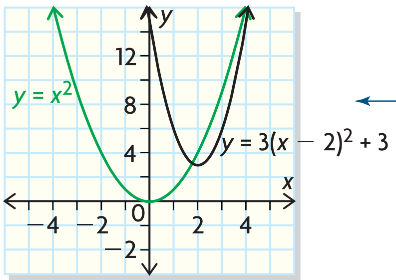
- **i)** The height of a stone that is thrown upward and falls to the ground, as a function of time
- **ii)** The height of a stone that is thrown upward and falls over a cliff, as a function of time
- **iii)** The percent of sale prices that a supermarket cashier can remember, as a function of time

APPLY the Math

EXAMPLE 2 Connecting the domain and range of a function to its graph

Find the domain and range of $y = 3(x - 2)^2 + 3$.

Indira's Solution



I graphed the function by translating $y = x^2$ to the right 2 units, vertically stretching by a factor of 3, and finally translating 3 units up.

The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all values of x. The domain is the set of all real numbers.

The range is $\{y \in \mathbf{R} \mid y \geq 3\}$.

The vertex is (2, 3).

Since the function is a parabola that opens upward, the y-coordinate of the vertex is a minimum value.

The range is all values of y greater than or equal to 3.

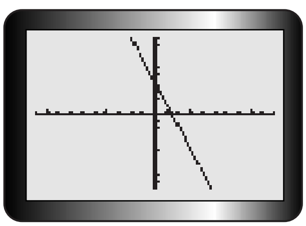
EXAMPLE 3 Connecting the domain and range to linear functions

Find the domain and range of each linear function.

a) $f(x) = -3x + 4$ **b)** $y = 5$

Paul's Solution

a)



I used a graphing calculator to graph the function.

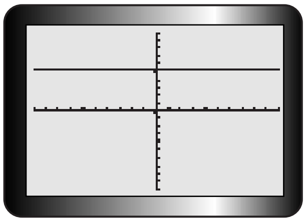
The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all real numbers.

The range is $\{y \in \mathbf{R}\}$.

Every real number corresponds to a value in the domain.

b)



I graphed the function on a graphing calculator.

The domain is $\{x \in \mathbf{R}\}$.

The function is defined for all real numbers.

The range is $\{y \in \mathbf{R} | y = 5\}$.

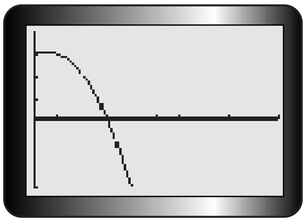
Every real number in the domain corresponds to the number 5 in the range.

Any time you work with a function that models a real-world situation, it is necessary to restrict the domain and range.

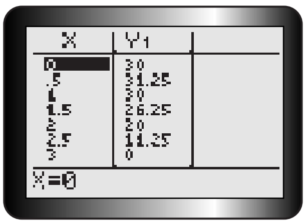
EXAMPLE 4 Placing restrictions on the domain and range

A baseball thrown from the top of a building falls to the ground below. The path of the ball is modelled by the function $h(t) = -5t^2 + 5t + 30$, where $h(t)$ is the height of the ball above ground, in metres, and t is the elapsed time in seconds. What are the domain and range of this function?

Kendall's Solution



Time is the independent variable, so the domain is all of the times the ball is in the air. I graphed the function on a graphing calculator, using only positive numbers for time.



I examined the resulting table of values.

Tech Support

For help on using the table of values, see Technical Appendix, B-6.

The domain is $\{t \in \mathbf{R} \mid 0 \leq t \leq 3\}$.

The baseball is thrown at $t = 0$ s and is in the air until it hits the ground at $t = 3$ s.

Height is the dependent variable, so the range is all of the heights the ball reaches during its flight.

The ball reaches its maximum height at $t = 0.5$, so

$$\begin{aligned} h(0.5) &= 5(0.5)^2 + 5(0.5) + 30 \\ &= 31.25 \text{ m} \end{aligned}$$

It reaches its minimum height of 0 m when it hits the ground at $t = 3$, so $h(3) = 0$.

Since the height cannot be negative, I disregarded the y-values below 0.

The range is
$\{h(t) \in \mathbf{R} \mid 0 \leq h(t) \leq 31.25\}$.

In Summary

Key Ideas

- The domain of a function is the set of values for which the function is defined. As a result, the range of a function depends on the defining equation of the function.
- When quadratic functions model real-world situations, the domain and range are often restricted to values that make sense for the situation. Values that don't make sense are excluded.

Need to Know

- Most linear functions have the set of real numbers as their domain and range. The exceptions are horizontal and vertical lines.
- The range of a horizontal line is the value of y.
- The domain of a vertical line is the value of x.
- Quadratic functions have the set of real numbers as their domain. The range depends on the location of the vertex and whether the parabola opens up or down.

CHECK Your Understanding

1. **a)** If $f(x)$ is a linear function with a positive or negative slope, what will the domain and range of this function be?

b) Consider linear relations that represent either horizontal lines, such as $y = 4$, or vertical lines, such as $x = 6$. What are the domain and range of these relations?

2. Find the domain and range of each function. Explain your answers.

a) $f(x) = -2(x + 3)^2 + 5$

b) $f(x) = 2x^2 + 4x + 7$

c) $f(x) = 3x - 4$

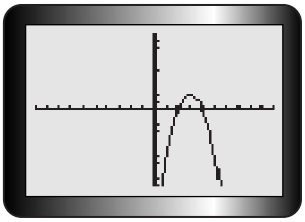
d) $x = 5$

3. The height of a flare is a function of the elapsed time since it was fired. An expression for its height is $f(t) = -5t^2 + 100t$. Express the domain and range of this function in set notation. Explain your answers.

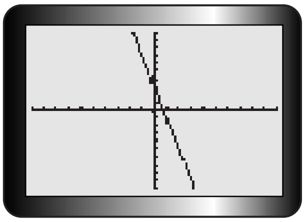
PRACTISING

4. For each graph, state the domain and range in set notation. The scale on both axes is 1 unit per tick mark in each calculator screen.

a)



b)



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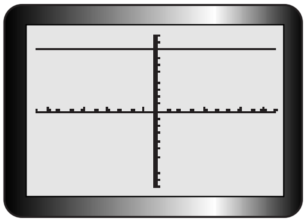

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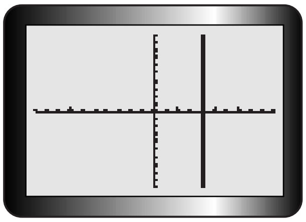

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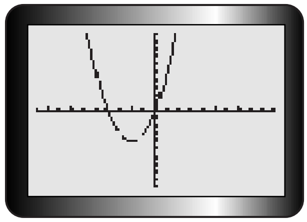

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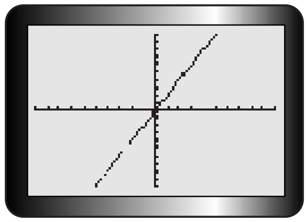


5. State the domain and range of the quadratic function represented by the following table of values.

x	1	2	3	4	5	6	7	8	9	10	11
y	12	20	27	32	35	36	35	32	27	20	12

6. **K** Use a graphing calculator or graphing software to graph each function. State its domain and range.

a) $f(x) = x^2 - 6$

b) $g(t) = 2 + t - 10t^2$

c) $f(x) = -2x^2 + 5$, where $x \geq 0$

d) $h(x) = 2(x - 1)^2 + 3$

e) $g(x) = x^2 - 6x + 2$

f) $h(x) = 3x^2 - 14x - 5$, where $x \geq 0$

7. **A** A pebble is dropped from a bridge into a river. The height of the pebble above the water after it has been released is modelled by the function $h(t) = 80 - 5t^2$, where $h(t)$ is the height in metres and t is time in seconds.

a) Graph the function for reasonable values of t.

b) Explain why the values you chose for t in part (a) are reasonable.

c) How high is the bridge? Explain.

d) How long does it take the pebble to hit the water? Explain.

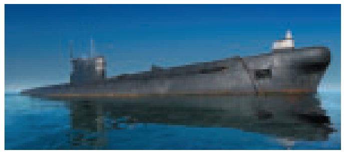
e) Express the domain and range in set notation.

8. The cost of a banquet is $550 for the room rental, plus $18 for each person served.
   a) Create a table of values and construct a scatter plot for this function.
   b) Determine the equation that models the function.
   c) State the domain and range of the function in this situation.

Tech Support

For help with scatter plots, see Technical Appendix, B-10.

9. A submarine at sea level descends 50 m every 5 min.
   T a) Determine the function that models this situation.
   b) If the ocean floor is 3 km beneath the surface of the water, how long will it take the submarine to reach the ocean floor.
   c) State the domain and range of this function in this situation.



10. A baseball is hit from a height of 1 m. The height of the ball is modelled by the function $h(t) = -5t^2 + 10t + 1$, where t is time in seconds.
    a) Graph the function for reasonable values of t.
    b) Explain why the values you chose for t in part (a) are reasonable.
    c) What is the maximum height of the ball?
    d) At what time does the ball reach the maximum height?
    e) For how many seconds is the ball in the air?
    f) For how many seconds is the ball higher than 10 m?
    g) Express the domain and range in set notation.

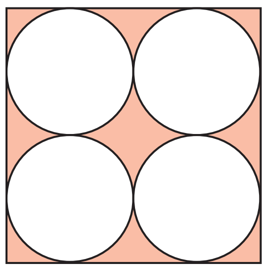
11. In the problem about selling raffle tickets in Getting Started on page 5, the student council wants to determine the price of the tickets. The council surveyed students to find out how many tickets would be bought at different prices. The council found that
    - if they charge $0.50, they will be able to sell 200 tickets; and
    - if they raise the price to $1.00, they will sell only 50 tickets.

    a) The formula for revenue, $R(x)$, as a function of ticket price, x, is $R(x) = -300x^2 + 350x$. Use a graphing calculator to graph the function.
    b) What ticket price should they charge to generate the maximum amount of revenue?
    c) State the range and domain of the function.

12. Sometimes when functions are used to model real-world situations, the
    C domain and range of the function must be restricted.
    a) Explain what this means.
    b) Explain why restrictions are necessary. Use an example in your explanation.

Extending

13. The picture shows four circles inside a square. Each small circle has a radius r. The area of the shaded region as a function is $A(r) = (16 - 4\pi)r^2$.
    a) What is the domain of the function?
    b) What is the range of the function?



Chapter Review

FREQUENTLY ASKED Questions

Q: What are some of the different kinds of transformations that can be applied to the function $f(x) = x^2$?

A: The transformations include:

1. horizontal translations: The graph of the function is shifted to the left or right.
2. vertical translations: The graph of the function is shifted up or down.
3. vertical stretches: The graph is stretched vertically. Each y-value on the graph is multiplied by a factor that is greater than 1.
4. vertical compressions: The graph is compressed vertically. Each y-value on the graph is multiplied by a factor that is between 0 and 1.
5. reflections: The graph is reflected in the x-axis when each y-value is multiplied by a negative factor.

Study *Aid*

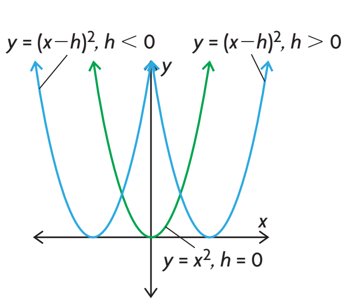
- See Lesson 1.5, Examples 1 to 4.
- Try Chapter Review Question 6.

Q: How is the form $f(x) = a(x - h)^2 + k$ of a quadratic function useful?

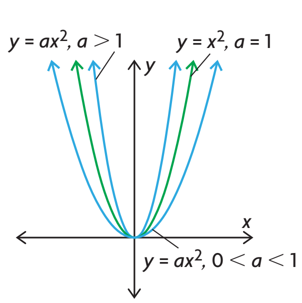
A: Every quadratic function can be written in the form $f(x) = a(x - h)^2 + k$.

The constants a, h, and k each change the location and/or shape of the graph of $f(x) = x^2$.

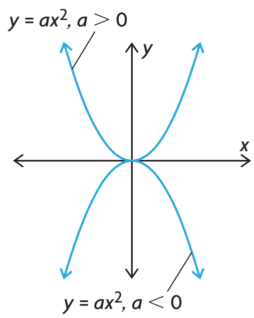
- Horizontal translations: The graph moves to the right when $h > 0$ and to the left when $h < 0$.
- Vertical stretches: The graph is stretched vertically when $a > 1$.
- Vertical compressions: The graph is compressed vertically when $0 < a < 1$.
- Vertical reflections: The graph is reflected in the x-axis when $a < 0$.
- Vertical translations: The graph moves up when $k > 0$ and down when $k < 0$.



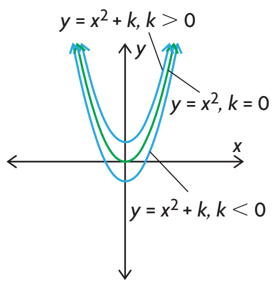
horizontal translations



vertical stretch/compressions



vertical reflections



vertical translations

Q: Why is the order in which I apply the transformations when graphing a quadratic function important?

A: Horizontal translations are applied first, vertical stretches and compressions second, reflections third, and then vertical translations. This order of transformations ensures that the y-values of $f(x) = x^2$ are transformed correctly by following the order of operations for every function of the form $f(x) = a(x - h)^2 + k$.

Study *Aid*

- See Lesson 1.6, Examples 1 to 4.
- Try Chapter Review Questions 7 to 10.

Q: How do you determine the domain and range of a linear and a quadratic function?

A: The domain of a function is the set of values for which the function is defined. As a result, the range of each function depends on the defining equation of the function.

Most linear functions have the set of real numbers as their domain and range. The exceptions are horizontal and vertical lines. When the line is horizontal, the range is the value of y. When the line is vertical, the domain is the value of x.

Quadratic functions have the set of real numbers for their domain. The range depends on the location of the vertex and whether the parabola opens up or down.

When linear and quadratic functions are used to model real-world situations, the domain and range are often restricted to values that make sense for the situation. Values that don't make sense are excluded.

Study *Aid*

- See Lesson 1.7, Examples 1 to 4.
- Try Chapter Review Questions 11 to 13.

PRACTICE Questions

Lesson 1.1

1. The data in the table show the average mass of a boy as he grows between the ages of 1 and 12. State the following:

a) domain

b) range

c) whether the relation is a function

Age (years)	1	2	3	4	5	6
Mass (kg)	11.5	13.7	16.0	20.5	23.0	23.0

Age (years)	7	8	9	10	11	12
Mass (kg)	30.0	33.0	39.0	38.5	41.0	49.5

Lesson 1.2

2. Determine, without graphing, which type of relationship (linear, quadratic, or neither) best models this table of values. Explain.

x	−1	0	1	2	3
y	1	2	−3	−14	−31

3. State the degree of each function and whether each is linear, or quadratic, or neither.

a) $f(x) = -8 + 3x$

b) $g(x) = 4x^2 - 3x + 5$

c) $y = (x - 4)(4x^2 - 3)$

Lesson 1.3

4. Evaluate the function $f(x) = 3x^2 - 3x + 1$ at the given values.

a) $f(-1)$ **b)** $f(3)$ **c)** $f(0.5)$

5. For each of the following, determine $f(3)$.

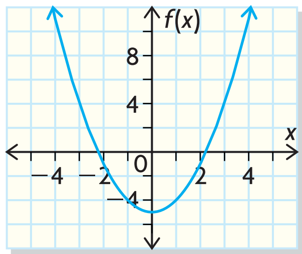
a) $f = \{(1, 2), (2, 3), (3, 5), (4, 5)\}$

b)

x	1	3	5	7
$f(x)$	2	4	6	8

c) $f(x) = 4x^2 - 2x + 1$

d)



Lesson 1.5

6. Use transformations to determine the vertex, axis of symmetry, and direction of opening of each parabola. Sketch the graph.

a) $y = x^2 - 7$

b) $y = -(x + 1)^2 + 10$

c) $y = -\frac{1}{2}(x + 2)^2 - 3$

d) $y = 2(x - 5)^2$

Lesson 1.6

7. Describe how the graph of $y = x^2$ can be transformed to the graphs of the relations from question 6.

8. a) Describe how the graph of $y = x^2$ can be transformed into the graph of the given quadratic function.

i) $y = 5x^2 - 4$

ii) $y = \frac{1}{4}(x - 5)^2$

iii) $y = -3(x + 5)^2 - 7$

b) List the domain and range of each function. Compare these with the original graph of $y = x^2$.

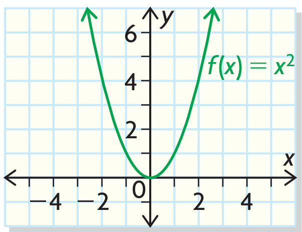
9. a) Describe the transformations to the graph of $y = x^2$ to obtain $y = -2(x + 5)^2 - 3$.

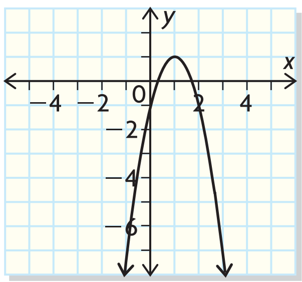
b) Graph $y = x^2$. Then apply the transformations in part (a) to graph $y = -2(x + 5)^2 - 3$.

10. The graphs of $f(x) = x^2$ (in green) and another parabola (in black) are shown.

a) Draw a combination of transformations that would produce the second parabola from the first.

b) Determine a possible equation for the second parabola.





11. On a calculator like the TI-83 Plus, enter these equations: $Y_1 = X^2$, $Y_2 = Y_1 - 4$, and $Y_3 = 2 * Y_2$.

a) Use an appropriate WINDOW setting to graph Y_1, Y_2, and Y_3. Write an equation in vertex form for Y_3.

b) Change Y_2 and Y_3 to $Y_2 = 2 * Y_1$ and $Y_3 = Y_2 - 4$. Find the coordinates of the vertex for Y_3. Write an equation in vertex form for Y_3.

c) Explain why the graph of Y_3 in part (a) is different from the one in part (b).

d) Describe the sequence of transformations needed to transform the graph of $y = x^2$ into the graph of $y = 2x^2 - 4$.

Lesson 1.7

12. A football is thrown into the air. The height, $h(t)$, of the ball, in metres, after t seconds is modelled by $h(t) = -4.9(t - 1.25)^2 + 9$.

a) How high off the ground was the ball when it was thrown?

b) What was the maximum height of the football?

c) How high was the ball at 2.5 s?

d) Is the football in the air after 6 s?

e) When does the ball hit the ground?

13. Clay shooting disks are launched from the ground into the air from a machine 12 m above the ground. The height of each disk, $h(t)$, in metres, is modelled by $h(t) = -5t^2 + 30t + 12$, where t is the time in seconds since it was launched.

a) What is the maximum height the disks reach?

b) At what time do the disks hit the ground?

c) Determine the domain and range of this model.

14. The height, h, in metres, of an object t seconds after it is dropped is $h = -0.5gt^2 + k$, where g is the acceleration due to gravity and k is the height from which the object is released. If an object is released from a height of 400 m, how much longer does it take to fall to a height of 75 m on the Moon compared with falling to the same height on Earth? The acceleration due to gravity is 9.8 m/s^2 on Earth and 1.6 m/s^2 on the Moon.

1 Chapter Self-Test

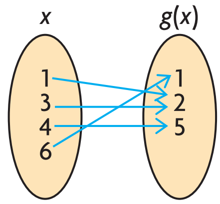
1. For each of the following relations, state

i) the domain and range

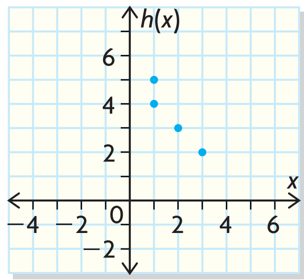
ii) whether or not it is a function, and justify your answer

a) $f = \{(1, 2), (3, 1), (4, 2), (7, 2)\}$

b)



c)



2. Define the term *function* and give an example and a non-example.

Time (s)	Height (m)
0	0
1	30
2	40
3	40
4	30
5	0

3. Use a difference table to determine whether the data in the table at the left represent a linear or quadratic relationship. Justify your decision.

4. If $f(x) = 3x^2 - 2x + 6$, determine

a) $f(2)$ **b)** $f(x - 1)$

5. $f(x) = 3(x - 2)^2 + 1$

a) Evaluate $f(-1)$.

b) What does $f(1)$ represent on the graph of f?

c) State the domain and range of the relation.

d) How do you know if f is a function from its graph?

e) How do you know if f is a function from its equation?

6. A function is defined by the equation $d(x) = 5(x - 3)^2 + 1$.

a) List the transformations to the graph of $f(x) = x^2$ to get $d(x)$.

b) What is the maximum or minimum value of the transformed function $d(x)$?

c) State the domain and range of $d(x)$.

d) Graph the function $d(x)$.

7. A football is kicked from a height of 0.5 m. The height of the football is modelled by the function $h(t) = -5t^2 + 18t + 0.5$, where t is time in seconds and $h(t)$ is height in metres.

a) Graph the function for reasonable values of t.

b) Explain why the values you chose for t in part (a) are reasonable.

c) What is the maximum height of the football?

d) At what time does the football reach the maximum height?

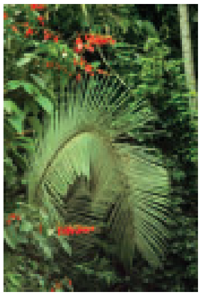
e) For how many seconds is the football in the air?

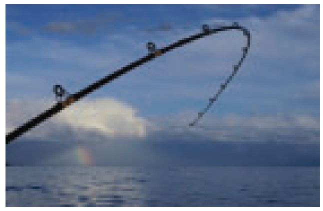
f) Express the domain and range in set notation.

1 Chapter Task

Using Transformations and Quadratic Function Models

Quadratic functions can be used as mathematical models of many real-life situations. Since the graphs of these functions are parabolas, many familiar examples can be found that appear to utilize this shape. Here are some of them:





? How can you use transformations to determine the equation of a quadratic function that models a parabola seen in a picture?

A. Use the Internet to find a picture that appears to have a parabola (or part of one) in it.

B. Copy the picture and paste it into *Geometer's Sketchpad*. Place a grid over top of your picture.

C. Create a quadratic function of the form $f(x) = ax^2$ by estimating an appropriate value for a. Adjust this value and re-graph your function until you are satisfied with the width of your parabola.

D. Create a quadratic function of the form $f(x) = a(x - h)^2 + k$ by estimating appropriate values for h and k. Adjust these values and re-graph your function until you are satisfied that the graph of your function closely matches that of the parabola in the photo. Print your final results.

E. List the transformations you used to create your graphical model as well as the final equation of your quadratic function.

F. State any restrictions on the domain and range that must be applied to ensure that your graph matches only the parabola shown in the photo.

G. Repeat parts A to F using a picture that contains a parabola that opens in the opposite direction to the one you originally found.

Tech Support

For help using *Geometer's Sketchpad* to graph functions, see Technical Appendix, B-19.

Task Checklist

- ✔ Did you show all your steps?
- ✔ Did you include a printout of your graph and photo?
- ✔ Did you support your choice of transformations used?
- ✔ Did you explain your thinking clearly?
- ✔ Did you restrict the domain and range?