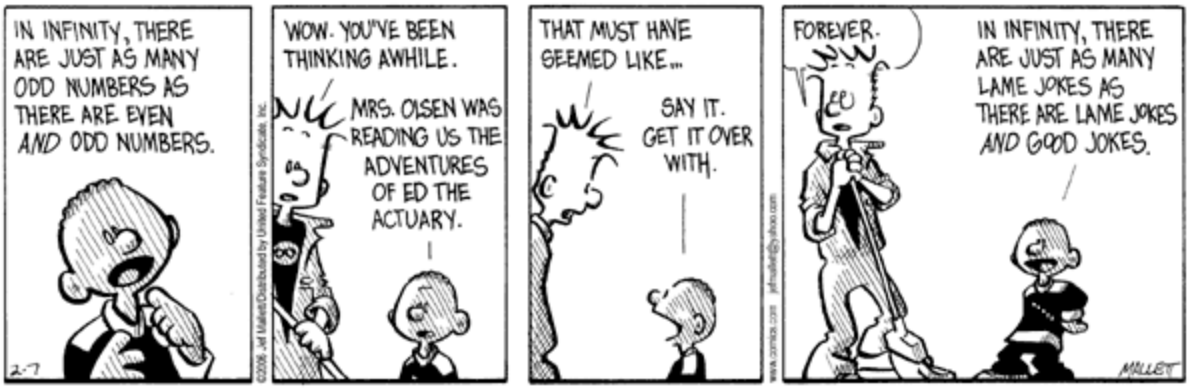
Functions & Applications 11

MCF3M

Course Notes

Unit 3: Quadratic Functions



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Homework

Contents with suggested problems from the Nelson Textbook. You are welcome to ask for help, from myself or your peers, with any of the following problems. They will be handed in on the day of the Unit Test as a homework check. Please post the work you do to OneNote, as you complete

Section 3.2 – Page 142

#3, 5, 10bcde, 11, 12, 13bcd, 14, 15

Section 3.3 – Page 150

#4, 6, 7-12, 16

Section 3.4 – Page 162

#4ace, 5ace, 6ace, 7ace, 8-14

Section 3.6 – Page 177

#5-11, 14

3.2 Relating the Standard and Factored Forms

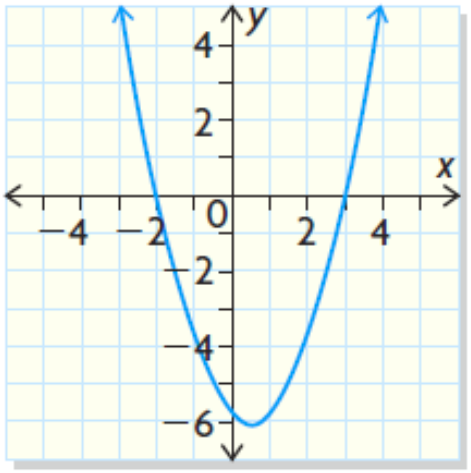
Learning Goal: We are learning to convert between the standard and factored forms of a quadratic function, and to identify the properties of each.

In this Chapter, we will be exploring the relationship between the Standard Form and the Factored Form of quadratic functions. The Standard Form is $f(x) = ax^2 + bx + c$ and the Factored Form is $f(x) = a(x-r)(x-s)$. These two forms are equivalent, meaning that they generate the exact same information or graph. Each letter (except *b*) represents important information about the parabola, but before we explore those properties, let's first do a little algebra.

Question 1: Convert $f(x) = 3x^2 + 18x - 48$ to the Factored Form.

Question 2: Covert $g(x) = -2(x-5)(x-3)$ to the Standard Form.

As you can see, switching form one form to another is just expanding and factoring, applying our skills from Chapter 2. Let's now look at the properties of parabolas.

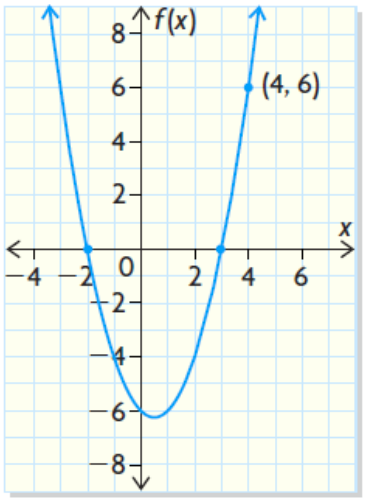


#10 from your text. Complete the table, then graph the function.

	Factored Form	Standard Form	Axis of Symmetry	Zeros	*y*-intercept	Vertex	Maximum or Minimum Value
a)	$R(x) = (40 - x)(10 + x)$	$R(x) = -x^2 + 30x + 400$					

#13 from your text. Complete the table.

	Zeros	Axis of Symmetry	Maximum or Minimum Value	Vertex	Function in Factored Form	Function in Standard Form
a)	2 and 8		6			



Find the equation of the parabola in both Factored form and Standard form.

The last thing for this lesson is a word problem. **DON'T OVERTHINK OR OVER COMPLICATE WORD PROBLEMS.** First of all, you will most likely be given a function to work with. Then, understand the **CONTEXT** of the problem.

The height of a football kicked from the ground is given by the function $h(t) = -5t^2 + 20t$, where $h(t)$ is the height in metres and t is the time in seconds from its release.

a) Write the function in factored form.
b) When will the football hit the ground?
c) When will the football reach its maximum height?
d) What is the maximum height the football reaches?
e) Graph the height of the football in terms of time without using a table of values.

Success Criteria

- I can convert standard form into factored form by factoring the function
- I can convert factored form into standard form by expanding the function
- I can identify the zeroes, vertex, max/min, axis of symmetry, and y-intercept of a quadratic function

3.3 Solving Quadratic Equations by Graphing

Learning Goal: We are learning to solve quadratic equations by using graphing technology.

Example 1: Solve $x^2 - 8x + 12 = -3$

Method 1: Move everything to one side, and then graph the new equation, looking for the zeros.

Method 2: Graph $f(x) = x^2 - 8x + 12$ and $g(x) = -3$, and find the points of intersection.

Example 2:

The population of an Ontario city is modelled by the function $P(t) = 0.5t^2 + 10t + 300$, where $P(t)$ is the population in thousands and t is the time in years. *Note*: $t = 0$ corresponds to the year 2000.

a) What was the population in 2000?
b) What will the population be in 2010?
c) When is the population expected to be 1 050 000?

Example 3:

The function $h(t) = 2 + 50t - 1.862t^2$, where $h(t)$ is the height in metres and t is time in seconds, models the height of a golf ball above the planet Mercury's surface during its flight.

a) What is the maximum height reached by the ball?
b) How long will the ball be above the surface of Mercury?
c) When will it reach a height of 200 m on the way down?

Success Criteria:

- I can solve a quadratic equation using graphing technology by
  - A) moving everything to one side and finding the zeroes of the equation
  - B) Graphing the two functions separately and seeing where they intersect
- I can state the solution to the equation by giving the x-coordinates
- I can use graphing technology to help me solve Word Problems involving quadratic functions

3.4 Solve Quadratic Equations by Factoring

Learning Goal: We are learning to solve quadratic equations algebraically.

Steps

1) Make one side of your equation equal to zero (stuff = zero)
2) **Factor**. The solutions will be the x values when each factor is set equal to zero

Example 1:
Solve $x^2 - 9x + 12 = -8$

Example 2:
Solve $16x^2 - 25 = 0$

Example 3:
Solve $2(x+3)^2 = 5(x+3)$

Example 4:

The profit of a skateboard company can be modelled by the function $P(x) = -63 + 133x - 14x^2$, where *P(x)* is the profit in thousands of dollars and *x* is the number of skateboards sold, also in thousands of dollars. When will the company break even, and when will it be profitable?

Example 5:

The path a dolphin travels when it rises above the ocean's surface can be modelled by the function $h(d) = -0.2d^2 + 2d$, where $h(d)$ is the height of the dolphin above the water's surface and *d* is the horizontal distance from the point where the dolphin broke the water's surface, both in feet. When will the dolphin reach a height of 1.8 feet?

Success Criteria

- I can solve quadratic equations by rearranging them so that "stuff = zero" and then factoring the new quadratic
- I can find the solutions by setting each factor equal to zero

3.6 Creating a Quadratic Model from Data

In this section, we get to use a fantastic piece of software which will allow us to take some data, and compute a quadratic regression model. Yes, it is as exciting as it sounds. A regression model is a very important thing in mathematics. You input data into a spreadsheet or other software, and you create a line or curve of best fit (a best estimate). With this line or curve, you also get an equation which allows you to make predictions or fill in the gaps.

Question: A ball is thrown into the air from the top of a building. The table of values gives the height of the ball at different times during the flight. What is a function that will model the data?

Time (s)	0	1	2	3	4	5
Height (m)	30	50	60	60	50	30

Step 1: Turn on computer and visit www.desmos.com. Click "Graphing Calculator"
Step 2: Click "+" and select "table"
Step 3: Input the *Time* values in the x-column and the *Height* values in the y-column. The first column must always represent the "x" axis, or the independent variable. Press "tab" to go from one cell to the next. Zoom out and you will see the points plotted!
Step 4: Select the input box under the table and type: $y_1 \sim a{x_1}^2 + bx_1 + c$
Step 5: The parabola will graph itself and you will see the a, b, and c values. That is your function!!

Now we can figure out the zeros, vertex, or anything else of significance!