Chapter 2 Problem Set – Polynomial Functions

2.1 Polynomial Functions: An Introduction #1 – 3 and #1, 2, 3cd, 5, 6 (3.1 in textbook)

1. Expand and simplify each of the following expressions.

a) $2x^2(3x - 11)$

b) $(x - 4)(x + 6)$

c) $4x(2x - 5)(3x + 2)$

d) $(5x - 4)(x^2 + 7x - 8)$

2. Factor each of the following expressions completely.

a) $x^2 + 3x - 28$

b) $2x^2 - 18x + 28$

3. Solve each of the following equations. Round your answer to two decimal places, if necessary.

a) $3x + 7 = x - 5$

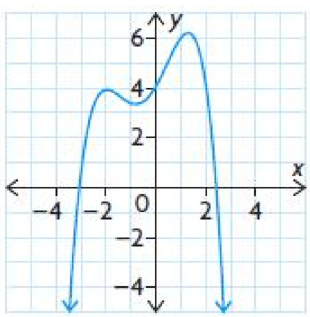
b) $(x + 3)(2x - 9) = 0$

c) $x^2 + 11x + 24 = 0$

d) $6x^2 + 22x = 8$

1. Determine which graphs represent polynomial functions. Explain how you know.

a)



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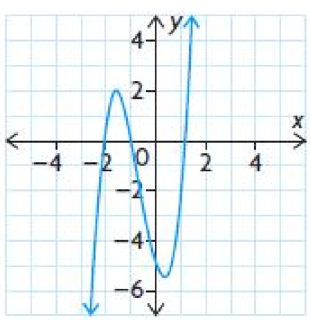

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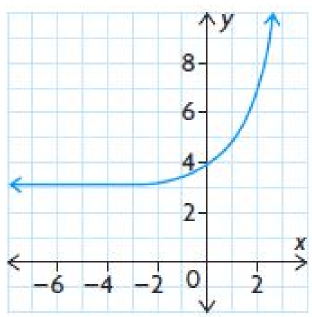

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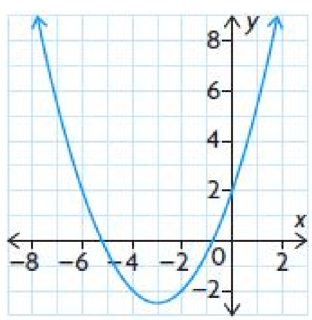

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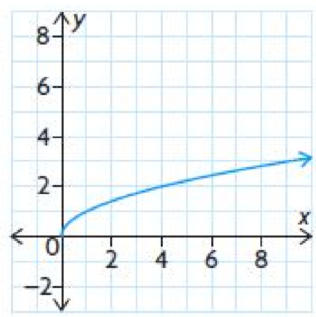

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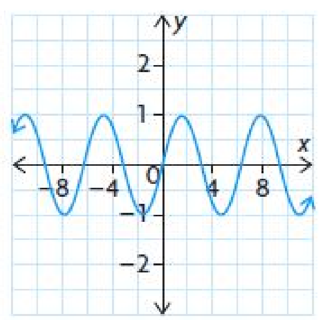


2. Determine whether each function is a polynomial function or another type of function. Justify your decision.

a) $f(x) = 2x^3 + x^2 - 5$

b) $f(x) = x^2 + 3x - 2$

c) $y = 2x - 7$

d) $y = \sqrt{x + 1}$

e) $y = \dfrac{x^2 - 4x + 1}{x + 2}$

f) $f(x) = x(x - 1)^2$

3. Use finite differences to determine the type of polynomial function that could model each relationship.

c) The volume of a box varies at different widths.

Width (cm)	1	2	3	4	5
Volume (cm^3)	200	225	250	275	300

d) The input for a function gives a certain output.

Input	0	1	2	3	4	5	6
Output	200	204	232	308	456	700	1064

5. Draw a graph of a polynomial function that satisfies all of the following characteristics:
- $f(-3) = 16$, $f(3) = 0$, and $f(-1) = 0$
- The y-intercept is 2.
- $f(x) \geq 0$ when $x < 3$.
- $f(x) \leq 0$ when $x > 3$.
- The domain is the set of real numbers.

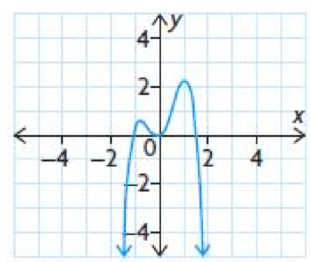
6. Explain why there are many different graphs that fit different combinations of the characteristics in question 5. Draw two graphs that are different from each other, and explain how they satisfy some, but not all, of the characteristics in question 5.

2.2 Characteristics of Polynomial Functions #1 – 5, 7, 8, 10, 11 (3.2 in textbook)

1. State the degree, leading coefficient, and end behaviours of each polynomial function.
   a) $f(x) = -4x^4 + 3x^2 - 15x + 5$
   b) $g(x) = 2x^5 - 4x^3 + 10x^2 - 13x + 8$
   c) $p(x) = 4 - 5x + 4x^2 - 3x^3$
   d) $h(x) = 2x(x - 5)(3x + 2)(4x - 3)$

2. a) Determine the minimum and maximum number of turning points for each function in question 1.
   b) Determine the minimum and maximum number of zeros that each function in question 1 may have.

3. For each of the following graphs, decide if
   a) the function has an even or odd degree
   b) the leading coefficient is positive or negative

i)


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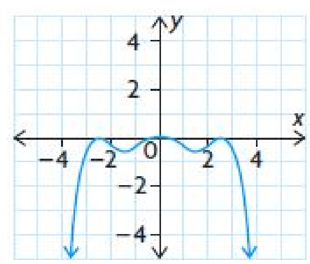

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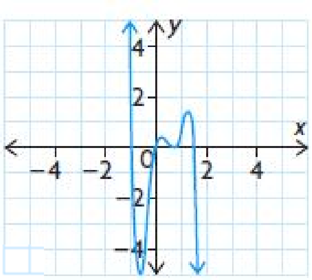

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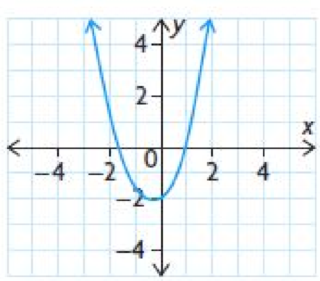

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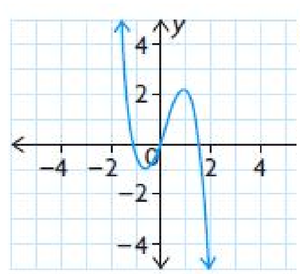

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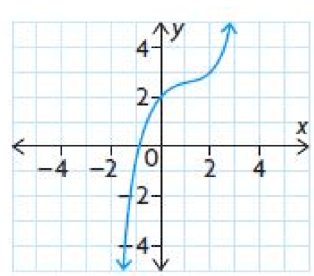

4. Describe the end behaviour of each polynomial function using the degree and the leading coefficient.
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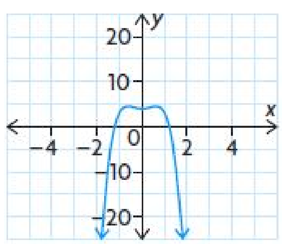
a) $f(x) = 2x^2 - 3x + 5$
b) $f(x) = -3x^3 + 2x^2 + 5x + 1$
c) $f(x) = 5x^3 - 2x^2 - 2x + 6$
d) $f(x) = -2x^4 + 5x^3 - 2x^2 + 3x - 1$
e) $f(x) = 0.5x^4 + 2x^2 - 6$
f) $f(x) = -3x^5 + 2x^3 - 4x$

5. Use end behaviours, turning points, and zeros to match each polynomial equation with the most likely graph below. Explain.

a) $y = 2x^3 - 4x^2 + 3x + 2$
b) $y = -4x^4 + 3x^2 + 4$
c) $y = x^2 + 3x - 5$
d) $y = x^4 - x^3 - 4x^2 + 5x$
e) $y = -2x^5 + 3x^4 + 6x^3 - 10x^2 + 2x + 5$
f) $y = 3x^3 + 5x^2 - 3x + 1$

A


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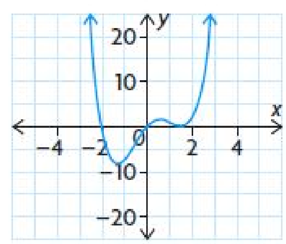

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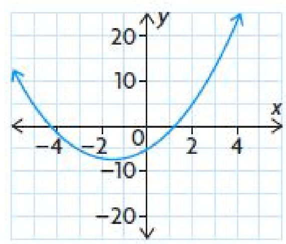

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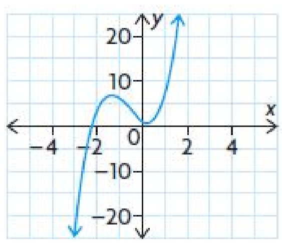

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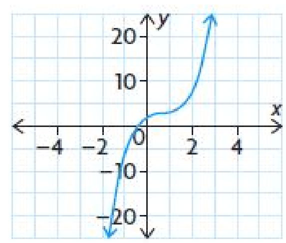

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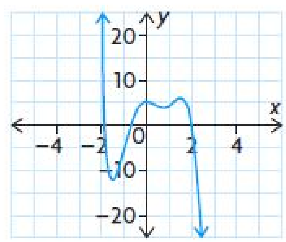


7. Sketch the graph of a polynomial function that satisfies each set of conditions.

a) degree 4, positive leading coefficient, 3 zeros, 3 turning points
b) degree 4, negative leading coefficient, 2 zeros, 1 turning point
c) degree 4, positive leading coefficient, 1 zero, 3 turning points
d) degree 3, negative leading coefficient, 1 zero, no turning points
e) degree 3, positive leading coefficient, 2 zeros, 2 turning points
f) degree 4, two zeros, three turning points, Range = $\{y \in \mathbf{R} \mid y \leq 5\}$

8. Explain why odd-degree polynomial functions can have only local maximums and minimums, but even-degree polynomial functions can have absolute maximums and minimums.
C

10. Sketch an example of a cubic function with a graph that intersects the x-axis at each number of points below.

a) only one point b) two different points c) three different points

11. Sketch an example of a quartic function with a graph that intersects the x-axis at each number of points below.

a) no points
b) only one point
c) two different points
d) three different points
e) four different points

2.3 Zeros of Polynomial Functions #1, 2, 4, 6, 8ab, 10, 12, 13b (3.3 in textbook)

1. Match each equation with the most suitable graph. Explain your reasoning.

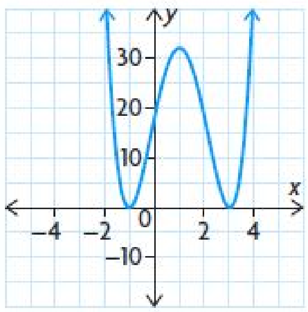
a) $f(x) = 2(x + 1)^2(x - 3)$

b) $f(x) = 2(x + 1)^2(x - 3)^2$

c) $f(x) = -2(x + 1)(x - 3)^2$

d) $f(x) = x(x + 1)(x - 3)(x - 5)$

A



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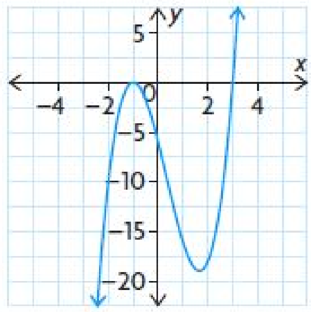

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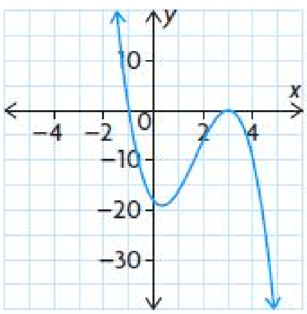

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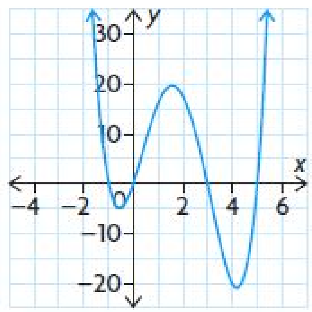


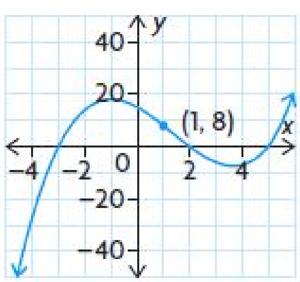
2. Sketch a possible graph of each function.

a) $f(x) = -(x - 4)(x - 1)(x + 5)$

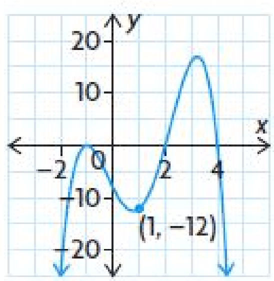
b) $g(x) = x^2(x - 6)^3$

4. Write the equation of each function.

a)



b)



6. Sketch the graph of each function.

a) $y = x(x - 4)(x - 1)$

b) $y = -(x - 1)(x + 2)(x - 3)$

c) $y = x(x - 3)^2$

d) $y = (x + 1)^3$

e) $y = x(2x + 1)(x - 3)(x - 5)$

f) $y = x^2(3x - 2)^2$

8. Sketch an example of a quartic function with the given zeros, and write the equation of the function. Then write the equations of two other functions that belong to the same family.

a) $-5, -3, 2, 4$

b) -2 (order 2), 3 (order 2)

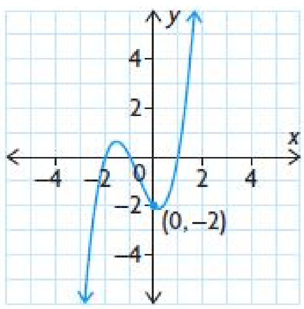
10. Sketch the graph of a polynomial function that satisfies each set of conditions.

a) degree 4, positive leading coefficient, 3 zeros, 3 turning points
b) degree 4, negative leading coefficient, 2 zeros, 1 turning point
c) degree 4, positive leading coefficient, 2 zeros, 3 turning points
d) degree 3, negative leading coefficient, 1 zero, no turning points

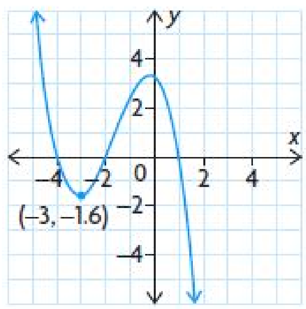
12. Determine the equation of the polynomial function from each graph.

K

a)



b)



13. b) Determine the cubic function that has zeros at -2, 3, and 4, if $f(5) = 28$.

2.4 Dividing Polynomials #2, 5, 6acdef, 10acef, 12, 13 (3.5 in textbook)

2. State the degree of the quotient for each of the following division statements, if possible.

a) $(x^4 - 15x^3 + 2x^2 + 12x - 10) \div (x^2 - 4)$
b) $(5x^3 - 4x^2 + 3x - 4) \div (x + 3)$
c) $(x^4 - 7x^3 + 2x^2 + 9x) \div (x^3 - x^2 + 2x + 1)$
d) $(2x^2 + 5x - 4) \div (x^4 + 3x^3 - 5x^2 + 4x - 2)$

5. Calculate each of the following using long division.

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a) $(x^3 - 2x + 1) \div (x - 4)$
b) $(x^3 + 2x^2 - 6x + 1) \div (x + 2)$
c) $(2x^3 + 5x^2 - 4x - 5) \div (2x + 1)$
d) $(x^4 + 3x^3 - 2x^2 + 5x - 1) \div (x^2 + 7)$
e) $(x^4 + 6x^2 - 8x + 12) \div (x^3 - x^2 - x + 1)$
f) $(x^5 + 4x^4 + 9x + 8) \div (x^4 + x^3 + x^2 + x - 2)$

6. Calculate each of the following using synthetic division.

a) $(x^3 - 7x - 6) \div (x - 3)$
b) $(2x^3 - 7x^2 - 7x + 19) \div (x - 1)$
c) $(6x^4 + 13x^3 - 34x^2 - 47x + 28) \div (x + 3)$
d) $(2x^3 + x^2 - 22x + 20) \div (2x - 3)$
e) $(12x^4 - 56x^3 + 59x^2 + 9x - 18) \div (2x + 1)$
f) $(6x^3 - 2x - 15x^2 + 5) \div (2x - 5)$

10. Determine whether each binomial is a factor of the given polynomial.

a) $x + 5, x^3 + 6x^2 - x - 30$

b) $x + 2, x^4 - 5x^2 + 4$

c) $x - 2, x^4 - 5x^2 + 6$

d) $2x - 1, 2x^4 - x^3 - 4x^2 + 2x + 1$

e) $3x + 5, 3x^6 + 5x^5 + 9x^2 + 17x - 1$

f) $5x - 1, 5x^4 - x^3 + 10x - 10$

12. a) $8x^3 + 10x^2 - px - 5$ is divisible by $2x + 1$. There is no remainder. Find the value of p.

b) When $x^6 + x^4 - 2x^2 + k$ is divided by $1 + x^2$, the remainder is 5. Find the value of k.

13. The polynomial $x^3 + px^2 - x - 2, p \in \mathbf{R}$, has $x - 1$ as a factor. What is the value of p?

2.5 The Factor Theorem #1, 2, 5, 6, 7abcd, 8ac, 9, 12 (3.6 in textbook)

1. a) Given $f(x) = x^4 + 5x^3 + 3x^2 - 7x + 10$, determine the remainder when $f(x)$ is divided by each of the following binomials, without dividing.

i) $x - 2$

ii) $x + 4$

iii) $x - 1$

b) Are any of the binomials in part a) factors of $f(x)$? Explain.

2. Which of the following functions are divisible by $x - 1$?

a) $f(x) = x^4 - 15x^3 + 2x^2 + 12x - 10$

b) $g(x) = 5x^3 - 4x^2 + 3x - 4$

c) $h(x) = x^4 - 7x^3 + 2x^2 + 9x$

d) $j(x) = x^3 - 1$

5. Determine whether $2x - 5$ is a factor of each polynomial.

a) $2x^3 - 5x^2 - 2x + 5$

b) $3x^3 + 2x^2 - 3x - 2$

c) $2x^4 - 7x^3 - 13x^2 + 63x - 45$

d) $6x^4 + x^3 - 7x^2 - x + 1$

6. Factor each polynomial using the factor theorem.

a) $x^3 - 3x^2 - 10x + 24$

b) $4x^3 + 12x^2 - x - 15$

c) $x^4 + 8x^3 + 4x^2 - 48x$

d) $4x^4 + 7x^3 - 80x^2 - 21x + 270$

e) $x^5 - 5x^4 - 7x^3 + 29x^2 + 30x$

f) $x^4 + 2x^3 - 23x^2 - 24x + 144$

7. Factor fully.

a) $f(x) = x^3 + 9x^2 + 8x - 60$

b) $f(x) = x^3 - 7x - 6$

c) $f(x) = x^4 - 5x^2 + 4$

d) $f(x) = x^4 + 3x^3 - 38x^2 + 24x + 64$

e) $f(x) = x^3 - x^2 + x - 1$

f) $f(x) = x^5 - x^4 + 2x^3 - 2x^2 + x - 1$

8. Use the factored form of $f(x)$ to sketch the graph of each function in question 7.

Only for 7a and 7c

9. The polynomial $12x^3 + kx^2 - x - 6$ has $2x - 1$ as one of its factors. Determine the value of k.

12. The function $f(x) = ax^3 - x^2 + bx - 24$ has three factors. Two of these factors are $x - 2$ and $x + 4$. Determine the values of a and b, and then determine the other factor.

2.6 Sums and Differences of Cubes #2aei, 3, 4 (3.7 in textbook)

2. Factor each of the following expressions.

a) $x^3 - 64$ e) $64x^3 - 125$ i) $216x^3 - 8$

3. Factor each expression.

a) $64x^3 + 27y^3$
b) $-3x^4 + 24x$
c) $(x + 5)^3 - (2x + 1)^3$
d) $x^6 + 64$

4. Factor.

K
a) $x^3 - 343$
b) $216x^3 - 1$
c) $x^3 + 1000$
d) $125x^3 - 512$
e) $64x^3 - 1331$
f) $343x^3 + 27$
g) $512x^3 + 1$
h) $1331x^3 + 1728$
i) $512 - 1331x^3$