Chapter 3 Problem Set – Polynomial Equations and Inequalities.

3.1 Solving Polynomial Equations #1, 2, 6, 7, 8, 10, 11, 12, 14, 15 (4.1 in textbook)

1. State the zeros of the following functions.

a) $y = 2x(x - 1)(x + 2)(x - 2)$
b) $y = 5(2x + 3)(4x - 5)(x + 7)$
c) $y = 2(x - 3)^2(x + 5)(x - 4)$
d) $y = (x + 6)^3(2x - 5)$
e) $y = -5x(x^2 - 9)$
f) $y = (x + 5)(x^2 - 4x - 12)$

2. Solve each of the following equations by factoring. Verify your solutions using graphing technology.

a) $3x^3 = 27x$
b) $4x^4 = 24x^2 + 108$
c) $3x^4 + 5x^3 - 12x^2 - 20x = 0$
d) $10x^3 + 26x^2 - 12x = 0$
e) $2x^3 + 162 = 0$
f) $2x^4 = 48x^2$

6. State the zeros of the following functions.

a) $f(x) = x(x - 2)^2(x + 5)$
b) $f(x) = (x^3 + 1)(x - 17)$
c) $f(x) = (x^2 + 36)(8x - 16)$
d) $f(x) = -3x^3(2x + 4)(x^2 - 25)$
e) $f(x) = (x^2 - x - 12)(3x)$
f) $f(x) = (x + 1)(x^2 + 2x + 1)$

7. Determine the roots algebraically by factoring.

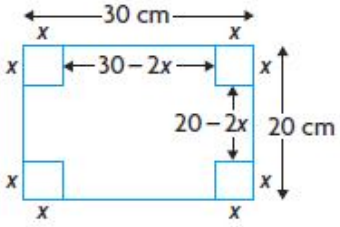
a) $x^3 - 8x^2 - 3x + 90 = 0$
b) $x^4 + 9x^3 + 21x^2 - x - 30 = 0$
c) $2x^3 - 5x^2 - 4x + 3 = 0$
d) $2x^3 + 3x^2 = 5x + 6$
e) $4x^4 - 4x^3 - 51x^2 + 106x = 40$
f) $12x^3 - 44x^2 = -49x + 15$

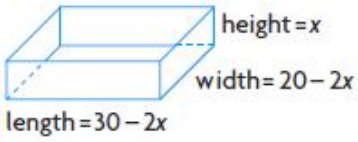
8. Use graphing technology to find the real roots to two decimal places.

a) $x^3 - 7x + 6 = 0$
b) $x^4 - 5x^3 - 17x^2 + 3x + 18 = 0$
c) $3x^3 - 2x^2 + 16 = x^4 + 16x$
d) $x^5 + x^4 = 5x^3 - x^2 + 6x$
e) $105x^3 = 344x^2 - 69x - 378$
f) $21x^3 - 58x^2 + 10 = -18x^4 - 51x$

10. An open-topped box can be created by cutting congruent squares from each of the four corners of a piece of cardboard that has dimensions of 20 cm by 30 cm and folding up the sides. Determine the dimensions of the squares that must be cut to create a box with a volume of 1008 cm^3.

This question is very similar to an optimization question you will see in Calculus! Awesome!!





11. The Sickle-Lichti family members are very competitive card players. They keep score using a complicated system that incorporates positives and negatives. Maya's score for the last game night could be modelled by the function $S(x) = x(x - 4)(x - 6), x < 10, x \in \mathbf{W}$, where x represents the game number.

a) After which game was Maya's score equal to zero?

b) After which game was Maya's score -5?

c) After which game was Maya's score 16?

d) Draw a sketch of the graph of $S(x)$ if $x \in \mathbf{R}$. Explain why this graph is *not* a good model to represent Maya's score during this game night.

12. The function $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$ can be used to calculate s, the height above a planet's surface in metres, where g is the acceleration due to gravity, t is the time in seconds, v_0 is the initial velocity in metres per second, and s_0 is the initial height in metres. The acceleration due to gravity on Mars is $g = -3.92\ \text{m/s}^2$. Find, to two decimal places, how long it takes an object to hit the surface of Mars if the object is dropped from 1000 m above the surface.

14. During a normal 5 s respiratory cycle in which a person inhales and then exhales, the volume of air in a person's lungs can be modelled by $V(t) = 0.027t^3 - 0.27t^2 + 0.675t$, where the volume, V, is measured in litres at t seconds.

a) What restriction(s) must be placed on t?

b) If asked, "How many seconds have passed if the volume of air in a person's lungs is 0.25 L?" would you answer this question algebraically or by using graphing technology? Justify your decision.

c) Solve the problem in part b).

15. Explain why the following polynomial equation has no real solutions: $0 = 5x^8 + 10x^6 + 7x^4 + 18x^2 + 132$

4.2 Linear Inequalities #1, 2, 4, 5, 7, 9, 13 (4.2 in textbook)

1. Solve the following inequalities graphically. Express your answer using set notation.

a) $3x - 1 \leq 11$

b) $-x + 5 > -2$

c) $x - 2 > 3x + 8$

d) $3(2x + 4) \geq 2x$

e) $-2(1 - 2x) < 5x + 8$

f) $\frac{6x + 8}{5} \leq 2x - 4$

2. Solve the following inequalities algebraically. Express your answer using interval notation.

a) $2x - 5 \leq 4x + 1$

b) $2(x + 3) < -(x - 4)$

c) $\frac{2x + 3}{3} \leq x - 5$

d) $2x + 1 \leq 5x - 2$

e) $-x + 1 > x + 1$

f) $\frac{x + 4}{2} \geq \frac{x - 2}{4}$

4. For each of the following inequalities, determine whether $x = 2$ is contained in the solution set.

a) $x > -1$
b) $5x - 4 > 3x + 2$
c) $4(3x - 5) \geq 6x$
d) $5x + 3 \leq -3x + 1$
e) $x - 2 \leq 3x + 4 \leq x + 14$
f) $33 < -10x + 3 < 54$

5. Solve the following algebraically. Verify your results graphically.

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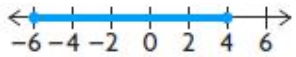
a) $2x - 1 \leq 13$
b) $-2x - 1 > -1$
c) $2x - 8 > 4x + 12$
d) $5(x - 3) \geq 2x$
e) $-4(5 - 3x) < 2(3x + 8)$
f) $\frac{x - 2}{3} \leq 2x - 3$

7. Solve the following inequalities algebraically.

a) $-5 < 2x + 7 < 11$
b) $11 < 3x - 1 < 23$
c) $-1 \leq -x + 9 \leq 13$
d) $0 \leq -2(x + 4) \leq 6$
e) $59 < 7x + 10 < 73$
f) $18 \leq -12(x - 1) \leq 48$

9. The following number line shows the solution to a double inequality.



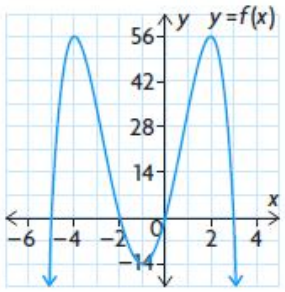
a) Write the solution using set notation.
b) Create a double inequality for which this is the solution set.

13. Some volunteers are making long distance phone calls to raise money for a charity. The calls are billed at the rate of \$0.50 for the first 3 min and \$0.10/min for each additional minute or part thereof. If each call cannot cost more that \$2.00, how long can each volunteer talk to a prospective donor?

4.3 Solving Polynomial Inequalities #2, 5, 6, 7, 10, 11, 12, 13 (4.3 in textbook)

2. For each graph shown, determine where $f(x) \leq 0$. Express your answers using interval notation.

a)



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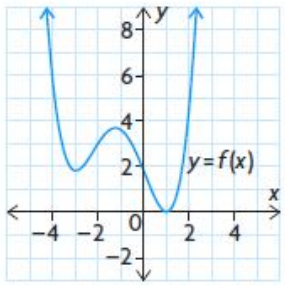

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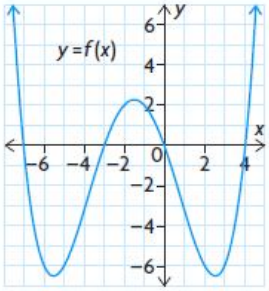

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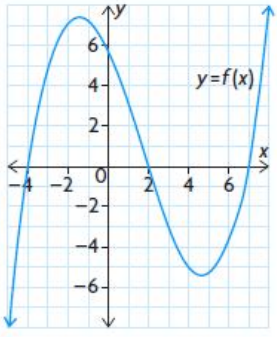
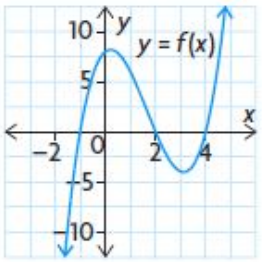
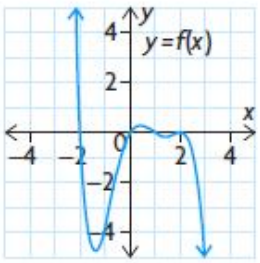


5. K For each of the following polynomial functions, state the intervals where $f(x) > 0$.

a)



c)



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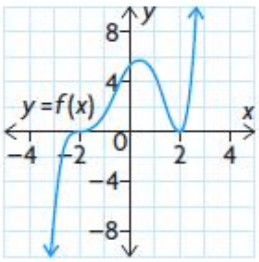
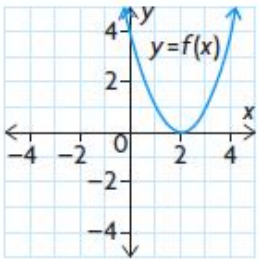


d)



6. Solve the following inequalities.

a) $(x - 1)(x + 1) > 0$

b) $(x + 3)(x - 4) < 0$

c) $(2x + 1)(x - 5) \geq 0$

d) $-3x(x + 7)(x - 2) < 0$

e) $(x - 3)(x + 1) + (x - 3)(x + 2) \geq 0$

f) $2x(x + 4) - 3(x + 4) \leq 0$

7. Solve the following inequalities algebraically. Confirm your answer with a graph.

a) $x^2 - 6x + 9 \geq 16$

b) $x^4 - 8x < 0$

c) $x^3 + 4x^2 + x \leq 6$

d) $x^4 - 5x^2 + 4 > 0$

e) $3x^3 - 3x^2 - 2x \leq 2x^3 - x^2 + x$

f) $x^3 - x^2 - 3x + 3 > -x^3 + 2x + 5$

10. Determine an expression for $f(x)$ in which $f(x)$ is a quartic function, $f(x) > 0$ when $-2 < x < 1$, $f(x) \leq 0$ when $x < -2$ or $x > 1$, $f(x)$ has a double root when $x = 3$, and $f(-1) = 96$.

11. The viscosity, v, of oil used in cars is related to its temperature, t, by the formula $v = -t^3 - 6t^2 + 12t + 50$, where each unit of t is equivalent to 50 °C.

a) Graph the function on your graphing calculator.

b) Determine the temperature range for which $v > 0$ to two decimal places.

c) Determine the temperature ranges for which $15 < v < 20$ to two decimal places.

12. A rock is tossed from a platform and follows a parabolic path through the air. The height of the rock in metres is given by $h(t) = -5t^2 + 12t + 14$, where t is measured in seconds.

a) How high is the rock off the ground when it is thrown?

b) How long is the rock in the air?

c) For what times is the height of the rock greater than 17 m?

d) How long is the rock above a height of 17 m?

13. A An open-topped box can be made from a sheet of aluminium measuring 50 cm by 30 cm by cutting congruent squares from the four corners and folding up the sides. Write a polynomial function to represent the volume of such a box. Determine the range of side lengths that are possible for each square that is cut out and removed that result in a volume greater than 4000 cm^3.