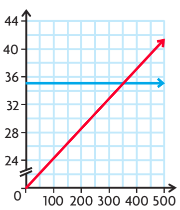
6. a) Answers may vary. For example, $2x + 1 > 17$
b) Answers may vary. For example, $3x - 4 \geq -16$
c) Answers may vary. For example, $2x + 3 \leq -21$
d) Answers may vary. For example, $-19 < 2x - 1 < -3$

7. a) $x \in \left(\frac{25}{2}, \infty\right)$
b) $x \in \left[-\frac{23}{8}, \infty\right)$
c) $x \in (-\infty, 2)$
d) $x \in (-\infty, 3]$

8. a) $\{x \in \mathbf{R} \mid -2 < x < 4\}$
b) $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$
c) $\{x \in \mathbf{R} \mid -3 \leq x \leq 5\}$
d) $\{x \in \mathbf{R} \mid -6 < x < -2\}$

9. a) The second plan is better if one calls more than 350 min per month.
b)



10. a) $-1 < x < 2$
b) $x \leq -\frac{3}{2}$ or $x \geq 5$
c) $x < -\frac{5}{2}$ or $1 < x < 7$
d) $x \leq -4$ or $1 \leq x \leq 5$

11. negative when $x \in (0, 5)$, positive when $x \in (-\infty, -2), (-2, 0), (5, \infty)$

12. $x \leq -3.81$

13. between January 1993 and March 1994 and between October 1995 and October 1996

14. a) average $= 7$, instantaneous $\doteq 8$
b) average $= 13$, instantaneous $\doteq 15$
c) average $= 129$, instantaneous $\doteq 145$
d) average $= -464$, instantaneous $\doteq -485$

15. positive when $-1 < x < 1$, negative when $x < -1$ or $x > 1$, and zero at $x = -1, 1$

16. a) $t \doteq 2.2$ s
b) -11 m/s
c) about -22 m/s

17. a) about 57.002
b) about 56.998
c) Both approximate the instantaneous rate of change at $x = 3$.

18. a) male:
$f(x) = 0.001x^3 - 0.162x^2 + 3.394x + 72.365$;
female:
$g(x) = 0.0002x^3 - 0.026x^2 + 1.801x + 14.369$
b) More females than males will have lung cancer in 2006.

c) The rate was changing faster for females, on average. Looking only at 1975 and 2000, the incidence among males increased only 5.5 per 100 000, while the incidence among females increased by 31.7.
d) Between 1995 and 2000, the incidence among males decreased by 6.1 while the incidence among females increased by 5.6. Since 1998 is about halfway between 1995 and 2000, an estimate for the instantaneous rate of change in 1998 is the average rate of change from 1995 to 2000. The two rates of change are about the same in magnitude, but the rate for females is positive, while the rate for males is negative.

Chapter Self-Test, p. 242

1. $1, \frac{3}{2}, -2$

2. a) positive when $x < -2$ and $0 < x < 2$, negative when $-2 < x < 0$ and $x > 2$, and zero at $-2, 0, 2$
b) positive when $-1 < x < 1$, negative when $x < -1$ or $1 < x$, and zero at $x = -1, 1$
c) -1

3. a) Cost with card: $50 + 5n$;
Cost without card: $12n$
b) at least 8 pizzas

4. a) $x < \frac{1}{2}$
b) $-2 \leq x \leq 1$
c) $-2 < x < -1$ or $x > 5$
d) $x \geq -3$

5. a) 15 m
b) 4.6 s
c) -3 m/s

6. a) about 5 b) $(1, 3)$ c) $y = 5x - 2$

7. Since all the exponents are even and all the coefficients are positive, all values of the function are positive and greater than or equal to 4 for all real numbers x.

8. a) $\{x \in \mathbf{R} \mid -2 \leq x \leq 7\}$
b) $-2 < x < 7$

9. 2 cm by 2 cm by 15 cm

Chapter 5

Getting Started, pp. 246–247

1. a) $(x - 5)(x + 2)$
b) $3(x + 5)(x - 1)$
c) $(4x - 7)(4x + 7)$
d) $(3x - 2)(3x - 2)$
e) $(a - 3)(3a + 10)$
f) $(2x + 3y)(3x - 7y)$

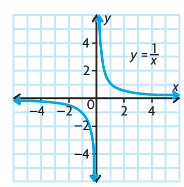
2. a) $3 - 2s$
b) $\frac{n^3}{3m}, m, n \neq 0$
c) $3x^2 - 4x - 1, x \neq 0$
d) $\frac{1}{5x - 2}, x \neq \frac{2}{5}$
e) $-\frac{x + 6}{3 + x}, x \neq -3, 3$
f) $\frac{a - b}{a - 3b}, a \neq -5b, \frac{3b}{2}$

3. a) $\frac{7}{15}$
b) $\frac{6}{x}, x \neq 0$
c) $\frac{-4x^2 + 20x - 6}{x - 3}, x \neq -2, 3$
d) $\frac{x^3 + 2x - 8x}{x^2 - 1}, x \neq -1, 0, 1, 3$

4. a) $1\frac{11}{21}$
b) $\frac{19x}{12}$
c) $\frac{4 + x}{x^2}, x \neq 0$
d) $\frac{3x - 6}{x^2 - 3x}, x \neq 0, 3$
e) $\frac{2x + 10 + y}{x^2 - 25}, x \neq 5, -5$
f) $\frac{-2a + 50}{(a + 3)(a - 5)(a + 3)}, x \neq -3, 4, 5$

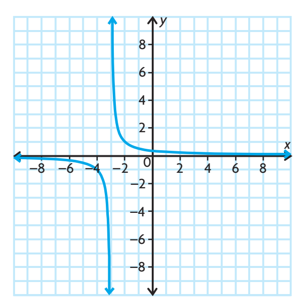
5. a) $x = 6$
b) $x = 2$
c) $x = 3$
d) $x = \frac{-12}{7}$

6.



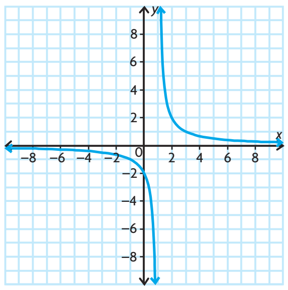
vertical: $x = 0$; horizontal: $y = 0$;
$D = \{x \in \mathbf{R} \mid x \neq 0\}$;
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

7. a) translated three units to the left

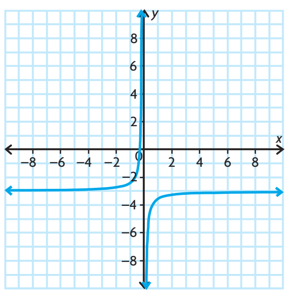


Answers

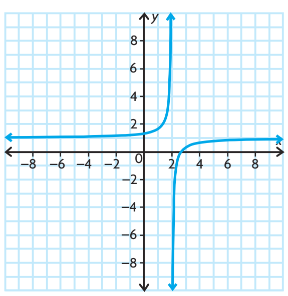
b) vertical stretch by a factor of 2 and a horizontal translation 1 unit to the right



c) reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, and a vertical translation 3 units down



d) reflection in the x-axis, vertical compression by a factor of $\frac{2}{3}$, horizontal translation 2 units right, and a vertical translation 1 unit up



8. Factor the expressions in the numerator and the denominator. Simplify each expression as necessary. Multiply the first expression by the reciprocal of the second.

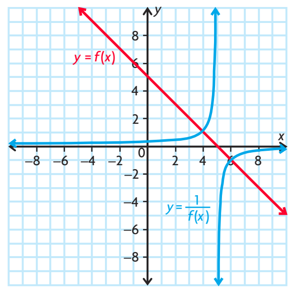
$$\frac{-3(3y-2)}{2(3y+2)}$$

Lesson 5.1, pp. 254–257

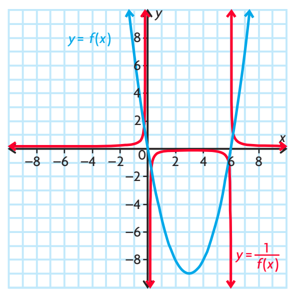
1. a) C; The reciprocal function is F.
b) A; The reciprocal function is E.
c) D; The reciprocal function is B.
d) F; The reciprocal function is C.
e) B; The reciprocal function is D.
f) E; The reciprocal function is A.

2. a) $x = 6$
b) $x = -\frac{4}{3}$
c) $x = 5$ and $x = -3$
d) $x = -\frac{5}{2}$ and $x = \frac{5}{2}$
e) no asymptotes
f) $x = -1.5$ and $x = -1$

3. a)



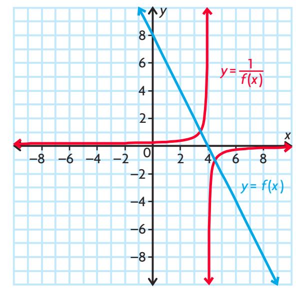
b)



4. a)

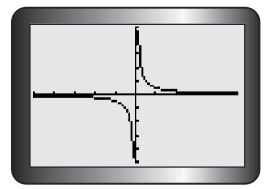
x	$f(x)$	$\frac{1}{f(x)}$
-4	16	$\frac{1}{16}$
-3	14	$\frac{1}{14}$
-2	12	$\frac{1}{12}$
-1	10	$\frac{1}{10}$
0	8	$\frac{1}{8}$
1	6	$\frac{1}{6}$
2	4	$\frac{1}{4}$
3	2	$\frac{1}{2}$
4	0	undefined
5	-2	$-\frac{1}{2}$
6	-4	$-\frac{1}{4}$
7	-6	$-\frac{1}{6}$

b)

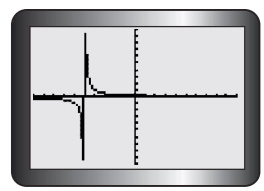


c) $f(x) = -2x + 8$, $y = \frac{1}{-2x+8}$

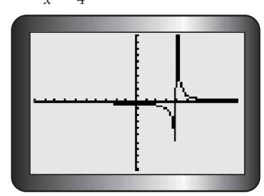
5. a) $y = \frac{1}{2x}$; vertical asymptote at $x = 0$



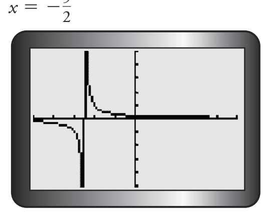
b) $y = \frac{1}{x+5}$; vertical asymptote at $x = -5$



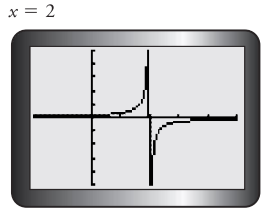
c) $y = \frac{1}{x-4}$; vertical asymptote at $x = 4$



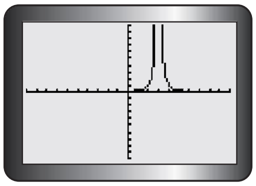
d) $y = \frac{1}{2x+5}$; vertical asymptote at $x = -\frac{5}{2}$



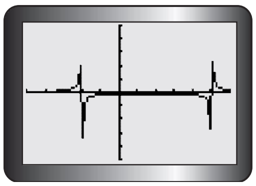
e) $y = \frac{1}{-3x+6}$; vertical asymptote at $x = 2$



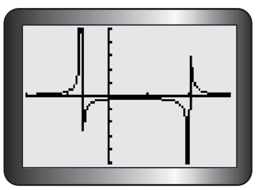
f) $y = \frac{1}{(x-3)^2}$; vertical asymptote at $x = 3$



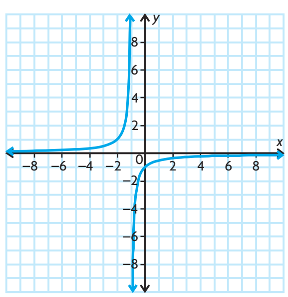
g) $y = \frac{1}{x^2 - 3x - 10}$; vertical asymptotes at $x = -2$ and $x = 5$



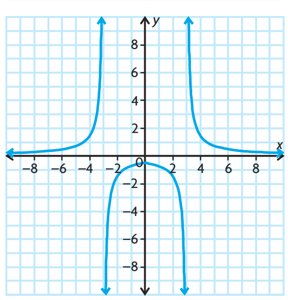
h) $y = \frac{1}{3x^2 - 4x - 4}$; vertical asymptotes at $x = -\frac{2}{3}$ and $x = 2$



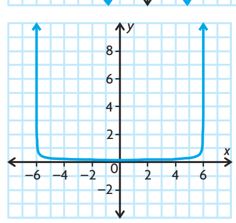
6. a)



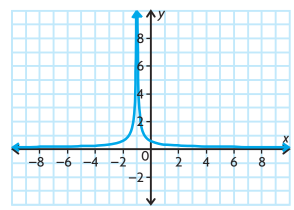
b)



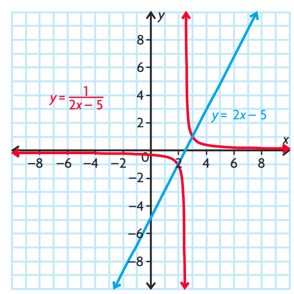
c)



d)

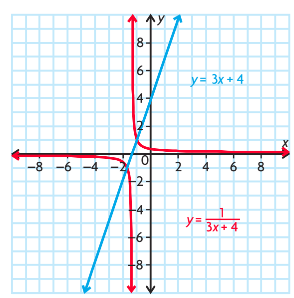


7. a)



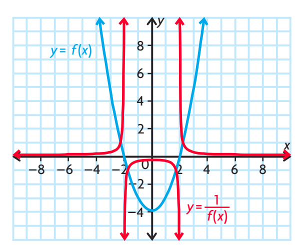
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

b)

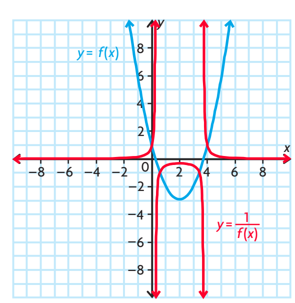


$D = \left\{x \in \mathbf{R} \mid x \neq -\frac{4}{3}\right\}$,
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

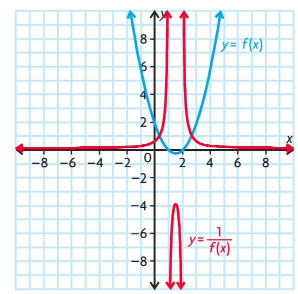
8. a)



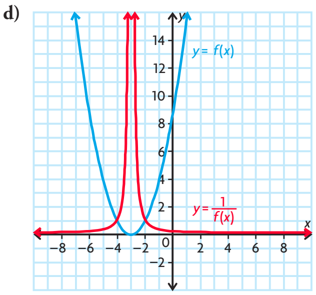
b)



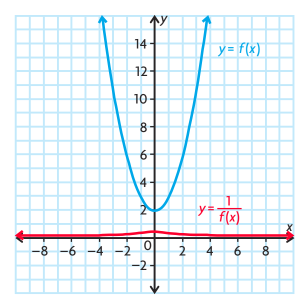
c)



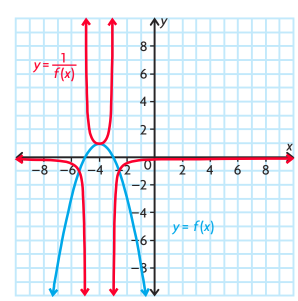
d)



e)

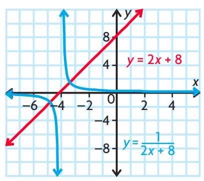


f)

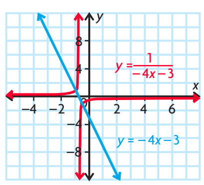


Answers

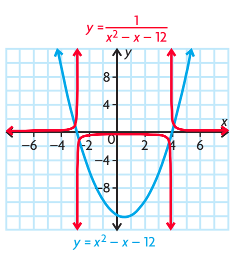
9. **a)** D = $\{x \in \mathbf{R}\}$
R = $\{y \in \mathbf{R}\}$
y-intercept = 8
x-intercept = −4
negative on $(-\infty, -4)$
positive on $(-4, -\infty)$
increasing on $(-\infty, \infty)$
equation of reciprocal = $\frac{1}{2x + 8}$



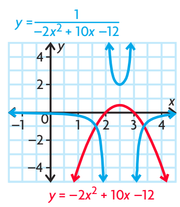
b) D = $\{x \in \mathbf{R}\}$
R = $\{y \in \mathbf{R}\}$
y-intercept = −3
x-intercept = $-\frac{3}{4}$
positive on $\left(-\infty, -\frac{3}{4}\right)$
negative on $\left(-\frac{3}{4}, \infty\right)$
decreasing on $(-\infty, \infty)$
equation of reciprocal = $\frac{1}{-4x - 3}$



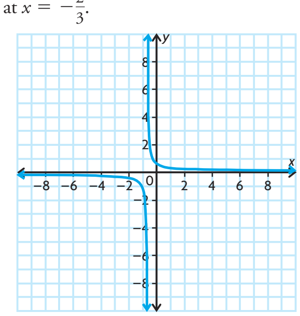
c) D = $\{x \in \mathbf{R}\}$
R = $\{y \in \mathbf{R} \mid y \geq -12.25\}$
y-intercept = 12
x-intercepts = 4, −3
decreasing on $(-\infty, 0.5)$
increasing on $(0.5, \infty)$
positive on $(-\infty, -3)$ and $(4, \infty)$
negative on $(-3, 4)$
equation of reciprocal = $\frac{1}{x^2 - x - 12}$



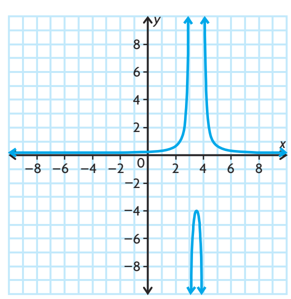
d) D = $\{x \in \mathbf{R}\}$
R = $\{y \in \mathbf{R} \mid y \leq 2.5\}$
y-intercept = −12
x-intercepts = 3, 2
increasing on $(-\infty, 2.5)$
decreasing on $(2.5, \infty)$
negative on $(-\infty, 2)$ and $(3, \infty)$
positive on $(2, 3)$
equation of reciprocal = $\frac{1}{-2x^2 + 10x - 12}$



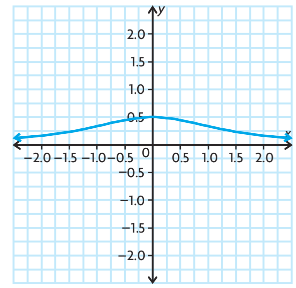
10. Answers may vary. For example, a reciprocal function creates a vertical asymptote when the denominator is equal to 0 for a specific value of x. Consider $\frac{1}{ax + b}$. For this expression, there is always some value of x that is $\frac{-b}{a}$ that will result in a vertical asymptote for the function. This is a graph of $y = \frac{1}{3x + 2}$ and the vertical asymptote is at $x = -\frac{2}{3}$.



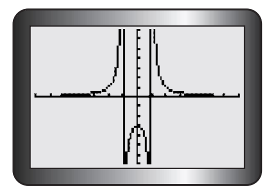
Consider the function $\frac{1}{(x - 3)(x - 4)}$. The graph of the quadratic function in the denominator crosses the x-axis at 3 and 4 and therefore will have vertical asymptotes at 3 and 4 in the graph of the reciprocal function.



However, a quadratic function, such as $x^2 + c$, which has no real zeros, will not have a vertical asymptote in the graph of its reciprocal function. For example, this is the graph of $y = \frac{1}{x^2 + 2}$.

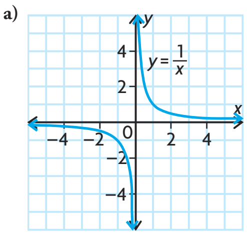


11. $y = \frac{3}{x^2 - 1}$



12. **a)** 500
b) $t = 2$
c) $t = 10\ 000$
d) If you were to use a value of t that was less than one, the equation would tell you that the number of bacteria was increasing as opposed to decreasing. Also, after time $t = 10\ 000$, the formula indicates that there is a smaller and smaller fraction of 1 bacteria left.
e) D = $\{x \in \mathbf{R} \mid 1 < x < 10\ 000\}$,
R = $\{y \in \mathbf{R} \mid 1 < y < 10\ 000\}$

13. **a)**

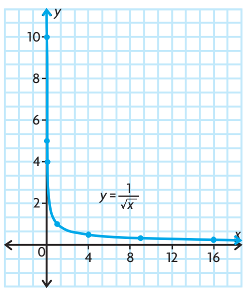


D = $\{x \in \mathbf{R} \mid x \neq -n\}$,
R = $\{y \in \mathbf{R} \mid y \neq 0\}$

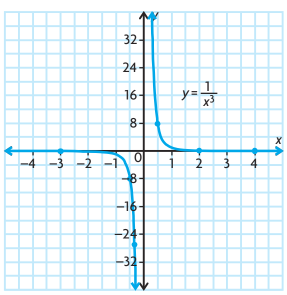
b) The vertical asymptote occurs at $x = -n$. Changes in n in the $f(x)$ family cause changes in the y-intercept—an increase in n causes the intercept to move up the y-axis and a decrease causes it to move down the y-axis. Changes in n in the $g(x)$ family cause changes in the vertical asymptote of the function—an increase in n causes the asymptote to move down the x-axis and a decrease in n causes it to move up the x-axis.
c) $x = 1 - n$ and $x = -1 - n$

14. Answers may vary. For example:
1) Determine the zero(s) of the function $f(x)$—these will be the asymptote(s) for the reciprocal function $g(x)$.
2) Determine where the function $f(x)$ is positive and where it is negative—the reciprocal function $g(x)$ will have the same characteristics.
3) Determine where the function $f(x)$ is increasing and where it is decreasing—the reciprocal function $g(x)$ will have opposite characteristics.

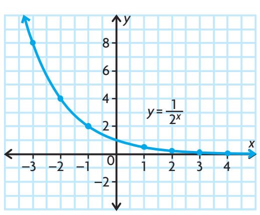
15. a)



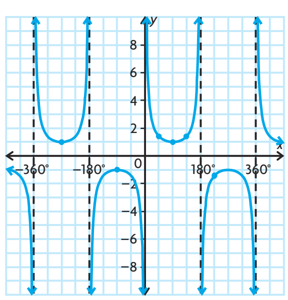
b)



c)



d)



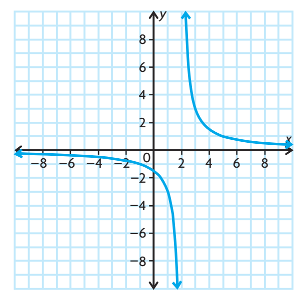
16. $y = \frac{1}{x + 4} - 1$

Lesson 5.2, p. 262

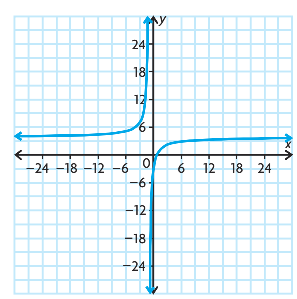
1. a) A; The function has a zero at 3 and the reciprocal function has a vertical asymptote at $x = 3$. The function is positive for $x < 3$ and negative for $x > 3$.
   b) C; The function in the numerator factors to $(x + 3)(x - 3)$. $(x - 3)$ factors out of both the numerator and the denominator. The equation simplifies to $y = x + 3$, but has a hole at $x = 3$.
   c) F; The function in the denominator has a zero at $x = -3$, so there is a vertical asymptote at $x = -3$. The function is always positive.
   d) D; The function in the denominator has zeros at $y = 1$ and $y = -3$. The rational function has vertical asymptotes at $x = 1$ and $x = -3$.
   e) B; The function has no zeros and no vertical asymptotes or holes.
   f) E; The function in the denominator has a zero at $x = 3$ and the rational function has a vertical asymptote at $x = 3$. The degree of the numerator is exactly 1 more than the degree of the denominator, so the graph has an oblique asymptote.
2. a) vertical asymptote at $x = -4$; horizontal asymptote at $y = 1$
   b) vertical asymptote at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$
   c) vertical asymptote at $x = 6$; horizontal asymptote at $y = 2$
   d) hole at $x = -3$
   e) vertical asymptotes at $x = -3$ and 5; horizontal asymptote at $y = 0$
   f) vertical asymptote at $x = -1$; horizontal asymptote at $y = -1$
   g) hole at $x = 2$
   h) vertical asymptote at $x = \frac{5}{2}$; horizontal asymptote at $y = -2$
   i) vertical asymptote at $x = -\frac{1}{4}$; horizontal asymptote at $y = 1$
   j) vertical asymptote at $x = 4$; hole at $x = -4$; horizontal asymptote at $y = 0$
   k) vertical asymptote at $x = \frac{3}{5}$; horizontal asymptote at $y = \frac{1}{5}$
   l) vertical asymptote at $x = 4$; horizontal asymptote at $y = -\frac{3}{2}$
3. Answers may vary. For example:
   a) $y = \frac{x - 1}{x^2 + x - 2}$
   b) $y = \frac{1}{x^2 - 4}$
   c) $y = \frac{x^2 - 4}{x^2 + 3x + 2}$
   d) $y = \frac{2x}{x + 1}$
   e) $y = \frac{x^3}{x^2 + 5}$

Lesson 5.3, pp. 272–274

1. a) A b) C c) D d) B
2. a) $x = 2$
   b) As $x \rightarrow 2$ from the right, the values of $f(x)$ get larger. As $x \rightarrow 2$ from the left, the values become larger in magnitude but are negative.
   c) $y = 0$
   d) As $x \rightarrow -\infty$ and as $x \rightarrow \infty$, $f(x) \rightarrow 0$.
   e) $D = \{x \in \mathbf{R} \mid x \neq 3\}$
   $R = \{y \in \mathbf{R} \mid y \neq 0\}$
   f) positive: $(2, \infty)$
   negative: $(-\infty, 2)$
   g)

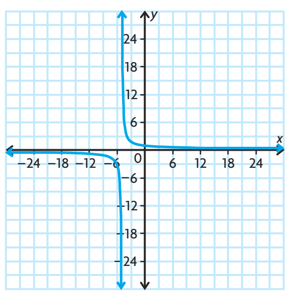


3. a) $x = -1$
   b) As $x \rightarrow -1$ from the left, $y \rightarrow \infty$. As $x \rightarrow -1$ from the right, $y \rightarrow -\infty$.
   c) $y = 4$
   d) As $x \rightarrow \pm\infty$, $f(x)$ gets closer and closer to 4.
   e) $D = \{x \in \mathbf{R} \mid x \neq -1\}$
   $R = \{y \in \mathbf{R} \mid y \neq 4\}$
   f) positive: $(-\infty, -1)$ and $\left(\frac{3}{4}, -\infty\right)$
   negative: $\left(-1, \frac{3}{4}\right)$
   g)



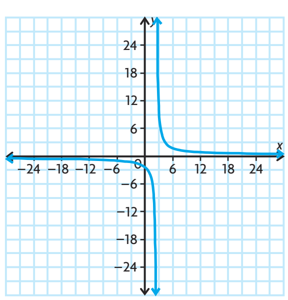
4. **a)** $x = -3$; As $x = -3$, $y = -\infty$ on the left.
As $x = -3$, $y = \infty$ on the right.
b) $x = 5$; As $x = 5$, $y = -\infty$ on the left.
As $x = 5$, $y = \infty$ on the right.
c) $x = \frac{1}{2}$; As $x = \frac{1}{2}$, $y = -\infty$ on the left.
As $x = \frac{1}{2}$, $y = \infty$ on the right.
d) $x = -\frac{1}{4}$; As $x = -\frac{1}{4}$, $y = -\infty$ on the left.
As $x = -\frac{1}{4}$, $y = \infty$ on the right.

5. **a)** vertical asymptote at $x = -5$
horizontal asymptote at $y = 0$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= \frac{3}{5}$
$f(x)$ is negative on $(-\infty, -5)$ and positive on $(-5, \infty)$.



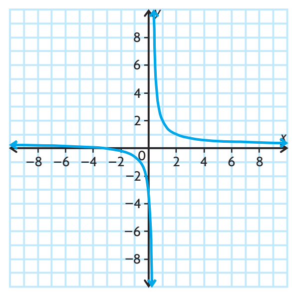
The function is decreasing on $(-\infty, -5)$ and on $(-5, \infty)$. The function is never increasing.

b) vertical asymptote at $x = \frac{5}{2}$
horizontal asymptote at $y = 0$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= -2$
$f(x)$ is negative on $\left(-\infty, \frac{5}{2}\right)$ and positive on $\left(\frac{5}{2}, \infty\right)$.



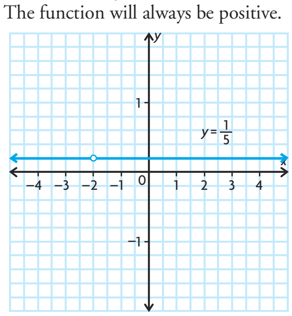
The function is decreasing on $\left(-\infty, \frac{5}{2}\right)$ and on $\left(\frac{5}{2}, \infty\right)$. The function is never increasing.

c) vertical asymptote at $x = \frac{1}{4}$
horizontal asymptote at $y = \frac{1}{4}$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{1}{4}\right\}$
$R = \left\{y \in \mathbf{R} \mid y \neq \frac{1}{4}\right\}$
x-intercept $= -5$
y-intercept $= -1$
$f(x)$ is positive on $(-\infty, -5)$ and $\left(\frac{1}{4}, \infty\right)$ and negative on $\left(-5, \frac{1}{4}\right)$.



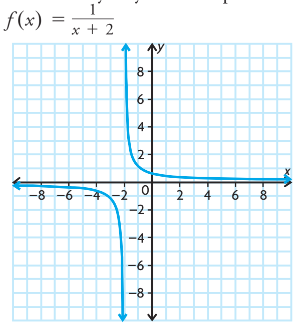
The function is decreasing on $\left(-\infty, \frac{1}{4}\right)$ and on $\left(\frac{1}{4}, \infty\right)$. The function is never increasing.

d) hole $x = -2$
$D = \{x \in \mathbf{R} \mid x \neq -2\}$
$R = \left\{y = \frac{1}{5}\right\}$
y-intercept $= \frac{1}{5}$
The function will always be positive.

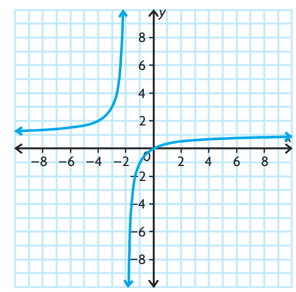


The function is neither increasing nor decreasing; it is constant.

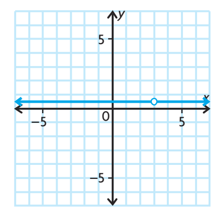
6. **a)** Answers may vary. For example:
$f(x) = \frac{1}{x + 2}$



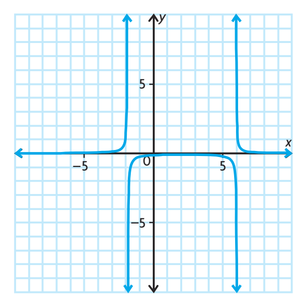
b) Answers may vary. For example:
$y = \frac{x}{x + 2}$



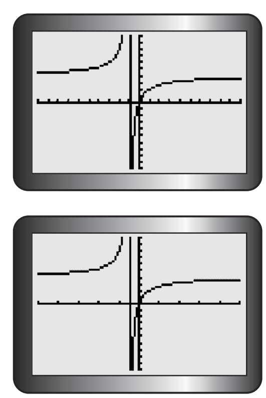
c) Answers may vary. For example:
$f(x) = \frac{x - 3}{2x - 6}$

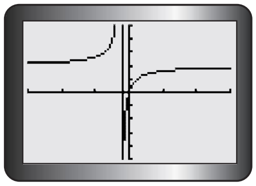


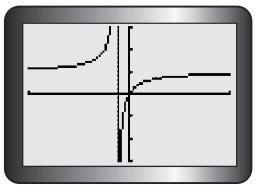
d) Answers may vary. For example:
$f(x) = \frac{1}{x^2 - 4x - 12}$



7. **a)**







The equation has a general vertical asymptote at $x = -\frac{1}{n}$. The function has a general horizontal asymptote at $y = \frac{8}{n}$. The vertical asymptotes are $-\frac{1}{8}, -\frac{1}{4}, -\frac{1}{2}$, and -1. The horizontal asymptotes are 8, 4, 2, and 1. The function contracts as n increases. The function is always increasing. The function is positive on $\left(-\infty, -\frac{17}{n}\right)$ and $\left(\frac{3}{10}, \infty\right)$. The function is negative on $\left(-\frac{17}{n}, \frac{3}{10}\right)$.

b) The horizontal and vertical asymptotes both approach 0 as the value of n increases; the x- and y-intercepts do not change, nor do the positive and negative characteristics or the increasing and decreasing characteristics.

c) The vertical asymptote becomes $x = \frac{17}{n}$ and the horizontal becomes $x = -\frac{10}{n}$. The function is always increasing. The function is positive on $\left(-\infty, \frac{3}{10}\right)$ and $\left(\frac{17}{n}, \infty\right)$. The function is negative on $\left(\frac{3}{10}, \frac{17}{n}\right)$. The rest of the characteristics do not change.

8. $f(x)$ will have a vertical asymptote at $x = 1$; $g(x)$ will have a vertical asymptote at $x = -\frac{3}{2}$. $f(x)$ will have a horizontal asymptote at $x = 3$; $g(x)$ will have a vertical asymptote at $x = \frac{1}{2}$.

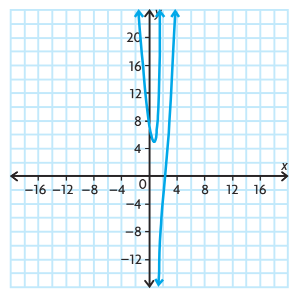
9. **a)** $27 500
b) $40 000
c) $65 000
d) No, the value of the investment at $t = 0$ should be the original value invested.
e) The function is probably not accurate at very small values of t because as $t \to 0$ from the right, $x \to \infty$.
f) $15 000

10. The concentration increases over the 24 h period and approaches approximately 1.89 mg/L.

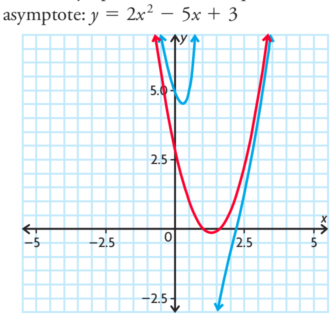
11. Answers may vary. For example, the rational functions will all have vertical asymptotes at $x = -\frac{d}{c}$. They will all have horizontal asymptotes at $y = \frac{a}{c}$. They will intersect the y-axis at $y = \frac{b}{d}$. The rational functions will have an x-intercept at $x = -\frac{b}{a}$.

12. Answers may vary. For example, $f(x) = \frac{2x^2}{2 + x}$.

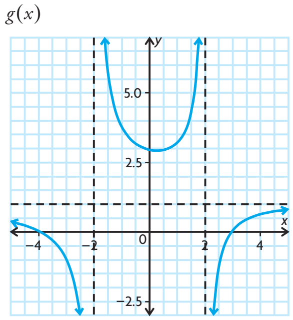
13. $f(x) = 2x^2 - 5x + 3 - \frac{2}{x - 1}$
As $x \to \pm\infty, f(x) \to \infty$.



vertical asymptote: $x = 1$; oblique asymptote: $y = 2x^2 - 5x + 3$



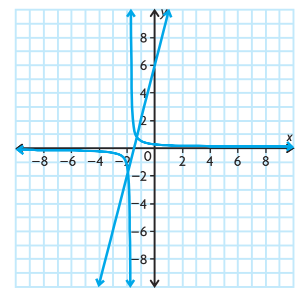
14. **a)** $f(x)$
b) $g(x)$ and $h(x)$
c) $g(x)$
d)



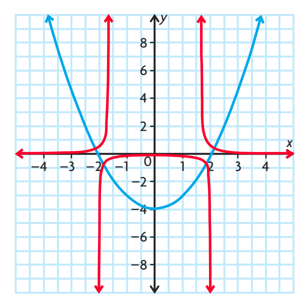
Mid-Chapter Review, p. 277

1. **a)** $\frac{1}{x - 3}$; $x = 3$
b) $\frac{1}{-4q + 6}$; $q = \frac{3}{2}$
c) $\frac{1}{z^2 + 4z - 5}$; $z = -5$ and 1
d) $\frac{1}{6d^2 + 7d - 3}$; $d = \frac{1}{3}$ and $-\frac{3}{2}$

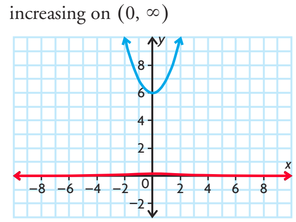
2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{x \in \mathbf{R}\}$; y-intercept $= 6$; x-intercept $= -\frac{3}{2}$; negative on $\left(-\infty, -\frac{3}{2}\right)$; positive on $\left(-\frac{3}{2}, \infty\right)$; increasing on $(-\infty, \infty)$



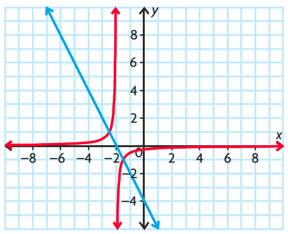
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > -4\}$; y-intercept $= -4$; x-intercepts are 2 and -2; decreasing on $(-\infty, 0)$; increasing $(0, \infty)$; positive on $(-\infty, -2)$ and $(2, \infty)$; negative on $(-2, 2)$



c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 6\}$; no x-intercepts; function will never be negative; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$



d) D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R}\}$; x-intercept $= -2$; function is always decreasing; positive on $(-\infty, -2)$; negative on $(-2, \infty)$

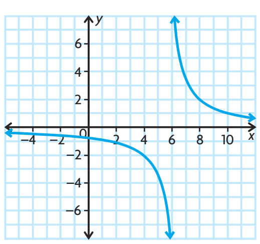


3. Answers may vary. For example: (1) Hole: Both the numerator and the denominator contain a common factor, resulting in $\frac{0}{0}$ for a specific value of x. (2) Vertical asymptote: A value of x causes the denominator of a rational function to be 0. (3) Horizontal asymptote: A horizontal asymptote is created by the ratio between the numerator and the denominator of a rational function as the function $\rightarrow \infty$ and $-\infty$. A continuous rational function is created when the denominator of the rational function has no zeros.

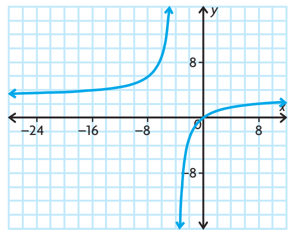
4. a) $x = 2$; vertical asymptote
   b) hole at $x = 1$
   c) $x = -\frac{1}{2}$; horizontal asymptote
   d) $x = 6$; oblique asymptote
   e) $x = -5$ and $x = 3$; vertical asymptotes

5. $y = \frac{x}{x-2}$, $y = 1$; $y = \frac{-7x}{4x+2}$, $y = \frac{-7}{4}$; $y = \frac{1}{x^2 + 2x - 15}$, $x = 0$

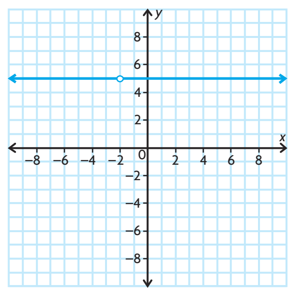
6. a) vertical asymptote: $x = 6$; horizontal asymptote: $y = 0$; no x-intercept; y-intercept: $-\frac{5}{6}$; negative when the denominator is negative; positive when the numerator is positive; $x - 6$ is negative on $x < 6$; $f(x)$ is negative on $(-\infty, 6)$ and positive on $(6, \infty)$; function is always decreasing



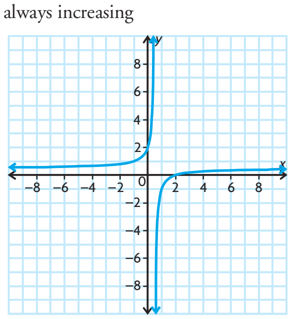
   b) vertical asymptote: $x = -4$; horizontal asymptote: $y = 3$; x-intercept: $x = 0$; y-intercept: $f(0) = 0$; function is always increasing; positive on $(-\infty, -4)$ and $(0, \infty)$; negative on $(-4, 0)$



   c) straight, horizontal line with a hole at $x = -2$; always positive and never increases or decreases



   d) vertical asymptote: $x = \frac{1}{2}$; horizontal asymptote: $y = \frac{1}{2}$; x-intercept: $x = 2$; y-intercept: $f(0) = 5$; function is always increasing



7. Answers may vary. For example: Changing the function to $y = \frac{7x+6}{x+1}$ changes the graph. The function now has a vertical asymptote at $x = -1$ and still has a horizontal asymptote at $y = 7$. However, the function is now constantly increasing instead of decreasing. The new function still has an x-intercept at $x = -\frac{6}{7}$, but now has a y-intercept at $y = 6$.

8. $n = \frac{1}{3}$; $m = 35$

9. Answers may vary. For example, $f(x) = \frac{4x+8}{x+2}$.
   The graph of the function will be a horizontal line at $y = 4$ with a hole at $x = -2$.

Lesson 5.4, pp. 285–287

1. 3; -2; Answers may vary. For example, substituting each value for x in the equation produces the same value on each side of the equation, so both are solutions.

2. a) $x = -3$ c) $x = -1$ and 2
   b) $x = 5$ d) $x = -4$

3. a) $f(x) = \frac{x-3}{x+3} - 2$
   b) $f(x) = \frac{3x-1}{x} - \frac{5}{2}$
   c) $f(x) = \frac{x-1}{x} - \frac{x+1}{x+3}$
   d) $f(x) = \frac{x-2}{x+3} - \frac{x-4}{x+5}$

4. a) $x = -9$ c) $x = 3$
   b) $x = 2$ d) $x = -\frac{1}{2}$

5. a) $x = 3$ d) $x = 0$
   b) $x = \frac{3}{4}$ e) $x = \frac{1}{4}$
   c) $x = -9$ f) $x = -23$

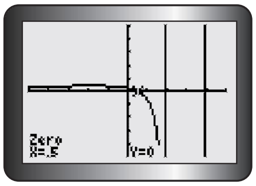
6. a) The function will have no real solutions.
   b) $x = 3$ and $x = -0.5$
   c) $x = -5$
   d) $x = 0$ and $x = -1$
   e) The original equation has no real solutions.
   f) $x = 5$ and $x = 2$

7. a) $x = 6$ d) $x = 3.25, 20.75$
   b) $x = 1.30, 7.70$ e) $x = -1.71, 2.71$
   c) $x = 10$ f) $x = -0.62, 1.62$

8. a) $\frac{x+1}{x-2} = \frac{x+3}{x-4}$
   Multiply both sides of the equation by the LCD, $(x-2)(x-4)$.
   $$(x-2)(x-4)\left(\frac{x+1}{x-2}\right)$$
   $$= (x-2)(x-4)\left(\frac{x+3}{x-4}\right)$$
   $(x-4)(x+1) = (x-2)(x+3)$
   Simplify. $x^2 - 3x - 4 = x^2 + x - 6$
   Simplify the equation so that 0 is on one side of the equation.
   $x^2 - x^2 - 3x - x - 4 + 6$
   $= x^2 - x^2 + x - x - 6 + 6$
   $-4x + 2 = 0$
   $-2(2x - 1) = 0$
   Since the product is equal to 0, one of the factors must be equal to 0. It must be $2x - 1$ because 2 is a constant.
   $$2x - 1 = 0$$
   $$2x - 1 + 1 = 0 + 1$$
   $$2x = 1$$
   $$\frac{2x}{2} = \frac{1}{2}$$
   $$x = \frac{1}{2}$$

b) $\dfrac{\frac{1}{2}+1}{\frac{1}{2}-2} = -1$ and $\dfrac{\frac{1}{2}+3}{\frac{1}{2}-4} = -1$

c)



9. $w = 9.271$

10. Machine A = 25.8 min; Machine B = 35.8 min

11. 75; $4.00

12. a) After 6666.67 s
b) The function appears to approach 9 kg/m^3 as time increases.

13. a) Tom = 4 min; Carl = 5 min; Paco = 2 min
b) 6.4 min

14. Answers may vary. For example, you can use either algebra or graphing technology to solve a rational equation. With algebra, solving the equation takes more time, but you get an exact answer. With graphing technology, you can solve the equation quickly, but you do not always get an exact answer.

15. $x = -3.80, -1.42, 0.90, 4.33$

16. a) $x = 0.438$ and 1.712
b) $(0, 0.438)$ and $(1.712, \infty)$

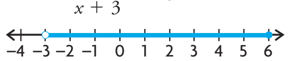
Lesson 5.5, pp. 295–297

1. a) $(\infty, 1)$ and $(3, \infty)$
b) $(-0.5, 1)$ and $(2, \infty)$

2. a) Solve the inequality for x.

$$\frac{6x}{x+3} \le 4$$
$$\frac{6x}{x+3} - 4 \le 0$$
$$\frac{6x}{x+3} - 4\frac{x+3}{x+3} \le 0$$
$$\frac{6x-4x-12}{x+3} \le 0$$
$$\frac{2x-12}{x+3} \le 0$$
$$\frac{2(x-6)}{x+3} \le 0$$

b)



c) $(-3, 6]$

3. a)
$$x + 2 > \frac{15}{x}$$
$$x + 2 - \frac{15}{x} > 0$$
$$\frac{x^2}{x} + \frac{2x}{x} - \frac{15}{x} > 0$$
$$\frac{x^2+2x-15}{x} > 0$$
$$\frac{(x+5)(x-3)}{x} > 0$$

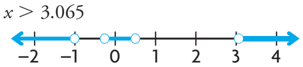
b) negative: $x < -5$ and $0 < x < 3$; positive: $-5 < x < 0, x > 3$
c) $\{x \in \mathbf{R} \mid -5 < x < 0 \text{ or } x > 3\}$ or $(-5, 0)$ or $(3, \infty)$

4. a) $5 < x < -4.5$
b) $-7 < x < -5$ and $x > -3$
c) $0 < x < 2$ and $x > 8$
d) $-6.8 \le x < -4$ and $x > 3$
e) $x < -1$ and $-\frac{1}{7} < x < 0$
f) $-1 < x < \frac{7}{8}$ and $x < 4$

5. a) $t < -3$ or $1 < t < 4$
b) $-3 \le t \le 2$ or $t > 4$
c) $-\frac{1}{2} < t < \frac{1}{3}$ or $t > \frac{1}{2}$
d) $t < -2$ and $-2 < t < 3$
e) $t < -5$ and $-2 < t < 0$
f) $-1 \le t < 0.25$ and $2 \le t < 9$

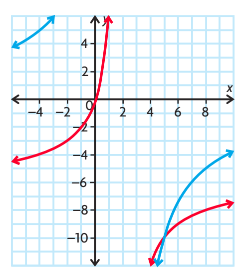
6. a) $x \in (-\infty, -6)$ or $x \in (-1, 4)$
b) $x \in (3, \infty)$
c) $x \in (-4, -2)$ or $x \in (-1, 2)$
d) $x \in (-\infty, -9)$ or $x \in [-3, -1)$ or $x \in [3, \infty)$
e) $x \in (-2, 0)$ or $x \in (4, \infty)$
f) $x \in (-\infty, -4)$ or $x \in (4, \infty)$

7. a) $x < -1, -0.2614 < x < 0.5, x > 3.065$
b)



c) Interval notation: $(-\infty, -1)$, $(-0.2614, 0.5)$, $(3.065, \infty)$
Set notation: $\{x \in \mathbf{R} \mid x < -1, -0.2614 < x < 0.5, \text{ or } x > 3.065\}$

8. a) $t < 2$ and $t > 5$.
b)



c) It would be difficult to find a situation that could be represented by these rational expressions because very few positive values of x yield a positive value of y.

9. The only values that make the expression greater than 0 are negative. Because the values of t have to be positive, the bacteria count in the tap water will never be greater than that of the pond water.

10. a) $\dfrac{(x^2-4x-5)}{2x} < 0$
b)

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$(x-5)$	−	−	−	+
$(x+1)$	−	+	+	+
$2x$	−	−	+	+
$\dfrac{(x-5)(x+1)}{2x}$	−	+	−	+

The inequality is true for $x < -1$ and $0 < x < 5$

11. when $x > 5$

12. a) The first inequality can be manipulated algebraically to produce the second inequality.
b) Graph the equation $y = \frac{x+1}{x-1} - \frac{x+3}{x+2}$ and determine when it is negative.
c) The values that make the factors of the second inequality zero are $-5, -2$, and 1. Determine the sign of each factor in the intervals corresponding to the zeros. Determine when the entire expression is negative by examining the signs of the factors.

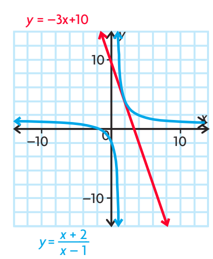
13. $[2, 4)$ and $(4, \infty)$

14. $14.48 < x < 165.52$ and $180 < x < 360$

15. $0 < x < 2$

Lesson 5.6, pp. 303–305

1. a) -0.5
b)



slope = −3

2. −3

3. −3

4. −1

5. a) 0.01
b) −0.3
c) −1.3
d) 6

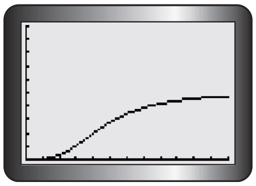
6. a) slope = 286.1; vertical asymptote: $x = -1.5$
b) slope = −2.74; vertical asymptote: $x = -5$
c) slope = 44.65; vertical asymptote: $x = -\frac{5}{3}$
d) slope = −1.26; vertical asymptote: $x = 6$

7. **a)** 0.01
b) 0.34

8. **a)** $R(x) = \dfrac{15x}{2x^2 + 11x + 5}$
b) 0.3, -0.03

9. **a)** \$5.67
b) -2

10. **a)** 68.46
b) 94.54
c)



The number of houses that were built increases slowly at first, but rises rapidly between the third and sixth months. During the last six months, the rate at which the houses were built decreases.

11. Answers may vary. For example: $14 \le x \le 15$; $x = 14.5$

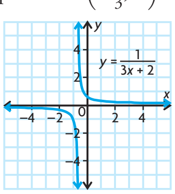
12. **a)** Find $s(0)$ and $s(6)$, and then solve $\dfrac{s(6) - s(0)}{6 - 0}$.
b) The average rate of change over this interval gives the object's speed.
c) To find the instantaneous rate of change at a specific point, you could find the slope of the line that is tangent to the function $s(t)$ at the specific point. You could also find the average rate of change on either side of the point for smaller and smaller intervals until it stabilizes to a constant. It is generally easier to find the instantaneous rate using a graph, but the second method is more accurate.
d) The instantaneous rate of change for a specific time, t, is the acceleration of the object at this time.

13. $y = -0.5x - 2.598$; $y = -0.5x + 2.598$; $y = 4x$

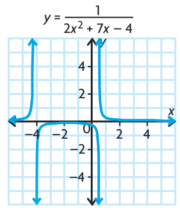
14. The instantaneous rate of change at $(0, 0) = 4$. The rate of change at this rate of change will be 0.

Chapter Review, pp. 308–309

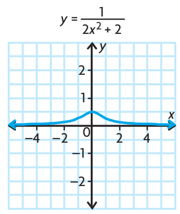
1. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; x-intercept $= -\frac{2}{3}$; y-intercept $= 2$; always increasing; negative on $\left(-\infty, -\frac{2}{3}\right)$; positive on $\left(-\frac{2}{3}, \infty\right)$



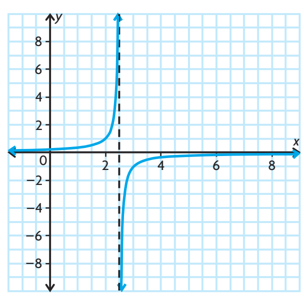
b) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > -10.125\}$; x-intercept $= 0.5$ and -4; positive on $(-\infty, -4)$ and $(0.5, \infty)$; negative on $(-4, 0.5)$; decreasing on $(-\infty, -10.125)$; increasing on $(-10.125, \infty)$



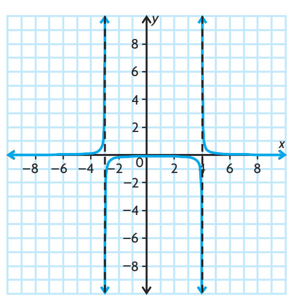
c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 2\}$; no x-intercepts; y-intercept $= 2$; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; always positive, never negative



2. **a)**



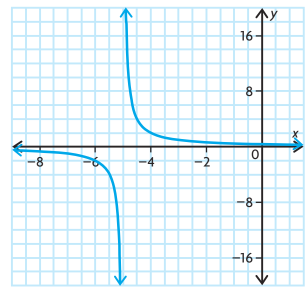
b)



3. **a)** $x = -17$
b) $x = -\frac{3}{5}$; horizontal asymptote; $y = \frac{2}{5}$
c) $x = 0.5$; hole at $x = -11$
d) $x = 1$; oblique asymptote; $y = 3x + 3$

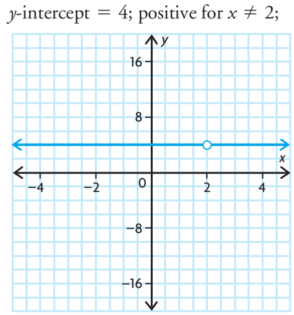
4. The locust population increased during the first 1.75 years, to reach a maximum of 1 248 000. The population gradually decreased until the end of the 50 years, when the population was 128 000.

5. **a)** x-intercept $= 2$: horizontal asymptote: $y = 0$; y-intercept $= \frac{2}{5}$: vertical asymptote: $x = -5$;



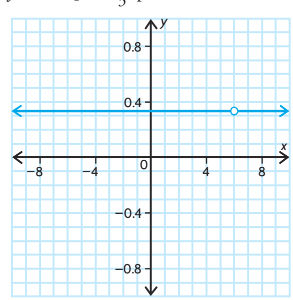
The function is never increasing and is decreasing on $(-\infty, -5)$ and $(-5, \infty)$.
$D = \{x \in \mathbf{R} \mid x \ne -5\}$;
negative for $x < -5$;
positive for $x > -5$

b) $D = \{x \in \mathbf{R} \mid x \ne 2\}$; no x-intercept; y-intercept $= 4$; positive for $x \ne 2$;



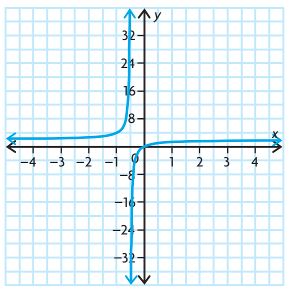
never increasing or decreasing

c) $D = \{x \in \mathbf{R} \mid x \ne 6\}$; no x-intercept; y-intercept $= \frac{1}{3}$; positive for $x \ne 6$;



never increasing or decreasing

d) $x = -0.5$; vertical asymptote: $x = -0.5$; $D = \{x \in \mathbf{R} \mid x \neq -0.5\}$; x-intercept $= 0$; y-intercept $= 0$; horizontal asymptote $= 2$; $R = \{y \in \mathbf{R} \mid x \neq 2\}$; positive on $x < -0.5$ and $x > 0$; negative on $-0.5 < x < 0$



The function is never decreasing and is increasing on $(-\infty, -0.5)$ and $(-0.5, \infty)$.

6. Answers may vary. For example, consider the function $f(x) = \frac{1}{x-6}$. You know that the vertical asymptote would be $x = 6$. If you were to find the value of the function very close to $x = 6$ (say $f(5.99)$ or $f(6.01)$) you would be able to determine the behaviour of the function on either side of the asymptote.

$$f(5.99) = \frac{1}{(5.99) - 6} = -100$$

$$f(6.01) = \frac{1}{(6.01) - 6} = 100$$

To the left of the vertical asymptote, the function moves toward $-\infty$. To the right of the vertical asymptote, the function moves toward ∞.

7. **a)** $x = 6$
b) $x = 0.2$ and $x = -\frac{2}{3}$
c) $x = -6$ or $x = 2$
d) $x = -1$ and $x = 3$

8. about 12 min

9. $x = 1.82$ days and 3.297 days

10. **a)** $x < -3$ and $-2.873 < x < 4.873$
b) $-16 < x < -11$ and $-5 < x$
c) $-2 < x < -1.33$ and $-1 < x < 0$
d) $0 < x < 1.5$

11. $-0.7261 < t < 0$ and $t > 64.73$

12. **a)** -6; $x = 3$
b) 0.2; $x = -2$ and $x = -1$

13. **a)** 0.455 mg/L/h
b) -0.04 mg/L/h
c) The concentration of the drug in the blood stream appears to be increasing most rapidly during the first hour and a half; the graph is steep and increasing during this time.

14. $x = 5$ and $x = 8$; $x = 6.5$

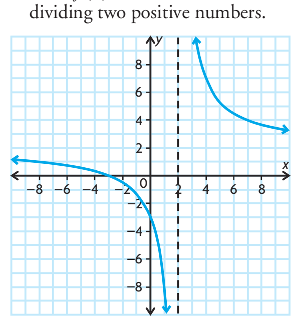
15. **a)** As the x-coordinate approaches the vertical asymptote of a rational function, the line tangent to graph will get closer and closer to being a vertical line. This means that the slope of the line tangent to the graph will get larger and larger, approaching positive or negative infinity depending on the function, as x gets closer to the vertical asymptote.
b) As the x-coordinate grows larger and larger in either direction, the line tangent to the graph will get closer and closer to being a horizontal line. This means that the slope of the line tangent to the graph will always approach zero as x gets larger and larger.

Chapter Self-Test, p. 310

1. **a)** B
b) A

2. **a)** If $f(n)$ is very large, then that would make $\frac{1}{f(n)}$ a very small fraction.
b) If $f(n)$ is very small (less than 1), then that would make $\frac{1}{f(n)}$ very large.
c) If $f(n) = 0$, then that would make $\frac{1}{f(n)}$ undefined at that point because you cannot divide by 0.
d) If $f(n)$ is positive, then that would make $\frac{1}{f(n)}$ also positive because you are dividing two positive numbers.

3.



4. 4326 kg; \$0.52/kg

5. **a)** Algebraic; $x = -1$ and $x = -3$
b) Algebraic with factor table
The inequality is true on $(-10, -5.5)$ and on $(-5, 1.2)$.

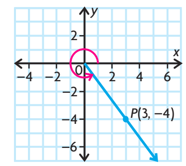
6. **a)** To find the vertical asymptotes of the function, find the zeros of the expression in the denominator. To find the equation of the horizontal asymptotes, divide the first two terms of the expressions in the numerator and denominator.
b) This type of function will have a hole when both the numerator and the denominator share the same factor $(x + a)$.

Chapter 6

Getting Started, p. 314

1. **a)** 28°
b) 332°

2. **a)**



$\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$,

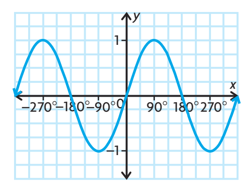
$\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

b) 307°

3. **a)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **e)** $-\sqrt{2}$
b) 0 **d)** $\frac{1}{2}$ **f)** -1

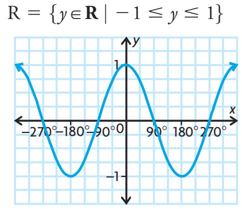
4. **a)** 60°, 300°
b) 30°, 210°
c) 45°, 225°
d) 180°
e) 135°, 315°
f) 90°

5. **a)**



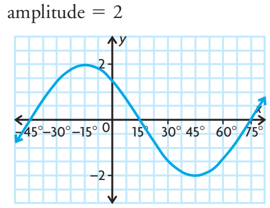
period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

b)



period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

6. **a)** period $= 120°$; $y = 0$; 45° to the left; amplitude $= 2$



Answers