Chapter 5 Problem Set – Trigonometric Functions

5.1 Radian Measure and Arc Length #2defh, 3 - 9 (pg 321 in textbook)

2. Sketch each rotation about a circle of radius 1.

d) $\frac{4\pi}{3}$ e) $\frac{5\pi}{3}$ f) $-\pi$ h) $-\frac{\pi}{4}$

3. Convert each angle from degrees to radians, in exact form.

a) $75°$ b) $200°$ c) $400°$ d) $320°$

4. Convert each angle from radians to degrees. Express the measure correct to two decimal places, if necessary.

a) $\frac{5\pi}{3}$ b) 0.3π c) 3 d) $\frac{11\pi}{4}$

5. a) Determine the measure of the central angle that is formed by an arc length of 5 cm in a circle with a radius of 2.5 cm. Express the measure in both radians and degrees, correct to one decimal place.

b) Determine the arc length of the circle in part a) if the central angle is 200°.

6. Determine the arc length of a circle with a radius of 8 cm if

a) the central angle is 3.5

b) the central angle is 300°

7. Convert to radian measure.

K a) $90°$ c) $-180°$ e) $-135°$ g) $240°$

b) $270°$ d) $45°$ f) $60°$ h) $-120°$

8. Convert to degree measure.

a) $\frac{2\pi}{3}$ c) $\frac{\pi}{4}$ e) $\frac{7\pi}{6}$ g) $\frac{11\pi}{6}$

b) $-\frac{5\pi}{3}$ d) $-\frac{3\pi}{4}$ f) $-\frac{3\pi}{2}$ h) $-\frac{9\pi}{2}$

9. If a circle has a radius of 65 m, determine the arc length for each of the following central angles.

a) $\frac{19\pi}{20}$ b) 1.25 c) $150°$

5.2 Trigonometric Ratios and Special Triangles (Part 1) #1b-f, 2bcd, 3 (pg 330 in textbook)

1. For each trigonometric ratio, use a sketch to determine in which quadrant the terminal arm of the principal angle lies, the value of the related acute angle, and the sign of the ratio.

a) $\sin \frac{3\pi}{4}$ d) $\sec \frac{5\pi}{6}$

b) $\cos \frac{5\pi}{3}$ e) $\cos \frac{2\pi}{3}$

c) $\tan \frac{4\pi}{3}$ f) $\cot \frac{7\pi}{4}$

2. Each of the following points lies on the terminal arm of an angle in standard position.

i) Sketch each angle.

ii) Determine the value of r.

iii) Determine the primary trigonometric ratios for the angle.

iv) Calculate the radian value of θ, to the nearest hundredth, where $0 \leq \theta \leq 2\pi$.

b) $(-12, -5)$ c) $(4, -3)$ d) $(0, 5)$

3. Determine the primary trigonometric ratios for each angle.

a) $-\frac{\pi}{2}$ c) $\frac{7\pi}{4}$

b) $-\pi$ d) $-\frac{\pi}{6}$

5.3 Trigonometric Ratios and Special Triangles (Part 2) #5, 7, 9 (pg 330 in textbook)

5. Determine the exact value of each trigonometric ratio.

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a) $\sin \frac{2\pi}{3}$ c) $\tan \frac{11\pi}{6}$ e) $\csc \frac{5\pi}{6}$

b) $\cos \frac{5\pi}{4}$ d) $\sin \frac{7\pi}{4}$ f) $\sec \frac{5\pi}{3}$

7. The terminal arm of an angle in standard position passes through each of the following points. Find the radian value of the angle in the interval $[0, 2\pi]$, to the nearest hundredth.

a) $(-7, 8)$ c) $(3, 11)$ e) $(9, 10)$

b) $(12, 2)$ d) $(-4, -2)$ f) $(6, -1)$

9. A leaning flagpole, 5 m long, makes an obtuse angle with the ground.

A

If the distance from the tip of the flagpole to the ground is 3.4 m, determine the radian measure of the obtuse angle, to the nearest hundredth.

5.4 Trigonometric Ratios and Special Triangles (Part 3) #6, 11, 16 (pg 331 in textbook)

6. For each of the following values of $\cos\theta$, determine the radian value of θ if $\pi \le \theta \le 2\pi$.

a) $-\frac{1}{2}$

b) $\frac{\sqrt{3}}{2}$

c) $-\frac{\sqrt{2}}{2}$

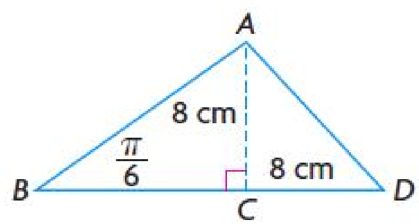
d) $-\frac{\sqrt{3}}{2}$

e) 0

f) -1

11. T A clock is showing the time as exactly 3:00 p.m. and 25 s. Because a full minute has not passed since 3:00, the hour hand is pointing directly at the 3 and the minute hand is pointing directly at the 12. If the tip of the second hand is directly below the tip of the hour hand, and if the length of the second hand is 9 cm, what is the length of the hour hand?

16. Determine the length of AB. Find the sine, cosine, and tangent ratios of $\angle D$, given $AC = CD = 8$ cm.



5.6 Transformations of Trigonometric Functions #1, 4, 6, 7, 8, 13, 14ab (pg 343 in textbook)

1. State the period, amplitude, horizontal translation, and equation of the axis for each of the following trigonometric functions.

a) $y = 0.5\cos(4x)$

b) $y = \sin\left(x - \frac{\pi}{4}\right) + 3$

c) $y = 2\sin(3x) - 1$

d) $y = 5\cos\left(-2x + \frac{\pi}{3}\right) - 2$

4. The following trigonometric functions have the parent function $f(x) = \sin x$. They have undergone no horizontal translations and no reflections in either axis. Determine the equation of each function.

a) The graph of this trigonometric function has a period of π and an amplitude of 25. The equation of the axis is $y = -4$.

b) The graph of this trigonometric function has a period of 10 and an amplitude of $\frac{2}{5}$. The equation of the axis is $y = \frac{1}{15}$.

c) The graph of this trigonometric function has a period of 6π and an amplitude of 80. The equation of the axis is $y = -\frac{9}{10}$.

d) The graph of this trigonometric function has a period of $\frac{1}{2}$ and an amplitude of 11. The equation of the axis is $y = 0$.

6. State the transformations that were applied to the parent function $f(x) = \sin x$ to obtain each of the following transformed functions. Then graph the transformed functions.

a) $f(x) = 4 \sin x + 3$

b) $f(x) = -\sin\left(\frac{1}{4}x\right)$

c) $f(x) = \sin(x - \pi) - 1$

d) $f(x) = \sin\left(4x + \frac{2\pi}{3}\right)$

7. The trigonometric function $f(x) = \cos x$ has undergone the following sets of transformations. For each set of transformations, determine the equation of the resulting function and sketch its graph.

a) vertical compression by a factor of $\frac{1}{2}$, vertical translation 3 units up

b) horizontal stretch by a factor of 2, reflection in the y-axis

c) vertical stretch by a factor of 3, horizontal translation $\frac{\pi}{2}$ to the right

d) horizontal compression by a factor of $\frac{1}{2}$, horizontal translation $\frac{\pi}{2}$ to the left

8. Sketch each graph for $0 \le x \le 2\pi$. Verify your sketch using graphing technology.

a) $y = 3 \sin\left(2\left(x - \frac{\pi}{6}\right)\right) + 1$

b) $y = 5 \cos\left(x + \frac{\pi}{4}\right) - 2$

c) $y = -2 \sin\left(2\left(x + \frac{\pi}{4}\right)\right) + 2$

d) $y = -\cos\left(0.5x - \frac{\pi}{6}\right) + 3$

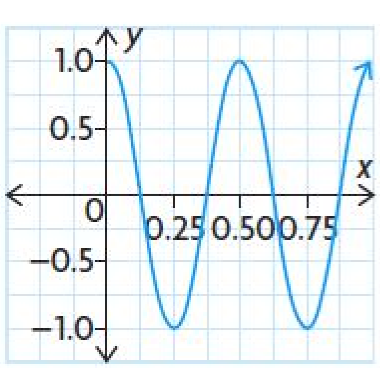
e) $y = 0.5 \sin\left(\frac{x}{4} - \frac{\pi}{16}\right) - 5$

f) $y = \frac{1}{2} \cos\left(\frac{x}{2} - \frac{\pi}{12}\right) - 3$

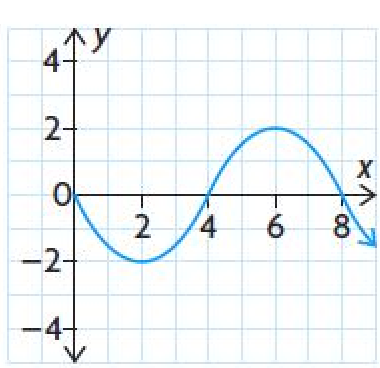
13. The graph of a sinusoidal function has been vertically stretched, vertically translated up, and horizontally translated to the right. The graph has a maximum at $\left(\frac{\pi}{13}, 13\right)$, and the equation of the axis is $y = 9$. If the x-axis is in radians, list one point where the graph has a minimum.

14. Determine a sinusodial equation for each of the following graphs.

a)



b)



5.7 Applications of Trigonometric Functions #4, 6, 9, 10 (pg 360 in textbook)

4. The height of a patch on a bicycle tire above the ground, as a function of time, is modelled by one sinusoidal function. The height of the patch above the ground, as a function of the total distance it has travelled, is modelled by another sinusoidal function. Which of the following characteristics do the two sinusoidal functions share: amplitude, period, equation of the axis?

6. **A** To test the resistance of a new product to temperature changes, the product is placed in a controlled environment. The temperature in this environment, as a function of time, can be described by a sine function. The maximum temperature is 120 °C, the minimum temperature is -60 °C, and the temperature at $t = 0$ is 30 °C. It takes 12 h for the temperature to change from the maximum to the minimum. If the temperature is initially increasing, what is the equation of the sine function that describes the temperature in this environment?

9. At one time, Maple Leaf Village (which no longer exists) had North America's largest Ferris wheel. The Ferris wheel had a diameter of 56 m, and one revolution took 2.5 min to complete. Riders could see Niagara Falls if they were higher than 50 m above the ground. Sketch three cycles of a graph that represents the height of a rider above the ground, as a function of time, if the rider gets on at a height of 0.5 m at $t = 0$ min. Then determine the time intervals when the rider could see Niagara Falls.

10. The number of hours of daylight in Vancouver can be modelled by a sinusoidal function of time, in days. The longest day of the year is June 21, with 15.7 h of daylight. The shortest day of the year is December 21, with 8.3 h of daylight.
    a) Find an equation for $n(t)$, the number of hours of daylight on the nth day of the year.
    b) Use your equation to predict the number of hours of daylight in Vancouver on January 30th.