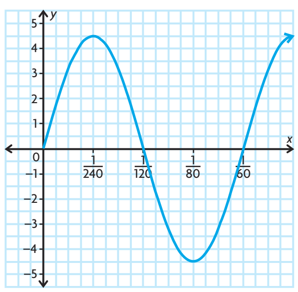
8. **a)** 2π radians
b) 2π radians
c) π radians

9. $y = 5 \sin\left(x + \frac{\pi}{3}\right) + 2$

10. $y = -3 \cos\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$

11. **a)** reflection in the x-axis, vertical stretch by a factor of 19, vertical translation 9 units down
b) horizontal compression by a factor of $\frac{1}{10}$, horizontal translation $\frac{\pi}{12}$ to the left
c) vertical compression by a factor of $\frac{10}{11}$, horizontal translation $\frac{\pi}{9}$ to the right, vertical translation 3 units up
d) reflection in the x-axis, reflection in the y-axis, horizontal translation π to the right

12. **a)**


b) $\frac{1}{60}$
c) $\frac{1}{240}$
d) $\frac{1}{80}$

13. **a)** 2π radians
b) 2π radians
c) π radians

14. **a)** the radius of the circle in which the bumblebee is flying
b) the time that the bumblebee takes to fly one complete circle
c) the height, above the ground, of the centre of the circle in which the bumblebee is flying
d) cosine function

15. $P(m) = 7250 \cos\left(\frac{\pi}{6}m\right) + 7750$

16. $h(t) = 30 \sin\left(\frac{5\pi}{3}t - \frac{\pi}{2}\right) + 150$

17. **a)** $0 < x < 5\pi$, $10\pi < x < 15\pi$
b) $2.5\pi < x < 7.5\pi$, $12.5\pi < x < 17.5\pi$
c) $0 < x < 2.5\pi$, $7.5\pi < x < 12.5\pi$

18. **a)** $x = 0, x = \frac{1}{2}$
b) $x = \frac{1}{8}, x = \frac{5}{8}$
c) $x = \frac{3}{8}, x = \frac{7}{8}$

19. **a)** $x = \frac{3}{4}$ s
b) the time between one beat of a person's heart and the next beat
c) 140
d) -129

Chapter Self-Test, p. 378

1. $y = \sec x$
2. $\sec 2\pi$
3. $y \doteq 108.5$
4. about 0.31 °C per day
5. $\frac{3\pi}{5}$, 110°, $\frac{5\pi}{8}$, 113°, and $\frac{2\pi}{3}$
6. $y = \sin\left(x + \frac{5\pi}{8}\right)$
7. $y \doteq -30$
8. **a)** $-3\cos\left(\frac{\pi}{12}x\right) + 22$
b) about 0.5 °C per hour
c) about 0 °C per hour

Cumulative Review Chapters 4–6, pp. 380–383

1. (d)	**9.** (c)	**17.** (d)	**25.** (b)
2. (b)	**10.** (c)	**18.** (b)	**26.** (d)
3. (a)	**11.** (d)	**19.** (b)	**27.** (a)
4. (c)	**12.** (a)	**20.** (b)	**28.** (c)
5. (a)	**13.** (d)	**21.** (d)	**29.** (b)
6. (b)	**14.** (c)	**22.** (c)	
7. (a)	**15.** (d)	**23.** (a)	
8. (c)	**16.** (a)	**24.** (d)	

30. **a)** If x is the length in centimetres of a side of one of the corners that have been cut out, the volume of the box is $(50 - 2x)(40 - 2x)x$ cm^3.
b) 5 cm or 10 cm
c) $x \doteq 7.4$ cm
d) $3 < x < 12.8$

31. **a)** The zeros of $f(x)$ are $x = 2$ or $x = 3$. The zero of $g(x)$ is $x = 3$. The zero of $\frac{f(x)}{g(x)}$ is $x = 2$. $\frac{g(x)}{f(x)}$ does not have any zeros.
b) $\frac{f(x)}{g(x)}$ has a hole at $x = 3$; no asymptotes. $\frac{g(x)}{f(x)}$ has an asymptote at $x = 2$ and $y = 0$.
c) $x = 1$; $\frac{f(x)}{g(x)}$: $y = x - 2$, $\frac{g(x)}{f(x)}$: $y = -x$

32. **a)** Vertical compressions and stretches do not affect location of zeros; maximum and minimum values are multiplied by the scale factor, but locations are unchanged; instantaneous rates of change are multiplied by the scale factor.
Horizontal compressions and stretches move locations of zeros, maximums, and minimums toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor.
Vertical translations change location of zeros or remove them; maximum and minimum values are increased or decreased by the amount of the translation, but locations are unchanged; instantaneous rates of change are unchanged.
Horizontal translations move location of zeros by the same amount as the translation; maximum and minimum values are unchanged, but locations are moved by the same amount as the translation; instantaneous rates of change are unchanged, but locations are moved by the same amount as the translation.
b) For $y = \cos x$, the answer is the same as in part **a)**, except that a horizontal reflection does not affect instantaneous rates of change. For $y = \tan x$, the answer is also the same as in part **a)**, except that nothing affects the maximum and minimum values, since there are no maximum or minimum values for $y = \tan x$.

Chapter 7

Getting Started, p. 386

1. **a)** 1
b) $-\frac{22}{7}$
c) 8 or -3
d) $\frac{2}{3}$ or $-\frac{5}{2}$
e) $-1 \pm \sqrt{2}$
f) $\frac{3 \pm \sqrt{21}}{6}$

2. To do this, you must show that the two distances are equal:

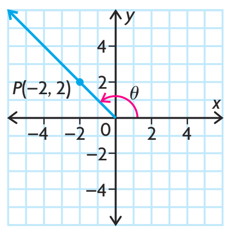
$$D_{AB} = \sqrt{(2 - 1)^2 + \left(\tfrac{1}{2} - 0\right)^2} = \frac{\sqrt{5}}{2};$$

$$D_{CD} = \sqrt{\left(0 - \tfrac{1}{2}\right)^2 + (6 - 5)^2} = \frac{\sqrt{5}}{2}.$$

Since the distances are equal, the line segments are the same length.

3. **a)** $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$, $\csc A = \frac{17}{8}$, $\sec A = \frac{17}{15}$, $\cot A = \frac{15}{8}$
b) 0.5 radians
c) 61.9°

4. a)



b) $\frac{\pi}{4}$ radians **c)** $\frac{3\pi}{4}$ radians

5. a) $A: \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$; $F: \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;

$B: \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $G: \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;

$C: \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $H: \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;

$D: \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; $I: \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;

$E: \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; $J: \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

b) i) $-\frac{\sqrt{2}}{2}$ **ii)** $-\frac{1}{2}$ **iii)** -1 **iv)** 2

6. a) If the angle x is in the second quadrant:

$\sin x = \frac{3}{5}$; $\cos x = -\frac{4}{5}$;

$\csc x = \frac{5}{3}$; $\sec x = -\frac{5}{4}$; $\cot x = -\frac{4}{3}$.

If the angle x is in the fourth quadrant:

$\sin x = -\frac{3}{5}$; $\cos x = \frac{4}{5}$; $\csc x = -\frac{5}{3}$;

$\sec x = \frac{5}{4}$; $\cot x = -\frac{4}{3}$

b) If x is in the second quadrant, $x = 2.5$. If x is in the fourth quadrant, $x = 5.6$.

7. a) true **b)** true **c)** false **d)** false **e)** true **f)** true

8.

Perform a vertical stretch/compression by a factor of $|a|$.

↓

Use $\left|\frac{1}{k}\right|$ to determine the horizontal stretch/compression.

↓

Use a and k to determine whether the function is reflected in the y-axis or the x-axis.

↓

Perform a vertical translation of c units up or down.

↓

Perform a horizontal translation of d units to the right or the left.

Lesson 7.1, pp. 392–393

1. a) Answers may vary. For example: $y = \cos(\theta + 2\pi)$, $y = \cos(\theta + 4\pi)$, $y = \cos(\theta - 2\pi)$

b) $y = \sin\left(\theta + \frac{\pi}{2}\right)$, $y = \sin\left(\theta - \frac{3\pi}{2}\right)$, $y = \sin\left(\theta + \frac{5\pi}{2}\right)$

2. a) $y = \csc\theta$ is odd, $\csc(-\theta) = -\csc\theta$; $y = \sec\theta$ is even, $\sec(-\theta) = \sec\theta$; $y = \cot\theta$ is odd, $\cot(-\theta) = -\cot\theta$

b) $y = \cot(-\theta)$ is the graph of $y = \cot\theta$ reflected across the y-axis; $y = -\cot\theta$ is the graph of $y = \cot\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \csc(-\theta)$ is the graph of $y = \csc\theta$ reflected across the y-axis; $y = -\csc\theta$ is the graph of $y = \csc\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \sec(-\theta)$ is the graph of $y = \sec\theta$ reflected across the y-axis. This results in the same graph as $y = \sec\theta$.

3. a) $\cos\frac{\pi}{3}$ **b)** $\sin\frac{\pi}{12}$ **c)** $\cot\frac{\pi}{8}$ **d)** $\sin\frac{3\pi}{16}$ **e)** $\cos\frac{3\pi}{8}$ **f)** $\cot\frac{\pi}{3}$

4. a) $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$;

$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$;

$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

b) $y = \tan\left(\frac{\pi}{2} - \theta\right) = \tan\left(-\left(\theta - \frac{\pi}{2}\right)\right)$; This is the graph of $y = \tan\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \cot\theta$.

$y = \csc\left(\frac{\pi}{2} - \theta\right) = \csc\left(-\left(\theta - \frac{\pi}{2}\right)\right)$; This is the graph of $y = \csc\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \sec\theta$.

$y = \sec\left(\frac{\pi}{2} - \theta\right) = \sec\left(-\left(\theta - \frac{\pi}{2}\right)\right)$; This is the graph of $y = \sec\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \csc\theta$.

5. a) $\sin\frac{\pi}{8}$ **b)** $-\cos\frac{\pi}{12}$ **c)** $\tan\frac{\pi}{4}$ **d)** $\cos\frac{\pi}{6}$ **e)** $-\sin\frac{3\pi}{8}$ **f)** $-\tan\frac{\pi}{3}$

6. a) Assume the circle is a unit circle. Let the coordinates of Q be (x, y). Since P and Q are reflections of each other in the line $y = x$, the coordinates of P are (y, x). Draw a line from P to the positive x-axis. The hypotenuse of the new right triangle makes an angle of $\left(\frac{\pi}{2} - \theta\right)$ with the positive x-axis. Since the x-coordinate of P is y, $\cos\left(\frac{\pi}{2} - \theta\right) = y$. Also, since the y-coordinate of Q is y, $\sin\theta = y$. Therefore, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$.

b) Assume the circle is a unit circle. Let the coordinates of the vertex on the circle of the right triangle in the first quadrant be (x, y). Then $\sin\theta = y$, so $-\sin\theta = -y$. The point on the circle that results from rotating the vertex by $\frac{\pi}{2}$ counterclockwise about the origin has coordinates $(-y, x)$, so $\cos\left(\frac{\pi}{2} + \theta\right) = -y$. Therefore, $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$.

7. a) true

b) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{\pi}{2}$, or 1. The right side is $-\sin\frac{\pi}{2}$, or -1.

c) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cos\pi$, or -1. The right side is $-\cos 5\pi$, or 1.

d) false; Answers may vary. For example: Let $\theta = \frac{\pi}{4}$. Then the left side is $\tan\frac{3\pi}{4}$, or $-\frac{\sqrt{2}}{2}$. The right side is $\tan\frac{\pi}{4}$, or $\frac{\sqrt{2}}{2}$.

e) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cot\frac{3\pi}{4}$, or -1. The right side is $\tan\frac{\pi}{4}$, or 1.

f) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{5\pi}{2}$, or 1. The right side is $\sin\left(-\frac{\pi}{2}\right)$, or -1.

Lesson 7.2, pp. 400–401

1. a) $\sin 3a$ **b)** $\cos 7x$

2. a) $\tan 60°$; $\sqrt{3}$ **b)** $\cos\frac{\pi}{3}$; $\frac{1}{2}$

3. a) $30° + 45°$ **b)** $30° - 45°$ **c)** $\frac{\pi}{6} - \frac{\pi}{3}$ **d)** $\frac{\pi}{4} - \frac{\pi}{6}$ **e)** $60° + 45°$ **f)** $\frac{\pi}{2} + \frac{\pi}{3}$

4. a) $\frac{\sqrt{2} + \sqrt{6}}{4}$ **b)** $\frac{\sqrt{2} + \sqrt{6}}{4}$ **c)** $2 + \sqrt{3}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **e)** $\frac{\sqrt{2} - \sqrt{6}}{4}$ **f)** $-2 + \sqrt{3}$

5. **a)** $-\frac{1}{2}$ **d)** $-\frac{1}{2}$
b) $-\frac{\sqrt{2}}{2}$ **e)** $\frac{\sqrt{3}}{3}$
c) 1 **f)** $-\frac{\sqrt{3}}{2}$

6. **a)** $-\sin x$ **d)** $\tan x$
b) $\sin x$ **e)** $-\sin x$
c) $-\sin x$ **f)** $-\tan x$

7. **a)** $\sin(\pi + x)$ is equivalent to $\sin x$ translated π to the left, which is equivalent to $-\sin x$.
b) $\cos\left(x + \frac{3\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{3\pi}{2}$ to the left, which is equivalent to $\sin x$.
c) $\cos\left(x + \frac{\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{\pi}{2}$ to the left, which is equivalent to $-\sin x$.
d) $\tan(x + \pi)$ is equivalent to $\tan x$ translated π to the left, which is equivalent to $\tan x$.
e) $\sin(x - \pi)$ is equivalent to $\sin x$ translated π to the right, which is equivalent to $-\sin x$.
f) $\tan(2\pi - x)$ is equivalent to $\tan(-x)$, which is equivalent to $\tan x$ reflected in the y-axis, which is equivalent to $-\tan x$.

8. **a)** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
b) $-2 + \sqrt{3}$ **e)** $-2 - \sqrt{3}$
c) $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **f)** $-2 - \sqrt{3}$

9. **a)** $\frac{63}{65}$ **d)** $\frac{56}{65}$
b) $-\frac{16}{65}$ **e)** $-\frac{16}{63}$
c) $-\frac{33}{65}$ **f)** $-\frac{56}{33}$

10. $\frac{323}{325}; \frac{323}{36}$

11. **a)** $\cos\left(\frac{\pi}{2} - x\right)$
$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$
$= (0)(\cos x) + (1)(\sin x)$
$= 0 + \sin x$
$= \sin x$
b) $\sin\left(\frac{\pi}{2} - x\right)$
$= \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$
$= (1)(\cos x) - (0)(\sin x)$
$= \cos x - 0$
$= \cos x$

12. **a)** 0 **b)** $-\sqrt{3}\sin x$

13. $\tan f$, $\cos f \neq 0$, $\cos g \neq 0$

14.
Write $\sin a$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $a \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos a$.
↓
Write $\sin b$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $b \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos b$.
↓
Use the formula $\cos(a + b) = \cos a\cos b - \sin a\sin b$ to evaluate $\cos(a + b)$.

15. See compound angle formulas listed on p. 399.
The two sine formulas are the same, except for the operators. Remembering that the same operator is used on both the left and right sides in both equations will help you remember the formulas.
Similarly, the two cosine formulas are the same, except for the operators. Remembering that the operator on the left side is the opposite of the operator on the right side in both equations will help you remember the formulas.
The two tangent formulas are the same, except for the operators in the numerator and the denominator on the right side. Remembering that the operators in the numerator and the denominator are opposite in both equations, and that the operator in the numerator is the same as the operator on the left side, will help you remember the formulas.

16. $2\sin\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$
$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{D}{2}\right) + \left(\cos\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)\left(\left(\cos\frac{C}{2}\right) \times \left(\cos\frac{D}{2}\right) + \left(\sin\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)$
$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2}\right) + \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2}\right) + \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\sin^2\frac{C}{2}\right) + \left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\sin^2\frac{D}{2}\right)\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2} + \sin^2\frac{D}{2}\right) + 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right) + 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)$
$= \sin\left(2\left(\frac{C}{2}\right)\right) + \sin\left(2\left(\frac{D}{2}\right)\right)$
$= \sin C + \sin D$

17. $\cot(x + y) = \frac{\cot x\cot y - 1}{\cot x + \cot y}$

18. Let $C = x + y$ and let $D = x - y$.
$\cos C + \cos D$
$= \cos(x + y) + \cos(x - y)$
$= \cos x\cos y - \sin x\sin y + \cos x\cos y + \sin x\sin y$
$= 2\cos x\cos y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C + \cos D$
$= 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$

19. Let $C = x + y$ and let $D = x - y$.
$\cos C - \cos D$
$= \cos(x + y) - \cos(x - y)$
$= \cos x\cos y - \sin x\sin y - (\cos x\cos y - \sin x\sin y)$
$= -2\sin x\sin y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C - \cos D$
$= -2\sin\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)$

Lesson 7.3, pp. 407–408

1. **a)** $\sin 10x$ **d)** $\tan 8x$
b) $\cos 2\theta$ **e)** $2\sin 2\theta$
c) $\cos 6x$ **f)** $\cos\theta$

2. **a)** $\sin 90°$; 1 **d)** $\cos\frac{\pi}{6}; \frac{\sqrt{3}}{2}$
b) $\cos 60°; \frac{1}{2}$ **e)** $\cos\frac{3\pi}{4}; -\frac{\sqrt{2}}{2}$
c) $\sin\frac{\pi}{6}; \frac{1}{2}$ **f)** $\sin 120°; \frac{\sqrt{3}}{2}$

3. a) $2 \sin 2\theta \cos 2\theta$
b) $2 \sin^2(1.5x) - 1$
c) $\dfrac{2 \tan(0.5x)}{1 - \tan^2(0.5x)}$
d) $\cos^2 3\theta - \sin^2 3\theta$
e) $2 \sin(0.5x) \cos(0.5x)$
f) $\dfrac{2 \tan(2.5\theta)}{1 - \tan^2(2.5\theta)}$

4. $\sin 2\theta = \dfrac{24}{25}$, $\cos 2\theta = -\dfrac{7}{25}$,
$\tan 2\theta = -\dfrac{24}{7}$

5. $\sin 2\theta = -\dfrac{336}{625}$, $\cos 2\theta = \dfrac{527}{625}$,
$\tan 2\theta = -\dfrac{336}{527}$

6. $\sin 2\theta = -\dfrac{120}{169}$, $\cos 2\theta = -\dfrac{119}{169}$,
$\tan 2\theta = \dfrac{120}{119}$

7. $\sin 2\theta = -\dfrac{24}{25}$, $\cos 2\theta = \dfrac{7}{25}$,
$\tan 2\theta = -\dfrac{24}{7}$

8. $a = \dfrac{1}{2}$

9. Jim can find the sine of $\frac{\pi}{8}$ by using the formula $\cos 2x = 1 - 2\sin^2 x$ and isolating $\sin x$ on one side of the equation. When he does this, the formula becomes $\sin x = \pm\sqrt{\dfrac{1 - \cos 2x}{2}}$. The cosine of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$, so $\sin \frac{\pi}{8} = \pm\sqrt{\dfrac{1 - \cos\frac{\pi}{4}}{2}}$
$= \pm\dfrac{\sqrt{2 - \sqrt{2}}}{2}$.
Since $\frac{\pi}{8}$ is in the first quadrant, the sign of $\sin \frac{\pi}{8}$ is positive.

10. Marion can find the cosine of $\frac{\pi}{12}$ by using the formula $\cos 2x = 2\cos^2 x - 1$ and isolating $\cos x$ on one side of the equation. When she does this, the formula becomes $\cos x = \pm\sqrt{\dfrac{1 + \cos 2x}{2}}$. The cosine of $\frac{\pi}{6}$ is $\frac{\sqrt{3}}{2}$, so $\cos \frac{\pi}{12} = \pm\sqrt{\dfrac{1 + \cos\frac{\pi}{6}}{2}}$
$= \pm\dfrac{\sqrt{2 + \sqrt{3}}}{2}$.
Since $\frac{\pi}{12}$ is in the first quadrant, the sign of $\cos \frac{\pi}{12}$ is positive.

11. a) $\sin 4x$
$= (2)(2 \sin x \cos x)(\cos 2x)$
$= (2)(2 \sin x \cos x)(1 - 2\sin^2 x)$
$= (4 \sin x \cos x)(1 - 2\sin^2 x)$
$= 4 \sin x \cos x - 8 \sin^3 x \cos x$

b) $\sin \dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\sin 4\left(\dfrac{2\pi}{3}\right) = 4 \sin \dfrac{2\pi}{3} \cos \dfrac{2\pi}{3} - 8 \sin^3 \dfrac{2\pi}{3} \cos \dfrac{2\pi}{3}$

$\sin \dfrac{8\pi}{3} = (4)\left(\dfrac{\sqrt{3}}{2}\right)\left(-\dfrac{1}{2}\right) - (8)\left(\dfrac{\sqrt{3}}{2}\right)^3\left(-\dfrac{1}{2}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - (-4)\left(\dfrac{3\sqrt{3}}{8}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{3\sqrt{3}}{2}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{6\sqrt{3}}{4}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} + -\dfrac{6\sqrt{3}}{4}$

$\sin \dfrac{8\pi}{3} = \dfrac{2\sqrt{3}}{4}$

$\sin \dfrac{8\pi}{3} = \dfrac{\sqrt{3}}{2}$

12. a) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2 \cos\theta \sin\theta$
$\sin 3\theta = (\sin 2\theta + \theta)$
$= (2 \cos\theta \sin\theta)(\cos\theta) + (\cos^2\theta - \sin^2\theta)(\sin\theta)$
$= 2\cos^2\theta \sin\theta + \cos^2\theta \sin\theta - \sin^3\theta$
$= 3\cos^2\theta \sin\theta - \sin^3\theta$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
$\sin 2\theta = 2 \cos\theta \sin\theta$
$\cos 3\theta = (\cos 2\theta + \theta)$
$= (\cos^2\theta - \sin^2\theta)(\cos\theta) - (2\cos\theta \sin\theta)(\sin\theta)$
$= \cos^3\theta - \cos x \sin^2\theta - 2\cos\theta \sin^2\theta$
$= \cos^3\theta - 3\cos\theta \sin^2\theta$

c) $\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$
$\tan 3\theta = (\tan 2\theta + \theta)$

$= \dfrac{\dfrac{2\tan\theta}{1 - \tan^2\theta} + \tan\theta}{1 - \left(\dfrac{2\tan\theta}{1 - \tan^2\theta}\right)\tan\theta}$

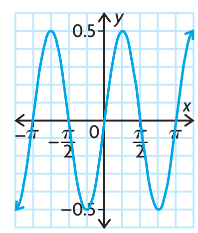
$= \dfrac{\dfrac{2\tan\theta + \tan\theta - \tan^3\theta}{1 - \tan^2\theta}}{\dfrac{1 - \tan^2\theta - 2\tan^2\theta}{1 - \tan^2\theta}}$

$= \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

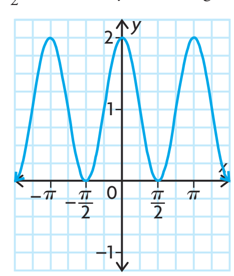
13. a) $-\dfrac{4\sqrt{2}}{9}$
b) $-\dfrac{7}{9}$
c) $\dfrac{\sqrt{3}}{3}$
d) $-\dfrac{10\sqrt{2}}{27}$

14.
Write $\sin a$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Choose the negative value of x since $a \in \left[\frac{\pi}{2}, \pi\right]$, and determine $\cos a$.
↓
Write $\cos a$ in terms of $\frac{x}{r}$.
↓
Use the formula $\sin 2a = 2 \sin a \cos a$ to evaluate $\sin 2a$.

15. a) Use the formula $\sin 2x = 2 \sin x \cos x$ to determine that $\sin x \cos x = \dfrac{\sin 2x}{2}$.
Then graph the function $f(x) = \dfrac{\sin 2x}{2}$ by vertically compressing $f(x) = \sin x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.

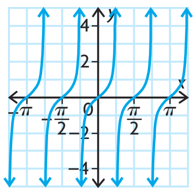


b) Use the formula $\cos 2x = 2\cos^2 x - 1$ to determine that $2\cos^2 x = \cos 2x + 1$.
Then graph the function $f(x) = \cos 2x + 1$ by horizontally compressing $f(x) = \cos x$ by a factor of $\frac{1}{2}$ and vertically translating it 1 unit up.



c) Use the formula $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$ to determine that $\dfrac{\tan x}{1 - \tan^2 x} = \dfrac{\tan 2x}{2}$.

Then graph the function $f(x) = \frac{\tan 2x}{2}$ by vertically compressing $f(x) = \tan x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.



16. **a)** $\frac{\tan^{-1} x}{2} = \tan^{-1} y$

b) $\frac{\cos^{-1} x}{2} = \cos^{-1} y$

c) $\frac{\cos^{-1} x}{2} = \csc^{-1} y$ or

$\frac{\cos^{-1} x}{2} = \sin^{-1}\left(\frac{1}{y}\right)$

d) $\frac{\sin^{-1} x}{2} = \frac{\sec^{-1} y}{4}$ or

$\frac{\sin^{-1} x}{2} = \frac{\cos^{-1}\left(\frac{1}{y}\right)}{4}$

17. **a)** $x = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

b) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{2}$

18. **a)** $\frac{2\tan\theta}{1 + \tan^2\theta}$

b) $\frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

c) $\tan\theta$

d) $\tan\theta$

Mid-Chapter Review, p. 411

1. **a)** $\cos\frac{31\pi}{16}$ **d)** $\cos\frac{7\pi}{5}$

b) $\sin\frac{2\pi}{9}$ **e)** $\sin\frac{2\pi}{7}$

c) $\tan\frac{19\pi}{10}$ **f)** $\tan\frac{7\pi}{4}$

2. $y = 6\sin x + 4$

3. **a)** $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

b) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

c) $\frac{1 + \tan x}{1 - \tan x}$

d) $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$

4. **a)** $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

b) $\frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$

c) $\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

d) $-\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$

5. **a)** $\sqrt{3}$ **c)** $\frac{1}{2}$

b) 0 **d)** 1

6. **a)** $\tan 2x$ **d)** $\cos x$

b) $\sin x$ **e)** $\sqrt{2}(\cos x - \sin x)$

c) $\sin x$ **f)** $\frac{\tan x - 1}{1 + \tan x}$

7. $2\sqrt{3}\cos\left(x + \frac{\pi}{3}\right)$

8. **a)** $-\frac{1}{2}$ **c)** $\frac{\sqrt{2}}{2}$

b) $-\frac{1}{2}$ **d)** -1

9. **a)** $-\frac{\sqrt{11}}{11}$ **c)** $\frac{2\sqrt{10}}{11}$

b) $-\frac{\sqrt{110}}{11}$ **d)** $\frac{9}{11}$

10. $\sin 2x = \frac{24}{25}$; $\cos 2x = \frac{7}{25}$

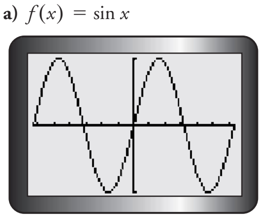
11. $\sin 2x = \frac{120}{169}$

12. $\tan 2x = \frac{24}{7}$

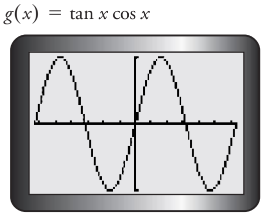
Lesson 7.4, pp. 417–418

1. Answers may vary. For example, $\sin\frac{\pi}{6} = \frac{1}{2}$; $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

2. **a)** $f(x) = \sin x$



$g(x) = \tan x \cos x$



b) $\sin x = \tan x \cos x$

c) $\tan x \cos x = \left(\frac{\sin x}{\cos x}\right)\cos x$

$= \frac{\sin x \cos x}{\cos x} = \sin x$

d) The identity is not true when $\cos x = 0$ because when $\cos x = 0$, $\tan x$, or $\frac{\sin x}{\cos x}$, is undefined.

3. **a)** C; $\sin x \cot x = \cos x$

b) D; $1 - 2\sin^2 x = 2\cos^2 x - 1$

c) B; $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

d) A; $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

4. **a)** $\sin x \cot x = \cos x$

LS $= \sin x \cot x$

$= (\sin x)\left(\frac{\cos x}{\sin x}\right)$

$= \frac{\sin x \cos x}{\sin x}$

$= \cos x$

= RS

b) $1 - 2\sin^2 x = 2\cos^2 x - 1$

$1 - 2\sin^2 x - 2\cos^2 x + 1 = 0$

$2 - 2\sin^2 x - 2\cos^2 x = 0$

$2 - 2(\sin^2 x + \cos^2 x) = 0$

$2 - 2(1) = 0$

$2 - 2 = 0$

$0 = 0$

c) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

LS $= (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x \cos x$
$+ \cos^2 x$

$= (\sin^2 x + \cos^2 x)$
$+ 2\sin x \cos x$

$= 1 + 2\sin x \cos x$

= RS

d) $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

RS $= \sin^2 x + \cos^2 x + \tan^2 x$

$= (\sin^2 x + \cos^2 x) + \tan^2 x$

$= 1 + \tan^2 x$

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

= LS

5. **a)** Answers may vary. For example, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$; $\frac{1}{\cos\frac{\pi}{6}} = \frac{2\sqrt{3}}{3}$.

b) Answers may vary. For example,

$1 - \tan^2\left(\frac{\pi}{4}\right) = 1 - (1)^2$

$= 1 - 1 = 0;$

$\sec^2\left(\frac{\pi}{4}\right) = (\sqrt{2})^2 = 2$

c) Answers may vary. For example,

$\sin\left(\frac{\pi}{2} + \pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1;$

$\cos\left(\frac{\pi}{2}\right)\cos\pi + \sin\left(\frac{\pi}{2}\right)\sin\pi$

$= (0)(-1) + (1)(0)$

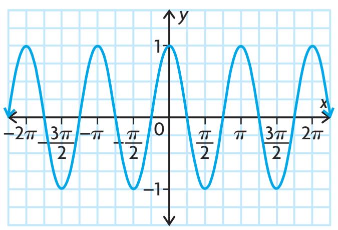
$= 0 + 0 = 0$

d) Answers may vary. For example,

$$\cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$1 + 2\sin^2\left(\frac{\pi}{3}\right) = 1 + (2)\left(\frac{\sqrt{3}}{2}\right)^2$$
$$= 1 + (2)\left(\frac{3}{4}\right)$$
$$= 1 + \frac{6}{4} = \frac{10}{4}$$
$$= \frac{5}{2}$$

6. Answers may vary. For example, cos 2*x*.



7. $$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\sec^2 x}$$
$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \cos^2 x$$
$$= \cos^2 x - \sin^2 x$$
$$= \cos 2x$$

8. $$\text{LS} = \frac{1 + \tan x}{1 + \cot x} \qquad \text{RS} = \frac{1 - \tan x}{\cot x - 1}$$
$$= \frac{1 + \tan x}{1 + \frac{1}{\tan x}} \qquad = \frac{1 - \tan x}{\frac{1}{\tan x} - 1}$$
$$= \frac{1 + \tan x}{\frac{\tan x + 1}{\tan x}} \qquad = \frac{1 - \tan x}{\frac{1 - \tan x}{\tan x}}$$
$$= \tan x \qquad = \tan x$$

Since the right side and the left side are equal, $\frac{1 + \tan x}{1 + \cot x} = \frac{1 - \tan x}{\cot x - 1}$

9. a) $$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta}$$
$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta)(\cos\theta + \sin\theta)}$$
$$= \frac{\cos\theta - \sin\theta}{\cos\theta}$$
$$= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$
$$= 1 - \tan\theta$$

b) $$\text{LS} = \tan^2 x - \sin^2 x$$
$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$
$$= \sin^2 x\left(\frac{1}{\cos^2 x} - 1\right)$$
$$= \sin^2 x(\sec^2 x - 1)$$
$$= \sin^2 x \tan^2 x$$
$$= \text{RS}$$

So $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$.

c) $$\tan^2 x - \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x$$
$$\tan^2 x - \cos^2 x + \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x + \cos^2 x$$
$$\tan^2 x = \frac{1}{\cos^2 x} - 1$$
$$\tan^2 x = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$
$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x}$$
$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$
$$\tan^2 x = \tan^2 x$$

d) $$\frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta}$$
$$= \frac{1 - \cos\theta}{(1 + \cos\theta)(1 - \cos\theta)} + \frac{1 + \cos\theta}{(1 - \cos\theta)(1 + \cos\theta)}$$
$$= \frac{1 - \cos\theta}{1 - \cos^2\theta} + \frac{1 + \cos\theta}{1 - \cos^2\theta}$$
$$= \frac{1 - \cos\theta + 1 + \cos\theta}{1 - \cos^2\theta}$$
$$= \frac{2}{1 - \cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$

10. a) $$\cos x \tan^3 x = \sin x \tan^2 x$$
$$\frac{\cos x \tan^3 x}{\tan^2 x} = \frac{\sin x \tan^2 x}{\tan^2 x}$$
$$\cos x \tan x = \sin x$$
$$\cos x\left(\frac{\sin x}{\cos x}\right) = \sin x$$
$$\sin x = \sin x$$

b) $$\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$$
$$\sin^2\theta + \cos^4\theta - \sin^4\theta = \cos^2\theta + \sin^4\theta - \sin^4\theta$$
$$\sin^2\theta + \cos^4\theta - \sin^4\theta = \cos^2\theta$$
$$\sin^2\theta + \cos^4\theta - \sin^4\theta - \sin^2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$$
$$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \cos^2\theta - \sin^2\theta$$
$$\cos^2\theta + \sin^2\theta = 1$$
$$1 = 1$$

c) $$(\sin x + \cos x)\left(\frac{\tan^2 x + 1}{\tan x}\right) = \frac{1}{\cos x} + \frac{1}{\sin x}$$
$$(\sin x + \cos x)\left(\frac{\sec^2 x}{\tan x}\right) = \frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x}$$
$$(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{1}{\tan x}\right) = \frac{\sin x + \cos x}{\cos x \sin x}$$
$$(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right) = \frac{\sin x + \cos x}{\cos x \sin x}$$
$$(\sin x + \cos x)\left(\frac{1}{\cos x \sin x}\right) = \frac{\sin x + \cos x}{\cos x \sin x}$$
$$\frac{\sin x + \cos x}{\cos x \sin x} = \frac{\sin x + \cos x}{\cos x \sin x}$$

d) $$\tan^2\beta + \cos^2\beta + \sin^2\beta = \frac{1}{\cos^2\beta}$$
$$\tan^2\beta + 1 = \frac{1}{\cos^2\beta}$$
$$\tan^2\beta + 1 = \sec^2\beta$$

Since $\tan^2\beta + 1 = \sec^2\beta$ is a known identity, $\tan^2\beta + \cos^2\beta + \sin^2\beta$ must equal $\frac{1}{\cos^2\beta}$.

e) $$\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x;$$
$$\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x = \sqrt{2}\cos x;$$
$$2\sin\frac{\pi}{4}\cos x = \sqrt{2}\cos x;$$
$$(2)\left(\frac{\sqrt{2}}{2}\right)(\cos x) = \sqrt{2}\cos x;$$
$$\sqrt{2}\cos x = \sqrt{2}\cos x$$

f) $$\sin\left(\frac{\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right) = -\sin x;$$
$$\sin\left(\frac{\pi}{2} - x\right)\left(\frac{\cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right) = -\sin x;$$
$$\left(\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x\right) \times \left(\frac{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}\right) = -\sin x;$$
$$((1)(\cos x) - (0)(\sin x)) \times \left(\frac{(0)(\cos x) - (1)(\sin x)}{(1)(\cos x) + (0)(\sin x)}\right) = -\sin x;$$
$$(\cos x - 0)\left(\frac{0 - \sin x}{\cos x + 0}\right) = -\sin x;$$
$$(\cos x)\left(-\frac{\sin x}{\cos x}\right) = -\sin x;$$
$$-\sin x = -\sin x$$

11. a) $$\frac{\cos 2x + 1}{\sin 2x} = \cot x$$
$$\frac{2\cos^2 x - 1 + 1}{2\sin x \cos x} = \cot x$$
$$\frac{2\cos^2 x}{2\sin x \cos x} = \cot x$$
$$\frac{\cos x}{\sin x} = \cot x$$
$$\cot x = \cot x$$

b)
$$\frac{\sin 2x}{1-\cos 2x} = \cot x$$
$$\frac{2\sin x\cos x}{1-(1-2\sin^2 x)} = \cot x$$
$$\frac{2\sin x\cos x}{1-1+2\sin^2 x}=\cot x$$
$$\frac{2\sin x \cos x}{2\sin^2 x}=\cot x$$
$$\frac{\cos x}{\sin x}=\cot x$$
$$\cot x = \cot x$$

c)
$$(\sin x+\cos x)^2 = 1+\sin 2x;$$
$$\sin^2 x + \sin x\cos x + \sin x \cos x + \cos^2 x = 1 + 2\sin x\cos x;$$
$$\sin^2 x + 2\sin x\cos x+\cos^2 x = 1+2\sin x\cos x;$$
$$(\cos^2 x+\sin^2 x)+2\sin x\cos x = 1+2\sin x\cos x;$$
$$1+2\sin x\cos x = 1+2\sin x\cos x$$

d)
$$\cos^4\theta - \sin^4\theta = \cos 2\theta$$
$$(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta) = \cos^2\theta-\sin^2\theta$$
$$(1)(\cos^2\theta-\sin^2\theta)=\cos^2\theta-\sin^2\theta$$
$$\cos^2\theta-\sin^2\theta=\cos^2\theta-\sin^2\theta$$

e)
$$\cot\theta-\tan\theta = 2\cot 2\theta$$
$$\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=2\frac{\cos 2\theta}{\sin 2\theta}$$
$$\frac{\cos^2\theta}{\sin\theta\cos\theta}-\frac{\sin^2\theta}{\cos\theta\sin\theta} = (2)\left(\frac{\cos 2\theta}{2\cos\theta\sin\theta}\right)$$
$$\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}=\frac{\cos 2\theta}{\cos\theta\sin\theta}$$
$$\frac{\cos 2\theta}{\cos\theta\sin\theta}=\frac{\cos2\theta}{\cos\theta\sin\theta}$$

f)
$$\cot\theta+\tan\theta = 2\csc 2\theta$$
$$\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}=2\frac{1}{\sin 2\theta}$$
$$\frac{\cos^2\theta}{\sin\theta\cos\theta}+\frac{\sin^2\theta}{\cos\theta\sin\theta}=(2)\left(\frac{1}{2\cos\theta\sin\theta}\right)$$
$$\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}=\frac{1}{\cos\theta\sin\theta}$$
$$\frac{1}{\cos\theta\sin\theta}=\frac{1}{\cos\theta\sin\theta}$$

g)
$$\frac{1+\tan x}{1-\tan x}=\tan\left(x+\frac{\pi}{4}\right)$$
$$\frac{1+\tan x}{1-\tan x}=\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}$$
$$\frac{1+\tan x}{1-\tan x}=\frac{\tan x + 1}{1-(\tan x)(1)}$$
$$\frac{1+\tan x}{1-\tan x}=\frac{1+\tan x}{1-\tan x}$$

h)
$$\csc 2x + \cot 2x = \cot x;$$
$$\frac{1}{\sin 2x}+\frac{1}{\tan 2x}=\cot x;$$
$$\frac{1}{2\sin x\cos x}+\frac{1}{\frac{2\tan x}{1-\tan^2 x}}=\cot x;$$
$$\frac{1}{2\sin x\cos x}+\frac{1-\tan^2 x}{2\tan x}=\cot x;$$
$$\frac{1}{2\sin x\cos x}+\frac{1-\tan^2x}{2\frac{\sin x}{\cos x}}=\cot x;$$
$$\frac{1}{2\sin x\cos x}+\frac{(\cos x)(1-\tan^2 x)}{2\sin x} = \frac{\cos x}{\sin x};$$
$$\frac{1}{2\sin x\cos x}+\frac{(\cos x)(1-\tan^2x)(\cos x)}{2\sin x\cos x}=\frac{(\cos x)(2\cos x)}{(\sin x)(2\cos x)};$$
$$\frac{1}{2\sin x\cos x}+\frac{(\cos^2 x)(1-\tan^2 x)}{2\sin x\cos x}=\frac{2\cos^2 x}{2\sin x\cos x};$$
$$\frac{1}{2\sin x\cos x}+\frac{\cos^2 x-(\tan^2 x)(\cos^2 x)}{2\sin x\cos x}=\frac{2\cos^2x}{2\sin x\cos x};$$
$$\frac{1}{2\sin x\cos x}+\frac{\cos^2x-\sin^2x}{2\sin x\cos x}=\frac{2\cos^2x}{2\sin x\cos x};$$
$$\frac{1+\cos^2x-\sin^2x}{2\sin x\cos x}=\frac{2\cos^2 x}{2\sin x\cos x};$$
$$\frac{1+\cos^2x-\sin^2x}{2\sin x\cos x}-\frac{2\cos^2x}{2\sin x\cos x}=\frac{2\cos^2x}{2\sin x\cos x}-\frac{2\cos^2x}{2\sin x\cos x};$$
$$\frac{1+\cos^2x-\sin^2x-2\cos^2x}{2\sin x\cos x}=0;$$
$$\frac{1-\sin^2x-\cos^2x}{2\sin x\cos x}=0;$$
$$\frac{1-(\sin^2x+\cos^2x)}{2\sin x\cos x}=0;$$
$$\frac{1-1}{2\sin x\cos x}=0;$$
$$\frac{0}{2\sin x\cos x}=0;$$
$$0=0$$

i)
$$\frac{2\tan x}{1+\tan^2 x}=\sin 2x$$
$$\frac{2\tan x}{\sec^2 x}=\sin 2x$$
$$\frac{2\tan x}{\frac{1}{\cos^2 x}}=\sin 2x$$
$$(2\tan x)(\cos^2 x)=\sin 2x$$
$$\left(\frac{2\sin x}{\cos x}\right)(\cos^2 x)=\sin 2x$$
$$\sin 2x = 2\sin x\cos x$$

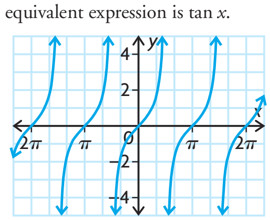
Since $\sin 2x = 2\sin x\cos x$ is a known identity, $\frac{2\tan x}{1-\tan^2 x}$ must equal $\sin 2x$.

j)
$$\sec 2t = \frac{\csc t}{\csc t - 2\sin t}$$
$$\frac{1}{\cos 2t}=\frac{\frac{1}{\sin t}}{\frac{1}{\sin t}-2\sin t}$$
$$\frac{1}{\cos 2t}=\frac{\frac{1}{\sin t}}{\frac{1}{\sin t}-\frac{2\sin^2 t}{\sin t}}$$
$$\frac{1}{\cos 2t}=\frac{\frac{1}{\sin t}}{\frac{1-2\sin^2 t}{\sin t}}$$
$$\frac{1}{\cos 2t}=\frac{1}{\sin t}\times\frac{\sin t}{1-2\sin^2 t}$$
$$\frac{1}{\cos 2t}=\frac{1}{1-2\sin^2 t}$$
$$\frac{1}{\cos 2t}=\frac{1}{\cos 2t}$$

k)
$$\csc 2\theta = \frac{1}{2}\sec\theta\csc\theta$$
$$\frac{1}{\sin 2\theta}=\left(\frac12\right)\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta}\right)$$
$$\frac{1}{\sin 2\theta}=\frac{1}{2\cos\theta\sin\theta}$$
$$\frac{1}{2\sin\theta\cos\theta}=\frac{1}{2\sin\theta\cos\theta}$$

l)
$$\frac{1}{\cos t}=\frac{2\sin t\cos t}{\sin t}-\frac{2\cos^2 t - 1}{\cos t}$$
$$\frac{\sin t}{\cos t\sin t}=\frac{2\sin t\cos^2 t}{\sin t\cos t}-\frac{(\sin t)(2\cos^2 t-1)}{\cos t\sin t}$$
$$\frac{\sin t}{\cos t\sin t}=\frac{2\sin t\cos^2 t}{\sin t\cos t}-\frac{2\cos^2 t\sin t-\sin t}{\cos t\sin t}$$
$$\frac{\sin t}{\cos t\sin t}=\frac{2\sin t\cos^2 t}{\sin t\cos t}-\frac{2\cos^2t\sin t-\sin t}{\sin t\cos t}$$
$$\frac{\sin t}{\cos t\sin t}=\frac{2\sin t\cos^2 t}{\sin t\cos t}+\frac{-2\sin t\cos^2 t+\sin t}{\sin t\cos t}$$
$$\frac{\sin t}{\cos t\sin t}=\frac{\sin t}{\cos t\sin t}$$

12. Answers may vary. For example, an equivalent expression is $\tan x$.



13. $\dfrac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

$\dfrac{\sin x + 2\sin x\cos x}{1 + \cos x + \cos 2x} = \tan x$

$\dfrac{\sin x(1 + 2\cos x)}{1 + \cos x + \cos 2x} = \tan x$

$\dfrac{\sin x(1 + 2\cos x)}{\cos x + (1 + \cos 2x)} = \tan x$

$\dfrac{\sin x(1 + 2\cos x)}{\cos x + 2\cos^2 x} = \tan x$

$\dfrac{\sin x(1 + 2\cos x)}{\cos x(1 + 2\cos x)} = \tan x$

$\dfrac{\sin x}{\cos x} = \tan x$

$\tan x = \tan x$

14.

Definition	**Methods of Proof**
A statement of the equivalence of two trigonometric expressions	Both sides of the equation must be shown to be equivalent through graphing or simplifying/rewriting.

Trigonometric Identities

Examples	**Non-Examples**
$\cos 2x + \sin^2 x = \cos^2 x$ $\cos 2x + 1 = 2\cos^2 x$	$\cos 2x - 2\sin^2 x = 1$ $\cot^2 x + \csc^2 x = 1$

15. She can determine whether the equation $2\sin x\cos x = \cos 2x$ is an identity by trying to simplify and/or rewrite the left side of the equation so that it is equivalent to the right side of the equation. Alternatively, she can graph the functions $y = 2\sin x\cos x$ and $y = \cos 2x$ and see if the graphs are the same. If they're the same, it's an identity, but if they're not the same, it's not an identity. By doing this she can determine it's not an identity, but she can make it an identity by changing the equation to $2\sin x\cos x = \sin 2x$.

16. a) $a = 2, b = 1, c = 1$
b) $a = -1, b = 2, c = -2$

17. $\cos 4x + 4\cos 2x + 3$; $a = 1$, $b = 4$, $c = 3$

Lesson 7.5, pp. 426–428

1. a) $\dfrac{\pi}{2}$ d) $\dfrac{7\pi}{6}$ or $\dfrac{11\pi}{6}$
b) $\dfrac{3\pi}{2}$ e) $0, \pi$, or 2π
c) $\dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$ f) $\dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

2. a) 0 or 2π d) $\dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$
b) π e) $\dfrac{\pi}{2}$ or $\dfrac{3\pi}{2}$
c) $\dfrac{\pi}{3}$ or $\dfrac{5\pi}{3}$ f) $\dfrac{\pi}{6}$ or $\dfrac{11\pi}{6}$

3. a) 2 c) $x = \dfrac{\pi}{3}$
b) quadrants I and II d) $x = \dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$

4. a) 2
b) quadrants II and III
c) 30°
d) $x = 150°$ or $210°$

5. a) 2
b) quadrants I and III
c) 1.22
d) $\theta = 1.22$ or 4.36

6. a) $\theta = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$
b) $\theta = \dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$
c) $\theta = \dfrac{\pi}{6}$ or $\dfrac{11\pi}{6}$
d) $\theta = \dfrac{4\pi}{3}$ or $\dfrac{5\pi}{3}$
e) $\theta = \dfrac{3\pi}{4}$ or $\dfrac{5\pi}{4}$
f) $\theta = \dfrac{\pi}{3}$ or $\dfrac{4\pi}{3}$

7. a) $\theta = 210°$ or $330°$
b) $\theta = 131.8°$ or $228.2°$
c) $\theta = 56.3°$ or $236.3°$
d) $\theta = 221.8°$ or $318.2°$
e) $\theta = 78.5°$ or $281.5°$
f) $\theta = 116.6°$ or $296.6°$

8. a) $x = 0.52$ or 2.62
b) $x = 0.52$ or 5.76
c) $x = 1.05$ or 5.24
d) $x = 3.67$ or 5.76

9. a) $x = 0.79$ or 3.93
b) $x = 0.52$ or 2.62
c) $x = 0$ or 6.28
d) $x = 3.67$ or 5.76
e) $x = 1.16$ or 5.12
f) $x = 1.11$ or 4.25

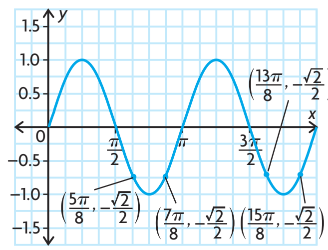
10. a) $x = 0.39, 1.18, 3.53$, or 4.32
b) $x = 0.13, 0.65, 1.70, 2.23, 3.27, 3.80, 4.84$, or 5.37
c) $x = 1.40, 1.75, 3.49, 3.84, 5.59$, or 5.93
d) $x = 0.59, 0.985, 2.16, 2.55, 3.73, 4.12, 5.304$, or 5.697
e) $x = 1.05, 2.09, 4.19$, or 5.24
f) $x = 1.05$

11. from about day 144 to about day 221

12. $1.86 \text{ s} < t < 4.14 \text{ s}$;
$9.86 \text{ s} < t < 12.14 \text{ s}$;
$17.86 \text{ s} < t < 20.14 \text{ s}$

13. $x = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$

14.



15. The value of $f(x) = \sin x$ is the same at x and $\pi - x$. In other words, it is the same at x and half the period minus x. Since the period of $f(x) = 25\sin\frac{\pi}{50}(x + 20) - 55$ is 100, if the function were not horizontally translated, its value at x would be the same as at $50 - x$. The function is horizontally translated 20 units to the left, however, so it goes through half its period from $x = -20$ to $x = 30$. At $x = 3$, the function is 23 units away from the left end of the range, so it will have the same value at $x = 30 - 23$ or $x = 7$, which is 23 units away from the right end of the range.

16. To solve a trigonometric equation **algebraically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5\cos x - 3 = 2$ would become $5\cos x = 5$, which would then become $\cos x = 1$. Next, apply the inverse of the trigonometric function to both sides of the equation. For example, the trigonometric equation $\cos x = 1$ would become $x = \cos^{-1} 1$. Finally, simplify the equation. For example, $x = \cos^{-1} 1$ would become $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.
To solve a trigonometric equation **graphically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5\cos x - 3 = 2$ would become $5\cos x = 5$, which would then become $\cos x = 1$. Next, graph both sides of the equation. For example, the functions $f(x) = \cos x$ and $f(x) = 1$ would both be graphed. Finally, find the points where the two graphs intersect. For example, $f(x) = \cos x$ and $f(x) = 1$ would intersect at $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.
Similarity: Both trigonometric functions are first isolated on one side of the equation.
Differences: The inverse of a trigonometric function is not applied in the graphical method, and the points of intersection are not obtained in the algebraic method.

17. $x = 0 + n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + n\pi$, where $n \in \mathbf{I}$

18. a) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{4}$
b) $x = \frac{\pi}{6}, \frac{\pi}{2}$, or $\frac{5\pi}{6}$

Lesson 7.6, pp. 435–437

1. a) $(\sin\theta)(\sin\theta - 1)$
b) $(\cos\theta - 1)(\cos\theta - 1)$
c) $(3\sin\theta + 2)(\sin\theta - 1)$
d) $(2\cos\theta - 1)(2\cos\theta + 1)$
e) $(6\sin x - 2)(4\sin x + 1)$
f) $(7\tan x + 8)(7\tan x - 8)$

2. a) $y = \pm\frac{\sqrt{3}}{3}, x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
b) $y = 0$ or -1, $x = 0, \pi, \frac{3\pi}{2}$, or 2π
c) $y = 0$ or $z = \frac{1}{2}, x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
d) $y = 0$ or $z = 1$, $x = 0, \pi$, or 2π

3. a) $y = \frac{1}{3}$ or $\frac{1}{2}$
b) $x = 1.05, 1.91, 4.37$, or 5.24

4. a) $\theta = 90°$ or $270°$
b) $\theta = 0°, 180°$, or $360°$
c) $\theta = 45°, 135°, 225°$, or $315°$
d) $\theta = 60°, 120°, 240°$, or $300°$
e) $\theta = 30°, 150°, 210°$, or $330°$
f) $\theta = 45°, 135°, 225°$, or $315°$

5. a) $x = 0°, 90°, 180°, 270°$, or $360°$
b) $x = 0°, 180°$, or $360°$
c) $x = 90°$ or $270°$
d) $x = 60°, 90°, 120°$, or $270°$
e) $x = 45°, 135°, 225°$, or $315°$
f) $x = 90°$ or $180°$

6. a) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
b) $x = \frac{3\pi}{2}$
c) $x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$, or 2π
d) $x = \frac{\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$
e) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$
f) $x = 0, \frac{3\pi}{2}$, or 2π

7. a) $\theta = \frac{\pi}{3}, \pi$, or $\frac{5\pi}{3}$
b) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$
c) $\theta = \pi$
d) $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
e) $\theta = \frac{\pi}{4}, 2.82, \frac{5\pi}{4}$, or 5.96
f) $\theta = 0.73, 2.41, 3.99$, or 5.44

8. a) $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$
b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$
c) $x = 0, 0.96\pi, 5.33$, or 2π
d) $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$
e) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}$, or $\frac{7\pi}{4}$
f) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π

9. a) $x = \frac{\pi}{3}, 1.98, 4.30$, or $\frac{5\pi}{3}$
b) $x = 0.45, \frac{2\pi}{3}, \frac{4\pi}{3}$, or 5.83
c) $x = \frac{\pi}{6}, 0.85, \frac{5\pi}{6}$, or 2.29
d) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

10. $x = 0.15, 1.02, 2.12$, or 2.99

11. $b = 1 + \sqrt{3}, c = \sqrt{3}$

12. $c = \frac{1}{2}$

13. $\frac{\pi}{3}\text{km} < d < \frac{2\pi}{3}\text{km}$, $\frac{4\pi}{3}\text{km} < d < \frac{5\pi}{3}\text{km}$

14. $x = 1.91$ or 4.37

15. a) $x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$
b) $x = \frac{3\pi}{4} + 2n\pi$ or $\frac{5\pi}{4} + 2n\pi$, where $n \in \mathbf{I}$

16. It is possible to have different numbers of solutions for quadratic trigonometric equations because, when factored, a quadratic trigonometric equation can be one expression multiplied by another expression or it can be a single expression squared. For example, the equation $\cos^2 x + \frac{3}{2}\cos x + \frac{1}{2}$ becomes $(\cos x + 1)\left(\cos x + \frac{1}{2}\right)$ when factored, and it has the solutions $\frac{2\pi}{3}$, π, and $\frac{4\pi}{3}$ in the interval $0 \le x \le 2\pi$. In comparison, the equation $\cos^2 x + 2\cos x + 1 = 0$ becomes $(\cos x + 1)^2$ when factored, and it has only one solution, π, in the interval $0 \le x \le 2\pi$. Also, different expressions produce different numbers of solutions. For example, the expression $\cos x + \frac{1}{2}$ produces two solutions in the interval $0 \le x \le 2\pi$ $\left(\frac{2\pi}{3} \text{ and } \frac{4\pi}{3}\right)$ because $\cos x = -\frac{1}{2}$ for two different values of x. The expression $\cos x + 1$, however, produces only one solution in the interval $0 \le x \le 2\pi$ (π), because $\cos x = -1$ for only one value of x.

17. $a = \frac{\pi}{4}, \frac{5\pi}{4}$

18. $x = 0.72, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 5.56

19. $x = 15°, 75°, 105°, 165°, 195°, 255°, 285°$, or $345°$

20. $\theta = 0.96$

Chapter Review, p. 440

1. a) Answers may vary. For example, $\sin\frac{7\pi}{10}$.
b) Answers may vary. For example, $\cos\frac{8\pi}{7}$.
c) Answers may vary. For example, $\sin\frac{6\pi}{7}$.
d) Answers may vary. For example, $\cos\frac{\pi}{7}$.

2. $y = 5\cos(x) - 8$

3. a) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$
b) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$
c) $\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}$
d) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

4. a) $-\frac{\sqrt{3}}{3}$ b) $-\frac{\sqrt{3}}{2}$

5. a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{\sqrt{2}}{2}$ d) $\sqrt{3}$

6. a) $\sin 2x = \frac{24}{25}, \cos 2x = \frac{7}{25}$, $\tan 2x = \frac{24}{7}$
b) $\sin 2x = -\frac{336}{625}, \cos 2x = -\frac{527}{625}$, $\tan 2x = \frac{336}{527}$
c) $\sin 2x = -\frac{120}{169}, \cos 2x = \frac{119}{169}$, $\tan 2x = -\frac{120}{119}$

7. a) trigonometric identity
b) trigonometric equation
c) trigonometric identity
d) trigonometric equation

8. $\frac{\cos^2 x}{\cot^2 x} = 1 - \cos^2 x$
$\frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} = 1 - \cos^2 x$
$\frac{(\cos^2 x)(\sin^2 x)}{\cos^2 x} = 1 - \cos^2 x$
$\sin^2 x = 1 - \cos^2 x$
$1 - \cos^2 x = 1 - \cos^2 x$

9. $\frac{2(\sec^2 x - \tan^2 x)}{\csc x} = \sin 2x \sec x$

$\frac{2(1)}{\csc x} = \sin 2x \sec x$

$\frac{2}{\csc x} = \sin 2x \sec x$

$2 \sin x = \sin 2x \sec x$

$\frac{2 \sin x \cos x}{\cos x} = \sin 2x \sec x$

$\frac{\sin 2x}{\cos x} = \sin 2x \sec x$

$\sin 2x \sec x = \sin 2x \sec x$

10. a) $x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

b) $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

c) $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

11. a) $y = -2$ or 2

b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

12. a) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

b) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π

c) $x = \frac{\pi}{4}, \frac{2\pi}{3}, \frac{4\pi}{3}$, or $\frac{7\pi}{4}$

d) $x = 0.95$ or 4.09

13. $x = \frac{\pi}{2}, \pi$, or $\frac{3\pi}{2}$

Chapter Self-Test, p. 441

1. $\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x = \cos x$

$\frac{1 - 2\sin^2 x}{\cos x + \sin x} + \sin x - \sin x$

$= \cos x - \sin x$

$\frac{1 - 2\sin^2 x}{\cos x + \sin x} = \cos x - \sin x$

$1 - 2\sin^2 x = (\cos x - \sin x) \times (\cos x + \sin x)$

$\cos 2x = (\cos x - \sin x) \times (\cos x + \sin x)$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos 2x$

2. all real numbers x, where $0 \le x \le 2\pi$

3. a) $x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$

b) $x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{3}$

c) $x = \frac{5\pi}{4}$ or $x = \frac{7\pi}{4}$

4. $a = 2, b = 1$

5. $t = 7, 11, 19$, and 23

6. Nina can find the cosine of $\frac{11\pi}{4}$ by using the formula $\cos(x + y) = \cos x \cos y - \sin x \sin y$. The cosine of π is -1, and the cosine of $\frac{7\pi}{4}$ is $\frac{\sqrt{2}}{2}$. Also, the sine of π is 0, and the sine of $\frac{7\pi}{4}$ is $-\frac{\sqrt{2}}{2}$. Therefore,

$\cos \frac{11\pi}{4} = \cos\left(\pi + \frac{7\pi}{4}\right)$

$= \left(-1 \times \frac{\sqrt{2}}{2}\right) - \left(0 \times -\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{2} - 0$

$= -\frac{\sqrt{2}}{2}$

7. $x = 3.31$ or 6.12

8. $-\frac{33}{65}, -\frac{16}{65}$

9. a) $-\frac{4\sqrt{5}}{9}$ c) $\sqrt{\frac{3 - \sqrt{5}}{6}}$

b) $\frac{1}{9}$ d) $\frac{22}{27}$

10. a) $x = -\frac{5\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}$, or $\frac{5\pi}{3}$

b) $x = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}$, or $\frac{4\pi}{3}$

c) $x = -\pi$ and π

Chapter 8

Getting Started, p. 446

1. a) $\frac{1}{5^2} = \frac{1}{25}$ d) $\sqrt[3]{125} = 5$

b) 1 e) $-\sqrt{121} = -11$

c) $\sqrt{36} = 6$ f) $\left(\sqrt[3]{\frac{27}{8}}\right)^2 = \frac{9}{4}$

2. a) $3^7 = 2187$ d) $7^4 = 2401$

b) $(-2)^2 = 4$ e) $8^{\frac{2}{3}} = 4$

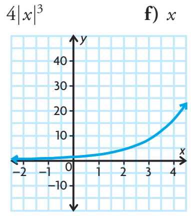
c) $10^3 = 1000$ f) $4^{\frac{1}{2}} = \sqrt{4} = 2$

3. a) $8m^3$ d) x^3y

b) $\frac{1}{a^8b^{10}}$ e) $-d^2c^2$

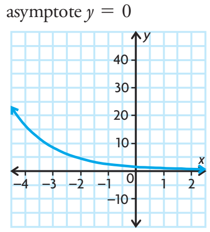
c) $4|x|^3$ f) x

4. a)



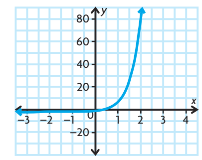
D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} | y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

b)



D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} | y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

c)



D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R} | y > -2\}$, y-intercept -1, horizontal asymptote $y = -2$

5. a) i) $y = \frac{x + 6}{3}$

ii) $y = \pm\sqrt{x + 5}$

iii) $y = \sqrt[3]{\frac{x}{6}}$

iv)

b) The inverses of (i) and (iii) are functions.

6. a) 800 bacteria

b) 6400 bacteria

c) 209 715 200

d) 4.4×10^{15}

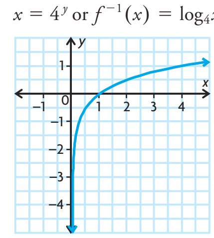
7. 12 515 people

8.

Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

Lesson 8.1, p. 451

1. a) $x = 4^y$ or $f^{-1}(x) = \log_4 x$



b) $x = 8^y$ or $f^{-1}(x) = \log_8 x$